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Optimizing Pricing and Inventory for Perishable Goods in Closed-Loop Supply Chains with SSMD Strategy

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Abstract *–This paper proposes a comprehensive bi-level pricing-inventory model for closed-loop supply chains managing perishable goods with price-sensitive demand. The study addresses a critical gap in the existing literature by integrating perishability constraints and pricing dynamics into a unified optimization framework. The proposed framework incorporates the Single Setup Multiple Delivery (SSMD) strategy, which enables manufacturers to optimize shipment frequencies and quantities, thereby minimizing inventory holding costs and mitigating perishability losses. Utilizing a hybrid mixed-integer optimization method that combines derivative-based techniques with integer search, the model identifies optimal pricing and inventory strategies to maximize supply chain profitability. A comprehensive sensitivity analysis is conducted to examine the interaction between manufacturer-retailer pricing strategies and evaluate the impact of key parameters on system performance. The results demonstrate the model's effectiveness in addressing critical challenges, including the trade-off between shipment costs and perishability, while underscoring its potential to enhance profitability in perishable goods supply chains. Finally, future research directions are outlined, focusing on the integration of real-time data and machine learning for dynamic decision-making.*

Keywords *– Pricing, Inventory control, Integrated supply chain, Perishable items, Price-dependent demand.*

I. INTRODUCTION

Efficient management of the supply chain is essential in today's highly competitive business environment (Ardestani et al., 2024) , especially when handling perishable goods. As markets become increasingly complex, organizations must adapt their relationships with suppliers and customers to enhance flexibility and respond to rapidly changing demands. This necessity has resulted in the development of integrated supply chain management strategies that promote collaboration among stakeholders, optimize inventory, and reduce costs across the network.

Perishable goods supply chains encounter distinct challenges due to the limited shelf life of products, elevated inventory costs, and substantial transportation losses. As consumer preferences evolve, these challenges have become increasingly pronounced, underscoring the necessity for innovative approaches to inventory and pricing management.

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Research indicates that effective inventory control and strategic pricing are essential for minimizing waste and maximizing profitability within the supply chains of perishable products (Song & Wu, 2023). Additionally, the demand for perishable goods is significantly influenced by their freshness, further enhancing the importance of inventory and pricing decisions (Mohammadi et al., 2023). Furthermore, rapid advancements in technology have facilitated more accurate pricing strategies and expedited consumer decision-making, highlighting the critical need for strategic management of perishable goods (Hou et al., 2024). Despite progress in supply chain research, many existing models do not adequately address the dynamic interplay between pricing, perishability, and multi-echelon networks, thereby creating a gap that this study seeks to fill. In this context, Agahgolnezhad Gerdrodbari et al. (2021) contribute significantly to the field by developing a green closed-loop supply chain model for the production and distribution of perishable products.

Previous studies have made significant advancements in enhancing supply chain management for perishable goods. For instance, Hemati et al. (2023) introduced a robust two-level model to optimize multi-tier supply chains under stochastic deterioration rates, while Mousazadeh & Pasha (2024) presented a multi-objective optimization model for the meat supply chain. Sadeghi et al. (2023) addressed an inventory optimization problem focusing on cost, profit, and shipment policies, thereby emphasizing operational efficiency. Their work underscored the importance of managing inventory levels and shipment schedules to reduce costs and enhance profitability, particularly for perishable items. However, these studies lack a comprehensive integration of pricing strategies and closed-loop supply chain dynamics, specifically in the context of perishable goods. Additionally, existing models often overlook the Single Setup Multiple Delivery (SSMD) strategy, which can significantly decrease inventory holding costs and perishability losses by optimizing shipment schedules.

To address these gaps, the present study aims to answer the following research questions: How can a comprehensive bi-level pricing-inventory model be developed for a closed-loop supply chain of perishable goods, incorporating price-dependent variable demand to optimize supply chain performance? How can the overall profit of the supply chain be increased while considering perishability and shipment strategies? How can the sensitivity of manufacturer pricing to retailer pricing strategies be examined, and what implications does this have for supply chain decision-making?

This study introduces an innovative bi-level pricing-inventory model designed to optimize supply chain performance for perishable goods. The key contributions are as follows: Development of a Novel Model: The proposed model integrates the SSMD strategy and closed-loop dynamics to manage perishable goods more effectively, addressing key limitations of previous studies. Optimized Decision-Making: By determining the optimal selling price, shipment frequency, and delivery quantities, the model aims to maximize the overall profit of the supply chain while minimizing waste and inventory costs. Sensitivity Analysis: The study explores the interplay between manufacturer and retailer pricing strategies, providing insights into their combined effects on supply chain profitability. Practical Implications: The model offers actionable recommendations for supply chain managers, particularly in industries with highly perishable goods such as dairy and pharmaceuticals.

The remainder of this paper is structured as follows: Section II provides a detailed literature review; Section III describes the problem and model formulation; Section IV outlines the solution method; Section V presents numerical examples; Section VI conducts sensitivity analysis; and Section VII concludes the study.

II. LITERATURE REVIEW

In today's volatile markets, supply chain integration has emerged as a critical strategy for organizations seeking to enhance competitiveness through improved supplier-customer coordination. While extensive research demonstrates that integrated supply chains can optimize inventory and reduce operational costs, significant gaps remain in addressing the complex challenges specific to perishable goods supply chains. The dynamic interplay between pricing strategies, product deterioration, and multi-echelon logistics creates unique complexities that conventional models often fail to capture.

The theoretical foundations of supply chain integration trace back to Clark (1958) pioneering work on hierarchical inventory models, which established the basis for modern supply chain management. Subsequent research by Yang & Wee (2000) quantitatively demonstrated the cost reduction potential of integrated buyer-seller systems. Rau et al. (2003) advanced the field by incorporating perishability factors into their models. However, these foundational studies were limited by their assumptions of static pricing structures, linear logistics models, and open-loop systems - limitations that become particularly problematic when applied to perishable goods supply chains.

Recent research has evolved along two primary trajectories. The operations-focused stream, exemplified by Sadeghi et al. (2023), has made significant advances in shipment policy optimization but typically treats demand as an exogenous variable. Conversely, sustainability-driven research such as Agahgolnezhad Gerdrodbari et al. (2021) has prioritized circular economy principles while often overlooking the critical role of dynamic pricing strategies. This disciplinary divide has left a significant research gap at the intersection of pricing optimization, logistics management, and perishability considerations.

Our study addresses this gap by developing a unified framework that simultaneously incorporates three critical dimensions: price-sensitive demand modeling, Single Setup Multiple Delivery (SSMD) logistics optimization, and closed-loop perishability management. This integrated approach builds upon the foundational work of earlier scholars while addressing their limitations through innovative modeling techniques and practical insights. The following sections will examine these dimensions in greater detail, beginning with an analysis of perishable inventory systems before proceeding to pricing-integration challenges and delivery strategy innovations. Table I presents a systematic comparison of these approaches, highlighting the distinctive contributions of our proposed model in relation to the existing literature.

A. Supply Chain and Perishable goods

Supply chain management for perishable goods has garnered significant attention due to challenges such as short product shelf life, demand variability, and perishability. Below, key studies in this area are reviewed in chronological order, highlighting advancements and limitations to demonstrate how the current study contributes to the field. Chen & Chang (2010) explored the interaction between pricing and inventory in perishable supply chains with exponential decay. However, their model is limited to single-level supply chains and does not account for dynamic multi-retailer networks or closed-loop considerations, which are critical aspects of this study. Hemmati et al. (2023) examined a green and integrated approach for perishable products. They explored strategies such as horizontal collaboration between distribution centers, emergency reserves, and backup sponsors to reduce barriers in the dairy industry.

Given the significance of supply chain management (SCM) in today's business landscape, which involves making timely and efficient decisions at strategic, tactical, and operational levels while considering economic and environmental factors, the perishable products industry is no exception. Consequently, Agahgolnezhad Gerdrodbari et al. (2021) developed a closed-loop supply chain for the production and distribution of perishable items, focusing on addressing economic and environmental concerns. Their bi-objective model, validated using the ε-constraint approach, demonstrated high efficiency in solving small-scale instances and provided valuable managerial insights through sensitivity analysis.

In healthcare systems, managing perishable products like red blood cell units (RBC) is critical for system optimization Khayat rasoli et al. (2019) aimed to enhance RBC unit consumption management by ensuring timely delivery and minimizing costs. They proposed a one-day supply and demand policy, where decisions for the next day were based on the previous day's inventory status. Using the Markov decision process method, they optimized the blood inventory supply chain, providing a sequential decision-making framework for medical centers such as hospitals and clinics. Yadav et al. (2025) further advanced this domain by integrating promotional efforts, green technology investment, and carbon emission penalties into a retail inventory model for perishable goods. Their study uniquely combined price-and-promotion-dependent demand with preservation technology, demonstrating that such investments not only extend product shelf life but also reduce total costs under sustainability constraints. Jia & Hu (2011) used game theory to analyze pricing strategies in supply chains with random demand. While their work highlighted the impact of pricing on supply chain performance, it did not integrate inventory management or closed-loop supply chain optimization, making it less applicable to the complex scenarios addressed in this study. Sadeghi et al. (2023) extended prior inventory models by addressing pricing dynamics and Stepwise Stock Management and Delivery (SSMD) strategies. An optimal integrated production-inventory model was proposed, incorporating a novel smoothing strategy and a heuristic algorithm to minimize costs. The results highlighted its superiority over traditional EOQ models, offering practical managerial insights for cost-effective decision-making. Hemati et al. (2023) developed a robust two-level model for optimizing multi-tier perishable supply chains under stochastic deterioration rates, specifically targeting the dairy industry. Although their model effectively handled uncertainty, it was specifically tailored to the dairy industry and lacked a comprehensive integration of pricing and delivery strategies. Komijani & Sajadieh (2024) developed an integrated decision-making model for production, inventory, and routing under uncertain conditions. Their focus on uncertainty modeling was valuable, but the absence of pricing strategies and closed-loop features limited its applicability to broader supply chain contexts, which this study addresses.

