



A SBM-NDEA Model for Evaluating the Efficiency of Cross-Docking Systems under Uncertainty: A Fuzzy Reasoning-Neutrosophic Programming Approach

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Abstract – While the evaluation of warehouses' efficiency through various methods has been increasingly prominent in recent times, this study stands out as the pioneering attempt to evaluate the efficiency of cross-docking systems. To this end, this study delves into a detailed analysis of a cross-docking system's structure, taking into account a wide array of factors that impact its operational performance, such as inbound and outbound doors, different modes of transportation, inspection processes, kitting activities, storage procedures, retrieval tasks, and staging operations. A comprehensive range of key performance indicators (KPIs) is recommended for every aspect related to digitization, automation, sustainability, resiliency, and lean principles. A new slack-based measure network data envelopment analysis (SBM-NDEA) model is developed to assess the efficiency of cross-docking systems with regard to undesirable factors. What is more, a novel hybrid uncertainty method is presented, which incorporates fuzzy reasoning techniques and Neutrosophic fuzzy programming to address uncertainties in the recommended KPIs and quantify qualitative assessments. Ultimately, a case study is analyzed to highlight the efficacy and validity of the developed model and uncertainty methodology. The findings reveal that the simultaneous and effective application of pre-distribution and post-distribution policies can boost the efficiency of the analyzed system by 33%. Furthermore, modifying the layout to remove unnecessary transportation activities can result in a 21% improvement in efficiency.

Keywords– Cross-docking, Network data envelopment analysis, Key performance indicator, Fuzzy sets.

I. INTRODUCTION

Cross-docking is a logistics concept that connects intermediate points in transportation systems. Upon the arrival of supplies via inbound trucks, they are consolidated, sorted based on their ultimate destination, and subsequently moved through a cross-dock for direct loading onto outbound vehicles (Vahdani and Zandieh, 2010; Stephan and Boysen, 2011; Vahdani and Shahramfard, 2019; Kargari Esfand Abad et al., 2019; Kiani Mavi et al., 2020; Torbali and Alpan, 2023). The outbound trucks travel directly to the next destination along the distribution route. This method differs from conventional warehouses as it reduces the quantity of stored items, enabling all deliveries to leave the cross-dock within a day on average (Mousavi et al., 2014; Abad et al., 2018; Acevedo-Chedid et al., 2023). A cross-docking system is

primarily designed to consolidate numerous smaller shipments from different shippers and receivers, which enables the movement of full truckloads (Vahdani et al., 2010; Reddy et al., 2017; Pan et al., 2021; Liu and Li, 2023). As a result, hub-and-spoke distribution networks supersede conventional point-to-point delivery systems, leading to cost savings in transportation (Javanmard et al., 2014; Mousavi and Vahdani, 2016, 2017; Liu and Li, 2023; Jabbouri et al., 2023). Indeed, the concept of cross-docking is increasingly being adopted across industries for its ability to consolidate cargo during transportation. Shippers benefit from reduced freight costs when transferring full trucks, while receiving departments also gain from fewer truck deliveries (Kuo, 2013; Ghomi et al., 2023). While it is beneficial for the environment to minimize truck deliveries and traffic in urban settings, the complexity of organizing the transport fleet increases when vehicles are routed through cross-docking (Vahdani, 2019; Mousavi et al., 2019; Benrqya and Jabbouri, 2023; Monaco and Sammarra, 2023).

Undeniably, it is essential to have methods in place to assess the accuracy and alignment of processes with the system's objectives. Key performance indicators (KPIs) serve as control tools to collect information on planned activities and their progress (Krauth et al., 2005; Pajić et al., 2021). Consequently, they can provide insights into the quality of the operational planning process. Additionally, the indicators facilitate the development of action plans and improve the control mechanisms. To realize this, they assess and amend the differences between actual occurrences and projected expectations. In conclusion, this contributes to an improvement in the overall performance of the system (Cosma et al., 2024).

Particularly, in the realm of warehousing systems and distribution center management, the primary KPIs are categorized into six general groups: general, cost, time, communication and information technology, social responsibility, and environment (Faveto et al., 2024). Nevertheless, given the numerous and diverse KPIs in use, it is essential to have a method that can consolidate them to create a streamlined measure for evaluating warehouse performance. As a result, data envelopment analysis (DEA) has emerged as a prominent and effective approach for this task (Cooper et al., 2011), utilizing KPIs as inputs and outputs (Faber et al., 2018; Dixit et al., 2020; Alidrisi, 2021; Nong, 2022).

