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A robust fuzzy stochastic programming model for a dual-channel closed-loop supply chain network design with delivery lead time sensitive customers and price-dependent uncertain demands

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Abstract *–The number of customers who want to make purchases online is growing daily due to the Internet's expanding functionality and e-commerce developments. This study offers a mixed-integer programming approach for developing a reliable dual-channel closed-loop supply chain network with hybrid uncertainties. The model attempts to maximize overall profit. In this study, it is considered that customer demand in the offline sales channel depends on the price offered by offline retailers. Additionally, it is considered that customer demand via the online channel is influenced by the lead time for product delivery. The hybrid uncertainties in the problem data were addressed here by utilizing a robust fuzzy stochastic programming optimization approach found in the present research literature. The numerical calculations demonstrate the accuracy of the model and the proposed solution method. Significant findings are obtained from sensitivity analyses of essential parameters. Outcomes indicate that the value of the objective function declines when the minimum level of satisfaction of the uncertainty parameters increases. Furthermore, as the delivery lead time rises from 0.1 to 0.6, the objective function increases by 24% and 25% at the minimum satisfaction levels of 0.6 and 0.8, respectively. Additionally, the results show that by increasing the number of price levels from 3 to 10, the objective function increases by 2.5% and 2.6% at the minimum satisfaction levels of 0.6 and 0.8, respectively.*

Keywords– *Closed-loop supply chain; Delivery lead time; Dual-channel; Pricing; Uncertainty.*

I. INTRODUCTION

Many countries have experienced significant economic growth recently, which has led to problems, including resource depletion and rising waste. As a result, it is now necessary to address the challenges of recycling raw materials from waste goods that have reached the end of their useful life. To accomplish this, researchers and practitioners have found that closed-loop supply chains are a practical instrument for reducing environmental pollution and decreasing worries about the consumption of raw materials (Gao et al., 2024a). Specifically, there are industries that have to comply with laws established by governments because of legislation, environmental concerns, and customer awareness. The tire industry is subject to regulations, requiring its supply chain to recycle used tires (Vali-Siar & Roghanian, 2022).

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The structure and design of the network are among the most essential planning aspects of supply chain management. A supply chain network not only directly affects a business's growth and profitability but also determines the physical configuration of the supply chain (Vali-Siar et al., 2022).

Due to the increasing popularity of online shopping driven by the growth of Internet usage and e-commerce, traditional closed-loop supply chains (CLSC) are transforming from only one channel to dual-channel (DC-CLSC). Compared to conventional closed-loop supply chains, this type of DC-CLSC is more complex and unpredictable because it incorporates both offline and online sales infrastructures (Gao et al., 2024a).

Revenue management decisions are made in SC planning to maximize a supply chain's profit or revenue by identifying, forecasting, and reacting to consumer behavior. These decisions comprise quantity and pricing options at the tactical planning level, as well as structural decisions at the strategic level (Talluri & Van Ryzin, 2006). Pricing decisions in supply chains serve two essential purposes since they impact customers' demand ranges: 1) to earn a profit from the sale of one unit of every item to consumers, and 2) to influence the quantity and capacity of supply chain locations to meet demand. Given the link between strategic design and pricing decisions, it appears vital to include these alternatives in an optimization framework (Fattahi et al., 2018).

One of the primary purposes of the present study is strategic supply chain planning when customer demand is dependent on delivery lead time. Serving a customer zone directly affects the delivery lead time for product delivery, which includes the amount of time needed for product preparation and delivery to customers. As a result, the facilities assigned to client zones meet customer demands (Fattahi et al., 2017).

Designing a closed-loop supply chain network in a practical decision-making environment can encounter uncertainties, particularly fuzzy and random ones. In the traditional supply chain network design problem, characteristics such as demand, cost, and capacity are assumed to be definite. Still, it is evident that these assumptions do not correlate to reality in practice (Farrokh et al., 2018). In supply chain network design, there are often two sorts of uncertainties. Whereas a well-planned supply chain network can potentially save a lot of expenditure, it can also increase a company's vulnerability to disruptions. Disruptions can result from both human-caused disasters, such as terrorist attacks and strikes, and natural events, such as earthquakes and floods (Sabouhi et al., 2020). Disruption risks have detrimental operational effects on transportation costs and order preparation delays, in addition to having a significant adverse impact on the economic performance of a company (e.g., lower productivity). To lessen and control the detrimental consequences of disruptions, reliable SCND models can be applied (Peng et al., 2011). The second type of uncertainty is operational or business-as-usual. This kind of uncertain situation is linked to the unpredictability of supply chain networks and affects numerous aspects, including customer demand, facility capacity, and manufacturing and transportation costs (Pishvaee & Torabi, 2010). To manage and control parameter uncertainty, the SCND scope employs robust, fuzzy, and stochastic programming. Research on CLSCND problems has extensively used randomness and epistemic uncertainty. When the distribution of parameters is known and they are essentially random, randomness uncertainty arises. This type of uncertainty is addressed via stochastic programming (Darvishi et al., 2020). Epistemic uncertainty occurs when parameters are ambiguous, imprecise, and vague; fuzzy programming addresses this type of uncertainty (Pishvaee & Torabi, 2010). Stochastic and fuzzy programming models can be integrated to address CLSCND problems across many real-world situations. A robust programming method is required to control and manage different sources of uncertainty and generate a robust solution, which is essential for long-term/strategic CLSCND decisions (Mousazadeh et al., 2018). By accounting for parameter uncertainty, DMs can use this method to adjust the degree of conservatism in the output results (Hatefi & Jolai, 2014). Finding the best solution that is not influenced by the fluctuations of uncertain parameters is the goal of applying a robust optimization technique (Pishvae et al., 2012).

Operational planning generally considers network flexibility; hence, the current problem concerns a network flexibility technique. Previous research suggests that network flexibility could be a significant factor in location decisions. Examples of network flexibility under various operating situations include outsourcing, hiring short-term employees, or renting equipment during periods of high demand (Yu & Solvang, 2020).

In this regard, the research's primary contributions that clearly set this work apart from related papers are as follows:

1. Developing a reliable supply chain model that optimizes both forward and reverse networks across two sales channels: online and offline. It maximizes the profit of the entire supply chain network.
2. The implementation of a novel mathematical model for the uncertain dual-channel closed-loop supply chain network design (DC-CLSCND) problem aims to support decision-making regarding aspects such as location, inventory quantities, network flexibility, production quantities, and recycling quantities.
3. There are two types of uncertainty, fuzzy and stochastic, in the design of a dual-channel closed-loop supply chain network, due to parameter uncertainty and uncertainty arising from the occurrence of disturbances, respectively. Therefore, both disruption and operational risks are involved in the decision-making environment of this study. A robust fuzzy stochastic approach (RFSP) is used to simultaneously deal with and manage these uncertainties.
4. In this research, we designed a problem in which customer demand for various products at offline retailers is influenced by the price of those products, establishing a linear price-demand relationship.
5. A dual-channel closed-loop supply chain network design problem is introduced, focusing on online retailer customers who are sensitive to product delivery lead time. There exists a linear relationship between customer demand and the delivery lead time for products at the online retailer.

This study's primary goal is to create a reliable, multiple-tier, multiple-period, multiple-product DC-CLSC network that accounts for demand, cost, capacity, and return rate uncertainties. It also considers how the offline retailer's customer demand is sensitive to pricing decisions and how the online retailer's customer demand depends on delivery lead time. Maximizing the overall profit is the objective function to address the problem. To maximize the overall profit, a mathematical model has been constructed in this study to optimize the DC-CLSC structure. A robust fuzzy stochastic programming approach is employed to control and manage uncertainty in the current DC-CLSC network; also, the capacity of distribution centers is affected by disturbances. This study addresses scenario-based uncertainty regarding the percentage of returned used products and the rate of products returned for recycling. It also explores hybrid fuzzy and stochastic scenario-based uncertainty related to customer demand, as well as fuzzy uncertainty affecting total costs and the capacity of supply chain centers. The format for the investigation reminder is as follows:

The literature on the study's topic is reviewed in Section II. The problem definition is stated in section III. Section IV presents the mathematical formulation and solution methodology. The numerical computations and their outcomes are covered in Section V, and Section VI contains conclusions and suggestions for additional research.

II. LITERATURE REVIEW

This section reviews the literature on lead time and pricing demand sensitivity, closed-loop supply chain network design with uncertainty, and dual-channel closed-loop supply chain networks. An evaluation of the relevant literature is followed by a summary of the results that emphasizes the current research gap.

A. Closed-loop supply chain network design in the presence of uncertainty

Fazli-Khalaf et al. (2017) have created a green CLSC that reduces the network's overall costs while minimizing the emission of dangerous gases. The suggested approach presents a scenario-based stochastic programming method to manage and mitigate the negative impacts of disruptions. Additionally, a hybrid robust fuzzy stochastic programming approach was used to address parameter uncertainty and the degree of risk aversion in output decisions. Jabbarzadeh et al. (2018) applied a resilience method to handle disruptions and a robust stochastic optimization model to establish a closed-loop supply chain network. In their proposed model, lateral transshipment was used as a reactive resilience strategy to manage operational risks and disruptions. A Lagrange relaxation algorithm has been proposed to solve the robust model effectively. In another study, a novel mathematical fuzzy-stochastic framework for a sustainable

CLSCND was developed (Yu & Solvang, 2020). With various kinds of uncertainty, this model seeks to balance environmental performance and cost-effectiveness in the supply chain network. Furthermore, to satisfy different customer requirements, the decision-making process illustrates the network's flexibility. This work uses the sample average approximation-based weighting method (SAAWM) to propose several Pareto optimal solutions for cost and carbon emissions according to various uncertainty conditions. To establish a closed-loop supply chain network (SCLSC) of water tanks that takes sustainability criteria into account, Nayeri et al. (2020) developed a multi-objective mathematical model. This SCLSC design challenge is intended for use in an uncertain environment. Therefore, a fuzzy robust optimization (FRO) approach has been used to address the problem's uncertainty. Next, a goal programming approach was applied to solve the suggested model. Mehrjerdi & Shafiee (2021) used the information-sharing supply chain strategy and the backup supplier resilience approach to create a resilient and sustainable closed-loop supply chain. To do this, they utilized linguistic variables and the assistance of professionals and scholars to investigate how supply chain techniques affect resilience criteria. The model for this problem was solved using the augmented ε-constraint (AUGMECON2) approach. Vali-Siar & Roghanian (2022) created a mixed-integer linear programming model to deal with the challenge of constructing a supply chain network that is both closed-loop and open-loop while taking resilience, responsiveness, and sustainability into account. The hybrid robust-stochastic optimization strategy has been utilized to control and manage the problem's uncertainty. Two methods have been devised to handle the issue in large dimensions: a constructive heuristic (CH) algorithm and a Lagrange relaxation (LR) approach. The analysis of this problem was based on an actual case study in the tire industry. In a different study, a flexible, stochastic, and fuzzy programming model with an emphasis on the *Me* measure was created utilizing robust optimization (Hosseini Dehshiri et al., 2023). Capacity, unmet demand, stochastic and fuzzy deviations, and violations of flexible constraints are all controlled and managed by the model. To take into account economic, environmental, and responsive objectives, a case study for a stone paper product was carried out by lowering the total cost and carbon emissions, in addition to processing and transportation time. Furthermore, the interactive fuzzy programming method is implemented to manage the multi-objective model, and the best-worst method (BWM) is utilized for estimating the weights of the objectives. In another study, Talebzadeh et al. (2023) addressed the design of a closed-loop supply chain network with multi-objective functions, including the minimization of costs, environmental impacts, and time delays in shipping raw materials and products, along with uncertainty in some parameters. By applying the method of Jimenez et al. (2007), the developed multi-objective fuzzy model was converted into a crisp model. To solve the multi-objective problem and evaluate the effectiveness of the proposed model, the Torabi-Hassini (TH) and augmented epsilon-constrained methods were used, such that the computational results showed that, overall, the application of the augmented epsilon-constrained method has better performance in terms of objective function values and solution time compared to the TH method at all confidence levels.