Pan & Shan (2024) designed a sustainable supply chain network for perishable products by integrating production, location, and inventory decisions using meta-heuristic algorithms, emphasizing sustainability. While their model focused on sustainability, it did not consider multi-retailer structures or dynamic pricing strategies, which are central to the current study. Biza et al. (2024) proposed a multi-echelon, multi-product, and multi-period mathematical model for agricultural supply chains, incorporating quantity discount policies and strategic decision-making. However, their work was limited to agricultural products and did not address closed-loop supply chains or price-sensitive demand, both of which are key features of this research. Mousazadeh & Pasha (2024) presented a multi-objective optimization model for meat supply chains, balancing cost, distribution time, and flexibility. While their model was effective for specific supply chains, it did not incorporate closed-loop structures or SSMD strategies, which are central to the current study.

Mohammadi et al. (2024) designed an intelligent supply chain model for perishable goods using IoT technologies. While their study leveraged advanced technologies for inventory management, it did not address pricing and delivery strategies or closed-loop dynamics, which are key contributions of this research. Souri & Fatemi Ghomi (2024) developed a multi-objective model to optimize perishable food supply chains under uncertainty, focusing on sustainability through cost, emission, and shipping time objectives. While their approach effectively addressed perishability and uncertainty, it did not incorporate SSMD strategies or consider the complexities of multi-retailer supply chains, which are key elements in our study. Jetto & Orsini (2024) proposed a dynamic, multi-stage supply chain model for handling perishability and uncertain demand. While their work effectively addressed uncertainty, it did not incorporate pricing strategies or closed-loop considerations, both of which are central to this study.

B. Perishable Supply Chain and Delivery Strategy

The supply chain models utilizing the Single Setup Multiple Deliveries (SSMD) strategy have been extensively examined. For instance, Chan et al. (2018) demonstrated that implementing the SSMD strategy could lead to significant reductions in transportation costs by optimizing shipment frequencies and quantities. While this research highlighted the efficiency of transportation cost management, it did not address the intricate pricing dynamics within a closed-loop supply chain, a gap that this paper aims to fill.

Sarkar et al. (2018) explored payment delays in a three-level supply chain, incorporating the SSMD strategy. Their model focused on minimizing total supply chain costs, including direct and indirect transportation, industrial carbon emissions, and payment delays, treating carbon emissions as a constant parameter. Although their work effectively integrated environmental considerations, it overlooked the interdependence of pricing strategies between manufacturers and retailers, which this study intends to address. Sadeghi (2019a) investigated scenarios where manufacturers could choose between Single Setup Single Delivery (SSSD) or SSMD policies, emphasizing decision-making flexibility. However, this research did not incorporate perishability or the complex interactions between price-dependent demand

and inventory levels, which are key components of this paper. Karthick & Uthayakumar (2023) analyzed a two-level supply chain involving a seller and buyer for perishable goods, advocating for the SSMD multi-shipment policy to minimize total costs and enhance product quality. While their study offered valuable insights into cost reduction and quality improvement, it did not explore the integration of price-sensitive demand or the impact of shipment policies on profitability within a closed-loop supply chain framework.

Iqbal et al. (2022) developed a centralized multi-echelon model for deteriorating products, optimizing preservation technology investment and unequal-sized deliveries under SSMD. Their findings demonstrated that synchronized cycles and advanced preservation significantly improve profitability by reducing deterioration costs—a critical advancement for perishable goods supply chains, though their model assumed fixed pricing.

Mozdgir Mobbarhan et al. (2022) examined a two-level supply chain characterized by discrete buyer demand and partial delays, assuming that retailers adopt the SSMD strategy. Their findings highlighted the operational benefits of SSMD. Still, they did not delve into the dynamic relationship between retailer and manufacturer pricing strategies or the effects of perishability, both of which are central to the current research. Sadeghi et al. (2023) established baseline inventory-pricing coordination, demonstrating that discrete demand modeling with multiple deliveries could increase profits by 16% over classical approaches. However, their infinite-horizon model with prohibited shortages ignored perishability - a limitation addressed by Karthick & Uthayakumar (2023) SSMD analysis for perishables.

Violi et al. (2024) addressed operational uncertainty in agri-food chains through an age-based inventory-routing model with stochastic demand. Their adoption of Conditional Value-at-Risk (CVaR) for risk measurement and dynamic planning over short-medium horizons provided a robust framework for real-world perishability challenges. While empirically validated in Italian agri-food cases, their model did not integrate preservation technologies or sustainability incentives explored in earlier works.

This study introduces several methodological advancements designed to address persistent limitations in the closed-loop supply chain (CLSC) literature for perishable goods. Our framework systematically resolves four key research gaps identified in recent scholarship:

- Considering variable demand dependent on price, thereby capturing real-world demand sensitivities and enabling better alignment of pricing strategies with market fluctuations.
- Replacing the conventional single-order delivery system with the SSMD strategy, which minimizes inventory holding costs and losses due to perishability.
- Incorporating the perishability of products as a percentage of retail inventory to facilitate more accurate inventory forecasts and reduce waste.
- Investigating the interdependence between manufacturer and retailer pricing strategies, a subject that has been relatively underexplored in previous studies.

To provide a comprehensive overview and classify relevant references, Table I highlights the distinctions between various research studies. This table illustrates the evolution of supply chain models from 2010 to 2024. Early studies primarily concentrated on isolated objectives, such as profit or cost, while more recent research, such as Sadeghi et al. (2023), integrates comprehensive objectives that include cost, profit, selling price, number of shipments, and quantity per shipment. The increasing emphasis on variable demand, multi-retail environments, and closed supply chains underscores the need for more realistic approaches to maximizing profitability in complex and dynamic settings. This paper builds upon these advancements to propose a more integrated and practical model for managing perishable goods in supply chains.

Table I. Summary of literature on the subject

Authors	Year	Objective		Decision Variables			Number of retailers		Closed-loop Supply Chain	Pricing	Demand		Shortage	SSMD
		Cost	Profit	Selling price	Quantity of delivery per shipment	Number of shipments per cycle	single	Multi			Constant	Variable		
Yang et al. (2010)	2010	-	✓	-	-	-	✓	-	✓	-	✓	-	-	-
Hsieh and Dye (2010)	2010	-	✓	✓	-	-	✓	-	-	✓	-	✓	✓	-
Roy et al. (2011)	2011	✓	-	-	✓	✓	✓	-	✓	-	✓	-	✓	✓
Yan et al. (2011)	2011	✓			✓	✓	✓	-	-	-	✓	-	-	✓
Maihami and Kamalabadi (2012)	2012	-	✓	✓	-	-	✓	-	-	✓	-	✓	✓	-
Sarkar (2013)	2013	✓			✓	✓	✓	-	-	-	✓	-	-	✓
Maihami and Karimi (2014)	2014	-	✓	✓	-	-	✓	-	-	✓	-	✓	✓	-
Sarkar et al. (2015)	2015	✓	-	-	✓	✓	✓	-	✓	-	✓	-	-	✓
Aljazzar et al. (2017)	2017	-	✓	-	✓	-	✓	-	-	-	✓	-	-	✓
Sarkar et al. (2018)	2018	✓	-	-	✓	✓	✓	-	-	-	✓	-	-	✓
Nugroho and Wee (2019)	2019	✓	-	-	✓	✓	✓	-	-	-	✓	-	-	✓
Sadeghi (2019b)	2019	-	✓	✓	-		✓	-	-	✓	-	✓	-	-
Sebatjane and Adetunji (2021)	2021	-	✓	-	✓	✓	✓	-	-	-	-	✓	-	✓
Hsiao et al. (2022)	2022	✓	-	-	✓	✓	✓	-	-	-	✓	-	-	✓
Hemati et al. (2023)	2023	✓	-	-	✓	-	-	✓	-	-	-	✓	-	-
Sadeghi et al. (2023)	2023	-	✓	✓	✓	✓	✓	-	-	✓	✓	-	-	✓
Pan and Shan (2024)	2024	✓	-	-	✓	-	-	✓	-	-	-	✓	-	-
Mousazadeh and Pasha (2024)	2024	✓	-	✓	✓	-	-	✓	-	-	-	✓	✓	-
Mohammadi et al. (2024)	2024	✓	-	-	✓	-	-	✓	-	-	✓	-	-	-
Jetto and Orsini (2024)	2024	✓	-	-	✓	-	-	-	-	-	-	✓	-	-
This paper			✓	✓	✓	✓	-	✓	✓	✓		✓	-	✓

The paper introduces a set of strategic innovations designed to address key gaps in the management of perishable goods within closed-loop supply chains, focusing on improving model realism and practical applicability:

- Demand Variability Based on Dynamic Pricing: By linking demand directly to pricing, the model captures real-world demand sensitivity, allowing supply chain managers to adjust pricing strategies to better align with fluctuating consumer demand, ultimately optimizing sales and reducing waste.
- SSMD Strategy: This approach introduces a flexible, multi-shipment model that enables manufacturers to deliver goods in staggered batches to retailers. It reduces inventory holding costs, minimizes spoilage, and offers greater adaptability in managing perishable goods, making it more efficient than traditional single-delivery models.
- Perishability Incorporated as a Dynamic Inventory Percentage: By calculating perishability as a function of inventory levels, the model reflects the progressive degradation of perishable items. This improvement enables more accurate inventory forecasts and waste reduction, adjusting for how perishability impacts inventory turnover at various stages.
- Sensitivity Analysis of Manufacturer-Retail Price Dynamics: The model explores the interdependence of manufacturer and retail pricing, providing insights into how retail price adjustments affect manufacturer pricing strategies. This dynamic analysis fills a gap in understanding the pricing power balance and its effect on profitability across the supply chain.

Together, these innovations provide a more realistic and actionable framework for inventory management, allowing supply chain managers to make informed, strategic decisions that improve profitability, minimize waste, and enhance operational efficiency in the management of perishable goods.

III. PROBLEM DESCRIPTION AND MODELING

The supply chain encompasses the cycle of product delivery to end users, including activities such as raw material procurement, production, distribution, and sales. Each level of the chain adds value, underscoring the significance of the lower-level supply chain. Minimizing inventory at this level is an effective approach to reducing costs. This paper focuses on the perishable nature of products and examines how inventory levels influence the rate of perishability. By reducing inventory, the quantity of perishable items can be diminished. Therefore, implementing a multi-delivery strategy within a two-level supply chain involving a manufacturer and multiple retailers can effectively reduce retailer inventory, leading to decreased holding and perishable costs. Additionally, this research adopts a closed-loop supply chain perspective, wherein remaining products can be reclaimed, repaired, modified, reintegrated into the production system, and resold rather than discarded. The manufacturer acquires necessary raw materials by either dismantling returned goods or purchasing new raw materials. Subsequently, the manufacturing process for the final product commences, utilizing these raw materials and components from the manufacturing department. The manufacturer then delivers the final product to retailers in fixed quantities and through multiple stages based on the retailers' order quantities, adhering to the SSMD strategy ordering policy.