The subsequent literature review reveals that issues related to cross-docking systems have not been adequately addressed in the existing studies, which predominantly concentrate on evaluating the efficiency of standard warehouses and distribution centers (Rodrigues et al., 2018). Furthermore, conventional DEA models have been utilized in these studies, treating warehouses and distribution centers as black boxes, despite their interconnected activities resembling a network, and the effectiveness of any individual component can influence the overall system efficiency (Kao, 2020; Jiang et al., 2021). Also, the presence of a greater number of internal processes and components in cross-docking systems compared to traditional warehouses exacerbates this issue. Neglecting to factor in internal operations during the efficiency assessment of these facilities may produce misleading results (Liang et al., 2022). Therefore, it is essential to employ network DEA (NDEA) models for assessing the efficiency of cross-docking systems, as traditional DEA models are not the right approach in this case.

Additionally, the consideration of undesirable factors has been lacking in the related studies using traditional DEA models to measure warehouse efficiency. This raises concerns about the validity of their findings, given that many have incorporated undesirable factors like pollution and waste. Within this framework, uncertainty constitutes a further challenge that has not been extensively addressed (Ardakani and Fei, 2020; Tavassoli et al., 2020; Singh et al., 2022; Gerami et al., 2023). In this regard, the precise values of several metrics used to derive KPIs, such as costs, energy consumption, and travel times, are inherently uncertain. This uncertainty affects the input and output parameters of DEA models. Furthermore, it is crucial to implement effective qualitative evaluation methods, as many factors cannot be quantitatively assessed.

As per the description provided, the primary goal of this study is to evaluate the efficiency of a cross-docking system using NDEA models. This involves considering various structural and procedural factors such as different types and quantities of inbound and outbound doors, diverse transportation equipment, various modes of product processing,

and workforces with specialized skills. On this matter, a broad range of KPIs is introduced to evaluate both the inputs and outputs of every sector within the cross-docking system. Some KPIs are sourced from existing literature, while others are established from the beginning. Noteworthy is the fact that these KPIs encompass a wide range of issues, including digitization, automation, sustainability, resilience, and lean principles. In what follows, a novel SBM-NDEA model is developed to assess the efficiency of cross-docking systems with respect to undesirable factors, where a suitable approach is implemented to tackle these factors in specific KPIs. Additionally, a hybrid uncertainty method is offered that combines fuzzy reasoning techniques and Neutrosophic fuzzy programming to tackle uncertainty in specific data and measure qualitative judgments. To sum up, our contributions in comparison to the relevant literature are detailed as follows:

- Offering an inclusive configuration of a cross-docking system to evaluate its efficiency, encompassing unloading and loading divisions, the inspection process, intra-warehouse transportation, sorting, value-added operations, and temporary storage.
- Tailoring and introducing the needed KPIs for assessing the efficiency of each division within the cross-docking system in terms of automation, digitization, resilience, sustainability, and lean aspects.
- Presenting a novel SBM-NDEA model to assess the efficiency of cross-docking systems with respect to undesirable factors.
- Offering a hybrid uncertainty method, encompassing a fuzzy reasoning technique and Neutrosophic fuzzy programming to cope with KPIs' uncertainty and quantify qualitative judgments.

The rest of this paper is prepared as follows: A comprehensive literature review is rendered in Section 2. The problem definition and formulation are presented in Section 3. The hybrid uncertainty method is explained in Section 4. Computational results are provided in Section 5. Finally, Section 6 provides conclusions and future directions.

II. LITERATURE REVIEW

The literature review in this section is organized into two distinct parts. The first part delves into KPIs that are essential for evaluating the performance of warehouses and distribution centers. The subsequent part addresses DEA models proposed for evaluating the efficiency of these facilities. It is noteworthy that the efficiency evaluation of cross-docking systems has not been addressed in the existing literature.

A. KPIs in warehouse performance assessment

Dotoli et al. (2015) proposed a systematic three-phase methodology aimed at enhancing warehouse efficiency. The initial phase involved the preparation of a detailed account of the logistics capabilities of the warehouse. The subsequent phase focused on identifying all activities that contribute to value addition. In the final phase, the discrepancies within the system were prioritized, and their effects on warehouse performance were analyzed. Chen et al. (2017) introduced a model for evaluating process performance that incorporates eight KPIs. These KPIs address various aspects such as quality, accuracy, cost, security, and the timeliness of operations. The authors highlighted the importance of integrating warehouse management performance indicators with the proposed KPIs to achieve a more comprehensive assessment of warehouse performance. Buonamico et al. (2017) proposed a set of seven KPIs to assess warehouse performance through the lens of leanness. These KPIs encompass just-in-time delivery, elimination of waste, striving for perfection and zero defects, implementation of lean tools, teamwork and collaboration, continuous enhancement, and management of suppliers. In addition, to evaluate the various indicators, a collection of sub-indices was introduced to allow for their quantitative assessment. When it became difficult to quantify some of these sub-indices due to reasons such as uncertainty or divergent viewpoints, the fuzzy logic method was applied to enable their quantification (AlAlawin et al., 2022).