Dehghani Sadrabadi et al. (2024) created a green and resilient closed-loop supply chain that addressed several interrelated interruptions while taking pricing into account using a multi-objective optimization approach. The fuzzy c-means clustering approach was used in this study to reduce the number of possible scenarios and cluster primary suppliers and target markets to decrease the problem's mathematical complexity. This research developed a novel robust optimization approach to cope with the hybrid uncertainty of the environment. Lastly, to assess the suggested model, a case study in the automotive battery production sector was performed. Han et al. (2024) designed a closed-loop supply chain network that accounts for demand uncertainty. A continuous approximation approach was used to solve the responsive CLSCN model. They also applied an inventory-sharing strategy in the continuous approximation approach to obtain a solution considering the uncertain demand in the network design. In this study, comparing the proposed model with the traditional responsive CLSCN model shows a significant reduction in costs.

B. Closed-loop supply chain network design utilizing the dual channels

Customers are increasingly shopping online because e-commerce and Internet services are developing more. In a study, a manufacturer sells new products through a retailer as well as directly from the online sales channel, so Liu et al. (2021) designed a novel CLSC framework for this problem. The manufacturer can also sell remanufactured or refurbished products directly online. They studied the optimal pricing and production methods in a two-channel closed-loop supply chain to create new and remanufactured goods. Channel competition, as well as new and remanufactured products, is considered. For the tire business, Fathollahi-Fard et al. (2021) suggested a dual-channel, multiple-product, multiple-period, multiple-echelon closed-loop SCND in an uncertain situation. This study employs a fuzzy methodology called the Jimenez method to handle uncertain parameters involving prices and demand. This research has shown that integrating novel metaheuristic algorithms, genetic metaheuristic algorithms (GA), and simulated annealing (SA) produces very effective solutions in a reasonable amount of time. Regarding a closed-loop supply chain, Gao et al. (2024a) developed two sales channels: an online retailer and an offline retailer. To tackle the difficulties that confront the home appliance business with uncertain demand, a multiple-period mathematical model has been formulated. Multiple modes of transportation, the retailer's regional protection policy, and the carbon emission restriction and trading policy are all involved in addressing this issue and the supply chain network design. They developed a robust data-driven optimization model to transform the robust optimization framework into a mathematically acceptable MILP model. They also proposed a two-stage adaptive genetic algorithm to manage the supply chain network's complexity and scale. Zhang et al. (2025) designed a robust dual-channel closed-loop supply chain network considering forward and reverse logistics, which includes two sales channels: online and offline. In the supply chain network design problem, they assumed the parameters of transportation costs and demand to be uncertain. They developed a distributionally robust optimization (DRO) model with ambiguous chance constraints (ACCs). They also created an accelerated Benders Decomposition (BD) algorithm to solve the proposed mixed-integer linear programming model for large-scale problems. Finally, a case study in the automotive industry confirmed the effectiveness of their proposed approach.

C. Pricing and lead time demand sensitivity in designing a supply chain network

Shen (2006) argued that customers' decisions about whether or not to receive services from a supply chain are only driven by product pricing. A credible price-demand relationship determines the price decision and the customers' demand. Fattahi et al. (2015) and Ahmadi-Javid & Hoseinpour (2015) utilized this method in a circumstance with deterministic decision-making. Fattahi et al. (2018) tackled the topic of redesigning a multi-period supply chain structure where price-dependent stochastic demand for various goods exists in consumer zones. To effectively make tactical decisions regarding product pricing and strategic redesign, they introduced an innovative multi-stage stochastic method. A scenario tree, which is a restricted set of possibilities, is used to describe the uncertainty in the expected demands of customer regions. A mixed-integer linear programming model was used to develop the multi-stage stochastic problem, which was solved at scale using the Benders decomposition method. Gao et al. (2024b) designed a closed-loop supply chain network that considers the influence of multiple factors. They designed a multi-product, multi-period closed-loop supply chain network under uncertainty in returns with the effects of various factors such as carbon emission policy, discounts on raw material procurement, and other constraints such as facility capacity. Also, in this research, customer demand depends on the greenness of the product as well as the price of the product. A two-stage distributionally robust chance-constrained optimization model is developed to deal with the uncertainty in the returns, which leads to a mixed-integer linear programming model. To deal with the complexity of the problem, the authors proposed an improved Benders decomposition (IBD) algorithm. In the numerical results section, they demonstrated the significant effectiveness of the IBD algorithm over the Benders decomposition algorithm. Sadeghi et al. (2024) presented a bi-level pricing-inventory model for managing closed-loop supply chains of perishable products, where demand is price-dependent. They integrated perishability limits and price dynamics into their proposed model. This research employs a mixed-integer optimization approach, which blends derivative-based methods with integer searching, to maximize supply chain profit. By doing so, the framework identifies the optimal inventory and pricing strategies. The results indicate that the model can resolve the trade-off between product perishability and transportation costs, thus creating the necessary potential to increase profitability in perishable goods supply chains.

Fattahi et al. (2017) created a supply chain network that is responsive and resilient to operational risks and disturbances for customers who are concerned about delivery lead time. They established a multiple-period supply chain (SC) network structure, where customers rely on the service centers based on their delivery lead time. Potential disruptions cause facilities' capacity to fluctuate randomly, and customer demands are stochastic. To determine the potential impact of disruptions on facility capacity, they developed a multi-stage stochastic programming model.

Table I. Classification and comparison of the present research with related studies in the literature

Paper	Supply chain features			Facility		Lead time sensitivity	Price dependency	Uncertain parameters				Methodology
	Forward SC	Closed-loop	Dual-channel	Reliable	Unreliable			Demand	Cost	Capacity	Return flows	
Fazli-Khalaf et al. (2017)		✓		✓	✓			✓	✓	✓	✓	Robust fuzzy stochastic prog. e-constraint & GAMS
Fattahi et al. (2017)	✓					✓		✓	✓			Multi-stage stochastic programming. GAMS
Jabbarzadeh et al. (2018)		✓						✓	✓		✓	Robust stochastic programming, Lagrange relaxation
Fattahi et al. (2018)	✓							✓				Multi-stage stochastic programming. Bender's decomposition
Yu & Solvang (2020)		✓						✓	✓	✓	✓	Lingo
Mehrjerdi & Shafiee (2021)		✓						✓	✓	✓	✓	e-constraint & GAMS
Vali-Siar & Roghanian (2022)		✓						✓	✓			Mixed robust-stochastic opt. Lagrange relaxation & heuristic algorithm
Talebzadeh et al. (2023)		✓						✓	✓	✓	✓	Applying the method of Jimenez, Torabi-Hassini (TH), and the Augmented Epsilon-Constrained methods
Hosseini Dehshiri et al. (2023)		✓						✓	✓	✓	✓	Flexible, probabilistic, and stochastic programming, GAMS
Gao et al. (2024b)		✓					✓				✓	A two-stage distributionally robust chance-constrained opt. IBD algorithm
Dehghani Sadrabadi et al. (2024)		✓										e-constraint & GAMS, c-means clustering algorithm
Han et al. (2024)		✓						✓				A continuous approximation approach by applying an inventory-sharing strategy
Sadeghi et al. (2024)		✓					✓					A mixed-integer optimization approach, which combines derivative-based methods with integer search
Gao et al. (2024a)		✓	✓					✓				Data-driven robust optimization, Two-stage adaptive genetic algorithm
Zhang et al. (2025)		✓	✓					✓	✓			Distributionally robust optimization model with ambiguous chance constraints and Bender's decomposition
This research		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	Robust fuzzy stochastic programming GAMS

Our research highlights several crucial aspects of closed-loop supply chain network design under uncertainty that have been largely overlooked in prior studies, as illustrated in Table I and our literature analysis. This research appears to be the first investigation to develop a dual-channel closed-loop supply chain structure that considers both disruption and operational risks. While Gao et al. (2024a) designed a dual-channel closed-loop supply chain that considers uncertainty only in the demand parameter, they did not account for disruptions. In this study, disruption is incorporated as an additional source of uncertainty. We also consider two types of uncertainty for the demand parameter: stochastic and fuzzy. Additionally, parameters such as costs and facility capacity are subject to fuzzy uncertainty, while the product return rate and the rate of used products sent to recycling centers are treated as stochastic uncertainty. The research employs a solution methodology that integrates stochastic programming, fuzzy programming, and robust programming to derive practical, acceptable solutions for real-world scenarios. This approach has not been previously documented in the literature concerning the design of dual-channel closed-loop supply chain networks. Furthermore, this study incorporates reliable centers to mitigate the effects of disruptions. However, most SCND models assume that customer demands are independent parameters, implying that supply chain configurations and decisions have no impact on the quantities they require (Fattahi et al., 2016). However, considering the present competitive business environment, this assumption is inappropriate. Based on strategic supply chain planning, this study shows how customers of online retailers are sensitive to delivery lead time. Additionally, this study assumes that the price offered by the retailer determines the uncertain demand of offline retailers. To the best of our knowledge, no research has demonstrated that, in a dual-channel closed-loop supply chain network design challenge, the uncertain demands of offline retailers are price-dependent, while the demands of online retailers depend on delivery lead time. As can be seen in the literature, Fattahi et al. (2017) designed a forward supply chain network with customers sensitive to delivery lead time, and Fattahi et al. (2018) also redesigned a forward supply chain network with price-dependent uncertain demand. The last two mentioned studies only include stochastic uncertainty and do not consider fuzzy uncertainty, while the present study considers both fuzzy and stochastic uncertainty. Thus, we developed a robust fuzzy stochastic model in which the demand from offline retailers' customers depends on price, while the demand from online retailers' customers is sensitive to delivery lead time.

III. PROBLEM DEFINITION

This study investigates a DC-CLSCND problem with several layers, periods, and products. The forward supply chain, which encompasses the facilities of suppliers, manufacturers, distributors, offline retailers, an online retailer, and customer segments, is one of the two distinct components of the CLSC, as shown in Fig. 1. Recycling facilities, a disposal center, and collection/inspection centers are all part of the network's reverse flow. Manufacturers can purchase their raw materials from recycling facilities and suppliers. The manufacturing facilities distribute their products to distribution centers and online retailers. Distribution centers transport goods to offline retailers. Selling goods to customers, handling returns, and gathering end-of-life items are the responsibilities of offline retailers. After being returned to offline retailers, goods are sorted and examined in collection/inspection facilities. Items that fulfill the recycling requirements are delivered to recycling facilities, while the others are taken to a disposal facility for proper destruction. The risk of disruption, which is defined as a stochastic scenario-based risk, is present in this supply chain network. In this DC-CLSC, only the distribution centers are vulnerable to disruption, and their capacity may be partially impacted. There are two types of distribution centers: reliable and unreliable. Reliable distribution centers can endure disturbances and preserve their capacity, while unreliable ones cannot. The remaining network infrastructure is expected to be reliable and resilient to interruptions. Both online and offline retailers address the demands of their respective customer zones. Since customers of the online retailer have a sensitivity to delivery lead time, the demands of customers in this study are dependent on the facility that serves them. The demand for different products from customer zones of offline retailers is influenced by the price per unit. A defined price-demand relationship indicates the sensitivity of varying customer regions to price changes.

