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Mining Project Portfolio Selection Based on an Interval Type-2 Fuzzy Entropy-MARCOS Method under Uncertainty

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***Abstract** –Mining project portfolio selection (PPS) is a complex, capital-intensive decision-making challenge, often exacerbated by conflicting criteria and high uncertainty. To address it, this study introduces a new integrated multi-criteria decision-making (MCDM) framework using trapezoidal Interval Type-2 Fuzzy Sets (TrIT2 FSs) to effectively capture the vagueness inherent in experts judgments. In this approach, we first utilize TrIT2 fuzzy numbers to quantify linguistic evaluations from decision-makers, followed by the entropy method to derive objective criteria weights. The MARCOS method is then employed to rank mining projects by evaluating their utility against ideal and anti-ideal solutions. The framework's efficacy is demonstrated through a real-world case study of major mining projects in Iran, assessed by a panel of deputy managers. The results identify Golgohar (A5) alternative as the top-performing project, while Zarshuran (A10) ranks lowest. Comparative analysis with TrIT2F-TOPSIS and TrIT2F-COPRAS methods confirms the stability and reliability of the rankings. Furthermore, sensitivity analyses across various strategic scenarios validate the robustness of the proposed model. Ultimately, the TrIT2F-Entropy-MARCOS framework offers a powerful, interpretable decision-support tool for strategic investment in the mining sector.*

***Keywords**– Project portfolio selection (PPS); Multi-criteria decision-making (MCDM); Group decision-making (GDM); Interval type-2 fuzzy sets (IT2FSs); Entropy weighting; MARCOS method.*

I. INTRODUCTION

A project portfolio encompasses a set of projects, programs, and initiatives managed simultaneously under a unified governance framework to achieve overarching strategic objectives (Khalilzadeh et al., 2026). From this perspective, projects are not evaluated in isolation; rather, their value is determined by their contribution to the portfolio as a whole, accounting for potential synergies, shared resource consumption, aggregate risk, and strategic alignment. Consequently, project portfolio management transcends the mere coordination of multiple projects; it involves critical decisions regarding project initiation, termination, and the optimal allocation of limited resources (Mohammed, 2023).

The necessity of rigorous project portfolio selection arises from the inherent limitations of organizational resources, including capital, human expertise, time, and infrastructure. Inadequate selection strategies often precipitate resource fragmentation, an accumulation of unfinished projects, and diminishing returns on investment (Jalilibal & Bozorgi-Amiri, 2022). Moreover, the dynamic nature of the business environment introduces significant uncertainty, as priorities

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are frequently subject to shifts in market conditions, regulatory frameworks, technological advancements, or financial climates. Under such volatility, effective portfolio selection allows organizations to balance short- and long-term objectives, mitigate risk, and maximize value from competing investment alternatives (Nascimento et al., 2025).

Project portfolio selection dynamics vary across industries, depending on risk profiles, time horizons, regulatory constraints, and value-creation mechanisms. For instance, the IT sector emphasizes agility, innovation, and market responsiveness; manufacturing prioritizes efficiency, process optimization, and cost containment; whereas energy and infrastructure are defined by high capital intensity, safety imperatives, and sustainability. Despite these differences, these sectors share a fundamental requirement: multi-criteria decision-making under resource constraints. This challenge is particularly acute in the mining industry, where projects are characterized by high upfront investment, prolonged life cycles, geological ambiguity, safety and environmental risks, and susceptibility to global commodity price volatility (Namazi et al., 2023).

The mining industry is one of the fundamental sectors supporting economic development, industrial production, and resource-based value creation. By supplying essential raw materials for industries, such as steel, construction, energy, manufacturing, and advanced technologies, mining plays a critical role in strengthening industrial supply chains and promoting long-term economic growth. At the same time, mining projects are characterized by substantial capital requirements, long development and operation periods, geological uncertainty, environmental and social impacts, and high exposure to fluctuations in global commodity markets. These characteristics make investment decisions in the mining sector highly complex and strategically important. Consequently, prioritizing and selecting mining projects requires systematic decision-making approaches that can consider multiple, often conflicting, technical, economic, environmental, and operational criteria under uncertainty (Alonso et al., 2025).

To navigate the complexities of portfolio selection, scholars and practitioners often employ weighting and ranking mechanisms for criteria and alternatives. Typically, decision criteria, such as economic value, risk, strategic alignment, and socioeconomic impacts, are first identified and weighted. Projects are then evaluated and ranked based on their performance across these criteria. This structured framework facilitates transparent comparisons among heterogeneous projects, especially when navigating trade-offs between conflicting objectives, such as profitability versus environmental preservation (Mohammed, 2023).

However, real-world portfolio selection is frequently impeded by imprecise or incomplete information. Assessments concerning risk, feasibility, or future economic potential often rely on subjective expert judgment rather than deterministic data. Fuzzy logic provides an effective methodology for quantifying linguistic assessments (e.g., “high,” “medium,” “low”), enabling decision models to systematically incorporate imprecision rather than disregarding it (Nascimento et al., 2025). In some contexts, uncertainty extends beyond mere value imprecision; the very definition of membership functions may be subject to debate. This necessitates the use of type-2 fuzzy sets, where membership degrees themselves are uncertain. Among these, interval type-2 fuzzy sets are particularly advantageous as they capture the dispersion of expert opinions and data fluctuations while maintaining manageable computational complexity. Consequently, they are well-suited for multi-criteria decision-making in high-risk, data-limited environments like mining (Sang & Liu, 2016).

In summary, project portfolio selection is a strategic multi-criteria challenge that is magnified in the mining sector due to elevated risk and capital intensity. By integrating weighting and ranking techniques with fuzzy logic, organizations can establish a robust decision-support framework that enhances result reliability under incomplete information. Accordingly, the following section presents a systematic literature review to identify critical selection criteria and examine the application of multi-criteria decision-making methods, specifically those utilizing interval type-2 fuzzy sets—thereby establishing the research gaps that motivate the proposed model.

Vijayakumar et al. (2022) investigated a project evaluation and selection problem with a particular emphasis on financial criteria. They employed a multi-criteria decision-making approach in which Fuzzy AHP was used to determine criteria weights, while Fuzzy TOPSIS was applied to rank investment alternatives. The case study evaluated and prioritized five investment alternatives. Jalilibal & Bozorgi-Amiri (2022) examined sustainability-related criteria for project portfolio selection. The relevant criteria were identified using a grounded theory approach. Subsequently, Fuzzy DEMATEL was applied to analyze the cause–effect relationships among the criteria, and ISM was used to structure and categorize them into different levels. The results highlighted the central role of profit, cost, and risk dimensions in construction projects. Ramedani et al. (2022) addressed a sustainable project portfolio selection problem under uncertainty by combining a multi-criteria decision-making approach with an optimization model. FBWM was used to determine the criteria weights, and FVIKOR was employed to rank the alternatives. In addition, a multi-objective fuzzy–robust–stochastic optimization model was developed to maximize profit while minimizing project delays.

Barzegari et al. (2023) proposed an interval-valued fuzzy decision method for supplier evaluation in oil production projects. Their approach integrates the strengths of MOOSRA, MOORA, and MULTIMOORA within an interval-valued fuzzy environment to better handle uncertainty in decision-makers' judgments. The proposed method was applied to a case study involving the assessment of ten potential suppliers for a Vietnamese petroleum company based on fifteen sub-criteria grouped into five main categories: reliability, capability, agility, effective asset management, and cost. The results showed that the IVF-MULTIMOOSRAL framework provides a comprehensive and reliable tool for supplier evaluation and ranking under uncertain conditions in the oil and gas industry. Mohammed (2023) studied project portfolio prioritization in an Iraqi oil company using a fuzzy multi-criteria decision-making approach. Fuzzy AHP was applied for criteria weighting, while Fuzzy TOPSIS was used to evaluate and rank the projects. In total, twenty projects were prioritized based on five key criteria. Namazi et al. (2023) proposed a strategic approach for managing and selecting R&D projects. The study used efficiency and uncertainty maps to analyze projects from a strategic perspective. Projects were mapped and classified into four strategic zones, supporting better alignment between project selection and organizational vision. The findings showed that the map-based approach can effectively structure strategic decision-making in R&D project portfolios. Zahmatkesh & Shakhsi-Niaei (2023) investigated sustainable project portfolio selection based on combined rankings under uncertainty. Several MCDM methods were integrated to generate a combined ranking of projects, and the selection problem was formulated using robust optimization. In the case study, the deterministic model selected six projects, whereas the robust model selected three projects. Although incorporating uncertainty reduced the objective function values, it improved the reliability of solutions under real-world conditions.

Zare Banadkouki & Mirabi (2024) examined project portfolio selection in the construction sector using fuzzy logic and TOPSIS. A fuzzy group decision-making approach was adopted to manage the evaluation process. Projects were assessed based on financial, technical, risk-related, environmental, and socio-political criteria. The case study focused on four dam construction projects, and the sensitivity analysis indicated that the resulting ranking was robust against changes in criteria weights. Er et al. (2024) investigated project portfolio selection criteria in the oil and gas refining industry. The study examined how refinery companies determine investment priorities under changing energy conditions, climate change pressures, and net-zero carbon commitments. Expert interviews were conducted with industry practitioners, and categorical content analysis was used to identify and validate the main project selection criteria. The importance weights of these criteria were then determined and incorporated into a fuzzy MULTIMOORA-based multi-criteria decision-making framework. The proposed decision-support tool was used to evaluate and rank projects, providing practical guidance for refinery companies in the portfolio selection process.

Nascimento et al. (2025) proposed a TOPSIS-based framework for selecting construction project portfolios in the public sector. The framework enables the simultaneous assessment of quantitative and qualitative criteria under organizational constraints. TOPSIS was integrated with integer linear programming to support the final portfolio selection. The approach elicits inputs through experts' intra-criteria assessment of alternatives and decision-makers' inter-criteria assessment of criteria. The structured process improves the feasibility and transparency of public-sector portfolio decisions under limited resources. Ghadi & Shahab (2025) proposed a comprehensive framework for project portfolio selection using data-driven multi-criteria decision-making methods. The study focused on integrating machine learning and big data analytics with MCDM methods to overcome the limitations of subjective expert weighting and reduce bias. Through a literature review and mathematical formulation, the authors showed that data-driven weighting can improve portfolio efficiency, reduce uncertainty, and better align project selection with organizational strategies. The study concluded that hybrid machine learning and MCDM models are necessary to enhance objectivity in complex decision-making environments.

Atoum et al. (2026) proposed a hybrid fuzzy multi-criteria decision-making framework for optimizing Agile scaling in industrial software projects. The study combined Fuzzy Analytic Hierarchy Process (FAHP) for criteria weighting under uncertainty with a tunable VIKOR–PROMETHEE ranking stage. The framework organized twenty-two criteria across organizational, project, group, and framework levels to support governance, scalability, adaptability, and auditability. Using two independent expert panels, the authors validated the model and showed that the hybrid approach achieved lower ranking error compared with single-method baselines. The study concluded that the proposed fuzzy MCDM framework can improve process governance by making scaling decisions more transparent, stable, and reproducible. Khalilzadeh et al. (2026) introduced an integrated method for construction project portfolio selection. K-means clustering was first used to group projects prior to decision analysis. SWARA was then applied to prioritize and weight the identified criteria, while MULTIMOORA was employed to rank and select projects. The results were compared with WASPAS for verification. The case study included five main criteria and eighteen sub-criteria and clustered twenty-five projects into four groups, showing that sub-criteria-based ranking can produce preferable results.

