Journal of Quality Engineering and Production Optimization

Volume 9, Issue 2, Summer & Autumn 2024

http://jqepo.shahed.ac.ir

Research Paper

DOI: 10.22070/jqepo.2025.19895.1291

Sustainable Distribution and Inventory Planning in Supply Chains under VMI Strategy for B2B and B2C Models Using IoT

Najmeh Bahrampour [1], Mehdi Seifbarghy [1*], Seyed Hamid Reza Pasandideh [2]

[1] *Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran*

[2] *Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran*

**** Corresponding Author:** Mehdi Seifbarghy (Email: m.seifbarghy@alzahra.ac.ir)*

Abstract *–This study explores optimizing sustainable supply chains by integrating vendor-managed inventory (VMI) and internet of things (IoT). The research focuses on business-to-business (B2B) and business-to-customer (B2C) models. While VMI is widely studied in B2B, its B2C application remains limited. This study examines the tire manufacturing sector, addressing significant environmental and safety concerns. A multi-level optimization framework is introduced to minimize costs, reduce carbon emissions and waste, and enhance customer safety. Customer safety is introduced as a novel social factor. The density-based spatial clustering of applications with noise (DBSCAN) algorithm clusters retailers, improving efficiency and reducing computational time. The framework serves very important customers under the VMI strategy, while normal customers are excluded. Empirical data from a tire manufacturer validates the framework using the Gurobi optimization package. The results demonstrate that applying VMI to all customers significantly increases service levels and the objective function value. Conversely, restricting VMI to B2B customers alone leads to a decline in both service levels and the objective function. Results confirm the scalability and efficiency of the model, with sensitivity analysis showing strong performance under varying parameters. This paper explores VMI in B2B and B2C models, offering insights into sustainable supply chain management*.

Keywords– *Internet of Thing, Supply chain, Sustainability, Vendor Managed Inventory*.

I. INTRODUCTION

A supply chain (SC) encompasses the flow of materials, information, and products from raw material procurement to final product delivery (Simchi-Levi et al., 1999). Optimizing SC processes is crucial for improving efficiency, reducing costs, and enhancing competitiveness in dynamic markets. However, challenges such as decentralized decision-making, conflicting objectives, and limited information sharing persist. Since SCs are interconnected, poor coordination often leads to inefficiencies, increased costs, and resource wastage (Kanda & Deshmukh, 2008).

Vendor-managed inventory (VMI) has emerged as a widely recognized solution to mitigate coordination issues in SCs. Under this strategy, the supplier manages the buyer's inventory (Waller et al., 1999). This strategy creates better alignment within the SC by leveraging real-time data sharing. Effective VMI implementation requires advanced

Manuscript Received: 17-June-2024 & Revised:20-August-2024 & Accepted: 1-October-2024
ISSN: 2423-3781

information technology systems to facilitate real-time communication among SC members. Technologies such as the Internet of Things (IoT) have emerged as crucial enablers of VMI. They offer the ability to collect and share data seamlessly, further enhancing decision-making and SC coordination.

The IoT revolution is reshaping SC management by providing the necessary infrastructure for real-time data collection and monitoring. IoT devices such as Radio Frequency Identification (RFID) tags and sensors enable continuous tracking of inventory levels and product conditions. This capability enhances continuous monitoring, allowing for tailored SC processes and cost reductions (Sallam et al., 2023). Moreover, IoT plays a vital role in supporting sustainable SCs by minimizing waste, lowering energy consumption, and optimizing resource allocation. As the world faces growing environmental challenges, SCs are under pressure to adopt sustainable practices. This is especially critical for industries like tire manufacturing, which have significant environmental footprints.

The growing awareness of environmental and social issues has led to a greater emphasis on sustainable SCs, particularly in tire manufacturing where end-of-life (EoL) tires pose significant environmental hazards, including: fires, soil contamination, and disease transmission. IoT technology enhances sustainability through improved product lifecycle tracking and automated regulatory compliance via RFID tagging. Beyond environmental benefits, IoT contributes to social sustainability via real-time tire monitoring, reducing accident rates, and enhancing passenger safety. Despite the significance of IoT and VMI, their B2C applications, especially for sustainability and coordination, is underexplored.

Although the VMI strategy has been extensively studied in Business-to-Business (B2B) contexts, its application to B2C contexts has not been fully explored. The use of IoT in B2C models provides unique opportunities for real-time inventory management at the consumer level. Most sustainability studies in SCs focus on the strategic level, with few addressing tactical or operational aspects (Barbosa-Póvoa et al., 2018). Job creation, closely linked to strategic decisions and readily quantifiable, is the most studied social criterion, as seen in the study of Hashemzahi et al. (2024). However, critical criteria like safety remain largely unexplored despite their significant importance. This study develops a comprehensive optimization framework that addresses existing gaps by integrating VMI strategies with IoT technologies to support sustainable SCs in the tire industry. The density-based spatial clustering of applications with noise (DBSCAN) algorithm is employed to enhance computational efficiency and scalability. By clustering retailers, DBSCAN reduces the number of entities in the optimization problem, streamlining the solution process. This approach minimizes computational complexity, making it particularly effective for large-scale SCs. The framework is solved with the Gurobi optimization solver, validated using real-world data, and its robustness is analyzed under varying operational conditions. The major contributions of this study include: 1) A proposed framework that optimizes production and distribution in a multi-level SC by incorporating VMI strategies across both B2B and B2C contexts. 2) The incorporation of omnichannel delivery to enhance customer service and flexibility. 3) The application of the DBSCAN clustering algorithm to improve computational efficiency and scalability in the optimization process. 4) A framework that integrates environmental, economic, and social sustainability objectives, emphasizing tire safety as a key social factor. 5) The introduction of VIP and normal customer categories to enable differentiated stockout management strategies and service levels.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature. Section 3 describes the problem and presents the mathematical modeling framework. Section 4 outlines the data and discusses the solution approach. Section 5 analyzes the results, and Section 6 concludes the study.

II. LITERATURE REVIEW

This section critically reviews the literature on four key areas: (A) supply chain management (SCM) with VMI strategy, (B) sustainable SCM with VMI, (C) IoT in SCs with VMI integration, and (D) Summary of the literature.

A. SCM under VMI strategy

VMI emerged in the 1980s as a strategy to improve SC coordination and has since been adopted by major corporations such as Walmart and Procter & Gamble (Waller et al., 1999). In VMI systems, suppliers manage retailer inventory using real-time data, which reduces overall inventory levels, lowers holding costs, and improves production planning (Waller et al., 1999; Yao et al., 2010). Additionally, VMI shortens delivery times and enhances customer service levels (Claassen et al., 2008). Building on these benefits, researchers have developed various SC models under the VMI strategy to optimize inventory, production, and distribution decisions.Various SC models under the VMI strategy have been explored, focusing on inventory, production, and distribution decisions. These models are classified into three categories: nonlinear programming (NLP), mixed-integer linear programming (MILP), and mixed-integer nonlinear programming (MINLP). Cetinkaya and Lee (2000) proposed an NLP model for coordinating inventory and transportation decisions in a single-supplier, multi-retailer VMI system. Gharaei et al. (2019) developed a MINLP model for a multi-product, three-level SC with VMI and consignment stock (VMI-CS), which integrated green policies, penalties, and quality control. Chaudhary et al. (2023) developed the MILP model to compare the VMI strategy with the information-sharing approach for inventory management, considering stochastic, non-stationary demand and service-level constraints. Solution methods for these problems include exact, heuristic, and metaheuristic algorithms. Gharaei et al. (2019) used outer-approximation, equality relaxation, and augmented penalty (OA/ER/AP) algorithm, an effective exact method for solving the problem. Sadeghi et al. (2015) reformulated an MILP model by incorporating replenishment frequency. They employed genetic algorithm (GA) and particle swarm optimization (PSO) algorithm to solve the model. Similarly, Kaasgari et al. (2017) developed an NLP model for managing the inventory of perishable products in a two-level SC comprising a supplier and multiple retailers, utilizing GA and PSO to solve the model. Lotfi et al. (2024) considered VMI-CS in their Viable SC model for healthcare inventory optimization, applying robust stochastic optimization and conditional value-at-risk (CVaR) analysis.

B. Sustainable SCM under VMI strategy

Sustainability considerations such as greenhouse gas (GHG) emissions, energy consumption, material usage, and hazardous waste management have been extensively examined in the context of VMI coordination mechanisms. Bazan et al. (2015) analyzed a two-level SC, evaluating GHG emissions from production and transportation by comparing classical coordination with VMI-CS for environmental and operational impacts. Expanding this work, Bazan et al. (2017) developed a closed-loop SC incorporating product remanufacturing and carbon emission constraints, while optimizing lot sizes and replenishment frequencies for cost minimization. Similarly, Marchi et al. (2019) analyzed carbon emissions and tax costs, identifying cost-minimizing strategies through a comparison of classical mechanisms and VMI-CS.