A. Assumptions

The dual-channel closed-loop supply chain network design model for the current problem has the characteristics and assumptions mentioned below:

1. There are fuzzy and stochastic uncertainties regarding the demand for each item across every time period.
2. The capacity of suppliers, production facilities, distribution facilities, collection and inspection facilities, and recycling facilities is constrained.
3. Disturbances partially disrupt distribution centers; however, they do not affect the capacity of the remaining facilities, which can sustain them.
4. Production centers are established using a variety of production technologies, each with its own costs. However, each production center is only established using one production technology.
5. In this supply chain, the characteristics concerning costs and capacities are regarded as fuzzy uncertainty.
6. The network does not cover the disposal center's location or the cost of waste transportation.
7. A multiple-period and multiple-product instance is considered for studying this problem.
8. In this study, online retailer customer demand is sensitive to product delivery lead time, and the relationship between demand and delivery lead time is linear.
9. In this research, product prices in offline retailers are defined as discrete and divided into a limited range of price levels. The relationship between customer demand and product prices in offline retailers is described as linear.

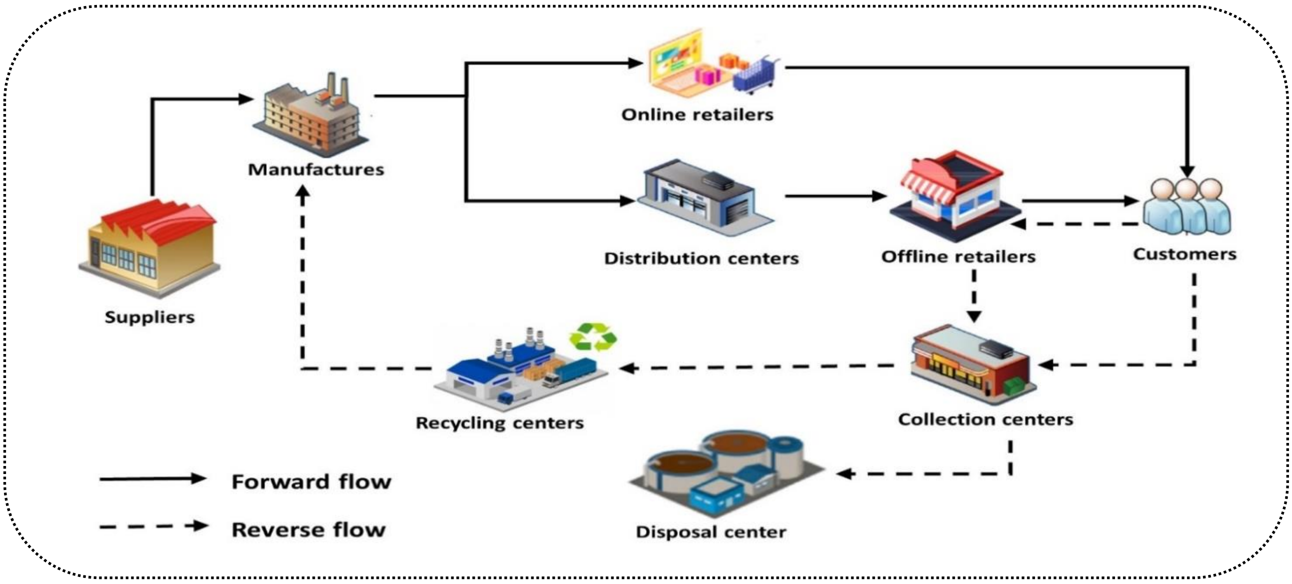


Fig. 1. The proposed dual-channel closed-loop supply chain network

B. Customers Sensitive to Delivery Lead Time

This study defines potential customer demand as stochastic and fuzzy. The two online and offline sales channels allow customers to fulfill their product demands. Additionally, the demand for the online sales channel appears to be sensitive to the lead time for customer product delivery. If the online retailer serves customer c, the percentage of this customer's demand for item p that is requested from this supply chain will be equal to $gama_{p,c}$ in the DC-CLSCND model created for this study. The goal of this parameter is to model how the facility where customers receive service affects their demands.

In this study, a linear function is used to indicate how sensitive customer demands are to delivery lead time. In Fig. 2, two factors, ***a*** and ***b***, illustrate the relationship between customer demand and delivery lead time. All customer demands are satisfied when the delivery lead time is less than ***a***, and they are zero when the delivery lead time is higher than ***b*** (Fattahi et al., 2017).

$$gama_{p,c} = \begin{cases} 0 & DLT_c \geq b_{c,p} \\ \frac{b_{c,p}-DLT_c}{b_{c,p}-a_{c,p}} & a_{c,p} < DLT_c < b_{c,p} \\ 1 & DLT_c \leq a_{c,p} \end{cases} \tag{1}$$

DLT_c represents the online retailer's delivery lead time for customer c. Fig. 2 indicates that expression (1) defines the parameter $gama_{p,c}$.

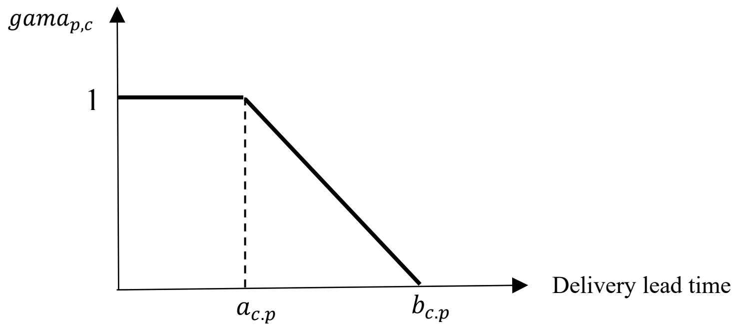


Fig. 2. The connection between customer demand and delivery lead time (Fattahi et al., 2017)

C. A strategy for modeling the relationship between demand and price

Talluri & Van Ryzin (2006) presented a broad method of price discretization for pricing problems. In many real-world situations, they highlighted the importance of discretizing prices and turning them into a limited range of price levels. This study suggests that product prices influence customer demand for offline retailers. A linear function has been used to model the price-demand connection for this purpose (Fattahi et al., 2018).

According to this method, a company's maximum and minimum prices for each product are PU and PL, respectively. As the linear function in Fig. 3 shows, demand reaches its maximum (DM) at the minimum price (PL) and falls to zero at the maximum price (PU) for offline retailers.

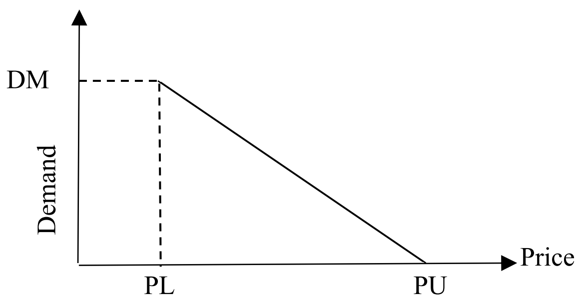


Fig. 3. Linear price-demand relationship (Fattahi et al., 2018)

As previously described, a collection of several price levels is defined as L for price discretization purposes. The price $PR_{c.p.l.t}$ is given to the customer $c \in C$ for a product unit $p \in P$ at the pricing level $l \in L$ during period $t \in T$, and its value is determined as follows:

$$PR_{p,c,l,t} = PL_{p,c,t} + \left(\frac{l-1}{|L|-1}\right)\left(PU_{p,c,t} - PL_{p,c,t}\right), \qquad c \in C, p \in P, l \in L, t \in T \tag{2}$$

Additionally, the following formulas are used to calculate the realized demand ($\widetilde{DEMF}_{p,c,l,t,s}$) and revenue ($Re_{p,c,l,t,s}$) from fulfilling offline retailer customer c's realized demand for product $p \in P$ and offering price level $l \in L$ during period $t \in T$ in scenario $s \in S$ ($\widetilde{DEM'}_{(p,c,t,s)}$ is the potential demand of the offline retailer):

$$\widetilde{DEMF}_{p,c,l,t,s} = \widetilde{DEM'}_{(p,c,t,s)}\left(\frac{PU_{p,c,t} - PR_{p,c,l,t}}{PU_{p,c,t} - PL_{p,c,t}}\right), \qquad c \in C, p \in P, l \in L, t \in T, s \in S \tag{3}$$

$$Re_{p,c,l,t,s} = \widetilde{DEMF}_{p,c,l,t,s} PR_{p,c,l,t}\ , \qquad c \in C, p \in P, l \in L, t \in T, s \in S \tag{4}$$

IV. MATHEMATICAL FORMULATION

This section presents the methods for handling the problem's uncertainties as well as the mathematical formulation of the dual-channel closed-loop supply chain network design problem that was defined in the previous section. We start by defining the notations required for mathematical modeling.

A. Notations

Sets:

p = (1,2,…, |P|)	Set of products, $p \in P$
j = (1,2,…, |J|)	Suppliers, $j \in J$
m = (1,2,…, |M|)	Set of possible sites for manufacturing facilities, $m \in M$
I = (1,2,…, |I|)	Set of possible distribution facility locations, $i \in I$
f = (1,2,…, |F|)	Set of offline retailers, $f \in F$
c = (1,2,…, |C|)	Set of customer regions, $c \in C$
k = (1,2,…, |K|)	Set of possible sites for facilities to collect and inspect, $k \in K$
r = (1,2,…, |R|)	Set of possible sites for recycling facilities, $r \in R$
t = (1,2,…, |T|)	Set of time periods, $t \in T$
s = (1,2,…, |S|)	Set of scenarios, $s \in S$
w = (1,2,…, |W|)	Set of technologies used in production, $w \in W$
l=(1,2,…, |L|)	Set of price levels, $l \in L$
o = (l,m,u)	Set of fuzzy numbers, indexed by *o*

Parameters:

$\widetilde{FC}_{(m.w)}$	Fixed cost of establishing production facility *m* with manufacturing technology *w*
$\widetilde{FC}_j$	Cost of selecting supplier *j*
$\widetilde{FC}_i$	Fixed cost of establishing unreliable distribution facility *i*
$\widetilde{FCR}_i$	Fixed cost of establishing reliable distribution facility *i*
$\widetilde{FC}_k$	Fixed cost of establishing collection/inspection facility *k*
$\widetilde{FC}_r$	Fixed cost of establishing recycling facility *r*
$\widetilde{tc}_p$	Transportation costs for a unit of a product type *p*
$\widetilde{tr}$	Transportation costs for a unit of raw materials
$dis_{(j.m)}$	The distance between supplier j and production facility *m*
$dis_{(m.i)}$	The distance between production facility *m* and distribution facility *i*
$diso_m$	The distance between production facility *m* and online retailer
$dis_{(i.f)}$	The distance between distribution facility *i* and offline retailer *f*
$dis_{(f.k)}$	The distance between offline retailer *f* and collection/inspection facility *k*
$dis_{(k.r)}$	The distance between collection/inspection facility *k* and recycling facility *r*
$dis_{(r.m)}$	The distance between recycling facility *r* and production facility *m*
$\widetilde{mc}_{(p.m.w)}$	Cost of manufacturing a product *p* by employing technology *w* at a production center *m*
$\widetilde{ch}_{(p.m)}$	Cost of holding Product *p* at production facility *m*
$\widetilde{RCA}_j$	Cost of acquiring raw materials through the supplier *j*
$\widetilde{cd}_i$	Cost for shipping each product unit to the distribution facility *i*
$\widetilde{cc}_p$	Cost of collecting and assessing one product unit *p*
$\widetilde{rc}_r$	Cost of recycling a single product unit at the recycling facility *r*
$\widetilde{dd}$	Cost of disposing of a single product item at the disposal facility
rmm_p	Percentage of waste raw materials produced during the manufacturing of a single product unit p
ω	The quantity of raw materials gained from a product unit's recycling
$\delta_{(p.s)}$	The rate of product *p* sent to the recycling facility in scenario *s*