Table I summarizes studies closely related to the present research in terms of theme, methodology, and application domain. These studies indicate that project portfolio selection has been widely examined using multi-criteria decision-making techniques, often integrated with fuzzy logic and optimization frameworks. Reviewing these publications provides a basis for identifying current research trends, commonly used methodologies, and the scope of the present study. Accordingly, Table I presents a consolidated overview of the relevant literature.

Table I. Literature review

No.	Author(s)	Year	PPS	Method			Uncertainty approach		
				Weighting method	Ranking method	GDM	Crisp	FSs	IT2FSs
1	Vijayakumar et al.	2022	*	AHP	TOPSIS			*	
2	Jalilibal & Bozorgi-Amiri	2022	*	DEMATEL				*	
3	Ramedani et al.	2022	*	BWM	VIKOR			*	
4	Barzegari et al.	2023			MULTIMOOSRAL			*	
5	Mohammed	2023	*	AHP	TOPSIS			*	
6	Namazi et al.	2023	*					*	
7	Zahmatkesh & Shakhsi-Niaei	2023	*	AHP	VIKOR				
8	Er et al.	2024	*		MULTIMOORA			*	
9	Zare Banadkouki & Mirabi	2024	*		TOPSIS			*	
10	Ghadi & Shahab	2025	*				*		
11	Nascimento et al.	2025	*		TOPSIS		*		
12	Atoum et al.	2026		AHP	VIKOR			*	
13	Khalilzadeh et al.	2026	*	SWARA	MULTIMOORA		*		
This Study			*	Entropy	MARCOS	*			*

A. Research gap

Although Ramedani et al. (2022) and Mohammed (2023) are among the closest studies to the present research, as they addressed project portfolio prioritization and selection under uncertainty using fuzzy MCDM techniques, several limitations remain. First, their approaches primarily focused on the application of existing fuzzy MCDM models rather than developing an integrated framework specifically tailored to project portfolio selection in highly uncertain and capital-intensive sectors. Second, most previous fuzzy-based approaches, particularly those based on crisp values or type-1 fuzzy sets, have limited capability to capture the uncertainty embedded not only in the evaluation data but also in the membership functions themselves. This limitation is important because expert judgments in project portfolio selection are often expressed through linguistic terms and are affected by ambiguity, hesitation, incomplete information, and differences in managerial perception.

To overcome these limitations, the present study develops an integrated Trapezoidal Interval Type-2 Fuzzy (TrIT2F)-Entropy-MARCOS framework for mining project portfolio selection under uncertainty. The use of trapezoidal interval type-2 fuzzy Sets enables a richer and more realistic representation of expert judgments by explicitly modeling the uncertainty associated with linguistic assessments. Furthermore, the entropy method is incorporated to determine objective criteria weights based on information dispersion. In contrast, the MARCOS method ranks alternatives according to their utility relative to ideal and anti-ideal solutions. This integrated structure is particularly suitable for the mining industry, where project selection is influenced by high investment costs, resource limitations, market fluctuations, operational risks, and uncertainty in technical and economic projections. Therefore, the proposed framework addresses an important methodological and practical gap in the literature by offering a more robust, reliable, and interpretable decision-support model for mining project portfolio selection.

The main innovation of this study lies in developing a new TrIT2F-Entropy-MARCOS framework for mining project portfolio selection under uncertainty. While the original MARCOS method evaluates alternatives based on their utility degrees relative to ideal and anti-ideal solutions, the proposed framework extends its applicability by integrating trapezoidal interval type-2 fuzzy sets to capture higher-order uncertainty in experts' judgments. In addition, the entropy method is incorporated to objectively determine criterion weights and reduce potential subjectivity in the weighting process. By combining these components within a group decision-making environment, this study provides a more robust and practical decision-support framework for selecting mining project portfolios compared with existing classical or type-1 fuzzy MCDM approaches.

The remainder of this paper is organized as follows. Section II presents the preliminaries, including the basic concepts of interval type-2 fuzzy Sets and the arithmetic operations of trapezoidal interval type-2 fuzzy numbers. Section III describes the proposed research methodology, including the entropy weighting method and the MARCOS approach in the trapezoidal interval type-2 fuzzy environment. Section IV presents the case study of mining project portfolio selection and discusses the computational results. Section V provides comparative and sensitivity analyses to evaluate the robustness and reliability of the proposed framework. Finally, Section VI concludes the paper and presents directions for future research.

II. PRELIMINARIES

In order to ensure a clear distinction between type-1 fuzzy sets and interval type-2 fuzzy sets, this section presents the essential definitions and notations required for the proposed TrIT2F-Entropy-MARCOS framework. A type-1 fuzzy set assigns a single membership degree to each element. In contrast, an interval type-2 fuzzy set allows the membership degree itself to be uncertain and represented within an interval bounded by upper and lower membership functions. This feature is particularly suitable for group decision-making problems in which expert evaluations are linguistic, subjective, and affected by uncertainty. Throughout this study, trapezoidal interval type-2 fuzzy numbers are employed to represent expert assessments. For notational consistency, $\tilde{A}$ is used only for type-1 fuzzy sets or fuzzy numbers when mentioned for conceptual comparison, while $\tilde{\tilde{A}}$ denotes interval type-2 fuzzy sets and trapezoidal interval type-2 fuzzy numbers. The definitions and arithmetic operations presented in this section are limited to those directly used in the proposed methodology.

A. BASIC CONCEPTS OF INTERVAL TYPE-2 FUZZY SETS

In this study, interval type-2 fuzzy sets are adopted to capture the uncertainty and ambiguity inherent in experts' judgments more effectively. Unlike conventional type-1 fuzzy sets, where each element is assigned a precise membership degree within the interval $[0, 1]$, type-2 fuzzy sets provide an additional degree of freedom by allowing the membership grades themselves to be uncertain. This feature makes them more suitable for complex decision-making environments (Keshavarz Ghorabaee et al., 2014).

Definition 1:

A type-2 fuzzy set $\tilde{A}$ can be viewed as an extension of a type-1 fuzzy set in which the membership function is itself fuzzy. Its mathematical representation is given by:

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{A}}(x,u) \leq 1\} \tag{1}$$

where X denotes the universe of discourse, $J_x$represents the primary membership domain associated with x, and $\mu_{\tilde{A}}(x,u)$ refers to the secondary membership function. The above definition may also be expressed in the following integral form:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_s} \mu_{\tilde{A}}(x,u)/(x,u) \tag{2}$$

where the double integral symbol indicates the union over all admissible values of x and u (Mendel, 2009).

Definition 2:

For practical applications, a simplified form of type-2 fuzzy sets, known as interval type-2 fuzzy sets, is often employed. In this case, all secondary membership grades are assumed to be equal to one. Accordingly, an interval type-2 fuzzy set can be written as:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u), J_x \subseteq [0,1] \tag{3}$$

This formulation preserves the capability of handling uncertainty while avoiding the excessive computational burden associated with general type-2 fuzzy sets (Sang & Liu, 2016).

Definition 3:

A fundamental concept in interval type-2 fuzzy theory is the Footprint of Uncertainty (FOU), which represents the range of uncertainty in the primary membership grades. The FOU is enclosed by two type-1 membership functions: the upper membership function (UMF) and the lower membership function (LMF). These two functions define the upper and lower bounds of the uncertain membership region and provide a more flexible structure for modeling ambiguous evaluations (Liao, 2015).

Definition 4:

To express linguistic assessments in a convenient numerical form, trapezoidal interval type-2 fuzzy numbers are used in this research. In such a representation, both the upper and lower membership functions are modeled as trapezoidal fuzzy numbers. A trapezoidal interval type-2 fuzzy number is denoted as:

$$\tilde{\tilde{A}} = \left(\tilde{A}^U, \tilde{A}^L\right) \tag{4}$$

where:

- the upper membership function is represented by

$$\left(a_1^u, a_2^u, a_3^u, a_4^u, H_1\left(\tilde{\tilde{A}}^U\right), H_2\left(\tilde{\tilde{A}}^U\right)\right) \tag{5}$$

- and the lower membership function is represented by

$$\left(a_1^L, a_2^L, a_3^L, a_4^L, H_1\left(\tilde{\tilde{A}}^L\right), H_2\left(\tilde{\tilde{A}}^L\right)\right) \tag{6}$$

This formulation makes it possible to describe uncertain expert opinions more realistically and is, therefore, well suited for the mining project portfolio selection problem addressed in this study (Sang & Liu, 2016).

B. Arithmetic Operations of Trapezoidal Interval Type-2 Fuzzy Numbers

To employ trapezoidal interval type-2 fuzzy numbers (IT2TrFNs) in decision-making models (e.g., mining project portfolio selection), the basic arithmetic operations on these numbers must be clearly defined. Since a structural illustration facilitates a better understanding of the upper and lower membership functions and the associated uncertainty region, the general form of an IT2 trapezoidal fuzzy number is first presented based on the main reference (Liao, 2015).

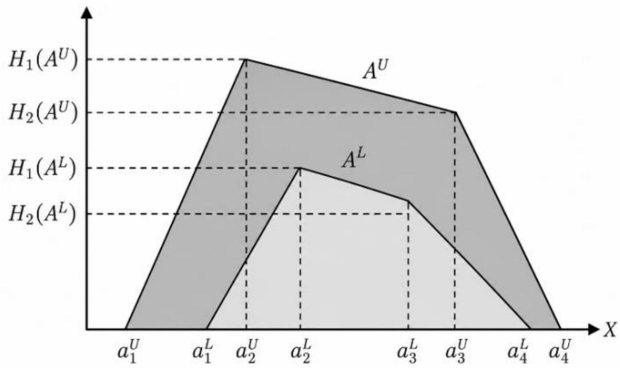

Fig. 1. A trapezoidal interval type-2 fuzzy number

As shown in Fig. 1, a trapezoidal interval type-2 fuzzy number is characterized by an upper membership function (UMF) and a lower membership function (LMF); the region between them forms the footprint of uncertainty (FOU). Based on this structure, the arithmetic operations required in subsequent computations are formulated below (Keshavarz Ghorabaee et al., 2014).