C. IoT in SCs and integrating of VMI and IoT

As SCs increasingly adopt digital technologies, the role of IoT in inventory management and product delivery has garnered attention, though studies remain limited. Szmerekovsky and Zhang (2008) pioneered the study of VMI with RFID technology, demonstrating its potential to enhance inventory management and SC efficiency by enabling real-time visibility of product movement and inventory levels. Their findings revealed that RFID optimizes inventory policies more effectively than traditional systems while maintaining cost efficiency. Fan et al. (2014) examined RFID in both centralized and decentralized SCs, focusing on a single retailer and supplier. They found that RFID improves SC transparency, inventory accuracy, and process efficiency while reducing stockout costs. Weißhuhn and Hoberg (2021) proposed an IoT-enabled smart system for VMI-based inventory and delivery planning. They used a simulation-based analytical model to optimize decisions in a two-level SC. Their findings highlighted IoT's potential to improve SC coordination and efficiency.

Bafandegan Emroozi et al. (2023) introduced a VMI model for perishable products in multi-supplier, multi-retailer SCs, leveraging IoT for quality monitoring and metaheuristics for optimization. Their approach reduced waste, improved quality, and enhanced sustainability while exploring IoT's potential in blockchain-enabled SC research.

Cammarano et al. (2023) analyzed the integration of blockchain with RFID, IoT, and VMI in the Parmigiano Reggiano SC, demonstrating that combining blockchain with VMI improved SC performance by reducing order preparation times, minimizing incomplete orders, and enhancing transparency.

Furthermore, in the realm of reverse logistics, Liu et al. (2018) proposed an IoT-based reverse logistics model that reduced costs and carbon emissions. Similarly, Paksoy et al. (2016) developed a mathematical model for closed-loop SCs using IoT data to optimize collection, transportation, and recycling. These studies highlight IoT's role in both forward and reverse logistics.

D. Summary of the literature

This study builds upon and distinguishes itself from prior works, as outlined in Table 1. The table compares SC structure, VMI type (B2B or B2C), SC levels, reverse flow inclusion, product type (single or multi-type), demand model, and uncertainty parameters. Additional comparisons include shortage management policy, fleet constraints, optimization model, solution method, sustainability metrics, and IoT implementation.

Weißhuhn and Hoberg (2021) developed an IoT-enabled VMI system for two-level B2C SCs using simulation-based models. This study extends their framework by integrating both B2B and B2C models into a multi-echelon SC framework with forward and reverse flows. It incorporates IoT applications for tire safety monitoring and employs DBSCAN clustering to group retailers effectively.

Additionally, the model uses MILP optimization via Gurobi to address three sustainability objectives. A key advancement is the introduction of customer segmentation (VIP and normal), enabling differentiated service levels and tailored stockout policies. The DBSCAN clustering further enhances scalability and computational efficiency. This research integrates B2B and B2C models with reverse logistics flows and IoT-driven data analytics while considering comprehensive sustainability factors, including economic, environmental, and social aspects. Customer safety is included as a critical social objective in the model. The study provides a hybrid MILP solution to address modern SC challenges effectively.

Table I. Summary of recent studies

Reference	VMI Strategy	Type of VMI	SC level	Reverse Flow	Product Type	Demand	Uncertainty	Shortage	Fleet	Fleet Cap	Model	Object Function	Multi Object	Solving Method	Sustainability	IOT
Szmerekovsky & Zhang (2008)	✓	B2B	SV-SB	-	S	S	✓	LS	-	-	NL	Max-P	-	EX	-	✓
Darwish & Odah (2010)	✓	B2B	SV-MB	-	S	D	-	-	-	-	NL	Mn -C	-	Heu	-	-
Sacone & Siri (2010)	✓	B2B	SS-SC	-	S	F	-	-	✓	✓	MILP	Mn -C	-	Heu	-	-
Braglia et al. (2014)	✓	B2B	SV-SB	-	S	S	✓	-	-	-	NL	Mn-C	-	Exc	-	-
Fan et al. (2014)	✓	B2B	SV-SB	-	S	S	✓	B	-	-	NL	Max-P	-	EX	-	
Bazan et al. (2015)	✓	B2B	SV-SB	-	S	D	-	-	✓	✓	MINLP	Mn -C	-	Exc	✓	-
Sadeghi et al. (2015)	✓	B2B	SV-MR	-	S	F	✓	B	-	-	NL	Mn -C	-	Meta	-	-
Escuín et al., (2017)	✓	B2B	SS-MB	-	S	S	✓	LS	-	-	NL	Mn -C	-	Heu	-	-
Bazan et al. (2017)	✓	B2B	SM-SR	✓	S	D	-	-	✓	✓	NL	Mn -C	-	Exc	✓	-
Kaasgari et al. (2017)	✓	B2B	SV-MR	-	S	S		N	-	-	NL	Mn -C	-	Meta	-	-

Continue Table I. Summary of recent studies

Reference	VMI Strategy	Type of VMI	SC level	Reverse Flow	Product Type	Demand	Uncertainty	Shortage	Fleet	Fleet Cap	Model	Object Function	Multi Object	Solving Method	Sustainability	IOT
Cai et al. (2017)	✓	B2B	MS-SR	-	S	S	✓	LS	-	-	NL	Mx-P	-	Exc	-	-
Chen (2019)	✓	B2B	SM-MR	-	S	F	-	N	✓	✓	MINLP	Mx- P	-	Meta	-	-
Gharaei et al. (2019)	✓	B2B	SV-MB	-	M	F	✓	-	-	-	MINLP	Mn -C	-	Exc	-	-
Weißhuhn & Hoberg (2021)	✓	B2C	SV-MB	-	S	S	✓	LS	-	-	MILP	Max-P	-	Heu	-	✓
Bafandegan Emroozi et al. (2023)	✓	B2B	SV-MB	-	S	S	✓	B	✓	✓	MILP	Mn -C,Mn- waste	-	Meta	✓	✓
This Paper	✓	B2B, B2C	SS-MR-MB	✓	M	D	-	B,LS	✓	✓	MILP	Mn -C	✓	Exc	✓	✓

SC level: SV (Single Vendor), SB (Single Buyer), MB (Multiple Buyer), SS (Single Supplier), MS (Multiple Supplier), MC (Multiple Customer), Product type: Single Product (S), Multiple Product (M), Demand (Deterministic (D), Stochastic (S), Fuzzy (F), Shortage (Backordered (B), Lost Sale (LS), Not allowed (N)), Model (Nonlinear (NL), Mixed Integer Linear Programing (MILP), Mixed Integer Non Linear Programing (MINLP)), Objective (Min Cost (Mn-C), Max Profit (Max-P)),Solving Method (Exact (Ex), Heuristic (Heu), Meta (Metaheuristic))

III. MODELING AND ANALYSIS

A. Problem statement

A multi-level SC encompasses a manufacturer, multiple retailers, final consumers, and collection centers. The given product is a tire with IoT sensors, which provides point-of-consumption (POC) data, including temperature, pressure, and mileage. The manufacturer produces and distributes different kinds of products weekly through retailers' diverse fleets of vehicles. Retailers deliver new products to customers and collect EoL products, which are then transferred to collection centers. An EoL product has completed its lifecycle after consumption.

Final customers are categorized into two groups: very important (VIP) and normal customers. The manufacturer manages VIP customer orders in a B2C context through a VMI strategy. VIP customers' product status is monitored using IoT sensors that track temperature, pressure, and distance data. The manufacturer places orders based on IoT POC data when products reach their replacement threshold. Normal customers place their orders themselves. The role of IoT technology for this customer category is limited to issuing alerts for product replacement, leaving the responsibility for ordering and product procurement to the customer.

Selling and delivering products from retailers to customers has been examined through an omnichannel approach, encompassing the following three modes: 1) Traditional purchase and in-store pickup at the retailer, 2) Online ordering and in-store pickup at the retailer, 3) Online ordering and delivery to the customer's address All three modes are available for normal customers, while VIP customers can only select from the second and third modes. Each customer can select from their available options.