$\alpha_{(p.t.s)}$	Percentage of product *p* returns during period *t* in scenario *s*
β_p	Percentage of online channel customers for product *p*
$\widetilde{oc}_{(p.c)}$	Cost of product *p* for customer *c* in the forward channel with flexible capacity
$\widetilde{orc}_{(p.c)}$	Cost of product *p* for customer *c* in the reverse channel with flexible capacity
$ol_{(p.t)}$	Total flexibility limit of product *p* in the forward channel during period *t*
$rol_{(p.t)}$	Total flexibility limit of product *p* in the reverse channel during period *t*
$\widetilde{DEM}_{(p,c,t,s)}$	Demand for product *p* from customer *c* during period *t* in scenario *s*
$ivo_{(p.m)}$	Initial inventory of product *p* at production plant *m*
$\widetilde{caph}_m$	Storage capacity of production facility *m*
$\widetilde{cap}_{(p.m)}$	Production capacity of plant *m* for product *p*
$\widetilde{cap}_j$	Capacity of supplier *j*
$\widetilde{cap}_i$	Capacity of distribution facility *i*
$\widetilde{cap}_k$	Capacity of collection/inspection facility *k*
$\widetilde{cap}_r$	Capacity of recycling facility *r*
$\widetilde{dco}_c$	Cost of delivering a product unit from an online retailer to a customer zone *c*
$\widetilde{dcl}_{(c.k)}$	Cost of transporting a returned product unit from customer zone *c* to the collection/inspection facility *k*
$\widetilde{rcp}_p$	Cost of purchasing one unit of used product *p*
$re_{(p.c.t)}$	Number of used products *p* in customer region *c* during period *t*
$\lambda i_{(i,s)}$	Percentage of capacity reduction of unreliable distribution center *i* due to disruption under scenario *s*
$PR_{(p,c,l,t)}$	Recommended pricing level *l* for one unit of product *p* during period *t* for a customer in zone *c* from an offline retailer
$\widetilde{DEMF}_{(p.c.l.t.s)}$	The offline retailer's demand from customer zone *c* for product *p* at price level *l* during period *t* in scenario *s*
$gama_{(p,c)}$	Proportion of the online retailer's demand for product *p* from customer *c* under delivery lead time
$po_{(p.c.t)}$	The price at which an online retailer sells a unit of product *p* to customer *c* during period *t*
π_s	Probability of disruption in scenario *s*

Variables:

OJ_j	If the supplier *j* is chosen, it is equal to 1; otherwise, it is equal to 0
$OM_{(m.w)}$	If production facility *m* is opened with technology *w,* it is equal to 1; otherwise, it is equal to 0
OI_i	If the unreliable distribution facility *i* is established, it is equal to 1; otherwise, it is equal to 0
OIR_i	If the reliable distribution facility *i* is established, it is equal to 1; otherwise, it is equal to 0
OK_k	If the Collection/inspection facility *k* is established, it is equal to 1; otherwise, it is equal to 0
ORR_r	If the Recycling facility *r* is established, it is equal to 1; otherwise, it is equal to 0
$delta_{(p,c,l,t,s)}$	If the level of price *l* is chosen for item p in customer region *c* during period t in scenario *s*, it is equal to 1; otherwise, it is equal to 0.
$yj_{(c.t)}$	If the online retailer serves customer *c* during period *t, i*t is equal to 1; otherwise, it is equal to 0.
$JM_{(j.m.t.s)}$	The amount of raw materials moved from supplier *j* to the production facility *m* during period *t* in scenario *s*
$MI_{(p.m.i.t.s)}$	The amount of product *p* moved from production facility *m* to distribution facility *i* during period *t* in scenario *s*
$MO_{(p,m,t,s)}$	The amount of product *p* moved from production facility *m* to online retailer during period *t* in scenario *s*
$IFF_{(p.i.f.t.s)}$	The amount of product *p* transported from distribution facility *i* to offline retailer *f* during period *t* in scenario *s*
$FK_{(p.f.k.t.s)}$	The amount of used product *p* returned to collection/inspection facility *k* by offline retailer f during period *t* in scenario *s*
$CK_{(p.c.k.t.s)}$	The amount of used product p that the customer *c* returns to the collection/inspection facility *k* during period *t* in scenario *s*
$KR_{(p.k.r.t.s)}$	The amount of used product *p* moved from collection/inspection facility *k* to recycling facility *r* during period *t* in scenario *s*
$KD_{(p.k.t.s)}$	The amount of used product p moved from collection/inspection facility *k* to disposal facility during period *t* in scenario *s*
$RM_{(r.m.t.s)}$	The amount of recycled raw materials at the recycling center *r* transferred to the production facility *m* during period *t* in scenario *s*
$nf_{(t.s)}$	The amount of returned products used for raw material recycling during period *t* in scenario s
$QO_{(p.c.t.s)}$	Amount of product p delivered to customer c through the forward channel during period *t* in scenario *s* with network flexibility (flexible capacity)
$QRO_{(p,c,t,s)}$	The amount of product *p* for customer *c* in the reverse channel during period *t* in scenario s with network flexibility (flexible capacity)

$ma_{(p.m.w.t.s)}$	Amount of product *p* produced during period *t* in production center *m* using technology *w* in scenario *s*
$iv_{(p.m.t.s)}$	The inventory quantity of product *p* in production center *m* in period t in scenario *s*
$g_{(p.f.c.t.s)}$	Quantity of product *p* sold by the offline retailer f to customer *c* during period *t* in scenario *s*
$gp_{(p.c.t.s)}$	Quantity of product *p* sold by the online retailer to customer *c* during period *t* in scenario *s*

B. Mathematical model

$$max\ Z = \sum_{S} \pi_s \left(REV(S)\right) - \left(FC + \sum_{S} \pi_s \left(IC_S + PC_S + TC_S + RC_S + DC_S + SC_S + MC_S + QC_S\right)\right) \tag{5}$$

s.t.

$$FC = \sum_{j} \widetilde{FC}_j\, OJ_j + \sum_{m,w} \widetilde{FC}_{(m,w)} OM_{(m,w)} + \sum_{k} \widetilde{FC}_k\, OK_k \quad + \sum_{r} \widetilde{FC}_r ORR_r\ + \sum_{i} \widetilde{FC}_i\, OI_i + \sum_{i} \widetilde{FCR}_i\, OIR_i \tag{6}$$

$$IC_s = \sum_{p,m,t} \widetilde{ch}_{(p,m)}\ iv_{(p,m,t,s)} \quad , \qquad \forall s \tag{7}$$

$$PC_s = \sum_{p,m,w,t} \widetilde{mc}_{(p,m,w)}\ ma_{(p,m,w,t,s)} \quad , \quad \forall s \tag{8}$$

$$TC_s = \sum_{j,m,t} \widetilde{tr} dis_{(j,m)} JM_{(j,m,t,s)} + \sum_{p,m,i,t} \widetilde{tc}_p dis_{(m,i)} MI_{(p,m,i,t,s)} + \sum_{p,i,f,t} \widetilde{tc}_p dis_{(i,f)} IFF_{(p,i,f,t,s)} + \sum_{p,f,k,t} \widetilde{tc}_p dis_{(f,k)} FK_{(p,f,k,t,s)} + \sum_{p,k,r,t} \widetilde{tc}_p dis_{(k,r)} KR_{(p,k,r,t,s)} + \sum_{r,m,t} \widetilde{tr} dis_{(r,m)} RM_{(r,m,t,s)} + \sum_{p,m,t} \widetilde{tc}_p diso_m MO_{(p,m,t,s)} + \sum_{p,c,t} \widetilde{dco}_c gp_{(p,c,t,s)} + \sum_{p,c,k,t} \widetilde{dcl}_{(c,k)} CK_{(p,c,k,t,s)} \quad , \quad \forall s \tag{9}$$

$$RC_s = \sum_{t} (\sum_{p,f,c} \widetilde{rcp}_p \alpha_{(p,t,s)} g_{(p,f,c,t,s)} + \sum_{p,f,c} \alpha_{(p,t,s)}\ g_{(p,f,c,t,s)} \qquad + \alpha_{(p,t,s)} gp_{(p,c,t,s)})\ \widetilde{cc}_p \qquad ,\forall s \tag{10}$$

$$DC_s = \sum_{p,k,t} \widetilde{dd}\ KD_{(p,k,t,s)} \quad , \qquad \forall s \tag{11}$$

$$SC_s = \sum_{j,m,t} \widetilde{RCA}_j\, JM_{(j,m,t,s)} \quad , \qquad \forall s \tag{12}$$

$$MC_s = \sum_{p,i,f,t} \widetilde{cd}_i\ IFF_{(p,i,f,t,s)} + \sum_{r,m,t} \widetilde{rc}_r\ RM_{(r,m,t,s)} \quad , \qquad \forall s \tag{13}$$

$$QC_s = \sum_{p,c,t} \widetilde{oc}_{(p,c)}\, QO_{(p,c,t,s)} + \sum_{p,c,t} \widetilde{orc}_{(p,c)}\, QRO_{(p,c,t,s)}\,, \qquad \forall s \tag{14}$$

$$REV(S) = \sum_{p,c,l,t} \widetilde{DEM}F_{(p,c,l,t,s)} PR_{(p,c,l,t)}\, delta_{(p,c,l,t,s)} + \sum_{p,c,t} gama_{(p,c)} \beta_p \widetilde{DEM}_{(p,c,t,s)}\, yj_{(c,t)} po_{(p,c,t)} \tag{15}$$

$$\sum_m JM_{(j,m,t,s)} \le \widetilde{cap}_j OJ_j \quad , \qquad \forall j,t,s \tag{16}$$

$$ma_{(p,m,w,t,s)} \le \widetilde{cap}_{(p,m)} OM_{(m,w)} \quad , \qquad \forall p,m,w,t,s \tag{17}$$

$$\sum_p iv_{(p,m,t,s)} \le \widetilde{caph}_m \sum_w OM_{(m,w)} \quad , \qquad \forall m,t,s \tag{18}$$

$$\sum_{p,f} FK_{(p,f,k,t,s)} + \sum_{p,c} CK_{(p,c,k,t,s)} \le \widetilde{cap}_k OK_k \quad , \qquad \forall k,t,s \tag{19}$$

$$\sum_m RM_{(r,m,t,s)} \le \widetilde{cap}_r ORR_r \quad , \qquad \forall r,t,s \tag{20}$$