1) Representation of Trapezoidal Interval Type-2 Fuzzy Numbers

Let $\tilde{\tilde{A}}_1$ and $\tilde{\tilde{A}}_2$ be two trapezoidal interval type-2 fuzzy numbers defined as:

$$\tilde{\tilde{A}}_1 = \left(\tilde{\tilde{A}}_1^U, \tilde{\tilde{A}}_1^L\right), \tilde{\tilde{A}}_2 = \left(\tilde{\tilde{A}}_2^U, \tilde{\tilde{A}}_2^L\right) \tag{7}$$

where the upper and lower trapezoidal components are given by:

$$\tilde{\tilde{A}}_1^U = \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{\tilde{A}}_1^U\right), H_2\left(\tilde{\tilde{A}}_1^U\right)\right) \tag{8}$$

$$\tilde{\tilde{A}}_1^L = \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{\tilde{A}}_1^L\right), H_2\left(\tilde{\tilde{A}}_1^L\right)\right) \tag{9}$$

$$\tilde{\tilde{A}}_2^U = \left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{\tilde{A}}_2^U\right), H_2\left(\tilde{\tilde{A}}_2^U\right)\right) \tag{10}$$

$$\tilde{\tilde{A}}_2^L = \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{\tilde{A}}_2^L\right), H_2\left(\tilde{\tilde{A}}_2^L\right)\right) \tag{11}$$

In what follows, the arithmetic operators are defined by combining the corresponding trapezoidal parameters in the upper and lower parts, while the associated heights are aggregated via the minimum operator (Liao, 2015).

2) Arithmetic Operators

Definition 6: Addition

The addition of two trapezoidal interval type-2 fuzzy numbers is obtained by summing the corresponding parameters of the UMFs and LMFs, while the heights are determined by the minimum of the respective heights (Keshavarz Ghorabaee et al., 2014):

$$\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = (a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min(H_1(\tilde{\tilde{A}}_1^U), H_1(\tilde{\tilde{A}}_2^U)), \min(H_2(\tilde{\tilde{A}}_1^U), H_2(\tilde{\tilde{A}}_2^U))); \ (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \min(H_1(\tilde{\tilde{A}}_1^L), H_1(\tilde{\tilde{A}}_2^L)), \min(H_2(\tilde{\tilde{A}}_1^L), H_2(\tilde{\tilde{A}}_2^L))) \tag{12}$$

Definition 7: Subtraction

The subtraction operator is defined by subtracting the parameters of the second fuzzy number from those of the first one, considering the reversed order of the second trapezoid's bounds. The heights are again aggregated using the minimum operator (Keshavarz Ghorabaee et al., 2014):

$$\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = (a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; min(H_1(\tilde{\tilde{A}}_1^U), H_1(\tilde{\tilde{A}}_2^U)), min(H_2(\tilde{\tilde{A}}_1^U), H_2(\tilde{\tilde{A}}_2^U))); \ (a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; min(H_1(\tilde{\tilde{A}}_1^L), H_1(\tilde{\tilde{A}}_2^L)), min(H_2(\tilde{\tilde{A}}_1^L), H_2(\tilde{\tilde{A}}_2^L))) \tag{13}$$

Definition 8: Multiplication

The multiplication of two trapezoidal interval type-2 fuzzy numbers is defined by combining the corresponding parameters in the upper and lower parts, while the resulting heights are obtained as the minimum of the associated heights (Keshavarz Ghorabaee et al., 2014):

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = (X_1^U, X_2^U, X_3^U, X_4^U; min(H_1(\tilde{\tilde{A}}_1^U), H_1(\tilde{\tilde{A}}_2^U)), min(H_2(\tilde{\tilde{A}}_1^U), H_2(\tilde{\tilde{A}}_2^U))) \ ; \ (X_1^L, X_2^L, X_3^L, X_4^L; min(H_1(\tilde{\tilde{A}}_1^L), H_1(\tilde{\tilde{A}}_2^L)), min(H_2(\tilde{\tilde{A}}_1^L), H_2(\tilde{\tilde{A}}_2^L))) \tag{14}$$

where

$$X_i^T = \begin{cases} min\left[a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right], & i = 1,2, \\ max\left[a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right], & i = 3,4. \end{cases} \quad T \in [U, L] \tag{15}$$

Definition 9: Division

The division of two trapezoidal interval type-2 fuzzy numbers is defined via ratios of the corresponding parameters. For each of the upper and lower parts, the first two parameters are determined by the minimum of all possible ratios, while the last two parameters are determined by the maximum of all possible ratios (Keshavarz Ghorabaee et al., 2014):

$$\tilde{\tilde{A}}_1 \oslash \tilde{\tilde{A}}_2 = (Y_1^U, Y_2^U, Y_3^U, Y_4^U; min(H_1(\tilde{\tilde{A}}_1^U), H_1(\tilde{\tilde{A}}_2^U)), min(H_2(\tilde{\tilde{A}}_1^U), H_2(\tilde{\tilde{A}}_2^U))) \ ; \ (Y_1^L, Y_2^L, Y_3^L, Y_4^L; min(H_1(\tilde{\tilde{A}}_1^L), H_1(\tilde{\tilde{A}}_2^L)), min(H_2(\tilde{\tilde{A}}_1^L), H_2(\tilde{\tilde{A}}_2^L))) \tag{16}$$

where

$$Y_i^T = \begin{cases} min\left(\frac{a_{1i}^T}{a_{2i}^T}, \frac{a_{1i}^T}{a_{2,5-i}^T}, \frac{a_{1,5-i}^T}{a_{2i}^T}, \frac{a_{1,5-i}^T}{a_{2,5-i}^T}\right), & i = 1,2 \\ max\left(\frac{a_{1i}^T}{a_{2i}^T}, \frac{a_{1i}^T}{a_{2,5-i}^T}, \frac{a_{1,5-i}^T}{a_{2i}^T}, \frac{a_{1,5-i}^T}{a_{2,5-i}^T}\right), & i = 3,4 \end{cases} \quad T \in \{U, L\} \tag{17}$$

For the division operator to be well-defined, the denominators must be nonzero:

$$a_{2j}^T \neq 0 \quad , \quad j = 1,2,3,4 \quad , T \in \{U, L\} \tag{18}$$

Definition 10: Scalar multiplication

Let k be a crisp scalar. For $k \geq 0$, scalar multiplication is defined by multiplying all parameters by k (Keshavarz Ghorabaee et al., 2014):

$$k\tilde{\tilde{A}}_1 = \left(ka_{11}^U, ka_{12}^U, ka_{13}^U, ka_{14}^U;\ H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right);\ \left(ka_{11}^L, ka_{12}^L, ka_{13}^L, ka_{14}^L;\ H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right),\ k \geq 0 \tag{19}$$

For $k \leq 0$, the order of the bounds is reversed:

$$k\tilde{A}_1 = \left(ka_{14}^U, ka_{13}^U, ka_{12}^U, ka_{11}^U;\ H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right);\ \left(ka_{14}^L, ka_{13}^L, ka_{12}^L, ka_{11}^L;\ H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right),\ k \leq 0 \tag{20}$$

Definition 11: Division by a crisp scalar

Let $l \neq 0$ be a crisp scalar. For $l > 0$, division by a scalar is defined as (Keshavarz Ghorabaee et al., 2014):

$$\tilde{\tilde{A}}_1/l = (a_{11}^U/l, a_{12}^U/l, a_{13}^U/l, a_{14}^U/l;\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U))\ ;\ (a_{11}^L/l, a_{12}^L/l, a_{13}^L/l, a_{14}^L/l;\ H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)), l > 0 \tag{21}$$

For $l < 0$, the order of the bounds is reversed:

$$\tilde{A}_1/l = (a_{14}^U/l, a_{13}^U/l, a_{12}^U/l, a_{11}^U/l;\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U))\ ;\ (a_{14}^L/l, a_{13}^L/l, a_{12}^L/l, a_{11}^L/l;\ H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)), l < 0 \tag{22}$$

III. PROPOSED METHODOLOGY

The MARCOS method was selected for ranking mining projects because of its strong ability to evaluate alternatives with respect to both ideal and anti-ideal reference solutions. Unlike some widely used MCDM methods, such as TOPSIS, COPRAS, and VIKOR, MARCOS considers the relationship of each alternative with both the best and worst solutions from the beginning of the decision-making process and calculates the utility degree of each alternative accordingly. This provides a more informative and compromise-based ranking structure. In the context of mining project portfolio selection, where projects are characterized by conflicting economic, technical, environmental, and risk-related criteria, such a mechanism is particularly useful. Moreover, when combined with trapezoidal interval type-2 fuzzy sets, MARCOS can effectively support decision-making under uncertainty by reflecting the imprecision and ambiguity of expert judgments. Therefore, the TrIT2F-MARCOS structure provides a robust and transparent ranking procedure for mining project portfolio selection.

A group multi-criteria decision-making framework is presented and implemented in a trapezoidal interval type-2 fuzzy environment. First, experts' evaluations are collected using trapezoidal interval type-2 fuzzy numbers, and the opinions are then aggregated to construct the group decision matrix. Next, the Shannon entropy method is applied to obtain objective criterion weights, and subsequently the MARCOS method is employed to prioritize and rank the alternatives under the interval type-2 fuzzy setting. This integrated approach enables handling the uncertainty inherent in experts' judgments while comparing alternatives with respect to their closeness to the ideal and anti-ideal reference solutions. Notably, the MARCOS method was originally introduced by Stević et al. (2020), and in this section its procedure is adapted to the group decision-making structure and the interval type-2 fuzzy environment. The overall workflow of the proposed approach is schematically illustrated in Fig. 2.

To model the uncertainty and ambiguity inherent in decision-making data, the interval type-2 fuzzy approach is employed. The proposed framework consists of two complementary parts. In the first part, based on Shannon entropy, the amount of information and dispersion of data are analyzed, and the weights of criteria are determined objectively. In the second part, the MARCOS method is applied in an interval type-2 fuzzy environment to evaluate the alternatives based on the weighted criteria and to obtain their final ranking.

- Section A: Shannon Entropy with an Interval Type-2 Fuzzy Approach
- Section B: MARCOS Method with an Interval Type-2 Fuzzy Approach

A. Interval Type-2 Fuzzy Shannon Entropy for Criteria Weighting

To determine the objective weights of the criteria C_j, the Shannon entropy method is employed in the interval type-2 trapezoidal fuzzy environment. The basic idea of this method is that a criterion with greater dispersion among the evaluations of alternatives contains more useful information and should therefore receive a higher weight. In contrast, a criterion with more uniform evaluations has lower discriminating power and consequently receives a smaller weight. In the proposed group decision-making framework, the individual IT2TrFN decision matrices provided by the decision-makers are first constructed and then aggregated into a collective IT2TrFN decision matrix. Based on this aggregated matrix, the divergence degree of each criterion and its corresponding objective weight are obtained.

Step 1. *Construct the individual fuzzy decision matrice*s

Let $D_k(k = 1,2,\dots,K)$ denote the k-th decision-maker. The IT2TrFN decision matrix provided by decision-maker D_k is denoted as:

$$\tilde{\tilde{X}}^{(k)} = \left[\tilde{\tilde{x}}_{ij}^{(k)}\right]_{m\times n}, k = 1,2,\dots,K \tag{23}$$

where $\tilde{\tilde{x}}_{ij}^{(k)}$ represents the interval type-2 trapezoidal fuzzy evaluation of alternative A_i with respect to criterion C_j given by decision-maker D_k.

Step 2. *Construct the group decision matrix*

Using the individual matrices, form the group decision information (group decision matrix) prior to aggregation.