Customers can only order and receive deliveries from a specific set of retailers, determined for each customer based on a predetermined maximum distance to retailers. To ensure every customer has at least one assigned retailer, the system implements a rule: if the retailer set is empty, the nearest retailer will be added. Stockouts occur when retailers are unable to fulfill customer orders, with different handling strategies applied to each customer category. Normal customers face stockouts if orders are not fulfilled within the designated lead time, primarily resulting in lost sales. The lead time varies by delivery method: zero periods for traditional purchases with in-store pickup and one period for online purchase with delivery. VIP customers must be supplied even during stockouts, with a penalty fee applied for each stockout day. However, a waiting period of up to one day is exempt from the penalty.

The model incorporates both environmental and social aspects. From an environmental perspective, it addresses carbon emissions from production and distribution, as well as EOL product recycling. A penalty is imposed if EOL products are not returned to recycling centers. Additionally, a carbon cap policy limits emissions across the SC, allowing for trading or purchasing additional capacity if the cap is exceeded.

The social aspect relates to customer safety, as every product has a safety level throughout its lifecycle. Failing to replace a product after its critical lifespan significantly decreases its safety, leading to lower customer satisfaction and delayed deliveries. So, safety cost is considered a social aspect of sustainability, and the manufacturer should pay a penalty for safety if VIP customers face stockout.

B. Model assumptions

This study assumes that real-time retailer inventory data is accessible through VMI systems and IoT technology. Products are shipped instantaneously from the manufacturer to retailers. Each customer, whether normal or VIP, places only one order within the planning horizon. The model also assumes an omnichannel distribution strategy, including in-store purchases, online orders with in-store pickup, and home delivery.

C. Indices and sets

The indices and sets used in modeling the problem are presented below.

Sets:
Normal Customers: $I'' = \{N' + 1, \dots, N''\}$
VIP Customers: $I' = \{1,2, \dots, N'\}$
Total Customer: $I = I' \cup I''$
Retailers: $J = \{1,2, \dots, M\}$
Customers Assignable to Retailer $CUS_j \subset I$
Retailers Assignable to Customer $RE_i \subset J$
Collection Centers: $OC = \{1,2,3, \dots, O\}$
Time Periods: $TP = \{1,2,3, \dots, T\}$
Sales Channels: $K = \{1,2,3\}$
Product Types: $PR = \{1,2,3, \dots, P\}$
Product Type for Customer i (single member): $PO_i \subset P$
Geographical Areas of Retailers: $GS = \{1,2,3, \dots G\}$
Retailers Located in Geographical Area g: $R_g \subset$ J
Vehicles from Manufacturer: $VP_= \{1,2, \dots., V\}$
Vehicles for Collection Centers: $VC = \{1,2, \dots, \mathrm{V}'\}$
Vehicles for Collection Center o (no overlap between subsets): $VEC_o \subset VC$
Channels Selected by Customer i: $KCUS_i \subset K$

Indices:
Customer Index: $i \in I$ Retailer Index: $j \in J$
Collection Center Index: $o \in OC$
Time Period Index: $t \in TP$
Sales Channel Index for Customer: $k \in K$
Product Type Index: $p \in PR$
Geographical Area Index for Retailers: $g \in GS$

Manufacturer's Vehicle Index: $v \in VP$
Collection Center's Vehicle Index: $v' \in \mathrm{VC}$

D. Variables and parameters

The parameters and variables employed in this model are outlined in Tables II and III.

Table II. Parameters

Holding Costs	
hm_p	Fixed holding cost per unit of product p for the manufacturer in each period.
hr_{pj}	Fixed holding cost per unit of product p for retailer j in each period.
Fixed Replenishment Costs	
sc_j	Fixed cost for online inventory monitoring of retailer j in each period for the manufacturer.
Emissions	
e_p	Carbon emissions per unit of product type p produced by the manufacturer.
cc	Excess carbon emission cost per unit for the manufacturer.
etm_v	Carbon emissions per unit distance by vehicle v.
$eto_{v'}$	Carbon emissions per unit distance by vehicle v'.
Customer-Related Parameters	
O_{it}	Order placed by customer i in period t (binary: 0 or 1).
cs	Fixed safety cost per unit of shortage for VIP customers in each period.
Collection Costs	
pnt	Penalty for uncollected units of consumed product.
Shortage Costs	
B_p	Shortage penalty cost per unit of product p for VIP customers in each period.
B'_p	Shortage penalty cost per unit of product p for normal customers in each period.
Product-Related Parameters	
Pr_P	Price per unit of product type p.
Production Costs	
c_p	Production cost per unit of product type p by the manufacturer.
Capacities	
Ce	Fixed carbon emission capacity across all periods for the manufacturer.
Cpm	Fixed production capacity for the manufacturer in each period.
Cwm	Fixed storage capacity for the manufacturer in each period.
Cwr_j	Fixed storage capacity for retailer j in each period.

Continue Table II. Parameters

Capacities	
Cvm_v	Capacity of vehicle v for the manufacturer.
$Cvo_{v'}$	Capacity of vehicle v' at the collection center.
Cur_j	Fixed storage capacity for consumed products at retailer j in each period.
Transportation Costs and Distances	
D_{ij}	Distance between retailer j and customer i.
Dr_{jo}	Distance between retailer j and collection center o.
Dm_g	Distance between the manufacturer and geographical area g.
ctm_v	Transportation cost per unit distance by vehicle v for the manufacturer.
$cto_{v'}$	Transportation cost per unit distance by vehicle v'.
ct	Fixed transportation cost per product per unit distance from any retailer to any customer for the manufacturer.
cvm_{pv}	Transportation cost per unit of product p for the manufacturer by vehicle v.
$cvc_{v'}$	Fixed transportation cost per unit of product for the collection center by vehicle v'.
Random Parameter	
α	Normally distributed random parameter (μ = mean, σ = variance)
Auxiliary Parameters	
M	A sufficiently large positive constant used to enforce logical constraints.

Table III. Variables

Collection Variables	
$\gamma_{jot}^{v'}$	Amount of product transported from retailer j to collection center o in period t by vehicle v'.
$\Lambda_{jot}^{v'}$	Binary variable, 1 if transportation occurs from retailer j to collection center o in period t by vehicle v': 0 otherwise.
Inventory Variables	
Im_{pt}	Inventory of product p at the manufacturer at the end of period t.
Ir_{pjt}	Inventory of product p at retailer j at the end of period t.
Ic_{jt}	Inventory of consumed products at retailer j in period t.
Shortage Variables	
L_{it}	Binary variable, 1 if VIP customer i experiences a shortage in period t; 0 otherwise.
Lp_{it}	Binary variable, 1 if normal customer i experiences a shortage in period t; 0 otherwise.
Production Variables	
xm_{pt}	Amount of product p produced in period t by the manufacturer.

Continue Table III. Variables

Distribution Variables	
ρg_{gt}^{v}	Binary variable, 1 if delivery of product from the manufacturer to at least one retailer in region g in period t by vehicle v occurs; 0 otherwise.
y_{pjt}^{v}	Amount of product p transported from the manufacturer to retailer j in period t by vehicle v.
ρ_{jt}^{v}	Binary variable, 1 if delivery of product from the manufacturer to retailer j in period t by vehicle v occurs; 0 otherwise.
z_{jipt}^{k}	Binary variable, 1 if delivery of product p to customer i by retailer j in period t via channel k occurs; 0 otherwise.
Ψ_{jit}	Binary variable, 1 if retailer j receives consumed product from customer i in period t; 0 otherwise.
τ_{jit}	Binary variable, 1 if transportation of product from retailer j to customer i in period t occurs; 0 otherwise.
W_{it}	Auxiliary binary variable,1 if a shortage condition is active for VIP customer i in period t; 0 otherwise.
Wp_{it}	Auxiliary binary variable, 1 if a shortage condition is active for normal customer i in period t; 0 otherwise.

E. Mathematical modeling of the problem

The problem objective includes three components: economic, environmental, and social objectives. The economic objective (Equation (1)) comprises 11 terms. The first two terms calculate manufactur’s production and inventory holding costs. The third and fourth terms calculate transportation costs from manufacturers to retailers, including both fixed transportation expenses per distance and variable costs per product. The fifth term covers the fixed costs of inventory screening and monitoring at each retailer, which add to the manufacturer's overall expenses. The sixth term addresses retailers' inventory holding costs. The seventh term calculates the manufacturer's delivery cost in the omnichannel approach, including online ordering and home delivery. The economic objective also incorporates collection center costs. The eighth and ninth terms address transportation expenses for moving EoL products from retailers to collection centers, covering fixed and variable costs. The final terms concern shortage costs, which depend on customer type. For the VIP customers, product unavailability leads to backorders, reflected in the tenth term as a delay penalty. For normal customers, shortage costs represent lost sales, as shown in the eleventh component.