Several assumptions are made in this problem formulation:

- The time horizon is infinite, with a focus on a single product.
- The problem involves a closed supply chain consisting of a manufacturer and multiple retailers.
- Delivery of goods to retailers occurs in stages, with a specific number of shipments each time (SSMD policy).
- The product is perishable, with its perishability represented as a constant inventory level.
- Demand is variable and influenced by the selling prices of the products.
- Shortages are not permitted.

A. Notations and Assumptions

In this study, notations are grouped into two categories, parameters and decision variables.

Manufacturer Notations

η	Production rate of the manufacturer.
K	The sum of all deliveries to the retailers in each cycle.
Q	Quantity produced in a manufacturer's production cycle.
P_1	Manufacturer's selling price per unit.
C_2	Cost to produce each unit of the product which produced by the manufacturer.
C_1	The purchase price of each returned item by the manufacturer.
C	Cost for raw material procurement by the manufacturer.
A	Fixed cost incurred per production cycle for the manufacturer
h	Cost of keeping each unit of goods in the producer's warehouse.
TP_m	Manufacturer profit.

Retailer Notations

β_i	Parameter depends on the demand price for retailer i.
α_i	Constant demand parameter for retailer i.
b	Cost for packaging and domestic shipment.
b_1	Cost associated with each shipment.
$D_i(p)$	Demand rate for retailer *i*
δ	Perishability rate of goods in the warehouse of retailer i.
n	Total retailers in the supply chain.
γ	A percentage of the retailer's selling price
ζ	A percentage of the manufacturer's selling price
$IR_i(t)$	Inventory level of retailer i
$Ir(t)$	Inventory level of returned goods
h	Holding cost for unit of goods in the producer's warehouse
h_1	Holding cost for unit of goods in the retailer's warehouse
A_i	The cost of ordering for retailer *i*
TP_i	The profit of retailer i
TP	Total profit

Decision Variable

m The number of shipments from the manufacturer to the retailers per unit cycle

K Quantity delivered in each shipment.

P Retailers selling price per unit

In this proposed model, manufacturers produce goods for multiple retailers. To determine the overall profit for the entire supply chain, it is essential to calculate the costs and revenues at each level and subsequently subtract the costs from the corresponding revenues. This approach enables the derivation of the combined profit function of the supply chain by summing the profit functions of each level, thereby facilitating an evaluation of the profitability and efficiency of the entire supply chain system.

B. Manufacturer Costs

In this supply chain model, the manufacturer operates under a make-to-order system, where production is initiated based on retailer orders, while the retailers replenish their stock according to

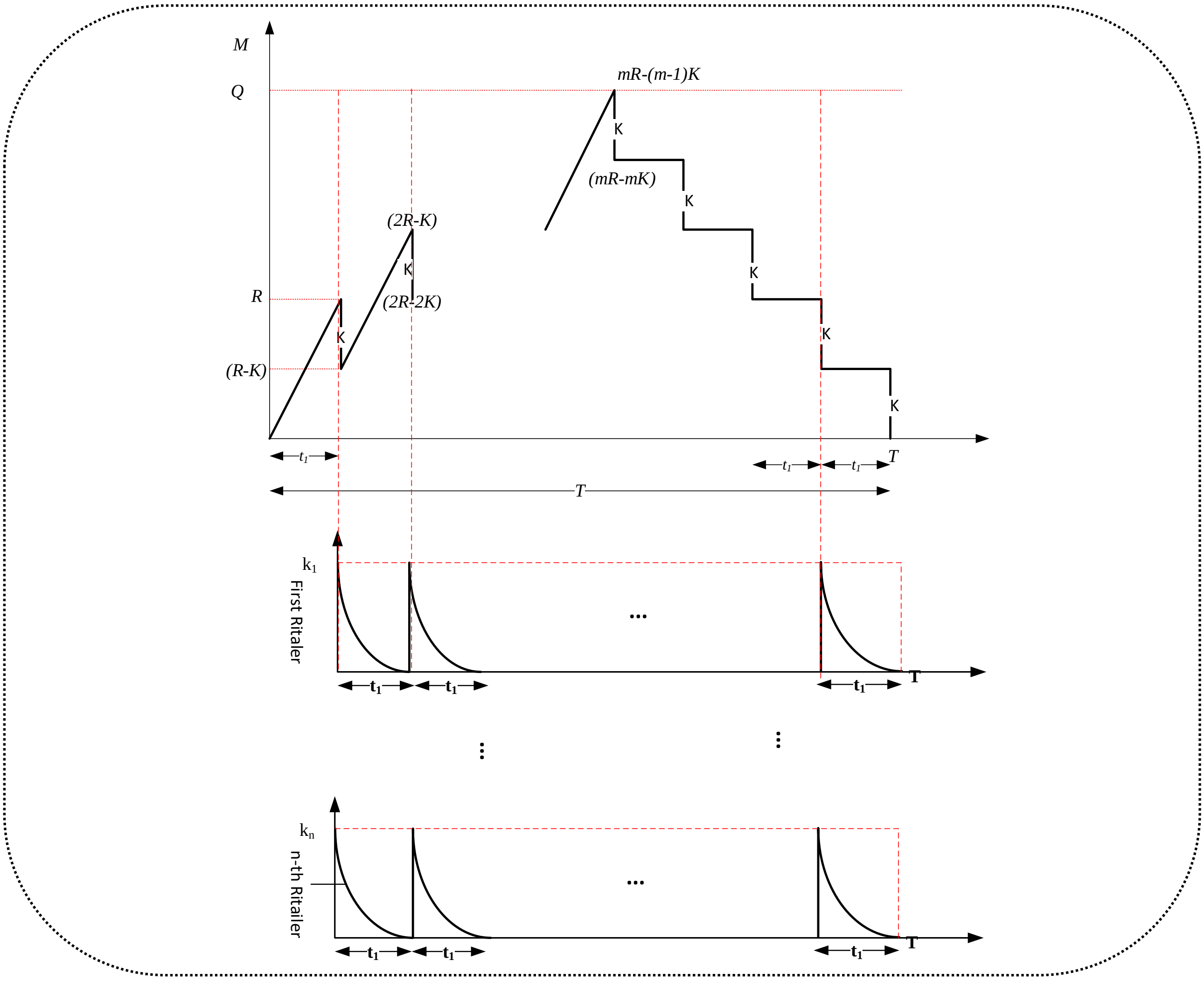

Fig. 1. Inventory level of the manufacturer with batch shipments to retailers

predetermined policies. During each replenishment cycle, the i-th retailer places an order for Q units of the product. To ensure efficient delivery and minimize product deterioration, the manufacturer ships the ordered quantity in m equal-sized batches, with each batch containing K units.

The manufacturer produces at a constant rate, denoted as η, aligning production closely with order requirements to avoid excess inventory. The amount allocated to each retailer is denoted as k_i, and the total quantity distributed to all retailers during a cycle is $= \sum_{i=1}^{n} k_i$. Shipments are dispatched at fixed intervals, represented by t_1.

The inventory dynamics of the manufacturer, illustrated in Fig. 1, demonstrate a stepwise reduction in inventory as K-unit batches are shipped to the retailers at regular intervals. The production rate R ensures that inventory levels are replenished sufficiently between shipments, where R is equal to or greater than K, maintaining a continuous flow of goods through the supply chain. The relationship between the production rate and the shipment interval is given as:

$$R = t_1 \times \eta \tag{1}$$

Here, t_1 represents the time interval between consecutive shipments, and η denotes the production rate per unit time. The variable K will be introduced in subsequent formulas to represent the quantity delivered in each shipment. The manufacturer produces R at each stage, with all stages equal to m, resulting in the following total production by the manufacturer:

$$Q = m \times R \tag{2}$$

As shown in Fig. 1, the manufacturer produces in m stages, and in each stage, R units of the product are produced. During this period, the manufacturer sends K units of the product to retailers m times. After each shipment, the remaining inventory level is equal to $m \times (R - K)$ units. To determine the total number of shipments in each cycle, the following equation applies:

$$m + \frac{m \times (R-K)}{K} \tag{3}$$

In $t = 0$, the warehouse inventory is zero, operations of production will begin, and the manufacturer's inventory level will begin to increase. In each time interval, the manufacturer produces R units of products and sends K units of them to retailers and others held in the warehouse. According to Fig. 1, the manufacturer's inventory is divided into two parts. In the first part, the manufacturer produces and sends the items to retailers in a certain quantity, in the second part the production operation is stopped and the remaining inventory is sent to the retailer in units of $\frac{m \times (R-K)}{K}$. The amount of inventory in the first part (S_1) and the second part (S_2) are calculated as follows:

$$S_1 = \frac{R \times t_1}{2} + \frac{(3R - 2K) \times t_1}{2} + \cdots + \frac{(mR - (m-1)K) \times t_1}{2} = \frac{2(m-1) \times R - 2(m-1) \times K}{2} \times t_1$$

$$S_2 = (K \times t_1) + (2K \times t_1) + \cdots + (mR - mK \times t_1) = \frac{m(R-K)}{K} \times t_1$$

Therefore, the manufacturer's holding cost can be determined as follows:

$$HC = h \times (S_1 + S_2) = h \times \left(\frac{m \times R(K - K \times m + m \times R) \times t_1}{2K}\right) \tag{4}$$

Production cost: Each cycle is divided into m stages, with R units of the product produced at each stage. If the cost of producing each unit of the product is denoted by C_2, then the total cost of producing the product in the cycle can be calculated as follows:

$$\text{P}R\,.C = C_2 \times m \times R \tag{5}$$

Setup and shipping cost: A fixed setup cost, referred to as A, is incurred for each production cycle, in addition to a fixed cost for packaging and domestic shipping associated with each shipment. The total amount of these costs for each cycle is calculated as follows:

$$SC = A + b \times \left(m + \frac{m \times (R - K)}{K}\right) \tag{6}$$

Raw material purchase cost: incurred by the manufacturer for each unit of raw materials is denoted as *C*. The quantity of raw materials required by the manufacturer is calculated by taking into account that certain materials are returned and reused after reconstruction. This quantity is determined as $(m \times R - \sum_{i=1}^{n} Ir_i)$. The total raw material purchase costs are calculated as follows:

$$C \times (m \times R - \sum_{i=1}^{n} Ir_i) \tag{6}$$

Where $(\sum_{i=1}^{n} Ir_i)$ is the amount of returned material.