Step 3. *Construct the aggregated decision matrix*

Aggregate the group decision matrix to obtain the collective IT2TrFN decision matrix:

$$\tilde{\tilde{X}} = \left[\tilde{\tilde{x}}_{ij}\right]_{m\times n} \tag{24}$$

where $\tilde{\tilde{x}}_{ij}$ denotes the aggregated IT2TrFN assessment of A_i under C_j.

Step 4. *Normalize the aggregated decision matrix*

Normalize the aggregated decision matrix to make the evaluations comparable across alternatives.

Step 5. *Obtain the normalized values* $\tilde{\tilde{p}}_{ij}$

Compute the normalized elements $\tilde{\tilde{p}}_{ij}$ from the normalized matrix.

Step 6. *Compute the entropy value*

Calculate the entropy value $\tilde{\tilde{E}}_j$ for each criterion C_j.

Step 7. *Determine divergence and compute the final weight*s

First, compute the degree of divergence:

$$\tilde{\tilde{d}}_j = 1 - \tilde{\tilde{E}}_j \tag{25}$$

Then, obtain the final objective weight of each criterion:

$$\tilde{\tilde{w}}_j = \frac{\tilde{\tilde{d}}_j}{\sum_{j=1}^{n} \tilde{\tilde{d}}_j} \tag{26}$$

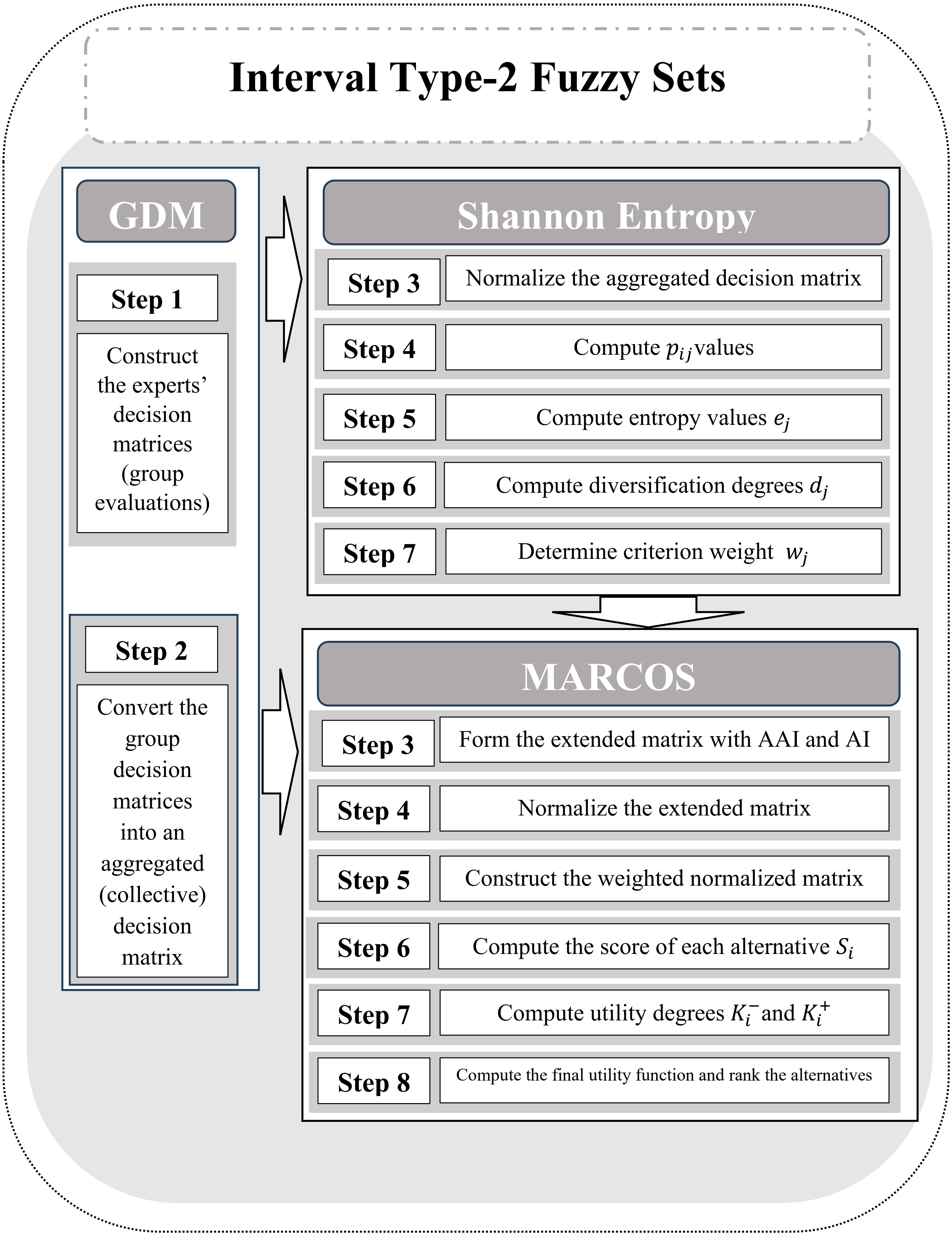

Fig. 2. Flowchart of the proposed method

B. Interval Type-2 Fuzzy MARCOS Method for Ranking Alternatives

The proposed TrIT2F-MARCOS method integrates trapezoidal interval type-2 fuzzy numbers with the MARCOS framework to effectively address the inherent uncertainty and ambiguity found in complex multi-criteria group decision-making problems. Within this methodology, let $A_i (i = 1,2, \dots, m)$ denote the set of alternatives, $C_j (j = 1,2, \dots, n)$ represent the evaluation criteria, and $E_k (k = 1,2, \dots, l)$ signify the group of decision experts, where l is the total number of experts involved in the evaluation. All assessments are expressed using TrIT2FNs, defined as $\tilde{\tilde{x}} = (\tilde{\tilde{x}}^U, \tilde{\tilde{x}}^L)$, where $\tilde{x}^U$ and $\tilde{x}^L$ represent the upper and lower trapezoidal membership functions, respectively. For each bound $T \in \{U, L\}$, the membership function is formally expressed as $\tilde{\tilde{x}}^T = (x_1^T, x_2^T, x_3^T, x_4^T; H_1(\tilde{\tilde{x}}^T), H_2(\tilde{\tilde{x}}^T))$, in which $x_1^T, x_2^T, x_3^T, x_4^T$ are the parameters of the trapezoid, and $H_1(\tilde{\tilde{x}}^T)$ and $H_2(\tilde{\tilde{x}}^T)$ correspond to the height parameters that characterize the intensity of the fuzzy uncertainty (Xu et al., 2023).

Step 1. *Construct the individual fuzzy decision matrices*

Each decision expert E_k provides an evaluation matrix using TrIT2FNs as follows:

$$\tilde{\tilde{X}}^k = \left[\tilde{\tilde{x}}_{ij}^k\right]_{m \times n} \tag{27}$$

where $\tilde{\tilde{x}}_{ij}^k$ denotes the evaluation of alternative A_i with respect to criterion C_j provided by expert E_k.

Each element is defined as:

$$\tilde{\tilde{x}}_{ij}^k = \left(\tilde{\tilde{x}}_{ij}^{k,U}, \tilde{\tilde{x}}_{ij}^{k,L}\right) \tag{28}$$

where, for each bound $T \in \{U, L\}$,

$$\tilde{\tilde{x}}_{ij}^{k,T} = \left(x_{ij1}^{k,T}, x_{ij2}^{k,T}, x_{ij3}^{k,T}, x_{ij4}^{k,T}; H_1(\tilde{\tilde{x}}_{ij}^{k,T}), H_2(\tilde{\tilde{x}}_{ij}^{k,T})\right) \tag{29}$$

Step 2. *Construct the aggregated group decision matrix*

The individual decision matrices provided by the decision maker are aggregated to obtain the group decision matrix. Since all experts are assumed to have equal importance in this study, their evaluations are aggregated using the arithmetic mean.

The group decision matrix is defined as:

$$\tilde{\tilde{X}} = \left[\tilde{\tilde{x}}_{ij}\right]_{m \times n} \tag{30}$$

Where

$$\tilde{\tilde{x}}_{ij} = \frac{1}{l} \sum_{k=1}^{l} \tilde{\tilde{x}}_{ij}^k \tag{31}$$

In this equation, l denotes the number of decision makers. To preserve the interval type-2 fuzzy structure, the upper and lower bounds are aggregated separately:

$$\tilde{\tilde{x}}_{ij}^T = \frac{1}{l} \sum_{k=1}^{l} \tilde{\tilde{x}}_{ij}^{k,T}, T \in \{U, L\} \tag{32}$$

Therefore, for each bound $T \in \{U, L\}$, the aggregated value is calculated as:

$$\tilde{\tilde{x}}_{ij}^T = \left(\frac{1}{l} \sum_{k=1}^{l} x_{ij1}^{k,T}, \frac{1}{l} \sum_{k=1}^{l} x_{ij2}^{k,T}, \frac{1}{l} \sum_{k=1}^{l} x_{ij3}^{k,T}, \frac{1}{l} \sum_{k=1}^{l} x_{ij4}^{k,T}; \frac{1}{l} \sum_{k=1}^{l} H_1\,(\tilde{\tilde{x}}_{ij}^{k,T}), \frac{1}{l} \sum_{k=1}^{l} H_2\,(\tilde{\tilde{x}}_{ij}^{k,T})\right) \tag{33}$$

Consequently, the group evaluation value of alternative A_i with respect to criterion C_j is expressed as:

$$\tilde{\tilde{x}}_{ij} = \left(\tilde{\tilde{x}}_{ij}^{U}, \tilde{\tilde{x}}_{ij}^{L}\right) \tag{34}$$

Step 3. *Determine the Anti-Ideal and Ideal Solution*s

In the MARCOS method, two reference alternatives are defined: the anti-ideal alternative AAI and the ideal alternative AI. For benefit-type criteria, the anti-ideal and ideal values are determined as:

$$\begin{aligned} \tilde{\tilde{x}}_{aaj} &= \min_{i} \tilde{\tilde{x}}_{ij}, \\ \tilde{\tilde{x}}_{aij} &= \max_{i} \tilde{\tilde{x}}_{ij}. \end{aligned} \tag{35}$$

For cost-type criteria, they are determined as:

$$\begin{aligned} \tilde{\tilde{x}}_{aaj} &= \max_{i} \tilde{\tilde{x}}_{ij}, \\ \tilde{\tilde{x}}_{aij} &= \min_{i} \tilde{\tilde{x}}_{ij}. \end{aligned} \tag{36}$$

Here, $\tilde{\tilde{x}}_{aaj}$ represents the anti-ideal value for criterion C_j, while $\tilde{\tilde{x}}_{aij}$ represents the ideal value for criterion C_j.

Step 4. *Construct the Extended Decision Matrix*

The extended decision matrix is constructed by adding the anti-ideal and ideal alternatives to the aggregated decision matrix:

$$\tilde{\tilde{X}}^{ext} = \begin{bmatrix} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{bmatrix}. \tag{37}$$

Therefore:

$$\tilde{\tilde{X}}^{ext} = \left[\tilde{\tilde{x}}_{ij}^{ext}\right]_{(m+2)\times n}. \tag{38}$$

The first row corresponds to the anti-ideal alternative, while the last row corresponds to the ideal alternative.