Laosirihongthong et al. (2018) proposed four primary criteria for assessing the performance of warehouses: accuracy, resource utilization, financial outcomes, and flexibility and responsiveness. They identified several indicators

for each criterion to facilitate a quantitative evaluation. Ultimately, they employed the fuzzy analytic hierarchy process (AHP) method to assign weights to these indicators. Ghaouta et al. (2018) provided a compilation of KPIs sourced from the literature, aimed at measuring the performance of warehouses in third-party logistics organizations. The selection process and the assignment of weights to these KPIs were based on expert evaluations. Kusrini et al. (2018) emphasized that the selection of KPIs for performance evaluation is contingent upon the specific industry being examined. They proposed a set of twenty-five KPIs tailored for assessing warehouse performance within the construction sector and utilized the AHP to determine the most critical indicators. Importantly, they applied Frazelle's framework as a foundation for the development of these KPIs. The Frazelle model identifies five primary processes within warehouses: receiving, putting away, storing, picking, and transportation. These processes serve as a framework for assessing warehouse performance through five key characteristics: financial metrics, productivity levels, utilization, quality standards, and cycle time efficiency. Kusrini et al. (2018) utilized the Frazelle model to analyze the operational performance of a retail warehouse. The AHP method was employed to assign weights to the KPIs, and a normalization process was subsequently applied to compute the warehouse's final performance evaluation.

Kusrini et al. (2019) introduced a set of KPIs focused on the sustainability of warehouses. They organized these evaluation metrics into three distinct categories: economic, environmental, and social. Importantly, the proposed KPIs were analyzed within the context of a leather warehouse. Liviu et al. (2009) identified five essential elements in the realm of warehouse management: optimal utilization of warehouse space, customer communication, quality standards, equipment efficiency, and cost management. They also delineated three categories of metrics to assess warehouse performance, which encompass inventory management, operational efficiency, and order fulfillment. Importantly, they offered a range of quantitative and measurable criteria for each of the aforementioned categories. Torabizadeh et al. (2020) presented a comprehensive set of thirty-three KPIs aimed at enhancing warehouse management through a sustainability lens. Their evaluation encompassed various dimensions, including sustainability, economic viability, environmental impact, social responsibility, operational efficiency, resource management, waste reduction, and overall environmental considerations. Additionally, structural equation modeling was utilized to ascertain the relative importance of these criteria.

Margareta et al. (2020) conducted an assessment of warehouse performance with a focus on sustainability. To achieve this, they analyzed 30 KPIs that influence warehouse efficiency as perceived by a third-party logistics provider. Furthermore, they employed the AHP to rank these KPIs. Bajec et al. (2020) utilized the fuzzy Delphi method alongside the best-worst approach to establish new KPIs aimed at assessing the performance of warehouses, with a particular emphasis on human and environmental considerations. Karim et al. (2021) gathered and refined a collection of KPIs found in existing literature concerning warehouse performance assessment. They organized these indicators according to four primary criteria: manpower, equipment, space, and information systems. However, to enable a quantitative assessment of many of these indicators, it is crucial to define appropriate KPIs. Islam et al. (2021) employed Frazelle's model to assess the operational efficiency of a clothing warehouse. They identified thirteen out of the twenty-five indicators proposed by Frazelle as the most significant, utilizing the AHP method. Additionally, they implemented a hybrid methodology to evaluate these KPIs through the particle swarm optimization meta-heuristic algorithm. Faveto et al. (2021) introduced a three-step framework for evaluating warehouse performance. The first step consisted of identifying KPIs documented in the literature. The second step involved ranking these KPIs according to their frequency of application in scholarly articles. The third and final step was to categorize the KPIs based on their areas of impact, which include economic, social, and environmental dimensions.

Demirkiran and Ozturkoglu (2022) introduced a collection of 30 KPIs aimed at assessing the performance of warehouses, with a significant emphasis on mobile and digital technologies. In a related study, Bernabei et al. (2022) identified 32 KPIs to evaluate warehouse performance, particularly in terms of resilience during the COVID-19 pandemic, incorporating 17 KPIs derived from earlier research. Falegnami et al. (2022) introduced a classification of KPIs found in the literature for assessing warehouse performance, which included various categories based on the industry under investigation. Iskandar and Sudiar (2022) conducted a classification of the existing KPIs pertinent to the assessment of warehouse performance, focusing specifically on environmental sustainability. They applied the AHP to

ascertain the relative importance of these KPIs. Additionally, to analyze the efficiency of warehouses and to classify the results, they implemented an objective matrix alongside a traffic light system.