Manufacturer's revenue: The manufacturer ships its products to retailers at a unit price of P_1. This paper assumes that the manufacturer's selling price is a function of the total selling price, representing a percentage of the retailer's price, and is defined as follows:

$$P_1 = \gamma \times P \tag{7}$$

The unit price of the producer products is P_1. However, it buys back perishable products from retailers at a lower price than the selling price. In this case, if C_1 is considered the purchase price of returned products, its value is expressed as a percentage of the manufacturer's sales price, and its relationship is as follows:

$$C_1 = \zeta \times P_1 \tag{8}$$

To ascertain the producer's income, it is essential to calculate the quantity of goods returned by retailers. This calculation serves as the basis for determining the producer's income and, ultimately, the profit. To facilitate this, the relationship between retailers will be analyzed to evaluate the integrated final profit of both the manufacturer and the retailers.

C. Retailer Costs

According to the principles of marketing and economic theory, it is widely recognized that as the price of a product increases, the demand for that product decreases. Therefore, the demand is directly influenced by the price. In this paper, it is assumed that the demand of the *i-th* retailer follows a non-increasing linear function with respect to the retailer's price, as represented by Eq. (10).

$$D_i(P) = \alpha_i - \beta_i \times P \tag{9}$$

In this problem, each stage involves the *i-th* retailer receiving $k_1$units of the product. This quantity represents the amount consumed by the retailer over time t_1. A certain percentage of this product decays, while the remaining units are sold. The decayed products are then returned to the manufacturer. Fig. 2 illustrates the inventory level of the i-th retailer.

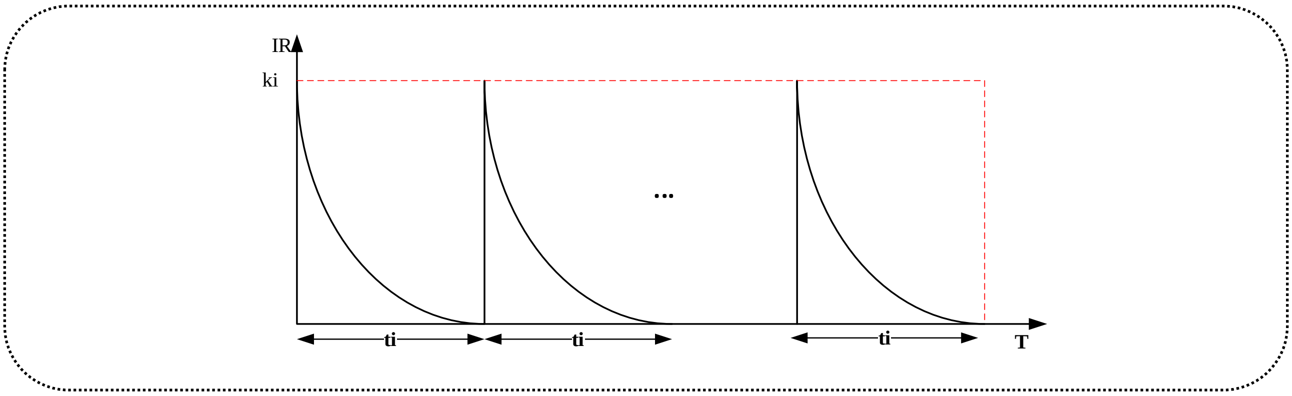

Fig. 2. The inventory level of i-th retailer

In this case, the sensitive inventory level over time for retailer i can be expressed as follows:

$$\frac{\partial IR_i(t)}{\partial t} = -\delta \times IR_i(t) - D_i(p) \tag{10}$$

At the initial moment, there are k_i units of inventory in the retailer's warehouse, and it is consumed during the interval (0, t_1). Considering the initial condition $IR_i(0) = k_i$ and solving the differential equation, the inventory level in this interval can be expressed as:

$$IR_i(t) = -\frac{e^{-t\times\delta}\left(-D_i(p) + D_i(p)\times e^{t\times\delta} - k_i\times\delta\right)}{\delta} \qquad 0 \le t \le t_1 \tag{11}$$

To simplify the above relationship using Taylor approximation, the following results are obtained.

$$IR_i(t) = k_i \times (1 - t \times \delta) - D_i(p) \times t \qquad 0 \le t \le t_1 \tag{12}$$

Based on the above relationship, it is clear that the amount of perishable product is a function of time. Through the applied estimation, this relationship has been obtained as a linear function of time, represented as:

$$Ir_i(\,t) = k_i \times t \times \delta \tag{13}$$

The amount for perishable goods of the i-th retailer during the interval $(0, t_1)$ can be determined as follows:

$$Ir_i = \frac{1}{2} \times t_1^2 \times k_i \times \delta \tag{15}$$

Holding cost: Each retailer's holding cost can be determined based on its inventory level relationship, as follows.

$$HC = h_1 \times \sum_{i=1}^{n}\left(\int_0^{t_1} IR_i(t)dt\right) = h_1 \times \sum_{i=1}^{n}\left(\int_0^{t_1} k_i \times (1 - t \times \delta) - D_i(p) \times t\right) \tag{16}$$

By simplifying, the retailer's holding cost is determined as:

$$HC = h_1 \times \left(K \times \left(t_1 - \frac{t_1^2}{2}\right) \times \delta - D(p) \times \frac{t_1^2}{2}\right) \times m + \frac{m\times(R-K)}{K} \tag{17}$$

The cost of buying goods from the manufacturer: The cost of purchasing goods from the manufacturer is determined by the quantity of goods received by the retailer, priced at P_1 per unit. The total cost incurred by the retailer for buying goods from the manufacturer during the cycle can be calculated as follows:

$$(P_1 \times K) \times m + \frac{m \times (R-K)}{K} \tag{18}$$

Ordering cost: Ordering cost is obtained from the total fixed ordering cost of retailers.

$$\sum_{i=1}^{n} A_i \tag{14}$$

The shipping cost of each receipt: The shipping cost of each receipt of goods by retailers is denoted as b_1. This cost is multiplied by the number of times the goods are received, resulting in the total shipping cost for the cycle, which can be calculated as follows:

$$b_1 \times \left(m + \frac{m \times (R-K)}{K}\right) \tag{15}$$

Retailer's revenue: The *i-th* retailer discards a portion of the k_i units it receives, which are then returned to the manufacturer. The manufacturer agrees to purchase the perished units from the retailer at a price of C_1. The remaining units are sold to customers at a price of *P*. Thus, the resulting income is calculated as follows:

$$P \times \left(\sum_{i=1}^{n} k_i - \frac{1}{2} \times t_1^2 \times \sum_{i=1}^{n} k_i \times \delta\right) + C_1 \times \frac{1}{2} \times t_1^2 \times \sum_{i=1}^{n} k_i \times \delta \tag{16}$$

Therefore, the income of retailers in a cycle is as follows:

$$\left(P \times \left(K - \frac{1}{2} \times t_1^2 \times K \times \delta\right) + C_1 \times \frac{1}{2} \times t_1^2 \times K \times \delta\right) \times \left(m + \frac{m \times (R-K)}{K}\right) \tag{17}$$

To calculate the manufacturer's revenue, the quantity of goods returned by retailers must be determined. Using this information, the producer's income and profit can be determined. The manufacturer's income is calculated as follows:

$$P_1 \times Q - C_1 \times \sum_{i=1}^{n} Ir_i \tag{18}$$

Finally, the producer's profit is calculated as follows:

$$TP_m = \left(P_1 \times Q - C_1 \times \sum_{i=1}^{n} Ir_i\right) - \left(\begin{array}{c} h \times \left(\frac{m \times R (K - K \times m + m \times R) \times t_1}{2K}\right) \\ + C_2 \times m \times R + A + b \times \left(m + \frac{m \times (R-K)}{K}\right) + C \times \left(m \times R - \sum_{i=1}^{n} Ir_i\right) \end{array}\right) \tag{19}$$

retailer i profit is as follows:

$$TP_i = \left(\left(P \times \left(K - \frac{1}{2} \times t_1^2 \times K \times \delta\right) + C_1 \times \frac{1}{2} \times t_1^2 \times K \times \delta\right) \times m + \frac{m \times (R-K)}{K}\right) - \left(\begin{array}{l} h_1 \times \sum_{i=1}^{n} \left(\int_0^{t_1} k_i \times (1 - t \times \delta) - D_i(p) \times t\right) \times m + \frac{m \times (R-K)}{K} + \\ (P_1 \times K) \times m + \frac{m \times (R-K)}{K} + \sum_{i=1}^{n} A_i + b_1 \times \left(m + \frac{m \times (R-K)}{K}\right) \end{array}\right) \tag{25}$$

Finally, the overall profit is computed as follows:

$$TP = \frac{1}{T}(TP_m + TP_i)$$
$$= \left(P_1 \times Q - C_1 \times \sum_{i=1}^{n} Ir_i\right) - \left(\begin{array}{c} h \times \left(\frac{m \times R(K - K \times m + m \times R) \times t_1}{2K}\right) \\ +C_2 \times m \times R + A + b \times \left(m + \frac{m \times (R-K)}{K}\right) + C \times \left(m \times R - \sum_{i=1}^{n} Ir_i\right) \end{array}\right) \quad (20)$$
$$+\left(\left(P \times \left(K - \frac{1}{2} \times t_1^2 \times K \times \delta\right) + C_1 \times \frac{1}{2} \times t_1^2 \times K \times \delta\right) \times m + \frac{m \times (R-K)}{K}\right)$$
$$-\left(\begin{array}{c} h_1 \times \sum_{i=1}^{n}\left(\int_0^{t_1} k_i \times (1 - t \times \delta) - D_i(p) \times t\right) \times m + \frac{m\times(R-K)}{K} + \\ (P_1 \times K) \times m + \frac{m\times(R-K)}{K} + \sum_{i=1}^{n} A_i + b_1 \times \left(m + \frac{m\times(R-K)}{K}\right) \end{array}\right)$$

IV. SOLUTION METHOD

To solve the problem, it is important to establish the concavity of the profit function. This is essential for determining the optimal solution. By proving concavity, it can be predicted that the integer solutions will be in the vicinity of the optimal point. With this understanding, a heuristic algorithm is proposed to search for the values of the decision variables around the optimal solution. This approach allows for a more efficient and effective exploration of the solution space.