Step 5. *Normalize the Extended Decision Matrix*

The normalization process is performed according to the type of each criterion. For benefit-type criteria, the normalized value is calculated as:

$$\tilde{\tilde{n}}_{ij} = \frac{\tilde{\tilde{x}}_{ij}^{ext}}{\tilde{\tilde{x}}_{aij}} \tag{39}$$

For cost-type criteria, the normalized value is calculated as:

$$\tilde{\tilde{n}}_{ij} = \frac{\tilde{\tilde{x}}_{aij}}{\tilde{\tilde{x}}_{ij}^{ext}}. \tag{40}$$

The normalized decision matrix is obtained as:

$$\tilde{\tilde{N}} = \left[\tilde{\tilde{n}}_{ij}\right]_{(m+2)\times n} \tag{41}$$

The normalization is performed separately for the upper and lower membership functions to preserve the interval type-2 fuzzy structure.

Step 6. *Construct the Weighted Normalized Matrix*

Let the weight vector of criteria be defined as:

$$\tilde{\tilde{W}} = \left(\tilde{\tilde{w}}_1, \tilde{\tilde{w}}_2, \dots, \tilde{\tilde{w}}_n\right), \tag{42}$$

The weighted normalized matrix is calculated as:

$$\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij}\right]_{(m+2)\times n} \tag{43}$$

where:

$$\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_j \otimes \tilde{\tilde{n}}_{ij}. \tag{44}$$

The multiplication is applied to all parameters of the upper and lower trapezoidal interval type-2 fuzzy numbers.

Step 7. *Calculate the Fuzzy Overall Score*

The fuzzy overall score of each alternative is obtained by summing the weighted normalized values across all criteria:

$$\tilde{\tilde{S}}_i = \sum_{j=1}^{n} \tilde{\tilde{v}}_{ij}\, , i = 0,1, \dots, m, m+1. \tag{45}$$

Here, $i = 0$ denotes the anti-ideal alternative AAI, while $i = m+1$ denotes the ideal alternative AI. Each $\tilde{\tilde{S}}_i$ is a TrIT2FN represented as:

$$\tilde{\tilde{S}}_i = (\tilde{\tilde{s}}_i^U, \tilde{\tilde{s}}_i^L) \tag{46}$$

where:

$$\begin{aligned} \tilde{\tilde{s}}_i^U &= (a_{1i}^U, a_{2i}^U, a_{3i}^U, a_{4i}^U; H_{1i}^U, H_{2i}^U), \\ \tilde{\tilde{s}}_i^L &= (a_{1i}^L, a_{2i}^L, a_{3i}^L, a_{4i}^L; H_{1i}^L, H_{2i}^L). \end{aligned} \tag{47}$$

To transform the fuzzy overall score into a crisp value, a centroid-based defuzzification approach is applied. Unlike conventional methods that mainly focus on the horizontal centroid coordinate, the proposed approach incorporates both the horizontal and vertical centroid coordinates. The horizontal centroid coordinate for the upper membership function can be calculated as:

$$C_{ix}^U = \frac{a_{1i}^U + a_{2i}^U + a_{3i}^U + a_{4i}^U}{4} \tag{48}$$

Similarly, for the lower membership function:

$$C_{ix}^L = \frac{a_{1i}^L + a_{2i}^L + a_{3i}^L + a_{4i}^L}{4} \tag{49}$$

The vertical centroid coordinate is calculated by considering the membership heights. For the upper membership function:

$$C_{iy}^{U} = \frac{H_{1i}^{U} + H_{2i}^{U}}{2} \tag{50}$$

For the lower membership function:

$$C_{iy}^{L} = \frac{H_{1i}^{L} + H_{2i}^{L}}{2} \tag{51}$$

Then, the integrated centroid value of the upper membership function is obtained as:

$$C_{i}^{U} = C_{ix}^{U} \times C_{iy}^{U} \tag{52}$$

Similarly, the integrated centroid value of the lower membership function is obtained as:

$$C_{i}^{L} = C_{ix}^{L} \times C_{iy}^{L} \tag{53}$$

Finally, the crisp score of alternative A_i is calculated as:

$$S_i = \frac{C_i^U + C_i^L}{2} \tag{54}$$

Equivalently:

$$S_i = \frac{1}{2}\left[\left(\frac{a_{1i}^U + a_{2i}^U + a_{3i}^U + a_{4i}^U}{4}\right)\left(\frac{H_{1i}^U + H_{2i}^U}{2}\right) + \left(\frac{a_{1i}^L + a_{2i}^L + a_{3i}^L + a_{4i}^L}{4}\right)\left(\frac{H_{1i}^L + H_{2i}^L}{2}\right)\right] \tag{55}$$

The use of the vertical coordinate C_{iy} allows the defuzzification process to capture the influence of the height parameters $H_1$and H_2. Therefore, the proposed score function reflects not only the horizontal spread of the fuzzy number but also the intensity of membership (Keshavarz Ghorabaee et al., 2014).

Step 8. *Calculate Utility Degrees and Final Utility Function*

After obtaining the crisp scores S_i, the utility degree of each alternative with respect to the anti-ideal solution is calculated as:

$$K_i^- = \frac{S_i}{S_{AAI}} \tag{56}$$

where S_{AAI} is the crisp score of the anti-ideal alternative. The utility degree of each alternative with respect to the ideal solution is calculated as:

$$K_i^+ = \frac{S_i}{S_{AI}} \tag{57}$$

where S_{AI} is the crisp score of the ideal alternative. The final utility function of each alternative is then calculated as:

$$f(K_i) = \frac{K_i^- + K_i^+}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} \tag{58}$$

where:

$$f(K_i^-) = \frac{K_i^+}{K_i^- + K_i^+},$$
$$f(K_i^+) = \frac{K_i^-}{K_i^- + K_i^+}. \tag{59}$$

Finally, alternatives are ranked in descending order of $f(K_i)$. The alternative with the highest value of $f(K_i)$is considered the best alternative. The final ranking of alternatives is determined as follows:

$$A_p \succ A_q \text{ if } f(K_p) > f(K_q) \tag{60}$$

Thus, a larger value of the final utility function indicates a more desirable alternative.

IV. CASE STUDY: SELECTION OF A MINING PROJECT PORTFOLIO

To demonstrate the applicability of the proposed method in a real-world decision-making context, a case study from the Iranian mining sector is considered. The data employed in this study were obtained from a state-run organization responsible for the management, development, and strategic planning of mining activities in Iran. Due to limitations in financial, technical, human, and operational resources, the organization is not capable of simultaneously supporting or investing in all mining projects. Therefore, identifying, evaluating, and prioritizing the mining projects with higher potential and strategic importance is essential. In this regard, 14 major mines in Iran are regarded as alternatives and are evaluated based on seven relevant criteria under uncertain conditions. The purpose of this case study is to examine the effectiveness and practical applicability of the proposed decision-making framework in supporting strategic project selection and mining portfolio formation.

In this study, seven criteria are considered to evaluate the mining alternatives. These criteria are selected to reflect the main technical, economic, operational, infrastructural, and environmental aspects of mining project selection. The first criterion is mineral reserve (C1), which represents the amount of exploitable mineral resources available in each mine. This criterion is considered a benefit criterion, since larger reserves indicate greater long-term production capacity and higher investment attractiveness. The second criterion is ore grade (C2), which refers to the quality of the ore and the concentration of valuable minerals within the deposit. A higher ore grade is preferred because it generally leads to higher economic value, improved processing efficiency, and greater profitability; therefore, it is also regarded as a benefit criterion. The third criterion is annual production (C3), which indicates the yearly production capacity of each mine. Higher annual production reflects stronger operational capability and better ability to meet market demand, making this criterion beneficial in nature. The fourth criterion is extraction cost (C4), which represents the costs associated with extracting the mineral material from the mine. Since lower extraction costs improve economic performance and project viability, this criterion is considered a cost criterion. The fifth criterion is distance to infrastructure (C5), which describes the distance of each mine from essential infrastructure such as roads, railways, power supply, water resources, processing facilities, and transportation networks. Shorter distances reduce development, transportation, and operational costs; thus, this criterion is treated as a cost criterion. The sixth criterion is environmental impact (C6), which reflects the potential negative effects of mining activities on the environment, including land degradation, air and water pollution, waste generation, and ecosystem disturbance. Since lower environmental impacts are more desirable, this criterion is considered a cost criterion. Finally, the seventh criterion is economic feasibility (C7), which represents the overall economic justification of each mining project by considering factors such as profitability, return on investment, operational costs, market potential, and the economic value of the deposit. Higher economic feasibility indicates a more attractive and viable project; therefore, this criterion is regarded as a benefit criterion. Overall, C1, C2, C3, and C7 are benefit criteria, whereas C4, C5, and C6 are cost criteria.

The alternatives considered in this study consist of 14 major mines located in different regions of Iran. These mines were selected based on the data received from the state-run organization and represent important mineral projects with strategic, economic, and operational significance. The considered alternatives are Sarcheshmeh (A01), Sungun (A02), Miduk (A03), Chadormalu (A04), Golgohar (A05), Sangan Iron Ore (A06), Choghart (A07), Angouran (A08), Mehdiabad (A09), Zarshuran (A10), Muteh (A11), Esfordi (A12), Jalalabad (A13), and Sechahun (A14). These alternatives cover different types of mineral resources and are distributed across various mining regions of the country. The geographical distribution and locations of the selected mining alternatives are illustrated in Fig. 3.