$$\sum_{p,f} IFF_{(p,i,f,t,s)} \le \widetilde{cap}_i \left[\left(\left(1 - \lambda i_{(i,s)}\right) OI_i \right) + OIR_i \right] \quad , \quad \forall i,t,s \tag{21}$$

$$\sum_f g_{(p,f,c,t,s)} + \left(1 - \beta_p\right) QO_{(p,c,t,s)} \ge \sum_l delta_{(p,c,l,t,s)} \widetilde{DEM}F_{(p,c,l,t,s)} \quad , \qquad \forall p,c,t,s \tag{22}$$

$$gp_{(p,c,t,s)} + \beta_p QO_{(p,c,t,s)} \ge gamma_{(p,c)}\, \beta_p \widetilde{DEM}_{(p,c,t,s)}\, yj_{(c,t)} \quad , \qquad \forall p,c,t,s \tag{23}$$

$$\sum_w OM_{(m,w)} \le 1 \quad , \qquad \forall m \tag{24}$$

$$\sum_j JM_{(j,m,t,s)} + \sum_r RM_{(r,m,t,s)} = \sum_p (1 - rmm_p) \sum_w ma_{(p,m,w,t,s)} \quad , \qquad \forall m,t,s \tag{25}$$

$$iv_{(p,m,t,s)} = ivo_{(p,m)} + \sum_{w} ma_{(p,m,w,t,s)} - MO_{(p,m,t,s)} - \sum_{i} MI_{(p,m,i,t,s)} = 1 \quad , \quad \forall m,p,s,t \tag{26}$$

$$iv_{(p,m,t,s)} = iv_{(p,m,t-1,s)} + \sum_{w} ma_{(p,m,w,t,s)} - MO_{(p,m,t,s)} - \sum_{i} MI_{(p,m,i,t,s)} > 1 \quad , \quad \forall m,p,s,t \tag{27}$$

$$\sum_{m} MI_{(p,m,i,t,s)} = \sum_{f} IFF_{(p,i,f,t,s)} \quad , \quad \forall p,i,t,s \tag{28}$$

$$\sum_{i} IFF_{(p,i,f,t,s)} = \sum_{c} g_{(p,f,c,t,s)} \quad , \quad \forall p,c,t,s \tag{29}$$

$$\sum_{m} MO_{(p,m,t,s)} = \sum_{c} gp_{(p,c,t,s)} \quad , \quad \forall p,t,s \tag{30}$$

$$\sum_{k} FK_{(p,f,k,t,s)} = \alpha_{(p,t,s)} \left(\sum_{c} g_{(p,f,c,t,s)} + (1-\beta_p) QO_{(p,c,t,s)} \right) \quad , \quad \forall p,f,t,s \tag{31}$$

$$\sum_{k} CK_{(p,c,k,t,s)} = \alpha_{(p,t,s)} \left(gp_{(p,c,t,s)} + \beta_p QO_{(p,c,t,s)} \right) \quad , \quad \forall p,c,t,s \tag{32}$$

$$\sum_{f} FK_{(p,f,k,t,s)} + \sum_{c} CK_{(p,c,k,t,s)} = \sum_{r} KR_{(p,k,r,t,s)} + KD_{(p,k,t,s)} \quad , \quad \forall p,k,t,s \tag{33}$$

$$\sum_{r} KR_{(p,k,r,t,s)} = \delta_{(p,s)} \left(\sum_{f} FK_{(p,f,k,t,s)} + \sum_{c} CK_{(p,c,k,t,s)} \right) \quad , \quad \forall p,k,t,s \tag{34}$$

$$\sum_{k,r,p} KR_{(p,k,r,t,s)} = nf_{(t,s)} \quad , \quad \forall t,s \tag{35}$$

$$\sum_{r,m} RM_{(r,m,t,s)} = \omega\, nf_{(t,s)} \quad , \quad \forall t,s \tag{36}$$

$$OI_i + OIR_i \leq 1 \quad , \quad \forall i \tag{37}$$

$$\sum_{i} OIR_i \geq 1 \tag{38}$$

$$\sum_k FK_{(p,f,k,t,s)} + \sum_k CK_{(p,c,k,t,s)} + QRO_{(p,c,t,s)} \geq re_{(p,c,t)} \ , \qquad \forall p,c,f,t,s \tag{39}$$

$$\sum_c QO_{(p,c,t,s)} \leq ol_{(p,t)} \ , \qquad \forall p,t,s \tag{40}$$

$$\sum_c QRO_{(p,c,t,s)} \leq rol_{(p,t)} \quad , \qquad \forall p,t,s \tag{41}$$

$$\sum_l delta_{(p,c,l,t,s)} = 1, \qquad \forall p,c,l,t,s \tag{42}$$

$$\begin{aligned} & OJ_j . OM_{(m.w)} . OI_i . OIR_i . OK_k . ORR_r \in \{0.1\} \\ & JM_{(j.m.t.s)} . MI_{(p.m.i.t.s)} . MO_{(p.m.t.s)} . IFF_{(p.i.f.t.s)} . \\ & g_{(p.f.c.t.s)} . gp_{(p.c.t.s)} . FK_{(p.f.k.t.s)} . CK_{(p.c.k.t.s)} . KR_{(p.k.r.t.s)} . KD_{(p.k.t.s)} . QO_{(p.c.t.s)} . \\ & QRO_{(p.c.t.s)} . ma_{(p.m.w.t.s)} . iv_{(p.m.t.s)} \geq 0 \end{aligned} \tag{43}$$

According to expression (5), the objective function is made up of ten terms that represent the costs and revenues for the supply chain network under consideration. Equations (6) through (14) thus represent the fixed costs of facility establishment and supplier selection, while the variable costs include keeping inventory within the production center, product manufacturing within the production center, product transfer between various supply chain participants, purchasing and collecting used products, waste disposal, purchasing raw materials from suppliers, distributing products in distribution centers, recycling raw materials in recycling centers, and deploying flexible capacity in the supply chain network. Finally, equation (15) describes supply chain network revenue, which includes revenue from product sales to both offline and online retailers. All of the cost parameters for the objective function are represented as triangular fuzzy numbers.

Constraints (16) through (21) stipulate that suppliers' maximum permitted capacities, production centers' manufacturing and inventory holding capacities, collection/inspection centers' capacities, recycling centers' capacities, and distribution centers' capacities cannot be exceeded. Network demand, which comes from two sales channels, online and offline, must be met through manufacturing products in the network and utilizing flexible capacity, as shown by constraints (22) and (23). Constraint (24) ensures that each newly established production center can only choose one production technology. According to constraint (25), suppliers and recycling facilities provide the raw materials required for product manufacturing.

Constraints (26) to (30) balance the forward flow of the supply chain network. According to constraint (31), the amount of used products moved from an offline retailer is equal to a percentage of the products sold to customers plus the same percentage of the flexible capacity used in the offline sales channel. Constraint (32) shows that the quantity of returned used products originally purchased through the online retailer is equal to a percentage of the products sold to customers plus the same percentage of the flexible capacity employed in the online sales channel. This quantity is transferred from customers to collection/inspection centers by third-party logistics. In the supply chain network's reverse flow, constraints (33) through (36) guarantee the balance of product flow. Constraint (37) guarantees that a distribution center can only be opened at any possible location using either a reliable or an unreliable facility. Constraint (38) ensures that at least one reliable distribution center is established. According to constraint (39), all returned products from the customer region must undergo appropriate processing. Constraints (40) and (41) illustrate the maximum amount of flexible capacity that can be used to assist the network in meeting customer demands in the forward and reverse flows, respectively. Each customer zone will receive a single price level for a single unit of any product in every scenario and time period, according to Constraint (42). Constraint (43) indicates the type of decision variables.

C. Methodology

In this research, an approach developed by Fazli-Khalaf et al. (2017) is used to model uncertain parameters and control the risk aversion of output solutions. In this section, the robust stochastic programming model is first presented. The fuzzy programming approach and its equivalent crisp model are described, and finally the hybrid robust fuzzy stochastic programming (RFSP) model is presented.

- ***Robust Stochastic Programming Model***

An efficient approach is scenario-based stochastic programming for modeling disturbances and parameter uncertainty. On the one hand, scenario-based stochastic programming could be considered less sensitive to deviations of the objective function and constraints. Based on the decision maker's risk aversion, a robust optimization approach may be helpful in this situation. Additionally, under various uncertain scenarios, the optimal solutions that are derived in this manner are less sensitive to parameter simulations (Fazli-Khalaf et al., 2017). Model (44) is described as a scenario-based stochastic programming model, and its robustness approach is displayed:

$$\begin{aligned}
&min\, Z = cx + \sum\nolimits_{s} p_s dy_s \\
&s.t. \\
&\quad Ax \le b \\
&\quad fy_s \ge h_s \\
&\quad x,\, y_s \ge 0
\end{aligned} \tag{44}$$

Two pillars guarantee the optimal solution derived from robust programming: optimality robustness and feasibility robustness. If the robust solution stays (almost) feasible when uncertain parameters change, it has the property of feasibility robustness; if it stays (near) optimal when uncertain parameters change, it has the property of optimality robustness (Pan & Nagi, 2010).

The variables x and y_s are decision variables in model (44), meaning that the random scenarios determine the value of y_s, while the scenarios do not affect x. The probability of each scenario occurring is represented by the parameter p_s. The sum of the values taken from the first-stage variables that are independent of the scenarios and the expected value taken from the second-stage variables under different scenarios should be used to optimize the objective function of the model, as mentioned above. The second constraint has an uncertain parameter, and the critical point is that this constraint must remain feasible for different scenario values. Applying the method of Leung et al. (2007) and Pan & Nagi (2010) to model (44) yields the robust stochastic programming approach presented in model (45):

$$\begin{aligned}
&min\, Z = cx + \sum_{s} p_s dy_s + \mu \sum_{s} p_s \left[\left(dy_s - \sum_{s'} p_{s'}\, dy_{s'} \right) + 2\theta_s \right] + \sum_{s} \delta \varepsilon_s \\
&s.t. \\
&\quad Ax \le b \\
&\quad fy_s + \varepsilon_s \ge h_s \\
&\quad dy_s - \sum_{s} p_s\, dy_s + \theta_s \ge 0 \\
&\quad x, y_s, \varepsilon_s \ge 0
\end{aligned} \tag{45}$$

The first and second terms of the objective function in model (45) express the same concepts as those presented in model (44). The optimality robustness in this model is maintained and controlled by the third term of the objective function. The objective function's total deviations from the expected value are minimized by the optimality robustness. Since the μ parameter determines the importance coefficient of the solution with regard to other parts of the objective function, the optimality robustness is often expressed as the penalty coefficient for deviations of the objective function. In order to determine the optimality robustness, the absolute value must be utilized because negative deviations may also occur. The absolute value is applied using the θ_s decision variable, and the third constraint is also applied to the model. The total penalty for violating constraints is calculated by the fourth term of the objective function. The penalty for violating the uncertain constraints is determined by the δ parameter. The decision variable of ε_s specifies the amount of violation of uncertain constraints. The feasibility robustness is determined and controlled by the last term.