Theorem 1: The profit function is concave if $X^t AX \leq 0$, where A is the Hessian matrix defined as Eq. (27), and X = [m, K, P]^T is also a vector of the decision variables.

$$\boldsymbol{A} = \begin{bmatrix} \frac{\partial^2 TP}{\partial m^2} & \frac{\partial^2 TP}{\partial m \partial K} & \frac{\partial^2 TP}{\partial m \partial P} \\ \frac{\partial^2 TP}{\partial K \partial m} & \frac{\partial^2 TP}{\partial K^2} & \frac{\partial^2 TP}{\partial K \partial P} \\ \frac{\partial^2 TP}{\partial P \partial m} & \frac{\partial^2 TP}{\partial P \partial K} & \frac{\partial^2 TP}{\partial P^2} \end{bmatrix} \quad (21)$$

The calculation of the elements of the Hessian matrix A was performed using Mathematica 11 software. Due to the large number of parameters in the matrix determinant, simplification is challenging. Therefore, the value of $X^{\mathrm{T}}AX \leq 0$ was computed using this software, yielding the following result:

$$-\frac{1}{K}\left(\begin{array}{l} \frac{2A\alpha(3\alpha - 2P\beta + K\delta)}{m\eta} + \frac{2\sum_{i=1}^{n} A_i \alpha(3\alpha - 2P\beta + K\delta)}{m\eta} \\ + \frac{1}{(\alpha - P\beta + K\delta)^4}\left(\begin{array}{c} K(\alpha - P\beta)\left(\begin{array}{l} \alpha^2(-K^2(c + h1 - 3P) + 8b2\alpha) \\ +P\alpha\left(-16b2\alpha + KP(-3K + 8(\zeta + \gamma)\alpha)\right)\beta \\ +P^2\left(8b2\alpha + KP(K - 16(\zeta + \gamma)\alpha)\right)\beta^2 + 8(\zeta + \gamma)KP^4\beta^3 \end{array}\right)\delta \\ +K^2\left(\begin{array}{c} 2cK^2\alpha^2 - h1K^2\alpha^2 + 12b2\alpha(\alpha - P\beta)^2 \\ +2KP^2\beta\left(6(\zeta + \gamma)(\alpha - P\beta)^2 + K(-2\alpha + P\beta)\right) \end{array}\right)\delta^2 \\ -K^3\left(K^2P^2\beta - 8(\alpha - P\beta)(b2\alpha + (\zeta + \gamma)KP^2\beta)\right)\delta^3 \\ +2K^4(b2\alpha + (\zeta + \gamma)KP^2\beta)\delta^4 + 2b\alpha(\alpha - P\beta + K\delta)^4 \\ +hK^2\left(\begin{array}{l} -(\alpha - P\beta + K\delta)\left(K\alpha^2\delta + m((\alpha - P\beta)^3 + KP\beta(-3\alpha + 2P\beta)\delta - K^2P\beta\delta^2)\right) \\ +m\alpha^2(\alpha - P\beta - 2K\delta)\eta \end{array}\right) \end{array}\right) \end{array}\right) \quad (22)$$

After simplifying Equation (28), the final value is obtained as follows:

$$X^t AX = -\frac{2A\alpha^3(3\alpha-2P\beta+K\delta)+2\sum_{i=1}^{n} A_i\alpha^3(3\alpha-2P\beta+K\delta)+m\eta\left(2(b+b2)\alpha^3+hK^2m\left(-\alpha^2+\alpha\eta+3P\beta\eta\right)\right)}{Km\alpha^2\eta} \tag{23}$$

Eq. (29) yields a negative result, indicating that the profit function is concave.

The main objective of this paper is to maximize profit by determining the values of *m*, *K* and *P*. Since the profit function is concave, the optimal values of these decision variables can be obtained by analyzing the first derivative with respect to them. In the following section, the profit function is extracted in terms of *m*, *K* and *P* and applied to present the optimization algorithm**.** This algorithm plays a significant role in finding the optimal profit values.

$$\frac{\partial TP}{\partial m} = \left\{ \frac{(\alpha - P\beta)\begin{pmatrix}2A(\alpha - P\beta + K\delta)^3 \\ +hK^2m^2(\alpha - P\beta + K\delta)\eta + (\alpha - P\beta + K\delta)^3\sum_{i=1}^{n} A_i\end{pmatrix}}{2Km^2(\alpha - P\beta + K\delta)^2\eta} \right\} \tag{24}$$

$$\frac{\partial TP}{\partial K} = \left\{ \frac{1}{2K^2m(\alpha - P\beta + K\delta)^3\eta}(\alpha - P\beta)\begin{pmatrix} 2A(\alpha - P\beta)(\alpha - P\beta + K\delta)^3 + 2A1n(\alpha - P\beta)(\alpha - P\beta + K\delta)^3 \\ +m\eta\begin{pmatrix} -h1K^2(\alpha - P\beta)(\alpha - P\beta + K\delta) + 2\begin{pmatrix}K^3(c-P)(\alpha - P\beta)\delta \\ +b(\alpha - P\beta + K\delta)^3 \\ +b2(\alpha - P\beta + K\delta)^3\end{pmatrix} \\ +hK^2\left((-1+m)(\alpha - P\beta)(\alpha - P\beta + K\delta) + m(-\alpha + P\beta + K\delta)\eta\right)\end{pmatrix}\end{pmatrix} \right\} \tag{31}$$

$$\frac{\delta TP}{\delta P} = \left\{ \frac{1}{2K}\left(\begin{matrix} \frac{2A\beta(2\alpha - 2P\beta + K\delta)}{m\eta} + \frac{2A1n\beta(2\alpha - 2P\beta + K\delta)}{m\delta} \\ + \frac{1}{(\alpha - P\beta + K\delta)^3} \\ \begin{pmatrix} 2(b+b2)\beta(\alpha - P\beta)^3 + 2K(\alpha - P\beta)^2\left((\alpha - P\beta)\left((\zeta+\gamma)\alpha + (c - 2(\zeta+\gamma)P)\beta\right) + 3(b+b2)\beta\delta\right) \\ +K^4\delta^2\left(\alpha(-1+2(\zeta+\gamma)\delta) + \beta\left(h + h1 - hm + 2P - 4(\zeta+\gamma)P\delta + c(-1+2\delta)\right)\right) \\ +K^2(\alpha - P\beta)(6(\zeta+\gamma)\alpha^2\delta + 6(c - 3(\zeta+\gamma)P)\alpha\beta\delta + 6\beta\delta(P(-c+2(\zeta+\gamma)P)\beta + (b+b2)\delta) - hm\beta\eta) \\ +K^3\delta\begin{pmatrix}\alpha^2(-1+6(\zeta+\gamma)\delta) + \alpha\beta(c + h + h1 - hm + P + 6(c - 3(\zeta+\gamma)P)\delta) \\ +\beta(-h1P\beta + h(-1+m)P\beta + 12(\zeta+\gamma)P^2\beta\delta + 2(b+b2)\delta^2 - cP\beta(1+6\delta) + hm\eta)\end{pmatrix} \end{pmatrix} \end{matrix} \right) \right\} \tag{32}$$

By simultaneously solving Equations (30)-(32), we can determine the continuous optimal values of the decision variables. However, accounting for the discrete nature of these variables, we develop a heuristic algorithm to solve the model. The proposed algorithm handles variable discreteness and is described below.

A. Solving algorithm

In the given problem, the values of *m* and *K* are integers, and the resulting value of *Q* will consequently be an integer. However, the selling price (*P*) is a continuous variable. To address the mixed-integer nature of the problem, we propose an approach that initially treats all variables as continuous. By leveraging optimization techniques from operations research, the optimal continuous solution is determined. This solution serves as an upper bound for the discrete problem. Considering that the integer values of the decision variables are expected to be continuous in the vicinity of the optimal solution, an algorithm is proposed to find the integer solution for the decision variables, including the optimal value of the selling price, the number of times of shipment, and the optimal value of each shipment. The algorithm aims to search for the optimal solution and identify an appropriate answer to the problem.

Step 1: The equations of the first derivative with respect to K, m, and P are solved simultaneously (Eq. (30-32)) and the optimal value of the price, the number of sending times, and the optimal value of each sending time are determined.

Step 2: If the optimal value of m and K are integers, the obtained solution is optimal. Otherwise, the algorithm proceeds to Step 3, where the neighborhood of the non-integer solutions is explored to find the closest integer solutions.

Step 3: Define the value of K_1 and K_2 as $k_1^* = [k^*]$, $k_2^* = [k^*] + 1$.

And, define the value of m_1 and m_2 as $m_1^* = [m^*], m_2^* = [m_2^*] + 1$, then go to step 4.

Step 4: Based on Step 3, the following four situations can be considered:

$$m^*{}_1 = [m_1^*], \quad K^*{}_1 = [K_1^*] \qquad m^*{}_3 = [m_1^*] + 1, \quad K^*{}_3 = [K_1^*]$$
$$m^*{}_2 = [m_1^*], \quad K^*{}_2 = [K_1^*] + 1 \qquad m^*{}_4 = [m_1^*] + 1, \quad K^*{}_4 = [K_1^*] + 1 \tag{25}$$

Step 5: Based on equation (32), the optimal value of the retailer's price is determined for the four states stated in the fourth step.

Step 6: Based on the values of Step 5, the total profit values are determined, and any mode that has the most profit is selected as the best solution.

To obtain more clarity about the proposed solution approach, it is completely represented in Fig. 3.