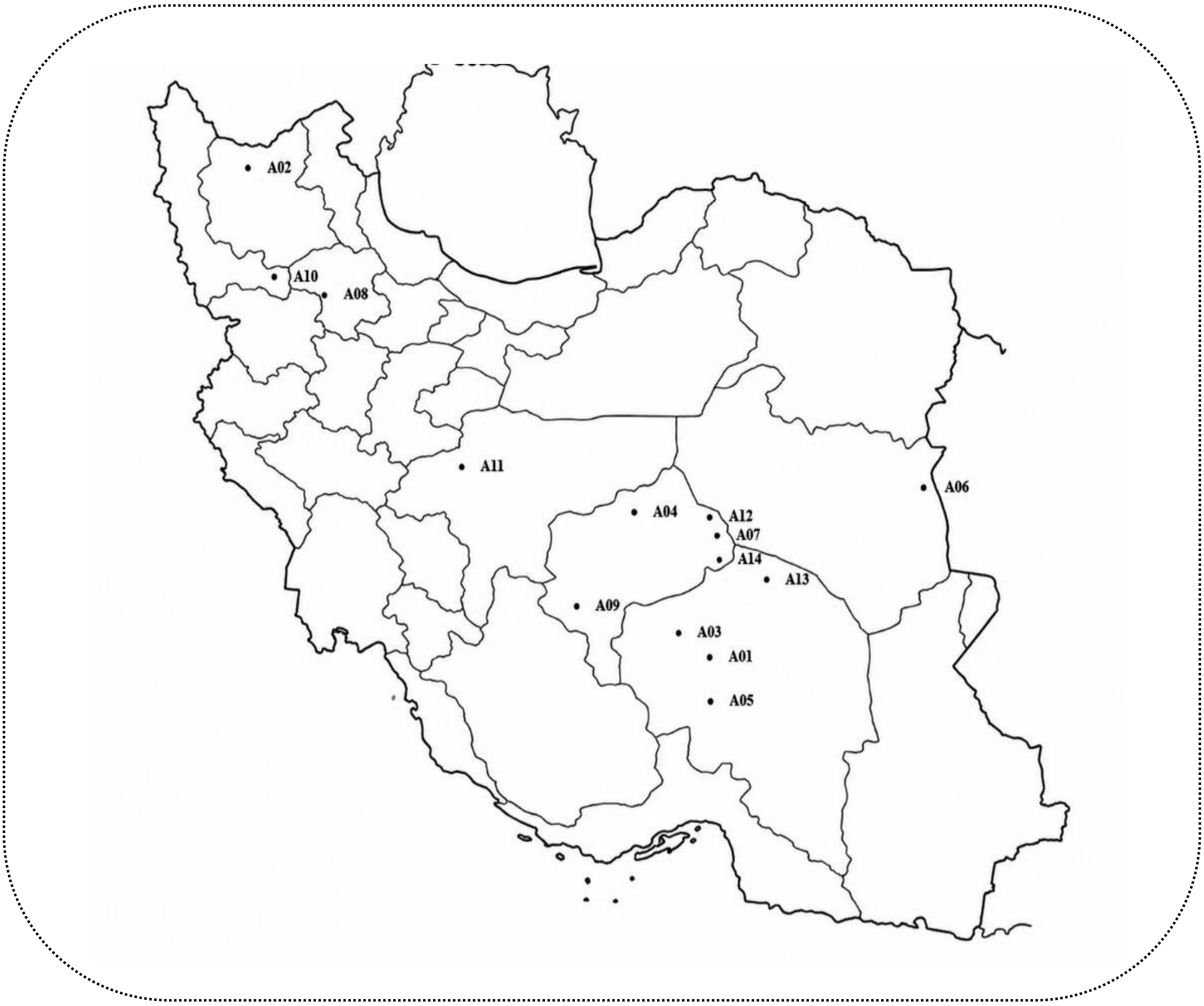

Fig. 3. Location of the selected mines

In this study, the performance of each alternative with respect to each evaluation criterion (C1–C7) is not entered directly as crisp numerical data; instead, it is elicited from the decision-makers through expert judgment using linguistic assessments. Accordingly, for each cell of the decision matrix (i.e., evaluating each mine under each criterion), the decision-makers assign one linguistic term from the predefined scale. These linguistic ratings are then converted into their corresponding interval type-2 fuzzy sets based on the linguistic scale adopted from Xu et al. (2023), as presented in Table II (e.g., VL, L, ML, M, MH, H, and VH), providing the required fuzzy inputs for subsequent computations and implementation of the proposed framework.

Table II. Linguistic terms and corresponding trapezoidal interval type-2 fuzzy sets adopted from Xu et al. (2023)

Linguistic terms	Interval type-2 fuzzy sets
Very low (VL)	((0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))
Low (L)	((0, 0.1, 0.15, 0.3; 1, 1), (0.05, 0.1, 0.15, 0.2; 0.9, 0.9))
Medium low (ML)	((0.1, 0.3, 0.35, 0.5; 1, 1), (0.2, 0.3, 0.35, 0.4; 0.9, 0.9))
Medium (M)	((0.3, 0.5, 0.55, 0.7; 1, 1), (0.4, 0.5, 0.55, 0.6; 0.9, 0.9))
Medium high (MH)	((0.5, 0.7, 0.75, 0.9; 1, 1), (0.6, 0.7, 0.75, 0.8; 0.9, 0.9))
High (H)	((0.7, 0.85, 0.9, 1; 1, 1), (0.8, 0.85, 0.9, 0.95; 0.9, 0.9))
Very high (VH)	((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))

In this study, the evaluations were provided by three decision-makers who are deputy managers of the aforementioned state-run organization and possess relevant expertise in mining project assessment and strategic planning. Each decision-maker independently assessed the 14 mining alternatives against the seven criteria using the linguistic scale defined earlier. The corresponding assessment scores for DM1, DM2, and DM3 are reported in Tables III, IV, and V, respectively.

Table III. Decision Maker 1 Ratings of the Alternatives Based on the Criteria

Alternative	C1	C2	C3	C4	C5	C6	C7
A1	VH	H	VH	M	ML	MH	VH
A2	H	H	H	MH	M	MH	H
A3	H	MH	H	M	ML	M	H
A4	VH	H	VH	ML	L	M	VH
A5	VH	H	VH	M	ML	M	VH
A6	VH	MH	H	MH	MH	M	H
A7	H	MH	MH	M	ML	ML	H
A8	MH	H	M	MH	M	MH	MH
A9	H	MH	MH	H	MH	H	MH
A10	MH	H	M	H	MH	H	MH
A11	M	MH	ML	MH	M	MH	M
A12	MH	M	M	M	ML	M	MH
A13	H	M	MH	M	M	ML	H
A14	MH	M	M	MH	M	ML	MH

Table IV. Decision Maker 2 Ratings of the Alternatives Based on the Criteria

Alternative	C1	C2	C3	C4	C5	C6	C7
A1	VH	VH	VH	M	ML	MH	VH
A2	H	H	H	M	M	MH	H
A3	H	MH	MH	M	ML	M	H
A4	VH	H	VH	ML	L	ML	VH

Continue Table IV. Decision Maker 2 Ratings of the Alternatives Based on the Criteria

Alternative	C1	C2	C3	C4	C5	C6	C7
A5	VH	H	VH	ML	ML	M	VH
A6	VH	MH	H	MH	H	M	H
A7	H	M	MH	M	ML	ML	MH
A8	MH	H	M	MH	M	H	MH
A9	H	MH	MH	H	MH	H	H
A10	MH	H	M	H	MH	H	MH
A11	M	MH	ML	MH	M	MH	M
A12	MH	M	M	M	ML	M	MH
A13	H	M	MH	ML	M	ML	H
A14	MH	M	M	M	M	ML	MH

Table V. Decision Maker 3 Ratings of the Alternatives Based on the Criteria

Alternative	C1	C2	C3	C4	C5	C6	C7
A1	VH	H	VH	MH	ML	H	VH
A2	H	H	H	MH	M	H	H
A3	H	MH	H	M	ML	MH	H
A4	VH	H	VH	ML	L	M	VH
A5	VH	H	VH	M	ML	M	VH
A6	VH	MH	H	H	MH	MH	H
A7	H	MH	MH	M	ML	M	H
A8	MH	H	M	MH	M	H	MH
A9	H	MH	MH	H	H	H	MH
A10	MH	H	M	H	MH	VH	MH
A11	M	MH	ML	MH	M	H	M
A12	MH	M	M	M	ML	M	MH
A13	H	M	MH	M	M	M	H
A14	MH	M	M	MH	M	M	MH

The final scores for each alternative are presented in Table VI, where the options have been sorted in descending order based on their results to clearly illustrate the final ranking.

Table VI. Final Score of Each Alternative

Alternative	K_i^-	K_i^+	$f(K^-)$	$f(K^+)$	$f(K_i)$	Rank
A5	26.850	0.899	0.032	0.968	0.895	1
A8	21.512	0.720	0.032	0.968	0.718	2
A2	20.950	0.702	0.032	0.968	0.700	3

Continue Table VI. Final Score of Each Alternative

Alternative	K_i^-	K_i^+	$f(K^-)$	$f(K^+)$	$f(K_i)$	Rank
A1	18.245	0.612	0.032	0.968	0.609	4
A4	16.550	0.555	0.032	0.968	0.552	5
A3	16.320	0.548	0.032	0.968	0.545	6
A6	15.840	0.532	0.032	0.968	0.528	7
A12	14.650	0.492	0.032	0.968	0.489	8
A13	14.380	0.482	0.032	0.968	0.479	9
A14	14.380	0.482	0.032	0.968	0.479	10
A11	14.050	0.471	0.032	0.968	0.468	11
A7	8.120	0.272	0.032	0.968	0.269	12
A9	6.880	0.231	0.032	0.968	0.228	13
A10	5.520	0.185	0.032	0.968	0.182	14

The final results indicate that Golgohar (A5), Angouran (A8), Sungun (A2), and Sarcheshmeh (A1) achieved the highest ranks, reflecting their stronger overall performance across the evaluated technical and economic criteria. This ordering is broadly consistent with real-world observations, as these mines are generally associated with larger or higher-quality resources, more stable production capacity, and better economic feasibility compared with many other alternatives. In this sense, the obtained ranking suggests that the proposed decision-making approach can realistically capture practical differences in operational and economic performance among the candidate mining projects.

V. RESULTS ANALYSIS AND VALIDATION

A. Results Analysis

To evaluate the validity and stability of the proposed method, the results obtained from the TrIT2F-MARCOS method are compared with two well-known and widely used multi-criteria decision-making methods, namely TrIT2F-TOPSIS (Liao, 2015) and TrIT2F-COPRAS (Keshavarz Ghorabaee et al., 2014). These methods were selected due to their broad application in ranking problems, their capability to handle both benefit and cost criteria, and their high acceptance in previous studies. This comparison provides a basis for examining the extent to which the proposed method can generate reliable rankings that are consistent with established reference methods. Therefore, the comparative analysis among these three methods is regarded as part of the model validation process and provides a framework for assessing the convergence or differences among the obtained rankings. The results of this comparison are presented in Table VII.

The comparative results indicate a high level of consistency among the three applied methods. In all three approaches, A5 is ranked as the best alternative, while A10 is placed in the last position. The stability of the first and last ranks across TrIT2F-MARCOS, TrIT2F-TOPSIS, and TrIT2F-COPRAS demonstrates that the proposed method produces results that are highly consistent with established MCDM techniques. Although minor differences can be observed in the ranking of some intermediate alternatives, these variations are expected due to the different computational structures of the methods. Overall, the agreement among the methods, particularly in identifying the best and worst alternatives, provides strong evidence for the validity and reliability of the proposed TrIT2F-MARCOS method.

Finally, the results show a high degree of closeness between the outputs of the TrIT2F-MARCOS and TrIT2F-

TOPSIS methods. This similarity can be attributed to the relatively close logic of these two methods, as both evaluate alternatives based on their relationship with ideal and anti-ideal solutions. However, the key distinction of the MARCOS method lies in its greater flexibility compared with TOPSIS. While TOPSIS mainly focuses on the distance of alternatives from the ideal and anti-ideal points, MARCOS provides a broader evaluation framework by considering the relative utility of each alternative with respect to both reference solutions. Therefore, although the closeness of the results confirms the validity of the proposed method, TrIT2F-MARCOS can offer a more flexible and comprehensive scoring process based on the overall utility of alternatives.