The environmental objective addresses costs from carbon emissions to be limited during production and transportation and penalties for uncollected EOL products: The first term of Equation (2) calculates excess carbon emission costs based on emissions exceeding the established cap. The second term quantifies penalties for uncollected EOL products, determined by the shortfall between delivered products and scheduled EOL collections. Safety costs represent the social objective function, including penalties for late deliveries to VIP customers, which underscore the manufacturer's responsibility for customer safety. This cost is reflected in Equation (3).

$$\begin{aligned} Ob1 = {} & \sum_{t=1}^{T}\sum_{p=1}^{P} xm_{pt} \times c_p + \sum_{t=1}^{T}\sum_{p=1}^{P} hm_p \times Im_{pt} \\ & + \sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{g=1}^{G} \rho g_{gt}^{v} \times Dm_g \times ctm_v + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{v=1}^{V}\sum_{p=1}^{P} y_{pjt}^{v} \times cvm_{pv} + + \sum_{t=1}^{T}\sum_{j=1}^{M} sc_{j} \\ & + \sum_{t=1}^{T}\sum_{j=1}^{M}\sum_{p=1}^{P} Ir_{pjt} \times hr_{pj} + \sum_{t=1}^{T}\sum_{j=1}^{M}\sum_{i\in CUS_j} \tau_{jit} \times D_{ij} \times ct + \sum_{t=1}^{T}\sum_{j=1}^{M}\sum_{o=1}^{O}\sum_{v'\in VEC_o} \Lambda_{jot}^{v'} \times Dr_{jo} \\ & \times cto_{v'} + \sum_{t=1}^{T}\sum_{j=1}^{M}\sum_{o=1}^{O}\sum_{v'\in VEC_o} \gamma_{jot}^{v'} \times cvc_{v'} + \sum_{t=3}^{T}\sum_{p\in PO_i}\sum_{i\in I''} L_{it} \times B_p \\ & + \sum_{t=3}^{T}\sum_{p\in PO_i}^{P}\sum_{i\in I'} Lpunique_i \times B'_p \end{aligned} \tag{1}$$

$$Ob2 = \left(\left(\left(\sum_{t=1}^{T} \sum_{p=1}^{P} xm_{pt} \times e_p + \sum_{t=1}^{T} \sum_{v=1}^{V} \sum_{g=1}^{G} \rho g_{gt}^{v} \times Dm_g \times etm_v + \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{o=1}^{O} \sum_{v' \in VEC_o} \Lambda_{jot}^{v'} \times Dr_{jo} \times eto_{v'} \right) - Ce \right) \times cc + \left(\sum_{t=1}^{T} \sum_{k \epsilon KCUS_i} \sum_{p \in PO_i} \sum_{i \epsilon CUS_j} \sum_{j=1}^{M} z_{jipt}^{k} - \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{o=1}^{O} \sum_{v' \in VEC_o} \gamma_{jot}^{v'} \right) \times pnt \right) \tag{2}$$

$$Ob3 = \sum_{t=1}^{T} \sum_{i \in I''} L_{it} \times cs \tag{3}$$

$$Ob = ob1 + ob2 + ob3 \tag{4}$$

Subject to:

$$\sum_{p=1}^{P} xm_{pt} \leq Cpm \;\; \forall\, t \epsilon TP \tag{5}$$

$$\sum_{p=1}^{P} Im_{pt} \leq Cwm \; \forall t \in TP \tag{6}$$

$$\sum_{p=1}^{P} Ir_{pjt} \leq Cwr_j \;\; \forall j \in J\,, t \in TP \tag{7}$$

$$\sum_{p=1}^{P} \sum_{j \in R_g} y_{pjt}^{v} \leq Cvm_v \; \forall\; g \in GS, t \in TP, v \in VP \tag{8}$$

$$\sum_{\boldsymbol{j=1}}^{\boldsymbol{J}} \gamma_{jot}^{v'} \leq Cvo_{v'} \;\; \forall o \in OC, t \in TP, v' \in VEC_o \tag{9}$$

$$\sum_{g=1}^{G} \rho g_{gt}^{v} \leq 1 \; \forall\; v \in VP, t \in TP \tag{10}$$

$$Im_{pt} = Im_{p(t-1)} + xm_{pt} - \sum_{j=1}^{M} \sum_{v=1}^{V} y_{pjt}^{v} \;\; \forall p \in PR, t \in TP \tag{11}$$

$$Ir_{pjt} = Ir_{pjt-1} + \sum_{v=1}^{V} y_{pjt}^{v} - \sum_{k \in KCUS_i} \sum_{i \in CUS_j} z_{jipt}^{k} \;\; \forall p \in PR, t \in TP, j \in J \tag{12}$$

$$Ic_{jt} = Ic_{j(t-1)} + \sum_{i\epsilon \boldsymbol{CUS_j}} \Psi_{ijt} - \sum_{o\in O} \sum_{v'\in VEC_o} \gamma_{jot}^{v'} \ \forall j\epsilon J\ , t \in TP \tag{13}$$

$$xm_{pt} + Im_{pt} \geq \ D_{pt} \ \ \forall\ t\epsilon TP, p \in PR \tag{14}$$

$$\sum_{p=1}^{P} y_{pjt}^{v} \leq M\rho_{jt}^{v} \ \ \forall j \in J, t \in TP, v \in VP \tag{15}$$

$$\gamma_{jot}^{v'} \leq M\Lambda_{jot}^{v'} \ \ \forall j \in J, o \in OC\ t \in TP\ , v' \in VEC_o \tag{16}$$

$$\rho g_{gt}^{v} \leq \sum_{j\epsilon R_g} \rho_{jt}^{v} \ \ \forall g \in GS, v \in VP, t \in TP \tag{17}$$

$$M \times \rho g_{gt}^{v} \geq \sum_{j\epsilon R_g} \rho_{jt}^{v} \ \ \forall g \in GS, v \in VP, t \in TP \tag{18}$$

$$\tau_{jit} \geq z_{jipt}^{3} \ \ \forall\ j \in J, \qquad i \in CUS_j, p \in PO_i, t \in TP \tag{19}$$

$$L_{it} + \sum_{j\in RE_i} z_{jipt}^{k} \leq 1\ \forall i \in I', p \in PO_i, t \in \mathrm{TP}, k \in\in KCUS_i \tag{20}$$

$$Lp_{it} + \sum_{j\in RE_i} z_{jipt}^{k} \leq 1\ \forall i \in I'', p \in PO_i, t \in \mathrm{TP}, k \in KCUS_i \tag{21}$$

$$\sum_{j\in RE_i} z_{jipt}^{k} \leq \sum_{te=0}^{t-1} o_{i(t-te)} \ \ \forall i \in I''\ p \in PO_i, t \in TP\ , k \in KCUS_i \tag{22}$$

$$\sum_{j\in RE_i} z_{jipt}^{k} \leq \sum_{te=0}^{1} o_{i(t-te)} \ \ \forall i \in I'\ p \in PO_i, t \in TP\ , k \in KCUS_{i_i} \tag{23}$$

$$\sum_{t=1}^{T} \sum_{j\in RE_i} z_{jipt}^{k} \leq 1\ \forall i \in I, p \in PO_i, t \in \mathrm{TP}\ , k \in KCUS_i \tag{24}$$

$$\Psi_{ijt} = E(\alpha) \times z_{jipt}^{k}\ \forall j\epsilon J\ , i\epsilon \boldsymbol{C}US_j, t\epsilon TP, p\epsilon Po_i, k \in KCUS_i \tag{25}$$

$$Ic_{jt} \leq Cur_j \ \ \forall j \epsilon J \ , t \in TP \tag{26}$$

$$L_{it} = (\sum_{t_e=0}^{t-1} O_{i(t-t_e)} - \sum_{j \in RE_i} \sum_{t_e=0}^{t-1} z_{jip(t-t_e)}^{k})^{+} \ \ \forall i \in I'', p \in PO_i, t = 2,..,T, k \in KCUS_i \tag{27}$$

$$L_{it} = \max\left(0, \sum_{te=0}^{t-1} O_{i(t-\mathrm{te})} - \sum_{j \in RE_i} \sum_{t_e=0}^{t-1} z_{jip(t-t_e)}^{k}\right) \forall i \in I'', p \in PO_i, t = 2,..,T, k \in KCUS_i \tag{28}$$