- ***Fuzzy Programming Model***

In this section, the fuzzy programming model presented by Liu & Liu (2002) and Pishvaee et al. (2012) is employed to deal with and control ambiguous parameters. Model (46) is solved using this fuzzy programming approach

$$
\begin{aligned}
& min\, Z = \tilde{c}x \\
& s.t. \\
& \quad Ax \le \tilde{b} \\
& \quad Dx \ge \tilde{f} \\
& \quad x \ge 0
\end{aligned}
\tag{46}
$$

The parameters c, b, and f are assumed to have fuzzy uncertainty with a triangular fuzzy distribution in this model. Model (47) is the crisp model that corresponds to model (46), and it is consistent with the fuzzy programming method that Liu & Liu (2002) and Pishvaee et al. (2012) proposed.

$$
\begin{aligned}
& min\, Z = \left[\frac{c^{(1)} + c^{(2)} + c^{(3)}}{3}\right] x \\
& s.t. \\
& \quad Ax \le \left[(2\alpha - 1)b^{(1)} + (2 - 2\alpha)b^{(2)}\right] \\
& \quad Dx \ge \left[(2\tau - 1)f^{(3)} + (2 - 2\tau)f^{(2)}\right] \\
& \quad x \ge 0
\end{aligned}
\tag{47}
$$

The average value of the fuzzy parameters is utilized to make the objective function deterministic. The parameters α and τ are used as the fuzzy parameters' satisfaction levels to make the constraints deterministic. According to the level of risk aversion, the decision maker determines the minimum satisfaction level of the uncertainty parameters $(0.5 < \alpha, \tau \le 1)$.

- ***Robust Fuzzy Stochastic Programming Model***

The following robust fuzzy stochastic programming model is derived from the fuzzy programming model and the robust scenario-based programming model described in the previous two sections (Fazli-Khalaf et al., 2017):

$$
\begin{aligned}
& min\, Z = \left[\frac{c^{(1)} + c^{(2)} + c^{(3)}}{3}\right] x + \sum_s p_s \left[\frac{d^{(1)} + d^{(2)} + d^{(3)}}{3}\right] y_s \\
& \qquad + \beta \sum_s p_s \left[\left(\left[\frac{d^{(1)} + d^{(2)} + d^{(3)}}{3}\right] y_s - \sum_{s'} p_{s'} \left[\frac{d^{(1)} + d^{(2)} + d^{(3)}}{3}\right] y_{s'}\right) + 2\theta_s\right] + \sum_s \delta\varepsilon_s \\
& s.t. \\
& \quad Ax \le \left[(2\alpha - 1)b^{(1)} + (2 - 2\alpha)b^{(2)}\right] \\
& \quad f y_s + \varepsilon_s \ge \left[(2\beth - 1){h_s}^{(3)} + (2 - 2\beth){h_s}^{(2)}\right] \\
& \quad \left[\frac{d^{(1)} + d^{(2)} + d^{(3)}}{3}\right] y_s - \sum_s p_s \left[\frac{d^{(1)} + d^{(2)} + d^{(3)}}{3}\right] y_s + \theta_s \ge 0 \\
& \quad x, y_s, \varepsilon_s \ge 0
\end{aligned}
\tag{48}
$$

V. COMPUTATIONAL RESULTS

This study's computations and evaluation of the suggested model and its outcomes were conducted using data from the literature. Seven customer zones, four potential suppliers, five potential production center locations, and twelve potential distributor locations have all been considered in this study. Additionally, there are six fixed-location offline retailers, five potential collection/inspection centers, and four potential recycling facility locations in the network.

In order to reduce computing complexity, this study has taken into account three scenarios. With equivalent probabilities of incidence of 0.25, 0.50, and 0.25, these three scenarios reflect the highest level of pessimism, the highest possible, and the highest optimism. They are frequently employed to create disruption scenarios within SCND situations. (See, for example, Nayeri et al., 2021; Paydar et al., 2017; Sazvar et al., 2022).

A. Report of results

The models used in this research to identify the optimal closed-loop supply chain structure decisions have been implemented in GAMS software version 24.1.2. The models have been run on a computer running Windows 10 (64-bit) with an Intel(R) Core (TM) i3-2120 CPU @ 3.30GHz and 4 GB of RAM.

B. Model performance analysis

The suggested model's performance must be evaluated by solving it with several levels of satisfaction while taking the supply chain's maximum profit into account. Fig. 4 demonstrates that when the minimum satisfaction level increases from 0.6 to 1, the objective function value (the supply chain profit) decreases. The reasoning is that as satisfaction levels grow, risk aversion increases. Increasing risk aversion leads to increased costs, and, as a result, the chain profit decreases.

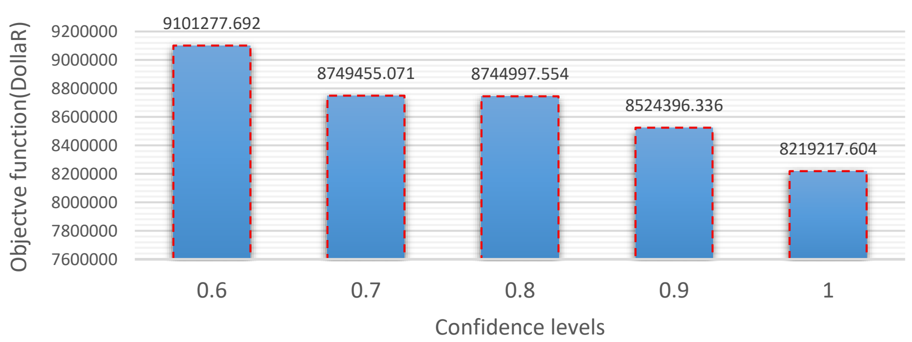


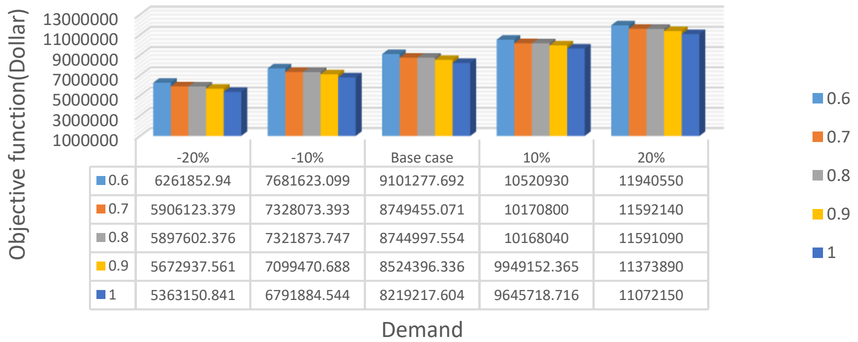
Fig. 4. Value of the objective function with different confidence levels

C. Sensitivity Analysis

The accuracy of the proposed model is verified through a sensitivity analysis of several significant parameters, assessing their impact on the objective function.

- ***Demand***

Fig. 5 displays the sensitivity analysis of the demand parameter under five different demand levels (i.e., -20%, -10%, base case, +10%, and +20%) and their impacts on the objective function. The numerical results presented in Fig. 5 indicate that supply chain profit increases as demand rises and that the objective function (total profit) decreases as satisfaction levels increase in each scenario considered. For instance, the objective function value (network profit) decreases by 31% when demand is 20% lower than the base case, at a minimum satisfaction level of 0.6. Additionally, the value of the objective function at the minimum satisfaction level of 0.6 increases by about 15.6% when demand increases by 10% compared to the base case. These findings confirm the accuracy of the model proposed in this study.



	-20%	-10%	Base case	10%	20%
0.6	6261852.94	7681623.099	9101277.692	10520930	11940550
0.7	5906123.379	7328073.393	8749455.071	10170800	11592140
0.8	5897602.376	7321873.747	8744997.554	10168040	11591090
0.9	5672937.561	7099470.688	8524396.336	9949152.365	11373890
1	5363150.841	6791884.544	8219217.604	9645718.716	11072150

Fig. 5. Analyzing the objective function's behavior in response to demand variations

Fig. 6. Sensitivity of optimal objective function value to delivery lead time variations (with satisfaction levels of 0.6 and 0.8)

- ***Delivery Lead Time***

Customer demand that depends on an online retailer's delivery lead time is one of the challenges addressed in this study. This section analyzes the objective function's sensitivity to changes in the delivery lead time. Fig. 6 demonstrates how delivery lead time variations affect the optimal objective function values.

The computational results in Fig. 6 show that, for delivery lead times from 0.1 to 0.6, with minimum satisfaction levels of 0.6 and 0.8, the optimal value of the objective function increases by 24% and 25%, respectively. This is because increased delivery time indicates less sensitivity of customers to delivery lead time, which reduces operational costs such as the cost of transporting products via third-party logistics, and consequently increases supply chain profit. Equation (1) describes the relationship between customer demand and delivery lead time. It is evident that keeping $\boldsymbol{a}$ constant and moving $\boldsymbol{b}$ closer to $\boldsymbol{a}$ increases the customer's sensitivity to delivery lead time.

- ***Price Discretization***

According to the study's assumptions, customer demand is influenced by the price offered by offline retailers, which is defined as a discrete variable.

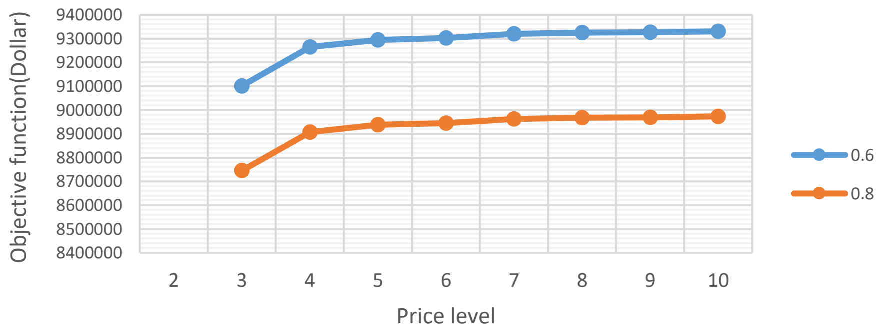


Fig. 7. Sensitivity of the optimal objective function value to the number of price levels (with satisfaction levels of 0.6 and 0.8)

As the number of price levels increases, it is evident that the discrete price-demand function approaches the continuous state and provides a more accurate approximation (Fattahi et al., 2018).

The objective function value has been determined for several different price levels in this study. As can be seen in Fig. 7, by increasing the number of price levels offered by offline retailers from 3 to 10 with minimum satisfaction levels of 0.6 and 0.8, the optimal value of the objective function increases by 2.5% and 2.6%, respectively.

- *Online sales ratio*

This section covers the sensitivity analysis of the online sales ratio parameter and its impact on the objective function. Fig. 8 displays the variations in profit, revenue, and costs for various online sales ratio values.

Due to a decrease in revenue and an increase in costs, supply chain profits decline as the ratio of online sales becomes greater, as shown in Fig. 8. Because online prices are lower than offline prices, the increase in the online sales proportion reduces supply chain revenues. Fig. 8 clearly indicates that overall costs, production costs, and transportation costs are all increasing as the proportion of online sales grows. In this case, the cost of using third-party logistics is usually the source of the increased transportation costs.