Table VII. Comparative analysis of the proposed TrIT2F-Entropy-MARCOS framework with existing MCDM methods

Alternative	TrIT2F-MARCOS		TrIT2F-TOSIS		TrIT2F-COPRAS	
	Score	Rank	Score	Rank	Score	Rank
A1	0.609	4	0.617	4	0.602	4
A2	0.700	3	0.694	3	0.724	2
A3	0.545	6	0.558	5	0.521	7
A4	0.552	5	0.551	7	0.563	5
A5	0.895	1	0.882	1	0.907	1
A6	0.528	7	0.557	6	0.536	6
A7	0.269	12	0.276	12	0.261	12
A8	0.718	2	0.731	2	0.708	3
A9	0.228	13	0.236	13	0.219	13
A10	0.182	14	0.190	14	0.176	14
A11	0.468	11	0.459	11	0.475	10
A12	0.489	8	0.496	8	0.492	8
A13	0.479	9	0.472	10	0.486	9
A14	0.479	10	0.484	9	0.469	11

B. Sensitivity Analysis

Sensitivity analysis based on criteria weights is conducted because, in multi-criteria decision-making problems, criteria weights represent the relative importance of each evaluation factor and may directly influence the final scores and ranking of alternatives. Since the determination of criteria weights is often affected by experts' judgments, managerial preferences, organizational policies, and environmental uncertainties, it is essential to examine whether the obtained results remain stable under different weighting conditions. In this study, two sensitivity analysis approaches were employed to evaluate the robustness of the results from different perspectives.

In the first approach, the weight of each criterion was changed individually to identify which criteria have the greatest influence on the ranking of mining projects and to assess the sensitivity of the model to variations in a single criterion. In the second approach, strategic scenarios were developed based on different decision-making priorities, such as technical, economic, infrastructural, and environmental orientations, to examine the behavior of the model under more realistic policy-based conditions. Therefore, the simultaneous use of these two approaches provides a more comprehensive assessment of ranking stability, helps identify influential criteria, and confirms the reliability of the proposed TrIT2F-MARCOS model.

1) Sensitivity Analysis Based on Criterion-wise Weight Variation

In the first sensitivity analysis approach, a criterion-wise weight variation procedure was applied to examine the influence of each individual criterion on the final ranking of mining projects. Since criteria weights represent the relative importance of evaluation factors in multi-criteria decision-making problems, changes in these weights may affect the final utility scores and ranking positions of the alternatives. Therefore, this analysis was conducted to assess the stability and robustness of the proposed TrIT2F-MARCOS model under different weighting conditions. In each scenario, the weight of one criterion was increased. In contrast, the weights of the remaining criteria were proportionally adjusted to ensure that the total sum of weights remained equal to one. The base scenario represents the original weighting structure and is used as the reference condition for comparison. Scenario S_C1 increases the importance of mineral reserve, S_C2 emphasizes ore grade, and S_C3 focuses on annual production. Scenario S_C4 assigns greater importance to extraction cost, S_C5 increases the importance of distance to infrastructure, and S_C6 emphasizes environmental impact. Since C4, C5, and C6 are cost criteria, alternatives with lower values in these criteria are expected to perform better when their weights are increased. Finally, scenario S_C7 increases the importance of economic feasibility. This approach makes it possible to identify the criteria with stronger effects on the ranking results and to evaluate whether the obtained ranking remains stable when the importance of individual criteria changes.

Table VIII presents the final utility scores of the mining project alternatives under the base condition and different criterion-wise weight variation scenarios. These scores show how the performance value of each alternative changes when the importance of an individual criterion is increased. In addition, Fig. 4 illustrates the corresponding rank variations of the alternatives across the same scenarios. By comparing the results in Table VIII and Fig. 4, it is possible to evaluate the stability of the ranking structure and identify which alternatives are more sensitive to changes in criteria weights. Overall, the combined interpretation of the table and figure provides a clear understanding of the robustness of the proposed TrIT2F-MARCOS model under different weighting conditions.

Table VIII. Final utility scores of alternatives under criterion-wise weight variation scenarios

Alternative	Base	S_C1	S_C2	S_C3	S_C4	S_C5	S_C6	S_C7
A1	0.742	0.766	0.735	0.758	0.731	0.748	0.724	0.761
A2	0.771	0.763	0.794	0.759	0.752	0.755	0.737	0.789
A3	0.684	0.681	0.706	0.672	0.695	0.699	0.676	0.691
A4	0.756	0.768	0.748	0.779	0.741	0.750	0.732	0.762
A5	0.823	0.847	0.815	0.841	0.811	0.829	0.801	0.852
A6	0.705	0.731	0.692	0.718	0.683	0.671	0.688	0.713
A7	0.591	0.604	0.583	0.596	0.602	0.587	0.599	0.584
A8	0.657	0.641	0.684	0.646	0.662	0.651	0.669	0.648
A9	0.628	0.651	0.642	0.616	0.601	0.592	0.614	0.639
A10	0.472	0.461	0.489	0.455	0.448	0.452	0.437	0.481
A11	0.523	0.507	0.532	0.511	0.548	0.536	0.519	0.515
A12	0.607	0.596	0.623	0.604	0.615	0.611	0.638	0.598
A13	0.641	0.658	0.631	0.653	0.649	0.667	0.662	0.626
A14	0.557	0.573	0.548	0.565	0.542	0.529	0.572	0.551

The results of the criterion-wise sensitivity analysis show that the overall ranking structure remains relatively stable across most scenarios. In particular, alternative A5 maintains the first rank in all scenarios, while A10 consistently remains the lowest-ranked alternative. This indicates that the best and weakest alternatives are not highly sensitive to changes in criteria weights. In scenario S_C1, where the weight of mineral reserve is increased, A4 and A1 improve to the second and third ranks, respectively, while A2 drops from the second to the fourth position. Moreover, A9 improves from the ninth to the seventh rank, whereas A13 slightly declines from the eighth to the ninth rank. In scenario S_C2, which emphasizes ore grade, the top-ranked alternatives remain mostly stable; however, A6 drops from the fifth to the seventh rank, and A13 shows a considerable decline from the eighth to the twelfth rank. In scenario S_C3, which increases the importance of annual production, only limited changes are observed: A4 improves from the third to the second rank, A2 moves from the second to the third rank, and A13 improves from the eighth to the seventh rank. In scenario S_C4, where extraction cost is given greater importance, the most notable improvement is observed for A13, which rises from the eighth to the fifth rank. In contrast, A6 declines from the fifth to the seventh rank, and A9 drops from the ninth to the eleventh rank. In scenario S_C5, which emphasizes distance to infrastructure, A13 again obtains the fifth rank, and A12 improves from the tenth to the ninth rank, while A6 and A9 experience rank declines. In scenario S_C6, where environmental impact is emphasized, the changes are relatively minor; A12 improves to the ninth rank, while A9 decreases from the ninth to the tenth position. Finally, in scenario S_C7, which increases the weight of economic feasibility, the leading alternatives preserve their high positions, and A13 improves from the eighth to the seventh rank, with only minor changes observed among the remaining alternatives. Overall, most rank variations occur among the middle-ranked alternatives, whereas the strongest and weakest alternatives, namely A5 and A10, remain unchanged across all scenarios. This confirms the acceptable robustness and stability of the proposed TrIT2F-MARCOS model against criterion-wise weight variations.

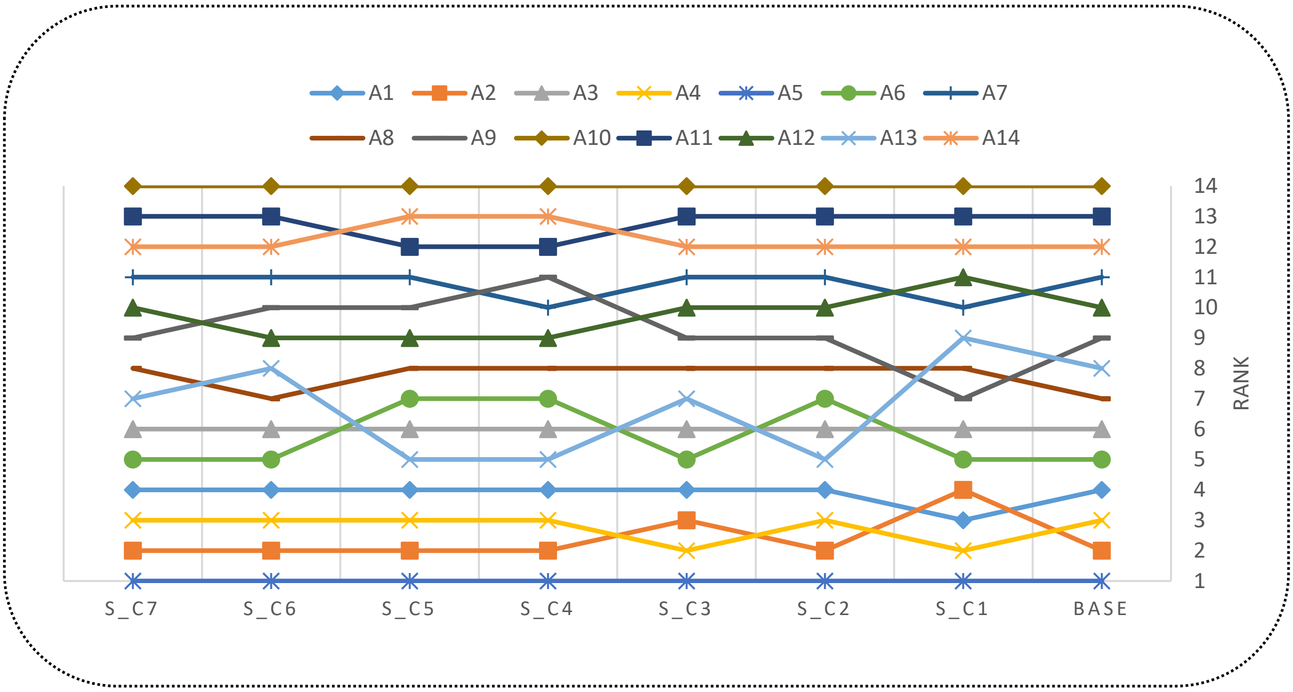

Fig. 4. Rank variations of alternatives under criterion-wise weight variation scenarios

2) Strategic Scenario-Based Sensitivity Analysis

In the second phase of the sensitivity analysis, a strategic scenario-based approach was applied to further examine the robustness and stability of the proposed TrIT2F-MARCOS model. While the previous sensitivity analysis investigated the effect of changing the weight of each criterion separately, this analysis evaluates the behavior of the model under broader managerial and strategic priorities. In real mining project selection problems, decision-makers usually do not focus on only one criterion; rather, they emphasize a group of related criteria according to organizational policies, investment priorities, operational limitations, or sustainability concerns. Therefore, this analysis provides a more realistic evaluation of the proposed model by assessing whether the final ranking of mining alternatives remains stable when the decision-making strategy changes.