$$L_{it} \geq 0 \ \forall i \in I'', t \in TP \tag{29}$$

$$L_{it} \geq \sum_{te=0}^{t-1} O_{i(t-te)} - \sum_{j \in RE_i} \sum_{t_e=0}^{t-1} z_{jip(t-t_e)}^{k} \ \ \forall i \in I'', p \in PO_i, t = 2,..,T \ k \in KCUS_i \tag{30}$$

$$L_{it} \leq \sum_{te=0}^{t-1} O_{i(t-te)} - \sum_{j \in RE_i} \sum_{t_e=0}^{t-1} z_{jip(t-t_e)}^{k} + M \times (1 - W_{it}) \forall i \in I'', p \in PO_i, t = 2,..,T \ k \in KCUS_i \tag{31}$$

$$L_{it} \leq M \times W_{it} \ \forall i \in I'', t = 2,..,T \tag{32}$$

$$Lp_{it} = (O_{i(t-1)} - \sum_{j \in RE_i} \sum_{t_e=0}^{1} z_{jip(t-t_e)}^{k})^{+} \ \ \forall i \in i', p \in PO_i, t \in 2,..,T, k \in KCUS_i \tag{33}$$

$$Lp_{it} = \max\left(0, O_{i(t-1)} - \sum_{j \in RE_i} \sum_{t_e=0}^{1} z_{jip(\mathrm{t}-t_e)}^{k}\right) \forall i \in i', p \in PO_i, t = 2,..,T, k \in KCUS_i \tag{34}$$

$$Lp_{it} \geq 0 \ \forall \ i \in i', t \in TP \tag{35}$$

$$Lp_{it} \geq O_{i(t-1)} - \sum_{j \in RE_i} \sum_{t_e=0}^{1} z_{jip(\mathrm{t}-t_e)}^{k} \ \ \forall i \in i', p \in PO_i, t = 2,..,T, k \in KCUS_i \tag{36}$$

$$Lp_{it} \leq O_{i(t-1)} - \sum_{j \in RE_i} \sum_{t_e=0}^{1} z_{jipt}^{k} + M \times (1 - Wp_{it}) \ \forall i \in i', p \in PO_i, t = 2,..,T, k \in KCUS_i \tag{37}$$

$$Lp_{it} \leq M \times Wp_{it} \ \forall \ i \in i', t = 2,..,T \tag{38}$$

$$Lpunique_i \geq Lp_{it} \;\; \forall\forall \; i \in i', t = 1, \dots, T \tag{39}$$

$$Lpunique_i \leq \sum_{t=1}^{T} Lp_{it} \quad \forall \; i \in i' \tag{40}$$

$$\gamma_{jot}^{v'}, Im_{pt}, Ir_{pjt}, Ic_{jt}, xm_{pt}, y_{pjt}^{v} \geq 0 \tag{41}$$

$$\Lambda_{jot}^{v'}, L_{it}, Lp_{it}, \rho_{gt}^{vr}, z_{jipt}^{k}, \Psi_{jit}, \tau_{jit}, Wp_{it}, W_{it}, Lpunique_i \in \{0,1\} \tag{42}$$

Constraint (5) ensures total production does not exceed the manufacturer's capacity. Constraint (6) limits total inventory to the available storage at the end of each period. Under the VMI strategy, Constraint (7) ensures that the manufacturer's inventory at retailer locations does not exceed the retailer's storage capacity. Transportation constraints are essential to the model. Constraints (8) and (9) limit the quantity of products transported from the manufacturer to regions and from retailers to collection centers based on vehicle capacity. Constraint (10) ensures that each vehicle visits only one geographical region per period.

The inventory balance constraint ensures that goods entering a warehouse equal those leaving plus inventory changes. Constraint (11) balances the manufacturer's inventory with production and dispatches to retailers. Constraint (12) balances retailer inventory with incoming products, prior stock, and customer deliveries. Constraint (13) maintains balance between the arrival EoL products and previous stock levels for each retailer. Constraint (14) ensures the manufacturer's production quantity and inventory meet demand in each period.

Several constraints govern the interaction between binary and continuous variables in the model. Constraint (15) ensures that dispatched quantities from the manufacturer to each retailer are positive only if transportation occurs. Constraint (16) ensures that quantities of EoL products dispatched from the retailer to the collection center are positive only if transportation is activated. Constraints (17) and (18) require each vehicle in a region deliver to at least one retailer. Constraint (19) ensures a retailer delivers to a customer within a specific period only if the delivery decision variable is active. Finally, Constraints (20) and (21) ensure that shortage and delivery variables cannot take positive values simultaneously for any customer. The following constraints regulate product delivery and demand fulfillment. Constraint (22) ensures VIP clients receive a product only if they placed an order in the current or previous period. Constraint (23) allows normal customers to receive a product if ordered within the last two periods, with a maximum one-period wait time. Constraint (24) ensures that each customer receives delivery through the designated channel only once per period.

EOL product collection constraints include: Constraint (25), which links collected EOL quantities to customer deliveries, approximated deterministically using a normal distribution for α, and Constraint (26), which enforces storage capacity limits for EOL products at retailers.

Customer shortage constraints (27–41) define and manage shortages for VIP and normal customers: Equation (27) calculates VIP shortages as the difference between ordered and delivered quantities within the lead time, penalizing delays beyond this threshold. Linearization is achieved through Equations (28–32) using binary variable W_{it}. Constraint (33) quantifies shortages for normal customers, linearized through Constraints (34–37) using binary variable Wp_{it}. Constraint (38) activates $Lpunique_i$. Constraints (39) and (40) restrict $Lpunique_i$ to at most one activation across periods. Constraints (41) and (42) define the model's specific properties of decision variables.

IV. RESULTS

A. Case study and input data

The input data for this study were obtained from a tire manufacturer in Iran, with the following specifications: Information regarding vehicle types, transportation costs, and loading and transportation expenses is provided in Appendix A (Tables 1 and 2). The product types considered in this study, along with their parameters such as price, production cost, and holding cost for each product, are detailed in Table 3 of Appendix A. Furthermore, parameters related to production and storage capacities for both the manufacturer and retailers are summarized in Tables 4 and 5 of Appendix A. A portion of the factory's total capacity is allocated to the production and storage of these specific product categories, while a proportion of the retailers' capacity is dedicated to handling these products. Emission parameters related to production and transportation are detailed in Table 6 of Appendix A. The company supplies a total of 290 retailers, whose geographical locations and coordinates are listed in Table 7 of Appendix A. Since collection centers for the products do not currently exist, hypothetical locations were determined based on the criterion that at least one collection center must be located within a a 100-kilometer radius of each retailer. Appendix A (Table 8) provides the details of these hypothetical collection centers. Furthermore, parameters such as penalties for delivery delays and customer safety costs, which are not currently available, were assumed to facilitate the analysis conducted in this study.

B. Model optimization and solution methods

The proposed optimization model was implemented in Python and solved using the Gurobi solver. Implementation details, including the associated code, are provided in Appendix B. The computational experiments were conducted on a system with a 7-core processor, 11th-generation CPU, and 16 GB of RAM.

C. Input Data Preparation

Retailers were clustered geographically, using the DBSCAN algorithm. This algorithm identifies clusters by analyzing the density of points within a defined area and can also detect noise points. Two key parameters for the DBSCAN algorithm are epsilon and min_samples. The epsilon parameter in the DBSCAN algorithm was set to 100 km to achieve a balance between clustering accuracy and retailer spatial distribution. Smaller values could result in fragmented clusters, while larger values might group geographically distant retailers, reducing analytical precision. This selection ensures meaningful and logically consistent cluster formation for the study. The minimum number of points required to form a cluster was set to 1. The Haversine formula was used to calculate the distance between two geographic locations, accounting for the Earth's curvature to provide accurate measurements. The clustering results, which group retailers into various geographic regions, are visualized in Figs. 1 and 2.

D. Solution method

The proposed model was solved using Gurobi, a powerful MILP solver that employs advanced algorithms like Branch-and-Bound, Cutting Planes, and Parallel Computing for efficient problem-solving (Bixby, 2012). The model was solved under different scenarios with baseline parameters detailed in Table IV.

To evaluate the performance of the proposed model, two key scenarios were tested: one excluding customer segmentation and the other without applying the clustering method. These scenarios were designed to quantify their respective impacts on: service levels, computational efficiency, and overall objective value.

The base scenario yields a total objective value of 79,616. When prioritizing only VIP customers (excluding normal customers), the service level improves by 6%, and the objective value rises to 98,737. Conversely, when all customers are treated as normal under a B2B VMI strategy, the objective value drops significantly alongside a 20% decline in service level. These results emphasize how customer prioritization enhances both profitability and service efficiency.The no-clustering scenario results in a slight increase in the objective value to 80,823. However, the solution time rises significantly to 639.2 seconds, nearly 200% longer than the base scenario. This increase occurs due to greater computational complexity, despite the decrease in simplex iterations to 134,314.