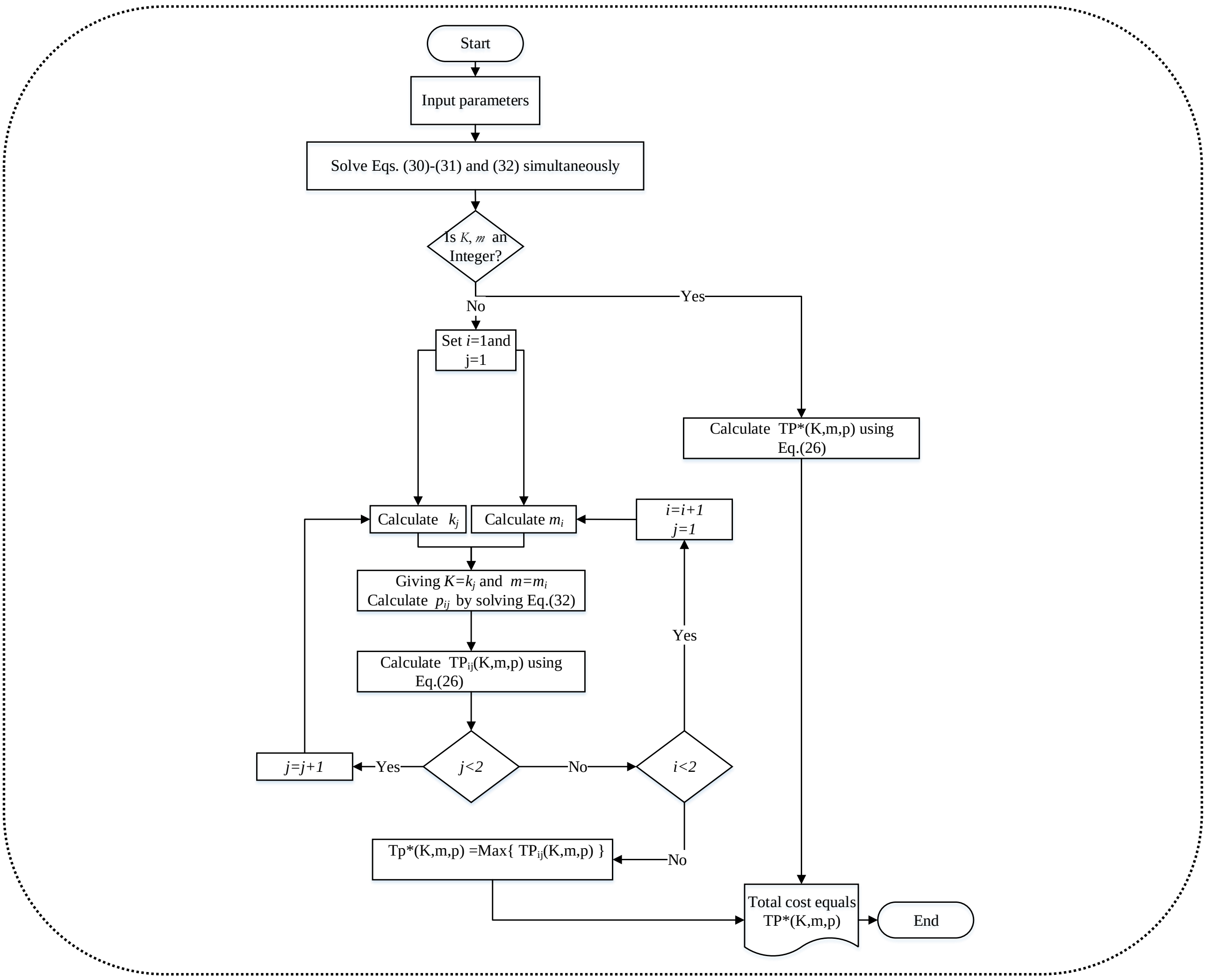

Fig. 3. The proposed solution approach

V. NUMERICAL EXAMPLES

Suppose the problem is considered with one manufacturer and three retailers (n=3). The manufacturer starts production to produce the final product at a rate of 150 units per cycle. The holding cost and production cost for each product unit in the cycle will be 5 and 10 monetary units, respectively. The producer starts production once per cycle, with a fixed start-up cost of 400 monetary units per production cycle. The producer pays 20 monetary units for the purchase of raw materials and 10 currency units for packaging and shipping costs for each shipment. The retailer's demand function is defined as D_i = 120 - 0.3×P. Due to its perishability, the product spoils at a rate of 0.01% of the retailer's inventory level. This amount is expressed in the form of returned goods at a cost of a percentage of the manufacturer's selling price, which is repurchased by the manufacturer. The unit holding cost per item per unit time for each retailer is 20 currency units. The cost of ordering for all three retailers will be 100, 190, and 180 monetary units, respectively, and the fixed shipping cost for each receipt of goods by the retailers will be 5 monetary units. The selling price of each unit of the producer's product is a function of the total selling price and is expressed as $P_1 = 0.7P$.

A. Solve the numerical example

Step 1: Equations (30), (31), and (32) are solved simultaneously, resulting the following values.

$$TP^*(m, K, P) = 18452.2 \qquad m^* = 8.46591 \quad K^* = 8.28661 \qquad P^* = 206.929 \tag{26}$$

If the optimal value of m is substituted into the profit function, the resulting graph, as shown in Fig. 4, illustrates the relationship between the profit function, the number of shipments (k), and the quantity per shipment (p). In this Fig, the horizontal axis represents the number of shipments (k), while the vertical axis denotes the shipment quantity (p). The profit function is depicted as a three-dimensional surface, highlighting the optimal points where specific combinations of shipment number and shipment quantity result in maximum profit.

Similarly, if the optimal value of k is substituted into the profit function, the graph in Fig. 5 demonstrates the relationship between the profit function, the retailer's selling price (p), and the shipment quantity (m). In this case, the horizontal axis represents the retailer's selling price, and the vertical axis corresponds to the shipment quantity. The three-dimensional surface clearly illustrates the optimal points where balancing the selling price and shipment quantity maximizes profit. An increase in selling price might boost profits but could reduce demand, while increasing shipment quantity might lower transportation costs but could increase storage and inventory risks.

Lastly, if the optimal value of the retailer's selling price (p) is substituted into the profit function, the graph in Fig. 6 shows the total profit function as a relationship between the number of shipments (m) and the selling price (k). Here, the horizontal axis indicates the number of shipments, and the vertical axis represents the selling price. The profit function, depicted as a three-dimensional surface, highlights the optimal points where the balance between the number of shipments and the selling price maximizes profit. Increasing the number of shipments might raise transportation costs but improve service quality while increasing the selling price could enhance profits but might reduce demand. These visual representations across all figures provide insights into determining optimal strategies for profit maximization.

Step 2: As seen from the non-integer values of m and K obtained in the first step, to find a discrete solution, the number of sending times and the volume for each sending time are calculated by searching the neighborhood of the numbers found in the first step, as shown in the third step.

Step 3: New values for the number of sending times and the volume of each sending time are calculated as follows:

$$K_1^* = [K^*] = 8 \quad K_2^* = [K^*] + 1 = 9 \qquad m_1^* = [m^*] = 8 \qquad m_2^* = [m^*] + 1 = 9 \tag{27}$$

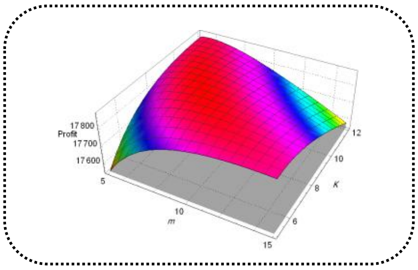

Fig. 4. Profit function with respect to different values of number of shipments and shipment quantity

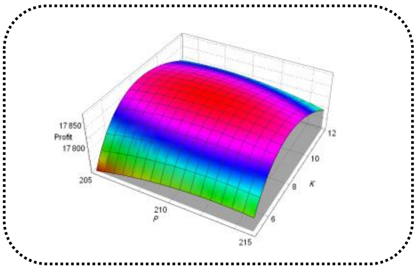

Fig. 5. Profit function with respect to different values of retailer selling price and shipment quantity

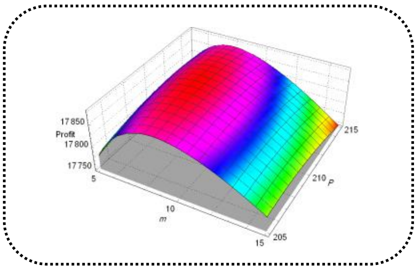

Fig. 6. Profit function with respect to different values of the number of shipments and retailer selling price

Step 4: Based on step 3, different values of *m* and *K* can be obtained for four situations:

State 1: $m_1^* = [m^*] = 8, K_1^* = [K^*] = 8$

State 2: $m_2^* = [m^*] = 8, K_2^* = [K_1^*] + 1 = 9$

State 3: $m_3^* = [m^*] + 1 = 9, K_3^* = [K^*] = 8$

State 4: $m_4^* = [m^*] + 1 = 9, K_4^* = [K^*] + 1 = 9$

Step 5: Retailer sales price values for different *m* and *K* values are calculated as follows.

$$P_1^* = \frac{207}{402P_2^*} = \frac{206}{76P_3^*} = \frac{206}{819P_4^*} = \frac{206}{183}$$

Step 6: Profit values for different *m*, *K*, and *P* values are calculated as follows.

$$TP_1^*(m_1^*, K_1^*, P_1^*) = \frac{18449}{8}, TP_2^*(m_2^*, K_2^*, P_2^*) = \frac{18451}{3}, TP_3^*(m_3^*, K_3^*, P_3^*) = \frac{18451}{9}, TP_4^*(m_4^*, K_4^*, P_4^*) = 18446$$

Based on the values obtained, the highest profit is associated with the third state, yielding the best solution:

$$TP_3^*(m_3^*, K_3^*, P_3^*) = \frac{18451}{9}, P_3^* = \frac{206}{819}, m_3^* = [m^*] + 1 = 9, K_3^* = [K^*] = 8$$

VI. SENSITIVITY ANALYSIS

In sensitivity analysis, changes in setup cost, manufacturer and retailer holding cost, transportation cost per shipment for manufacturer and retailer, production cost, and raw material purchase cost will be examined to find the optimal solution for the problem. One parameter will be adjusted while the others are held constant. The results will be displayed in Table II.