In this regard, five strategic scenarios were designed to reflect different managerial perspectives in the mining project selection process. The first scenario represents a technical-oriented strategy, in which greater importance is assigned to criteria related to mineral reserve, ore grade, and annual production capacity. The second scenario reflects an economic-oriented strategy by emphasizing extraction cost and economic feasibility. The third scenario focuses on infrastructure and environmental considerations, giving higher priority to distance from infrastructure and environmental impact. The fourth scenario represents a benefit-oriented strategy, in which all benefit-type criteria, including mineral reserve, ore grade, annual production capacity, and economic feasibility, are emphasized. Finally, the fifth scenario follows a cost-oriented strategy, where the cost-type criteria, namely extraction cost, distance to infrastructure, and environmental impact, receive greater importance. These scenarios make it possible to evaluate the consistency of the ranking results under different strategic decision-making conditions.

The results of the strategic scenario-based sensitivity analysis are presented in Table IX and Fig. 5. Table IX reports the final utility scores of each alternative under the original condition and the five strategic scenarios. These scores indicate the relative desirability of each alternative under different decision-making priorities and allow the performance of the mining projects to be compared across technical-oriented, economic-oriented, infrastructure–environmental, benefit-oriented, and cost-oriented scenarios. In addition, Fig. 5 illustrates the ranking position of each alternative in the different scenarios, making it possible to clearly observe rank variations compared with the original results. Accordingly, the combined interpretation of Table IX and Fig. 5 provides a basis for evaluating the robustness of the ranking results and identifying alternatives that are either stable or sensitive to changes in strategic decision-making priorities.

Table IX. Utility scores of alternatives under strategic sensitivity scenarios

Alternative	Original	SS1 Technical	SS2 Economic	SS3 Infrastructure–Environmental	SS4 Benefit-oriented	SS5 Cost-oriented
A1	0.712	0.729	0.704	0.699	0.728	0.702
A2	0.745	0.734	0.756	0.722	0.754	0.695
A3	0.638	0.646	0.641	0.630	0.649	0.626
A4	0.731	0.742	0.731	0.713	0.742	0.688
A5	0.884	0.891	0.879	0.872	0.893	0.868
A6	0.689	0.677	0.681	0.641	0.684	0.640
A7	0.661	0.668	0.659	0.649	0.669	0.648
A8	0.654	0.658	0.652	0.655	0.660	0.657
A9	0.621	0.633	0.616	0.604	0.629	0.598
A10	0.472	0.478	0.469	0.461	0.476	0.458
A11	0.603	0.612	0.620	0.612	0.610	0.609
A12	0.592	0.600	0.590	0.681	0.596	0.722
A13	0.648	0.641	0.692	0.746	0.645	0.748
A14	0.579	0.586	0.581	0.575	0.584	0.571

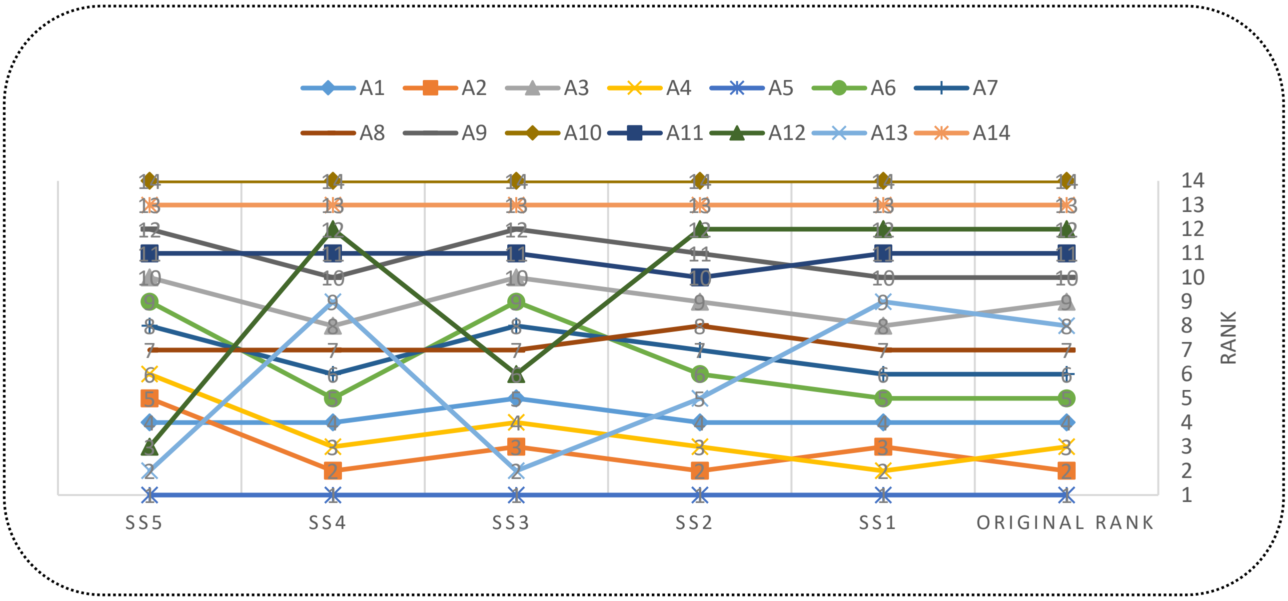

Fig. 5. Ranking of alternatives under strategic sensitivity scenarios

The results of the strategic scenario-based sensitivity analysis demonstrate that the proposed TrIT2F-MARCOS model has an acceptable level of robustness and reliability against structural changes in decision-making priorities. A key feature of this sensitivity analysis is that, unlike single-criterion weight variation, it does not examine the effect of increasing only one individual criterion; rather, it evaluates the behavior of alternatives under more realistic groups of managerial priorities. The results show that A5 maintains the highest utility score and the first rank across all scenarios, indicating its stable superiority from different technical, economic, benefit-oriented, and cost-oriented perspectives. Conversely, A10 remains the lowest-ranked alternative in all scenarios, confirming that its weak performance is not affected by changes in strategic priorities. However, the most notable rank variations occur among the middle-ranked alternatives. In particular, A13 and A12 show considerable improvements under the infrastructure–environmental and cost-oriented scenarios, suggesting that their performance is strongly dependent on the adopted strategic viewpoint. Therefore, this analysis not only confirms the stability of the best and weakest alternatives but also reveals the model's ability to distinguish the behavior of middle-ranked alternatives under different decision-making conditions, making it a useful tool for strategic decision support in mining project selection.

The two sensitivity analyses provide complementary insights into the robustness of the proposed TrIT2F-MARCOS model. The first sensitivity analysis, based on criterion-wise weight variation, examines the direct effect of increasing the importance of each criterion individually and therefore provides a detailed understanding of how every single criterion influences the final ranking. In contrast, the second analysis, based on strategic scenarios, evaluates the model under broader and more realistic decision-making conditions by emphasizing groups of related criteria according to technical, economic, infrastructure–environmental, benefit-oriented, and cost-oriented priorities. The comparison of the two analyses shows that both approaches confirm the overall stability of the ranking results, particularly because A5 remains the best alternative and A10 remains the weakest alternative across all scenarios. However, the criterion-wise analysis is more suitable for identifying the influence of individual criteria. In contrast, the strategic scenario-based analysis is more effective in revealing how alternatives respond to changes in managerial priorities. Overall, the consistency between the two analyses strengthens the validity of the proposed model and confirms that the final ranking is not highly dependent on a specific weighting structure.

Based on the results of the two sensitivity analyses, it can be concluded that the economic–cost-related dimension plays the most influential role in differentiating mining projects. While the strongest and weakest alternatives, namely A5 and A10, maintained their positions across all scenarios, the observed rank variations among the middle-ranked alternatives indicate that these alternatives are more sensitive to changes in economic, cost-related, and, to some extent, infrastructure–environmental priorities. In particular, the rank changes of alternatives such as A12 and A13 under the strategic scenarios show that criteria related to cost, economic return, and project implementation conditions can play an important role in distinguishing closely competing alternatives. Therefore, based on the sensitivity analysis outputs, the economic-cost-related dimension can be considered the most influential factor in the evaluation and selection of mining projects, as it contributes most to the rank variability of competitive alternatives and supports final managerial decision-making.

VI. CONCLUSION

In this study, a multi-criteria decision-making framework based on the TrIT2F-MARCOS method was proposed for evaluating and prioritizing mining projects under uncertainty. The selection of mining projects is inherently complex, capital-intensive, and multidimensional, requiring the simultaneous consideration of technical, economic, infrastructural, environmental, and managerial criteria. Moreover, much of the information used in such decisions is derived from expert judgments expressed through linguistic variables, which are often associated with ambiguity, subjectivity, and uncertainty. Therefore, the use of trapezoidal interval type-2 fuzzy sets in this study enabled a more accurate representation of uncertainty in human assessments. In the proposed framework, the linguistic evaluations of decision-makers were converted into fuzzy numbers and then processed using the MARCOS logic, in which alternatives are ranked according to their utility degree and their relationship with ideal and anti-ideal solutions. This feature allowed the proposed model not only to generate a final ranking but also to provide an interpretable assessment of the utility level of each mining project.

The results obtained from the implementation of the model demonstrated that the TrIT2F-MARCOS method has a strong capability to differentiate mining projects and identify the most preferable alternatives. Based on the findings, Golgohar (A5) achieved the highest utility score and was identified as the most suitable project for investment and development. This result reflects the strong performance of this alternative across the considered evaluation criteria. In contrast, Zarshuran (A10) obtained the lowest rank, indicating its relatively weaker performance compared with the other alternatives. The results also showed that the differences among alternatives were not driven by a single criterion, but rather by the combined effect of technical, economic, operational, and environmental factors. Accordingly, the proposed framework provided a comprehensive and structured evaluation of the mining projects and prioritized them based on their overall utility.

To examine the validity and reliability of the results, the ranking obtained from the proposed method was compared with two other decision-making methods, namely TrIT2F-TOPSIS and TrIT2F-COPRAS. The comparative results indicated a high level of convergence among the rankings produced by these methods, particularly for the alternatives placed at the top and bottom positions. This finding confirms that the results of the proposed model are not dependent on a single computational technique and have an acceptable level of stability. Nevertheless, the MARCOS method, due to its simultaneous consideration of ideal and anti-ideal solutions and its utility function calculation, provides a more interpretable and flexible evaluation of alternatives. Therefore, the TrIT2F-MARCOS approach can be regarded as an effective and reliable method for complex decision-making problems, like mining project selection, where uncertainty, diverse criteria, and expert-based judgments are involved.

Finally, the results of the two sensitivity analyses further confirmed the robustness of the proposed model. The criterion-wise sensitivity analysis showed that changing the weight of each criterion individually had a limited effect on the overall ranking structure. In contrast, the strategic scenario-based analysis demonstrated that the model remained stable under different managerial priorities, including technical, economic, infrastructure–environmental, benefit-oriented, and cost-oriented perspectives. In both analyses, A5 maintained the first position and A10 remained in the last position, indicating the stability of the results in identifying the best and weakest projects. However, the observed rank variations among the middle-ranked alternatives revealed that economic–cost-related criteria play a more influential role in distinguishing closely competing projects. From a managerial perspective, this implies that attention to costs, economic returns, and project implementation conditions can be decisive in the final selection of mining projects. Future studies are recommended to extend the proposed framework by incorporating larger expert panels, real operational data, dynamic weighting schemes, or hybrid approaches, such as fuzzy methods combined with machine learning, to enhance the accuracy and applicability of decision support in mining project selection.

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