These findings highlight trade-offs between model complexity, efficiency, and service levels, underscoring the need for balanced segmentation and clustering to optimize SC performanc.

Table IV. Parameters and Results of Solving the Baseline Problem

Baseline Problem Parameters			
Initial inventory of retailers (per product)	0	Number of normal customers and VIPs	250,250
Carbon emission limit for production per period (kg)	22000	Number of each type of manufacturer vehicles	5
Carbon emission limit for transportation (kg)	20000	Number of each type of collection center vehicles	2
Penalty for each uncollected product	30	Initial inventory of the manufacturer (per product)	10
Safety penalty per product (percent of product's price)	20%	Number of periods considered	T=5
Emission cost	0.05	Shortage cost for normal customers (percent of product's price)	1%
Fixed screening cost	1	Shortage cost for VIP customers (percent of product's price)	10%

Results of Solving						
Number of Variables		Initial Heuristic	Simplex Iterations	Solution Time (s)	Objective Value	MIPGAP
Continious	Binary					
476050	293650	95410.40	1189090	250	79827	4.00%

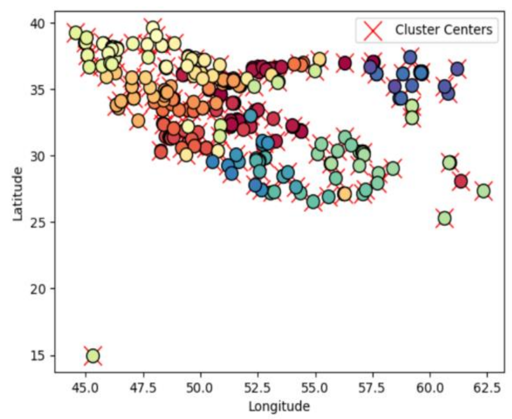

Fig. 1. Group retailers into various geographic regions

Fig. 2. Group retailers in Iran Map

E. Sensitive analysis of model parameters

Optimization models play a pivotal role in addressing SC complexities, yet their performance depends significantly on key parameter variations. This section examines the impact of these variations on computational efficiency and solution quality, providing insights for achieving an optimal balance between accuracy and computational feasibility.

- **MIP GAP Variation**: As shown in Fig. 3, reducing the MIP gap improves objective value. However, Fig. 4 shows that it significantly increases solution time (correlation: 0.99). For example, reducing MIP gap from 10% to 2.5% enhances the objective value from 82,430 to 79,299 but leads to an exponential rise in solution time (126 to 26,021 seconds). Moderate MIP gap values (3%–5%) offer a balanced trade-off between solution quality and computational efficiency.
- **Variation in Number of Customers:** Table VI illustrates the impact of customer count on model performance. As shown in Fig. 5 and Fig. 6, increasing the number of customers raises model complexity, which is reflected in higher variable counts and greater computational effort. For instance, with 5,000 customers, the solution time rises to 15,674 seconds, and the objective value reaches 840,395. These results demonstrate the framework's ability to scale efficiently while maintaining a balance between computational time and solution quality.

Table V. Results of Solving the Problem with Different MIP Gaps

MIP GAP%	Simplex Iterations	Solution Time (s)	Objective Value
9.23	136023	126	82430
7.88	230101	285	82258
5.99	288673	415	81046
4.99	381704	695	80579
3.89	443873	928	79819
3.50	641856	1275	79773
3.00	1189090	4242	79616
2.50	4128336	26021	79299
2.40	7895510	41977	79252

Table VI. Results of Solving the Problem with Different Numbers of Customers

Number of Customers	Numver of Continous Variable	Number of Binary Variable	Initial Heuristic	Simplex Iterations	Solution Time (s)	Objective Value
100	454320	245790	10774	1619435	22897	10743
200	460255	258760	39726	989921	2207	28487
500	476050	293650	95410	3552756	10858	79611

Continue Table VI. Results of Solving the Problem with Different Numbers of Customers

Number of Customers	Numver of Continous Variable	Number of Binary Variable	Initial Heuristic	Simplex Iterations	Solution Time (s)	Objective Value
700	483900	311550	111760	1801580	7032	96259
1000	498320	343690	155760	2458834	3360	137181
1600	527260	408170	249303	798728	6986	205633
3000	597375	563800	549549	2048637	6278	476868
4000	647330	674710	819052	4301632	12650	648928
5000	698800	788650	1118280	1971828	15674	840395

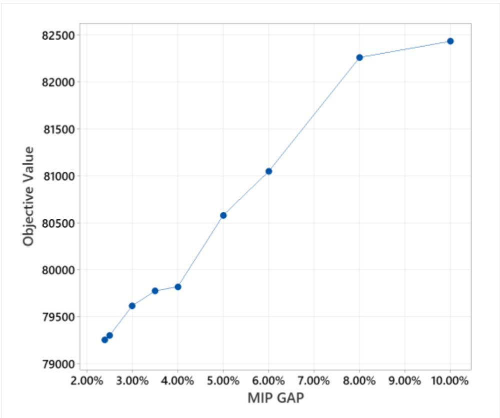

Fig. 3. Variation in solution time with respect to MIP gap

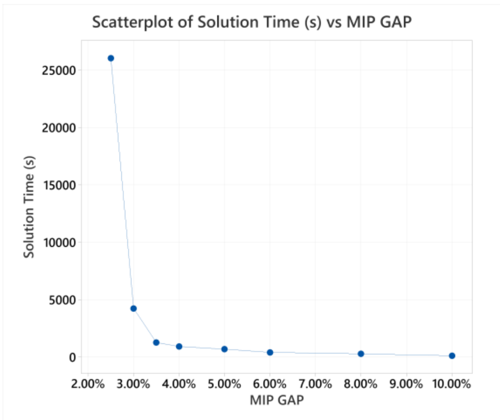

Fig. 4. Variation in objective value with respect to MIP gap

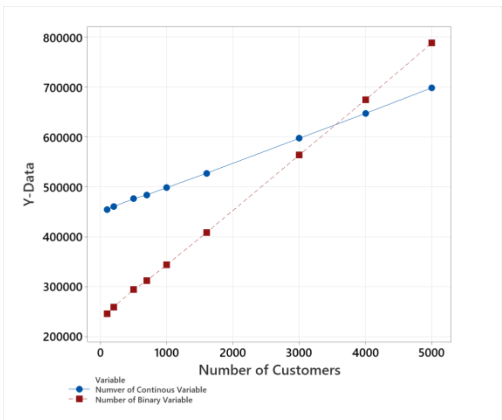

Fig. 5. Variation of binary and continuous variable with respect to Number of Customers

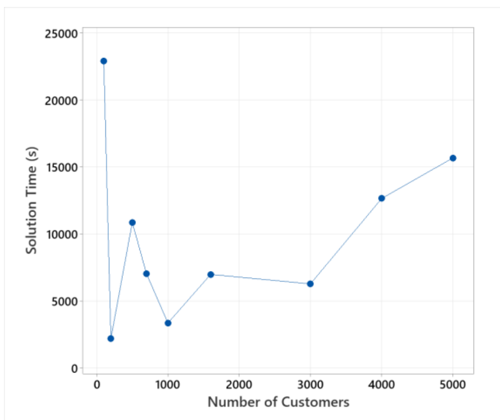

Fig .6. Variation of solution time with respect to Number of Customers

- **Safety Cost Variation:** As shown in Table VII increasing in safety cost for VIP customers increases both solution time (Fig. 7) and objective value (Fig. 8). The shortest solve time (241 seconds) occurs at a penalty of 10, and the longest (18,767 seconds) occurs at a penalty of 25. Penalties between 10 and 15 balance objective value improvement with computational efficiency, while penalties above 20 show diminishing returns.
- **Emission cost variation:** Computational analysis shows the model performs effectively at carbon emission costs of 0.01–0.05, remaining feasible up to 0.1, though with increased computational effort (see Fig. 9). Beyond this threshold, solution divergence occurs. Table VIII and Fig. 10 illustrate an inverse relationship between emission costs and objective values, with 0.05 being the optimal balance for efficiency and feasibility.