D. Performance Comparison of the Deterministic and RFSP Models

The performance analysis of the RFSP model is presented in this section. Two minimal satisfaction levels (i.e., 0.6 and 0.8) are considered when solving the model to achieve this goal. This is done by comparing the outcomes of the deterministic model with the RFSP model at the two satisfaction levels. Five realizations are created and solved for each model to achieve this. The deterministic model must be specified and solved with respect to the nominal values of the uncertain parameters. The nominal values of the uncertain parameters can be determined in relation to their average values (i.e., $\bar{c} = (c^l + c^m + c^u)/3$). Then, the uncertain parameters need to be realized, which means they must be generated at random around the triangular fuzzy distribution's extreme points (i.e., uniform~$[c^l, c^u]$). The objective function of each of these models is then obtained by solving them with respect to these five different realizations. Furthermore, the mean and standard deviation of the objective functions of the deterministic model and the RFSP model are compared. Table II presents the numerical outcomes of this comparison. In comparison to the other model, the one with a higher mean and a smaller standard deviation performs better.

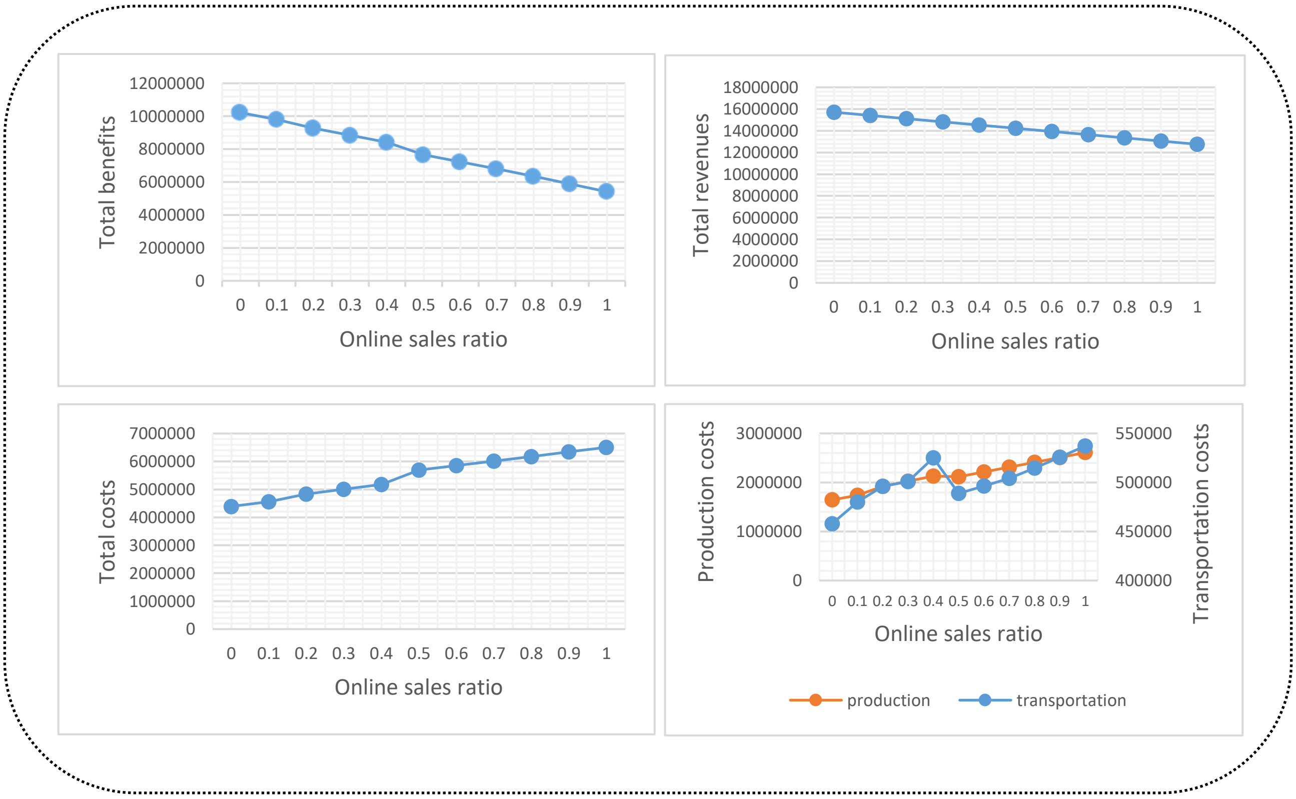


Fig. 8. Sensitivity analysis regarding the online sales ratio (with satisfaction levels of 0.6)

Table II. Comparison of results from solving the deterministic and rfsp models

Realizations	Deterministic model	RFSP model (satisfaction level=0.6)	RFSP model (satisfaction level=0.8)
No. 1	7073865.576	7713509.422	6688517.748
No. 2	7981092.478	7915259.034	7473190.893
No. 3	9368861.156	9219561.802	9021714.378
No. 4	7782657.698	7925407.503	7015194.157
No. 5	6904453.814	7080093.477	6237987.293
Mean	7822186.144	7970766.248	7287320.894
Standard Deviation	977398.7947	778477.3378	1069624.652

E. Analysis of the Robust Optimization Parameters

This section focuses on the sensitivity analysis of the model's robustness parameters. In order to accomplish this, the model is solved using various robustness coefficient values, and the outcomes are investigated. The objective function's behavior is depicted in Fig. 9 as a function of variations in the optimality robustness coefficient (for a feasibility robustness coefficient of 0.7 and a minimum satisfaction level of 0.6). For this analysis, the objective function is separately examined based on both profits and penalty costs.

As illustrated in Fig. 9, the profit declines and penalty cost rises as the optimality robustness coefficient in the objective function increases.

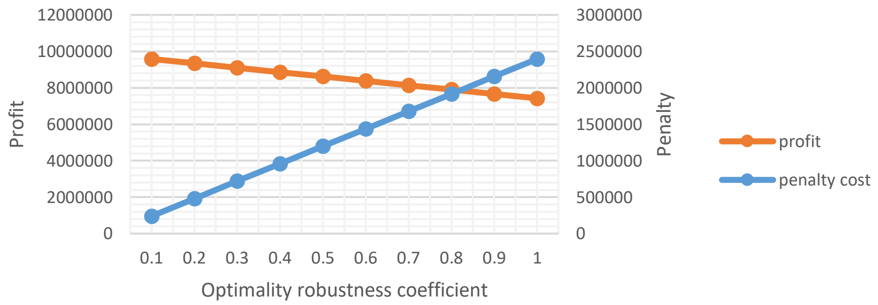


Fig. 9. Sensitivity analysis regarding the optimality robustness coefficient

The behavior of the objective function in response to variations in the feasibility robustness coefficient is depicted in Fig. 10 from another perspective (for an optimality robustness coefficient of 0.3 and a minimum satisfaction level of 0.6). This sensitivity analysis is also evaluated here, taking into account changes in the feasibility robustness and the corresponding inverse behavior in profits and penalties. Fig. 10 illustrates that profit decreases and penalty increases as the feasibility robustness coefficient increases.

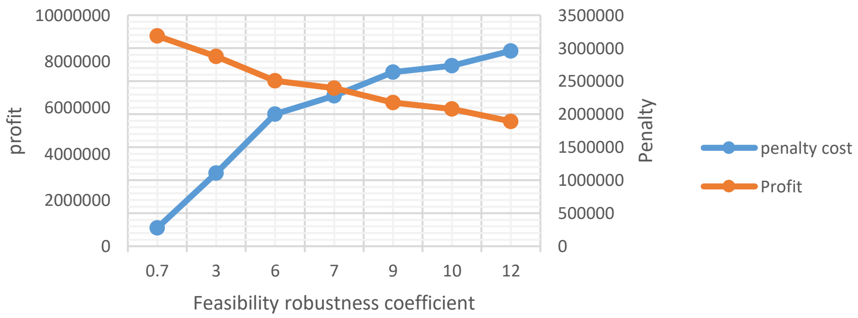


Fig. 10. Sensitivity analysis regarding the feasibility robustness coefficient

VI. CONCLUSIONS

The design of a reliable dual-channel, multiple-period, multiple-product closed-loop supply chain network including hybrid uncertainties is the main emphasis of the current study. We proposed a scenario-based MILP model to maximize the supply chain network's total profit. The traditional sales channel (offline retailers) as well as the Internet sales channel (online retailer) are the two sales channels taken into consideration in this study for selling products to customers. Additionally, it is assumed in this supply chain network that customers' demand in the offline sales channel (offline retailers) is dependent on the price that the retailer offers. This assumption is applied to the problem by defining the price as discrete, which means that the higher the offered price level, the lower the customer demand. Furthermore, it is assumed in this problem that customer demand in the online sales channel (online retailer) is sensitive to delivery lead time. As a result, the relationship between delivery lead time and demand is defined as a linear function, implying that the longer the delivery lead time, the lower the customer demand.

We utilized a robust fuzzy stochastic programming method derived from the research literature to handle the mixed uncertainty in the problem parameters. Sensitivity analysis was then carried out on several significant parameters to validate the model, and the findings demonstrated that the model performed correctly. The results showed that by increasing the delivery lead time from 0.1 to 0.6, at the minimum satisfaction levels of 0.6 and 0.8, the objective function value increases by 24% and 25%, respectively. It was also observed that, by increasing the number of price levels offered by offline retailers from 3 to 10, at the minimum satisfaction levels of 0.6 and 0.8, the objective function value increases by 2.5% and 2.6%, respectively. The proposed model was also examined across different satisfaction levels, and the results showed that as the minimum satisfaction level increases from 0.6 to 1, the objective function value decreases, confirming the model's correct performance. In addition, this research examined parameters related to the model robustness. The results of this section showed that as the optimality robustness coefficient and the feasibility robustness coefficient increase, the objective function value decreases and the penalty cost increases. Next, we solved the problem using both the deterministic and RFSP counterpart models (the RFSP counterpart was solved at two minimum satisfaction levels of 0.6 and 0.8). We then compared the results of these two models using two criteria: mean and standard deviation. The RFSP counterpart with a minimum satisfaction level of 0.6 has the best performance, as it has the lowest standard deviation and the highest mean.

Considering the complexity of the problem in large dimensions, the development of heuristic or meta-heuristic algorithms could be beneficial for future research. Given the problem's disruption, resilience strategies can be applied to tackle the adverse effects of disruptions, which can also be suggested as a topic for future research. Additionally, supply chain responsiveness can be evaluated by considering a responsive supply chain, which may also be an appropriate option for further study. Finally, it should be noted that products such as tires can benefit from the closed-loop supply chain developed in this problem.