Sensitivity analysis of the fixed setup cost parameter is being investigated while keeping the other parameters of the current problem constant. According to Table II, as the fixed setup cost increases, the frequency of product receipt in each cycle also increases. This, in turn, leads to an increase in the order quantity. A higher order quantity results in product accumulation, subsequently increasing the inventory level and holding costs within the system. Ultimately, this trend leads to a decrease in optimal profit.

Table II. Sensitivity analysis based on the holding cost of retailer

	#	Percentage of Changes	Model Parameters									Decision Variables					Integrated Profit
			Setup Cost	Manufacturer's Holding Cost	Retailer's Holding Cost	Manufacturer's Transportation Cost	Retailer's Transportation Cost	Production Cost	Raw Material Cost	Perishability rate	Demand sensitivity to price	Number of shipments	Shipment Quantity	Retail Price	Manufacturer's Price	Cost of Returned Goods	
Manufacturer Setup Cost	1	-100	0	5	20	10	5	10	20	0.01	0.3	7	8	209.936	146.955	73.478	18023.70
	2	-80	80	5	20	10	5	10	20	0.01	0.3	7	8	209.62	146.776	73.388	17990.60
	3	-60	160	5	20	10	5	10	20	0.01	0.3	7	8	209.99	146.993	73.497	17959.60
	4	-40	240	5	20	10	5	10	20	0.01	0.3	8	8	209.696	146.787	73.394	17930.50
	5	-20	320	5	20	10	5	10	20	0.01	0.3	8	8	209.966	146.976	73.488	17903.40
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.378	17876.60
	7	20	480	5	20	10	5	10	20	0.01	0.3	9	8	209.889	146.922	73.461	17852.40
	8	40	560	5	20	10	5	10	20	0.01	0.3	9	8	210.127	147.889	73.544	17828.30
	9	60	640	5	20	10	5	10	20	0.01	0.3	10	8	209.776	146.846	73.422	17805.20
	10	80	720	5	20	10	5	10	20	0.01	0.3	10	8	209.991	146.994	73.497	17783.40
	11	100	8800	5	20	10	5	10	20	0.01	0.3	10	8	210.204	146.143	73.571	17761.8
Producer's holding cost	1	-100	400	0	20	10	5	10	20	0.01	0.3	17	9	209.584	146.6836	73.3418	18200.7
	2	-80	400	1	20	10	5	10	20	0.01	0.3	16	9	209.301	146.5109	73.25535	18150.3
	3	-60	400	2	20	10	5	10	20	0.01	0.3	12	9	209.653	146.7571	73.37855	18093.6
	4	-40	400	3	20	10	5	10	20	0.01	0.3	10	9	209.651	146.7557	73.37785	18010.3
	5	-20	400	4	20	10	5	10	20	0.01	0.3	10	8	209.648	146.7536	73.3768	17939.3
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17876.6
	7	20	400	6	20	10	5	10	20	0.01	0.3	8	8	209.819	146.8733	73.43665	17820.3
	8	40	400	7	20	10	5	10	20	0.01	0.3	7	8	210.183	147.1281	73.56405	17767.6
	9	60	400	8	20	10	5	10	20	0.01	0.3	7	8	209.823	146.8761	73.43805	17718.1
	10	80	400	9	20	10	5	10	20	0.01	0.3	6	8	210.476	147.3332	73.6666	17671.1
	11	100	400	10	20	10	5	10	20	0.01	0.3	6	8	210.168	147.1176	73.5588	17620.8
Retailers holding cost	1	-100	400	5	0	10	5	10	20	0.01	0.3	4	17	209.4	146.58	73.29	17988.5
	2	-80	400	5	4	10	5	10	20	0.01	0.3	5	14	209.45	146.615	73.3075	17958.4
	3	-60	400	5	8	10	5	10	20	0.01	0.3	6	11	209.54	146.678	73.339	17934.0

Continue Table II. Sensitivity analysis based on the holding cost of retailer

	#	Percentage of Changes	Model Parameters									Decision Variables					Integrated Profit
			Setup Cost	Manufacturer's Holding Cost	Retailer's Holding Cost	Manufacturer's Transportation Cost	Retailer's Transportation Cost	Production Cost	Raw Material Cost	Perishability rate	Demand sensitivity to price	Number of shipments	Shipment Quantity	Retail Price	Manufacturer's Price	Cost of Returned Goods	
Retailers holding cost	4	-40	400	5	12	10	5	10	20	0.01	0.3	7	10	209.54	146.678	73.339	17913.1
	5	-20	400	5	16	10	5	10	20	0.01	0.3	8	9	209.509	146.713	73.3565	17892.8
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17876.6
	7	20	400	5	24	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17860.6
	8	40	400	5	28	10	5	10	20	0.01	0.3	10	7	209.866	146.9062	73.4531	17846.5
	9	60	400	5	32	10	5	10	20	0.01	0.3	10	7	209.123	147.0861	73.54305	17832.6
	10	80	400	5	36	10	5	10	20	0.01	0.3	11	6	210.259	147.1813	73.59065	17817.8
	11	100	400	5	40	10	5	10	20	0.01	0.3	11	6	210.259	147.1813	73.59065	17806.8
Producer's transportation cost	1	-100	400	5	20	0	5	10	20	0.01	0.3	14	2	209.542	146.6794	73.3397	17946.9
	2	-80	400	5	20	2	5	10	20	0.01	0.3	12	6	209.45	146.615	73.3075	17942.3
	3	-60	400	5	20	4	5	10	20	0.01	0.3	11	6	209.976	146.9832	73.4916	17923.7
	4	-40	400	5	20	6	5	10	20	0.01	0.3	10	7	209.704	146.7928	73.3964	17907.1
	5	-20	400	5	20	8	5	10	20	0.01	0.3	9	8	209.579	146.7053	73.35265	17890.8
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17876.6
	7	20	400	5	20	12	5	10	20	0.01	0.3	8	9	209.653	146.7571	73.37855	17863.1
	8	40	400	5	20	14	5	10	20	0.01	0.3	8	9	209.716	146.8012	73.4006	17850.4
	9	60	400	5	20	16	5	10	20	0.01	0.3	7	10	209.85	146.895	73.4475	17839.0
	10	80	400	5	20	18	5	10	20	0.01	0.3	7	10	209.907	146.9349	73.46745	17827.6
	11	100	400	5	20	20	5	10	20	0.01	0.3	6	11	210.19	146.133	73.5665	17816.3
	1	-100	400	5	20	10	0	10	20	0.01	0.3	10	7	209.663	146.7641	73.38205	17915.3
	2	-80	400	5	20	10	1	10	20	0.01	0.3	10	7	209.704	146.7928	73.3964	17907.1
	3	-60	400	5	20	10	2	10	20	0.01	0.3	10	7	209.744	146.8208	73.4104	17899.0
	4	-40	400	5	20	10	3	10	20	0.01	0.3	9	8	209.579	146.7053	73.35265	17890.8
	5	-20	400	5	20	10	4	10	20	0.01	0.3	9	8	209.614	146.7298	73.3649	17883.7
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17876.6

Continue Table II. Sensitivity analysis based on the holding cost of retailer

	#	Percentage of Changes	Model Parameters									Decision Variables					Integrated Profit
			Setup Cost	Manufacturer's Holding Cost	Retailer's Holding Cost	Manufacturer's Transportation Cost	Retailer's Transportation Cost	Production Cost	Raw Material Cost	Perishability rate	Demand sensitivity to price	Number of shipments	Shipment Quantity	Retail Price	Manufacturer's Price	Cost of Returned Goods	
Retail transportation cost	7	20	400	5	20	10	6	10	20	0.01	0.3	8	9	209.621	146.7347	73.36735	17869.5
	8	40	400	5	20	10	7	10	20	0.01	0.3	8	9	209.653	146.7571	73.37855	17863.1
	9	60	400	5	20	10	8	10	20	0.01	0.3	8	9	209.684	146.7788	73.3894	17855.8
	10	80	400	5	20	10	9	10	20	0.01	0.3	7	10	209.794	146.8558	73.4279	17850.4
	11	100	400	5	20	10	10	10	20	0.01	0.3	7	10	209.822	146.8754	73.4377	17844.7
Production and raw material purchase cost	1	-100	400	5	20	10	5	0	0	0.01	0.3	9	8	206.819	144.7723	72.3866	18450.9
	2	-80	400	5	20	10	5	2	4	0.01	0.3	9	8	207.386	145.1702	72.5851	18336.1
	3	-60	400	5	20	10	5	4	8	0.01	0.3	9	8	207.952	145.5664	72.7832	18220.7
	4	-40	400	5	20	10	5	6	12	0.01	0.3	9	8	208.518	145.9626	72.9813	18105.7
	5	-20	400	5	20	10	5	8	16	0.01	0.3	9	8	209.084	146.3588	73.1794	17990.9
	6	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.755	73.3775	17876.6
	7	20	400	5	20	10	5	12	24	0.01	0.3	8	8	210.801	147.5607	73.78035	17762.6
	8	40	400	5	20	10	5	14	28	0.01	0.3	8	8	211.367	147.9569	73.97845	17649.2
	9	60	400	5	20	10	5	16	32	0.01	0.3	8	8	211.933	148.3531	74.17655	17536.2
	10	80	400	5	20	10	5	18	36	0.01	0.3	8	8	212.499	148.7493	74.37465	17423.6
	11	100	400	5	20	10	5	20	40	0.01	0.3	8	8	213.064	149.1448	74.5724	17311.2
Price elasticity of demand	1	-50	400	5	20	10	5	10	20	0.01	0.15	9	8	409.823	286.8761	143.43805	38251.2
	2	-30	400	5	20	10	5	10	20	0.01	0.21	9	8	295.47	206.829	103.4145	26604.5
	3	-10	400	5	20	10	5	10	20	0.01	0.27	9	8	231.908	162.3356	81.1678	20138.3
	4	0	400	5	20	10	5	10	20	0.01	0.3	9	8	209.65	146.775	73.3775	17876.6
	5	10	400	5	20	10	5	10	20	0.01	0.33	8	8	192.016	134.4112	67.2056	16027.2
	6	30	400	5	20	10	5	10	20	0.01	0.39	8	8	163.968	114.7776	57.3888	13184.7
	7	50	400	5	20	10	5	10	20	0.01	0.45	8	8	143.38	100.366	50.183	11102.9