Table VII. Sensitivity Analysis of Safety Cost Impact on Solution Performance

Safety cost	Initial Heuristic	Simplex Iterations	Solution Time (s)	Objective Value
5	66934	169271	388	52769
10	75034	133461	241	63941
15	84513	242386	506	73953
20	96940	418956	679	79773
25	83512	5408578	18767	83501
30	87790	1083255	2539	85279
35	87656	1372483	4354	85778

Table VIII. Sensitivity Analysis of Emission Cost Impact on Solution Performance

Emission cost	Initial Heuristic	Simplex Iterations	Solution Time (s)	Objective Value
0.01	99876	445808	674	85342
0.02	98329	464012	798	84551
0.05	95410	819109	1033	79919
0.1	91933.2	431025	1714	77041
0.5	66003	*	*	*
1	37410	*	*	*
5	-188754	536375	2800	-452535
10	-439712	221486	558	-991928
20	-1050130	66969	179	-2039839
50	-2886890	65788	53	-5320635

*Not Found in limited time to algorithm

- **Fleet Size variation:** Table IX illustrates the impact of fleet size variations on model performance. Fig. 11 demonstrates an exponential growth in computational time with increasing fleet sizes, while Fig. 12 shows the associated improvement in objective value. Additionally, the analysis reveals that mixed fleet configurations (e.g., [5, 5, 5, 10, 10]) offer a better balance, while larger homogeneous fleets show diminishing returns in objective value with increased computation time.

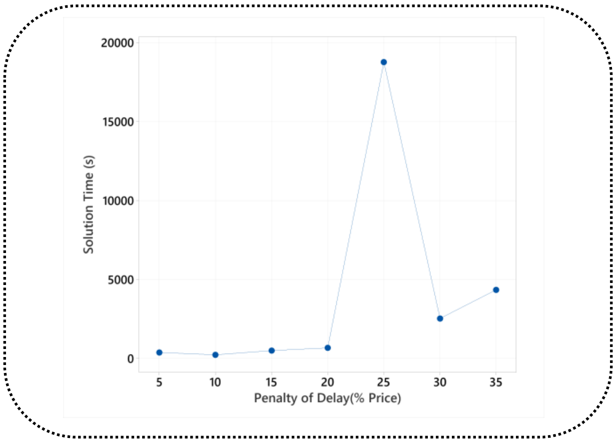

Fig. 7. Variation of Solution Time with respect to Penalty of Delay

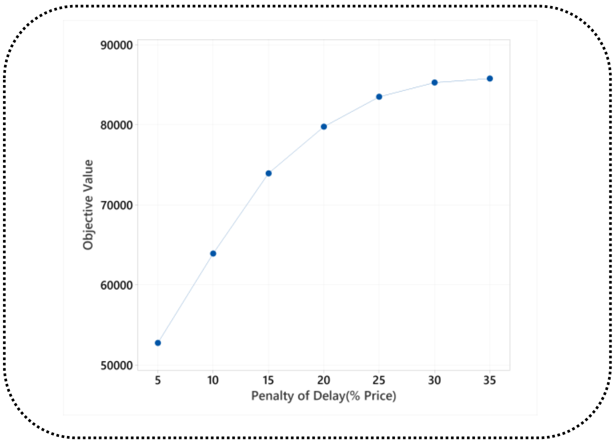

Fig. 8. Variation of Object Value with respect to Penalty of Delay

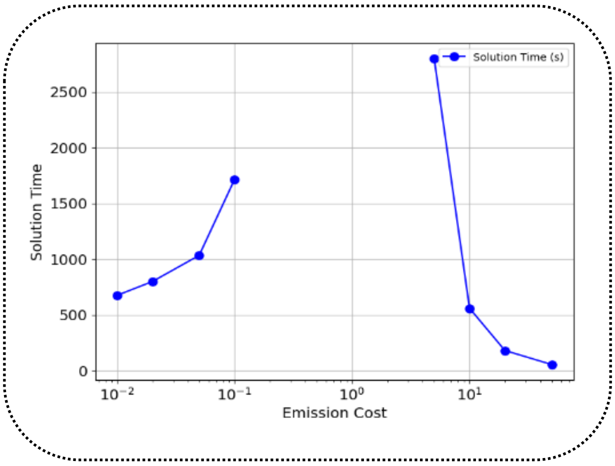

Fig. 9. Variation of Solution Time with respect to Emission Cost

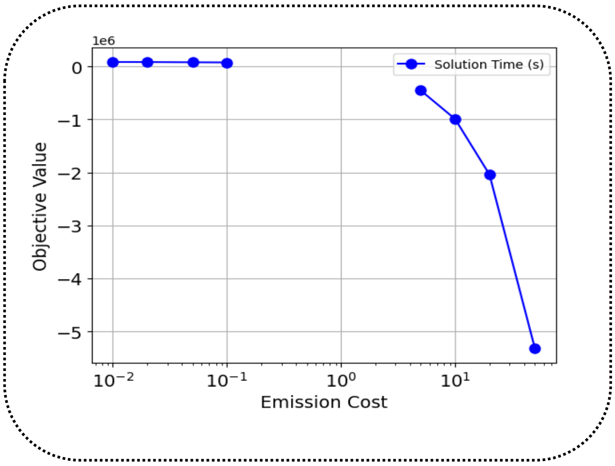

Fig. 10. Variation of Object Value with respect to Emission Cost

Table IX. Results of Fleet Size Variation Analysis

Number of each type of Vehicle	Fleet size	Binary	Continous	Initial Heuristic	Simplex Iterations	Solution Time (S)	Objective Value
[2,2,2,2,2]	10	230350	258550	89459	317088	432	80223
[3,3,3,3,3]	12	240900	294800	90317	272566	355	79576
[5,5,5,5,0]	20	251450	331050	92481	394519	602	80180
[5,5,5,5,5]	25	262000	367300	92635	325085	458	79272
[5,5,5,5,10]	30	272550	403550	92648	363076	624	78410
[5,5,5,10,10]	35	283100	439800	91628	352859	748	79148
[5,5,10,10,10]	40	293650	476050	92669	386797	745	78530
[5,10,10,10,10]	45	304200	512300	91369	417872	1109	78665
[10,10,10,10,10]	50	548550	314750	95725	843195	4721	79282
[10,10,10,10,20]	60	621050	335850	95249	836052	6528	79140

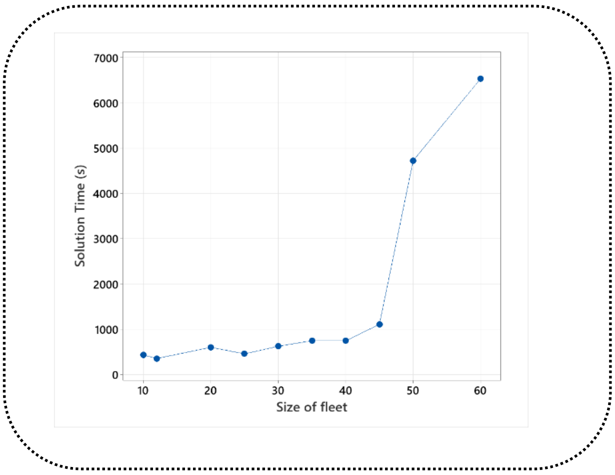

Fig. 11. Variation of Solution Time with respect to Fleet size

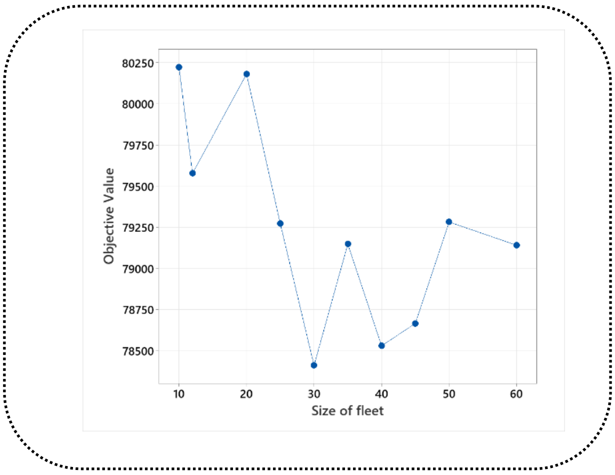

Fig. 12. Variation of Objective Value with respect to Fleet size

V. CONCLUSION

This study presents an integrated framework for optimizing sustainable SCs by combining VMI strategy with IoT technologies. The framework addresses both B2B and B2C models, extending the application of VMI to B2C SCs, a critical yet underexplored area in the literature. Sustainability studies in SCs often focus on the strategic level and social criteria like safety remain overlooked despite their importance. By considering customer safety as a key factor in social sustainability, the study fills these critical gaps. The multi-objective optimization model balances economic, environmental, and social objectives. It aims to reduce operational costs, minimize carbon emissions, and enhance customer safety. By leveraging IoT for real-time monitoring, the framework ensures timely product delivery to maintain customer safety, particularly for VIP customers under the VMI strategy. Safety is treated as a social objective and calculated based on delivery delays. Furthermore, the framework incorporates a penalty system for EoL product collection. The novel categorization of customers into VMI and normal segments allows for customized stockout strategies, while integrating safety parameters reflects a comprehensive approach to sustainability.