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APPENDIX

A. Robust Fuzzy Stochastic Counterpart Model

$$\max Z = REV(S) - \left(FC + \sum_{S} \pi_s \left(IC_S + PC_S + TC_S + RC_S + DC_S + SC_S + MC_S + QC_S \right) \right) - \mu \sum_{s} \pi_s \left(\{ww_s - vv\} + 2\theta_s \right) - \sum_{p.c.t.s} \upsilon\, dr1_{(p.c.t.s)} - \sum_{p.c.t.s} \upsilon\, dr2_{(p.c.t.s)}$$

s.t.

$$FC = \sum_{j} \left[\frac{FC_j^l + FC_j^m + FC_j^u}{3} \right] OJ_j + \sum_{m.w} \left[\frac{FC_{(m.w)}^l + FC_{(m.w)}^m + FC_{(m.w)}^u}{3} \right] OM_{(m.w)} + \sum_{k} \left[\frac{FC_k^l + FC_k^m + FC_k^u}{3} \right] OK_k + \sum_{r} \left[\frac{FC_r^l + FC_r^m + FC_r^u}{3} \right] ORR_r + \sum_{i} \left[\frac{FC_i^l + FC_i^m + FC_i^u}{3} \right] OI_i + \sum_{i} \left[\frac{FCR_i^l + FCR_i^m + FCR_i^u}{3} \right] OIR_i$$

$$IC_s = \sum_{p.m.t} \left[\frac{ch_{(p.m)}^l + ch_{(p.m)}^m + ch_{(p.m)}^u}{3} \right] iv_{(p.m.t.s)}, \qquad \forall s$$

$$PC_s = \sum_{p.m.w.t} \left[\frac{mc_{(p.m.w)}^l + mc_{(p.m.w)}^m + mc_{(p.m.w)}^u}{3} \right] ma_{(p.m.w.t.s)}, \quad \forall s$$

$$TC_s = \sum_{j.m.t} \left[\frac{tr^l + tr^m + tr^u}{3} \right] dis_{(j.m)} JM_{(j.m.t.s)} + \sum_{p.m.i.t} \left[\frac{tc_p^l + tc_p^m + tc_p^u}{3} \right] dis_{(m.i)} MI_{(p.m.i.t.s)} + \sum_{p.i.f.t} \left[\frac{tc_p^l + tc_p^m + tc_p^u}{3} \right] dis_{(i.f)} IFF_{(p.i.f.t.s)} + \sum_{p.f.k.t} \left[\frac{tc_p^l + tc_p^m + tc_p^u}{3} \right] dis_{(f.k)} FK_{(p.f.k.t.s)} + \sum_{p.k.r.t} \left[\frac{tc_p^l + tc_p^m + tc_p^u}{3} \right] dis_{(k.r)} KR_{(p.k.r.t.s)} + \sum_{r.m.t} \left[\frac{tr^l + tr^m + tr^u}{3} \right] dis_{(r.m)} RM_{(r.m.t.s)} + \sum_{p.m.t} \left[\frac{tc_p^l + tc_p^m + tc_p^u}{3} \right] diso_m MO_{(p.m.t.s)} + \sum_{p.c.t} \left[\frac{dco_c^l + dco_c^m + dco_c^u}{3} \right] gp_{(p.c.t.s)} + \sum_{p.c.k.t} \left[\frac{dcl_{(c.k)}^l + dcl_{(c.k)}^m + dcl_{(c.k)}^u}{3} \right] CK_{(p.c.k.t.s)}, \qquad \forall s$$

$$RC_s = \sum_{t} \Bigg(\sum_{p.f.c} \left[\frac{rcp_p^l + rcp_p^m + rcp_p^u}{3} \right] \boldsymbol{\alpha}_{(\boldsymbol{p.t.s})} g_{(p.f.c.t.s)} + \sum_{p.f.c} \boldsymbol{\alpha}_{(\boldsymbol{p.t.s})}\, g_{(p.f.c.t.s)} + \boldsymbol{\alpha}_{(\boldsymbol{p.t.s})} gp_{(p.c.t.s)} \Bigg) \left[\frac{cc_p^l + cc_p^m + cc_p^u}{3} \right], \forall s$$

$$DC_s = \sum_{p.k.t} \left[\frac{dd^l + dd^m + dd^u}{3} \right] KD_{(p.k.t.s)}, \qquad \forall s$$

$$SC_s = \sum_{j.m.t} \left[\frac{RCA_j^l + RCA_j^m + RCA_j^u}{3}\right] JM_{(j.m.t.s)}, \qquad \forall s$$

$$MC_s = \sum_{p.i.f.t} \left[\frac{cd_i^l + cd_i^m + cd_i^u}{3}\right] IFF_{(p.i.f.t.s)} + \sum_{r.m.t} \left[\frac{rc_r^l + rc_r^m + rc_r^u}{3}\right] RM_{(r.m.t.s)} \quad , \qquad \forall s$$

$$QC_s = \sum_{p.c.t} \left[\frac{oc_{(p.c)}^l + oc_{(p.c)}^m + oc_{(p.c)}^u}{3}\right] QO_{(p.c.t.s)} + \sum_{p.c.t} \left[\frac{orc_{(p.c)}^l + orc_{(p.c)}^m + orc_{(p.c)}^u}{3}\right] QRO_{(p.c.t.s)}, \qquad \forall s$$

$$REV(S) = \sum_{p.c.l.t} [\frac{DEMF_{(p.c.l.t.s)}^l + DEMF_{(p.c.l.t.s)}^m + DEMF_{(p.c.l.t.s)}^{lu}}{3}] \, PR_{(p.c.l.t)} \, delta_{(p.c.l.t.s)}$$
$$+ \sum_{p.c.t} gamma_{(p.c)} \beta_p [\frac{DEM_{(p.c.t.s)}^{(l)} + DEM_{(p.c.t.s)}^{(m)} + DEM_{(p.c.t.s)}^{(u)}}{3}] \, yj_{(c.t)} po_{(p.c.t)}, \forall s$$

s.t:

$$\sum_m JM_{(j.m.t.s)} \le \left(\left[(2\varphi 1 - 1)cap_j^{(l)} + (2 - 2\varphi 1)cap_j^{(m)}\right] OJ_j\right), \qquad \forall j.t.s$$

$$ma_{(p.m.w.t.s)} \le \left(\left[(2\varphi 3 - 1)cap_{(p.m)}^{(l)} + (2 - 2\varphi 3)cap_{(p.m)}^{(m)}\right] OM_{(m.w)}\right), \qquad \forall p.m.w.t.s$$

$$\sum_p iv_{(p.m.t.s)} \le \left(\left[(2\varphi 4 - 1)cap_m^{(l)} + (2 - 2\varphi 4)cap_m^{(m)}\right] \sum_w OM_{(m.w)}\right), \qquad \forall m.t.s$$

$$\sum_{p.f} FK_{(p.f.k.t.s)} + \sum_{p.c} CK_{(p.c.k.t.s)} \le \left[(2\varphi 6 - 1)cap_k^{(l)} + (2 - 2\varphi 6)cap_k^{(m)}\right] OK_k, \qquad \forall k.t.s$$

$$\sum_m RM_{(r.m.t.s)} \le \left[(2\varphi 7 - 1)cap_r^{(l)} + (2 - 2\varphi 7)cap_r^{(m)}\right] ORR_r, \qquad \forall r.t.s$$

$$\sum_{p.f} IFF_{(p.i.f.t.s)} \le \left[(2\varphi 8 - 1)cap_i^{(l)} + (2 - 2\varphi 8)cap_i^{(m)}\right] \left[\left(\left(1 - \lambda i_{(i.s)}\right) OI_i\right) + OIR_i\right], \qquad \forall i.t.s$$

$$\sum_f g_{(p.f.c.t.s)} + \left(1 - \beta_p\right) QO_{(p.c.t.s)} + dr1_{(p.c.t.s)}$$
$$\ge \sum_l delta_{(p.c.l.t.s)} \left[(2 - 2\varphi 9) DEMF_{(p.c.l.t.s)}^{(u)} + (2\varphi 9 - 1) DEM_{(p.c.l.t.s)}^{(m)}\right], \quad \forall p.c.t.s$$

$$gp_{(p.c.t.s)} + \beta_p QO_{(p.c.t.s)} + dr2_{(p.c.t.s)}$$
$$\ge gamma_{(p.c)} \beta_p \left[(2 - 2\varphi 10) DEM_{(p.c.t.s)}^{(u)} + (2\varphi 10 - 1) DEM_{(p.c.t.s)}^{(m)}\right] yj_{(c.t)}, \qquad \forall p.c.t.s$$

$$ww_s = REV(S) - (IC_S + PC_S + TC_S + RC_S + DC_S + SC_S + MC_S + QC_S), \qquad \forall s$$

$$vv = \sum_s \pi_s \left(REV(S)\right) - \sum_s \pi_s \left(IC_S + PC_S + TC_S + RC_S + DC_S + SC_S + MC_S + QC_S\right)$$

Constraints (24)-(43).

B. Problem Data

Table BI. Fuzzy parameter values and their corresponding ranges

Parameter	Range		
	Parameters ($\varphi = \varphi^{(l)}, \varphi^{(m)}, \varphi^{(u)}$)		
	$\varphi^{(l)}$	$\varphi^{(m)}$	$\varphi^{(u)}$
$\widetilde{FC}_{(j)}$	U (8000,10000)	U (10000,12500)	U (12500,15625)
$\widetilde{FC}_{(m.w)}$	U (384000,480000)	U (480000,600000)	U (600000,750000)
$\widetilde{FC}_i$	U (2400,3000)	U (3000,3750)	U (3750,4688)
$\widetilde{FCR}_i$	U (4000,5000)	U (5000,6250)	U (6250,7812)
$\widetilde{FC}_k$	U (2400,3000)	U (3000,3750)	U (3750,4688)
$\widetilde{FC}_r$	U (64000,80000)	U (80000,100000)	U (100000,125000)
$\widetilde{tc}_p$	U (0.004, 0.005)	U (0.005,0.00625)	U (0.00625,0.007812)
$\widetilde{tr}$	U (0.004, 0.005)	U (0.005,0.00625)	U (0.00625,0.007812)
$\widetilde{mc}_{(p.m.w)}$	U (26,32)	U (32,40)	U (40,50)
$\widetilde{ch}_{(p.m)}$	U (0.0036, 0.0045)	U (0.0045,0.0056)	U (0.0056,0.0070)
$\widetilde{RCA}_j$	U (8,10)	U (10,12.5)	U (12.5,15.6)
$\widetilde{cd}_i$	U (0.0036, 0.0045)	U (0.0045,0.0056)	U (0.0056,0.0070)
$\widetilde{cc}_p$	U (0.036, 0.045)	U (0.045,0.056)	U (0.056,0.070)
$\widetilde{rc}_r$	U (0.15,0.3)	U (0.3,0.6)	U (0.6,1.2)
$\widetilde{dd}$	U (0.00036, 0.00045)	U (0.00045,0.00056)	U (0.00056,0.00070)
$\widetilde{oc}_{(p.c)}$	U (36.8,46)	U (46,57.5)	U (57.5,71.9)
$\widetilde{orc}_{(p.c)}$	U (17.6,22)	U (22,27.5)	U (27.5,34.8)
$\widetilde{cap}_{(p.m)}$	U (4000,8000)	U (8000,16000)	U (16000,32000)
$\widetilde{caph}_m$	U (3000,6000)	U (6000,12000)	U (12000,24000)
$\widetilde{cap}_j$	U (8000,10000)	U (10000,12500)	U (12500,15625)
$\widetilde{cap}_i$	U (64000,80000)	U (80000,100000)	U (80000,125000)
$\widetilde{cap}_k$	U (56000,70000)	U (70000,87500)	U (87500,109375)
$\widetilde{cap}_r$	U (9000,18000)	U (18000,36000)	U (36000,72000)

Table BII. Fuzzy demand parameters under scenarios

Scenario	Range	Demand of customer *c* in scenario *s* for product *p*
1	$\varphi^{(l)}$	U (4000,5000)
	$\varphi^{(m)}$	U (5000,6250)
	$\varphi^{(u)}$	U (6250,7812)
2	$\varphi^{(l)}$	U (5600,7000)
	$\varphi^{(m)}$	U (7000,8750)
	$\varphi^{(u)}$	U (8750,10937)
3	$\varphi^{(l)}$	U (6400,8000)
	$\varphi^{(m)}$	U (8000,10000)