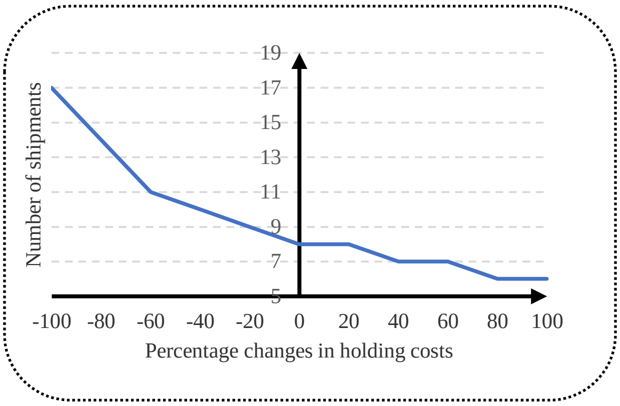

Fig. 7. Sensitivity of the optimal order quantity to changes in holding costs

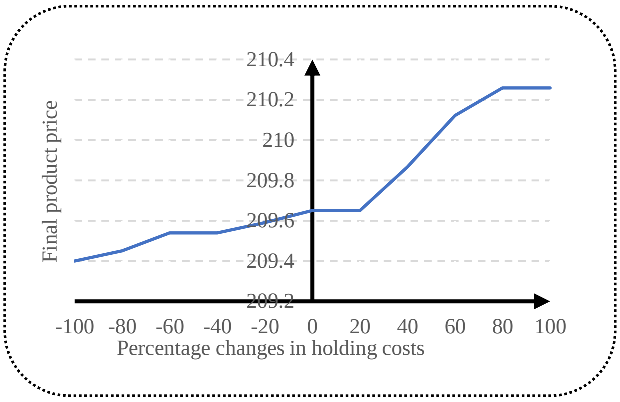

Fig. 8. Sensitivity of the total product price to changes in holding costs

According to Table II, all parameters of the problem have been considered except for the holding cost, and the problem has been investigated under different values of manufacturer and retailer holding costs. The results obtained show that as the cost of holding increases, the number of shipments from the manufacturer to the retailers increases, while the volume of products in each shipment decreases (see Fig. 6). This leads to a reduction in the amount of the order. Since the cost of holding is added to the initial purchase price, it affects the final price of the product, resulting in an increase in the final price (see Fig. 7). Demand is dependent on the final price of the product, so as the price increases, demand decreases, ultimately reducing the profit of the chain (see Fig. 12).

Fig. 9. Sensitivity of the number of receipts and the optimal amount per shipment to changes in the cost per shipment

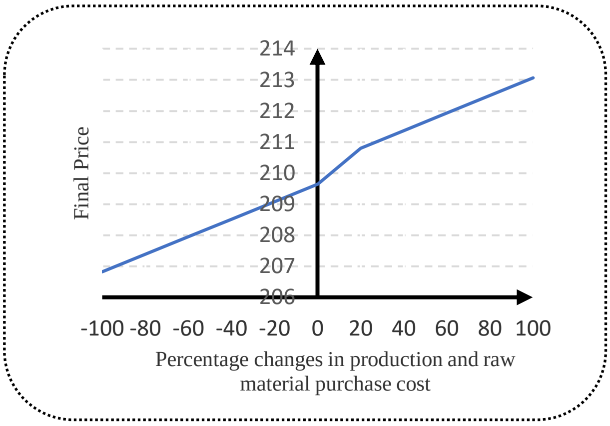

Fig. 10. Sensitivity of the final product price to changes in production cost and raw material purchase cost

According to the investigation, the sensitivity analysis of transportation costs shows that as the cost per shipment increases, the optimal number of shipments from the manufacturer to the retailers decreases. However, the volume and quantity of each shipment increase Fig. 8, leading to an increase in the amount of ordering in each cycle. Therefore, the optimal profit amount will decrease Fig. 12. Next, changes in production costs and initial purchase costs will be discussed based on the results from Table II. It is known that as initial purchase costs and production costs increase, the finished product price increases (Fig. 9). Since demand is assumed to be dependent on the finished product price, this leads to a decrease in demand. The decrease in retailer demand will result in fewer orders, leading to a decrease in retailer profit due to lower product sales. Additionally, the manufacturer's price and the price of returned products are dependent on the final sale price, and these prices will increase with the retailer's final price, resulting in a decrease in the system's integrated profit Fig. (12).

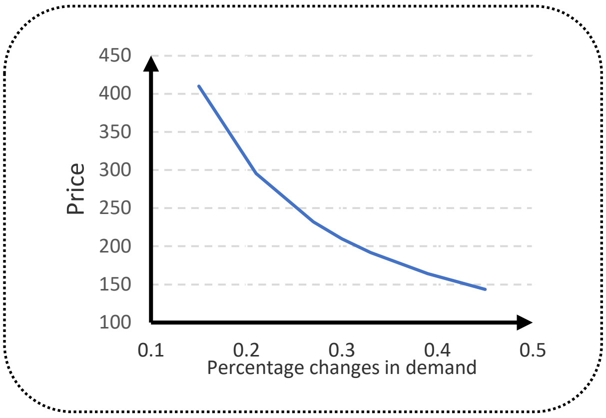

Fig. 11. Sensitivity of the total product price to changes in demand

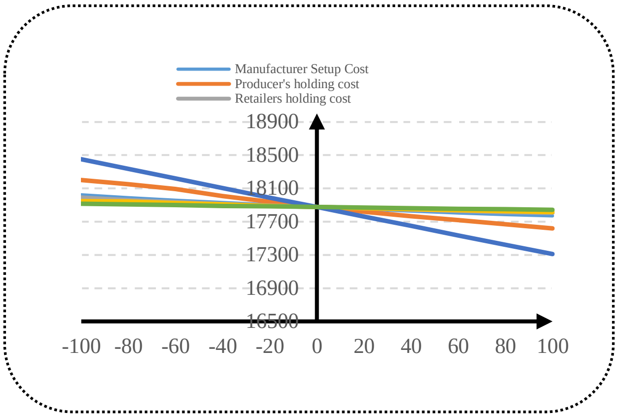

Fig. 12. Sensitivity of integrated system profit to changes in model parameters

As mentioned earlier, demand is dependent on the cost price of the product. Therefore, as the cost price of the products, increases, the demand decreases, leading to a reduction in orders from retailers. Consequently, it is evident that the total product price decreases due to its sensitivity to changes in demand according to Fig. 11. Additionally, the sensitivity chart of the integrated profit concerning the model parameters is displayed in Fig. 12.

Managerial insights derived from the results of this research can provide practical guidance for managers seeking to apply these findings to real-world scenarios. Key insights include:

With the increase in startup costs, the frequency of resource receipts in each cycle also rises. Proper cost management is essential to prevent resource maintenance issues and significantly reduce maintenance expenses. Managers should carefully plan the timing of order receipts to optimize operations. Sensitivity analysis of transportation costs reveals that as the cost per shipment increases, the quantity of each shipment decreases. To manage costs effectively, a balance must be maintained between various cost components, such as transportation and setup costs, to maximize overall profitability. Managers should evaluate which costs to adjust to achieve optimal financial outcomes.

An increase in shipment volume and frequency leads to higher order costs on each cycle. Striking a balance between shipment frequency and volume, while considering prior managerial insights, is crucial for achieving maximum profit and operational efficiency. In industries dealing with perishable goods, such as dairy and pharmaceuticals, time sensitivity is a critical factor. Managers must adopt optimal strategies for ordering and transportation to prevent product expiration and avoid unnecessary cost escalations. For instance, implementing advanced technologies such as temperature tracking systems and demand forecasting tools can enhance the efficiency and reliability of supply chains in these sectors.

Modern technologies, including big data analytics and artificial intelligence, play a vital role in decision-making. These technologies enable managers to improve demand forecasting, optimize transportation routes, and streamline order scheduling, particularly in the context of perishable goods supply chains. Their integration into operational strategies can significantly enhance overall supply chain performance.

VII. CONCLUSION

The need to increase profits in perishable supply chains has prompted agricultural project managers to optimize these chains more effectively. As a result, the perishable supply chain, recognized as a vital component of the economy, has garnered significant interest in the field of system optimization. However, a review of the literature reveals that researchers have primarily focused on a single retailer in the perishable supply chain. In reality, supply chains in this sector consist of more than one retailer, and considering the closed-loop nature of the perishable supply chain is crucial due to environmental impacts and recycling and waste management costs. This study addresses these gaps by developing a pricing-inventory model in a closed-loop supply chain for perishable goods. The proposed model optimizes the selling price, delivery quantity per shipment, and number of shipments to maximize the overall profitability of the supply chain.

The results demonstrate that optimal pricing strategies effectively enhance profitability, while sensitivity analysis reveals the impact of key parameters on supply chain performance. For instance, as the fixed setup cost increases, the order quantity rises, and as holding costs grow, the number of shipments from manufacturers to retailers increases. Similarly, higher transportation costs lead to fewer shipments, validating the proposed model. This study contributes by integrating multiple factors affecting perishable goods supply chains into a comprehensive mathematical model, providing valuable insights for supply chain managers to make informed decisions on pricing, inventory, and delivery strategies. These findings advance both theoretical understanding and practical application in the field.

The proposed model has several limitations. First, it relies heavily on historical data for decision-making, which limits its ability to adapt in real-time and respond to sudden changes in demand and supply chain conditions. Additionally, the model is designed as a single-objective optimization that focuses solely on maximizing supply chain profitability, without accounting for other important factors such as customer satisfaction, service levels, and operational efficiency.

Based on these limitations, future research should focus on developing adaptive models that incorporate real-time data and advanced technologies such as machine learning and Internet of Things (IoT) to enhance decision-making under dynamic conditions. Additionally, integrating robust or stochastic optimization techniques can help address uncertainties in demand, transportation, and supply availability. Furthermore, future studies should explore multi-objective optimization approaches that balance profitability with key performance indicators such as customer satisfaction, service levels, and operational efficiency to provide a more comprehensive and practical decision-making framework.

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