The framework has been validated using real-world data from the tire manufacturing sector, demonstrating its scalability and relevance for industries with significant environmental impacts. Empirical results from Gurobi confirm its effectiveness in reducing costs, minimizing emissions, and enhancing safety. Additionally, the use of clustering algorithms, such as DBSCAN, improves computational efficiency, while sensitivity analysis underscores the framework's robustness under different conditions.

Despite the promising results, this study acknowledges certain limitations. First, the model assumes static customer demand, which may limit its ability to adapt to unforeseen disruptions, such as changes in SC shifting customer preferences. Future research should focus on enhancing the framework's adaptability by incorporating additional real-time data streams, including SC disruptions and customer behavior patterns. Additionally, while validated in the tire industry, further studies are needed to generalize the framework to other sectors and geographical regions.

This study bridges a critical gap in sustainable SC management by presenting a unified framework integrating VMI and IoT technologies for both B2B and B2C models. The framework delivers actionable insights for high-environmental-impact industries, providing a clear a pathway to align economic, environmental, and social objectives. By prioritizing customer safety, timely delivery, and EoL product collection, the framework advances sustainability and operational efficiency.

REFERENCES

Bafandegan Emroozi, V., Modares, A., Roozkhosh, P., & Agarwal, R. (2024). A Novel Integrated Supply Chain Model to Manage Perishable Products Demand and Quality by Applying IoT in Vendor-Managed Inventory. *Process Integration and Optimization for Sustainability*, 1-29.

Barbosa-Póvoa, A. P., da Silva, C., & Carvalho, A. (2018). Opportunities and challenges in sustainable supply chain: An operations research perspective. *European journal of operational research*, *268*(2), 399-431.

Bazan, E., Jaber, M. Y., & Zanoni, S. (2015). Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions. *Applied Mathematical Modelling*, *39*(17), 5131-5151.

Bazan, E., Jaber, M. Y., & Zanoni, S. (2017). Carbon emissions and energy effects on a two-level manufacturer-retailer closed-loop supply chain model with remanufacturing subject to different coordination mechanisms. *International Journal of Production Economics*, *183*, 394-408.

Bixby, R. E. (2012). A brief history of linear and mixed-integer programming computation. *Documenta Mathematica*, *2012*, 107-121.

Braglia, M., Castellano, D., & Frosolini, M. (2014). Safety stock management in single vendor–single buyer problem under VMI with consignment stock agreement. *International Journal of Production Economics*, *154*, 16-31.

Cai, J., Tadikamalla, P. R., Shang, J., & Huang, G. (2017). Optimal inventory decisions under vendor managed inventory: Substitution effects and replenishment tactics. *Applied Mathematical Modelling*, *43*, 611-629.

Cammarano, A., Varriale, V., Michelino, F., & Caputo, M. (2023). Blockchain as enabling factor for implementing RFID and IoT technologies in VMI: A simulation on the Parmigiano Reggiano supply chain. *Operations Management Research*, *16*(2), 726-754.

Cetinkaya, S., & Lee, C.-Y. (2000). Stock replenishment and shipment scheduling for vendor-managed inventory systems. *Management science*, *46*(2), 217-232.

Chaudhary, R., Mittal, M., & Jayaswal, M. K. (2023). A sustainable inventory model for defective items under fuzzy environment. *Decision Analytics Journal*, *7*, 100207.

Chen, Z. (2019). Optimization of production inventory with pricing and promotion effort for a single-vendor multi-buyer system of perishable products. *International Journal of Production Economics*, *209*, 285-301.

Claassen, M. J., Van Weele, A. J., & Van Raaij, E. M. (2008). Performance outcomes and success factors of vendor managed inventory (VMI). *Supply Chain Management: An International Journal*, *13*(6), 406-414.

Darwish, M. A., & Odah, O. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research*, *204*(3), 473-484.

Escuín, D., Polo, L., & Ciprés, D. (2017). On the comparison of inventory replenishment policies with time-varying stochastic demand for the paper industry. *Journal of Computational and Applied Mathematics*, *309*, 424-434.

Fan, T.-J., Chang, X.-Y., Gu, C.-H., Yi, J.-J., & Deng, S. (2014). Benefits of RFID technology for reducing inventory shrinkage. *International Journal of Production Economics*, *147*, 659-665.

Gharaei, A., Karimi, M., & Shekarabi, S. A. H. (2019). An integrated multi-product, multi-buyer supply chain under penalty, green, and quality control polices and a vendor managed inventory with consignment stock agreement: The outer approximation with equality relaxation and augmented penalty algorithm. *Applied Mathematical Modelling*, *69*, 223-254.

Hashemzahi, P., Mohammadi, M., & Atabaki, M. S. (2024). Multi-objective Design of Sustainable Closed-loop Supply Chain Considering Social Benefits: Metaheuristic Optimization Approaches. *Journal of Quality Engineering and Production Optimization*, *9*(1), 149-186.

Kaasgari, M. A., Imani, D. M., & Mahmoodjanloo, M. (2017). Optimizing a vendor managed inventory (VMI) supply chain for perishable products by considering discount: Two calibrated meta-heuristic algorithms. *Computers & Industrial Engineering*, *103*, 227-241.

Kanda, A., & Deshmukh, S. (2008). Supply chain coordination: perspectives, empirical studies and research directions. *International journal of production Economics*, *115*(2), 316-335.

Liu, S., Zhang, G., & Wang, L. (2018). IoT-enabled dynamic optimisation for sustainable reverse logistics. *Procedia CIRP*, *69*, 662-667.

Lotfi, R., MohajerAnsari, P., Nevisi, M. M. S., Afshar, M., Davoodi, S. M. R., & Ali, S. S. (2024). A viable supply chain by considering vendor-managed-inventory with a consignment stock policy and learning approach. *Results in Engineering*, *21*, 101609.

Marchi, B., Zanoni, S., Zavanella, L., & Jaber, M. (2019). Supply chain models with greenhouse gases emissions, energy usage, imperfect process under different coordination decisions. *International Journal of Production Economics*, *211*, 145-153.

Paksoy, T., Karaoğlan, İ., Gökçen, H., Pardalos, P. M., & TORĞUL, B. (2016). An experimental research on closed loop supply chain management with internet of things. *Journal of Economics Bibliography*, *3*(1S), 1-20.

Sacone, S., & Siri, S. (2010). Optimal Vendor-Managed Inventory policies in distribution systems with discrete-event processes. *IFAC Proceedings Volumes*, *43*(12), 444-449.

Sadeghi, J., Taghizadeh, M., Sadeghi, A., Jahangard, R., & Tavakkoli-Moghaddam, R. (2015). Optimizing a vendor managed inventory (VMI) model considering delivering cost in a three-echelon supply chain using two tuned-parameter meta-heuristics. *International Journal of System Assurance Engineering and Management*, *6*, 500-510.

Sallam, K., Mohamed, M., & Mohamed, A. W. (2023). Internet of Things (IoT) in supply chain management: challenges, opportunities, and best practices. *Sustainable Machine Intelligence Journal*, *2*, (3): 1-32.

Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (1999). Designing and managing the supply chain: Concepts, strategies, and cases. McGraw-hill New York.

Szmerekovsky, J. G., & Zhang, J. (2008). Coordination and adoption of item-level RFID with vendor managed inventory. *International Journal of Production Economics*, *114*(1), 388-398.

Waller, M., Johnson, M. E., & Davis, T. (1999). Vendor-managed inventory in the retail supply chain. *Journal of business logistics*, *20*, 183-204.

Weißhuhn, S., & Hoberg, K. (2021). Designing smart replenishment systems: Internet-of-Things technology for vendor-managed inventory at end consumers. *European Journal of Operational Research*.

Yao, Y., Dong, Y., & Dresner, M. (2010). Managing supply chain backorders under vendor managed inventory: An incentive approach and empirical analysis. *European Journal of Operational Research*, *203*(2), 350-359.

VI. APPENDIX

Adress of Appendix A: https://drive.google.com/file/d/1jSVGHLGD4q6p7oxhUfHtJQAZ670hdSOw/view?usp=drive_link

Adress of Appendix B: https://drive.google.com/file/d/1HGXor1B8GcsM_MqC6qgRz4max1qgx2a5/view?usp=drive_link