Minashkina and Happonen (2023) analyzed KPIs outlined in prior research to measure warehouse performance. The indicators included employee safety, sustainability practices, technological integration, and environmental impacts. Baglio et al. (2023) presented the essential criteria from the perspective of a third-party logistics provider, which are required for a warehouse to achieve optimal performance. These criteria include a convenient location, loading dock facilities, standardized layout, suitable height to enhance the movement of goods, well-designed interior spaces, mezzanine levels, and additional value-added services. Faveto et al. (2024) conducted a comprehensive analysis of seventy KPIs derived from 203 studies to assess warehouse performance. These indicators were systematically organized into six distinct categories: general, time, cost, information and communication technology, environment, social responsibility, and human resources. Subsequently, the indicators were prioritized according to their perceived significance, utilizing metrics such as questionnaire responses, relative frequency, citation-weighted frequency, singularity index, and annual weighted frequency.

B. DEA models in the assessment of warehouse performance

Johnson and McGinnis (2010) offered a two-phase methodology for assessing the performance of warehouses. The initial phase employed a traditional DEA model to evaluate the efficiency of warehouses, taking into account inputs such as labor, space, capital investment in equipment, and inventory levels. The outputs of the DEA model included value-added services, storage capacity, accumulation, pallets, returns, and components. Subsequently, a regression analysis was conducted to identify the key factors that significantly impact warehouse performance. Faber et al. (2018) introduced a two-phase methodology for assessing the performance of warehouses. Initially, they employed a linear regression model to analyze the relationship between the structure of warehouse management and the prevailing conditions within the warehouses. This analysis took into account five primary factors: the complexity of operations, demand forecasting capabilities, comprehensive planning, decision-making rules, and the intricacy of control mechanisms. In the subsequent phase, a traditional DEA model was utilized to measure the efficiency of the warehouses, considering inputs such as workforce, warehouse size, automation levels, and the number of stored items, while outputs included flexibility, operational processes, and the number of orders fulfilled. Dixit et al. (2020) utilized a traditional DEA model to assess the operational efficiency of a pharmaceutical warehouse. The inputs for the DEA included factors such as the storage capacity of the warehouse, temperature control capabilities, the number of qualified personnel, and the associated operating costs. In contrast, the outputs measured by the DEA comprised the filling rate, variety of medicines, volume of medicines, inventory turnover ratio, time efficiency, and energy consumption.

Alidrisi (2021) proposed a dual-stage strategy that integrates the conventional DEA model with the PROMETHEE II approach for the evaluation of distribution center efficiency. This strategy utilized the PROMETHEE II method to assess effectiveness, while the DEA model was tasked with measuring efficiency. Specifically, the PROMETHEE II method facilitated the ranking of distribution centers, whereas the DEA model was responsible for ranking Decision-Making Units (DMUs). To combine the findings from both methods, the results from the PROMETHEE II method were normalized and then multiplied by the DEA model's results. Gafner et al. (2021) utilized a traditional DEA model to assess the operational efficiency of a grocery store. In this evaluation, inputs included warehouse space, labor expenses, and both fixed and variable material costs. The outputs measured in the DEA framework comprised the number of orders processed, average loading, delayed orders, the count of stores serviced by the warehouse, inventory levels, and the quantity of incorrectly processed items. Balk et al. (2021) applied the cross-efficiency technique as an enhancement of the conventional DEA model to evaluate warehouse efficiency. The inputs analyzed were the number of full-time staff, the storage area of the warehouse, the total number of products stored, and the level of automation utilized. The outputs measured comprised the number of orders processed, the count of specialized processes, the flexibility in order management, and the error rate in order processing.

Kusrini et al. (2022) employed a conventional DEA model to evaluate warehouse efficiency. Their evaluation was based on five performance indicators: cost, productivity, utilization, quality, and cycle time. The researchers classified three KPIs—receiving, putting away, and storage—as inputs, while order pickup and shipping were treated as outputs in the DEA model. It is important to highlight that these inputs and outputs were selected based on the highest weights derived from the AHP applied to twenty-five KPIs identified in the existing literature. Nong (2022) offered a two-phase methodology for assessing the efficiency of retail warehouses within the fashion sector. Initially, the Delphi method was employed to identify the key inputs and outputs relevant to the DEA model. Subsequently, a traditional DEA model was utilized to measure the warehouse's efficiency, taking into account workforce, operating costs, and warehouse size as inputs, while sales revenue and customer count were regarded as outputs.

C. Research gaps

A review of the literature reveals that two principal strategies have been adopted for evaluating the efficiency and performance of warehouses and distribution centers. The initial strategy focuses on the creation and implementation of diverse KPIs in numerous studies, which include factors related to environmental considerations, sustainability, automation, intelligence, digital technologies, energy usage, renewable energy sources, lean methodologies, and the principles of Industry 4.0. What is more, a limited subset of these studies has classified warehouses as DMUs, employing specific KPIs identified in the literature as the inputs and outputs associated with these DMUs. Following this classification, conventional DEA models have been utilized to evaluate the efficiency of warehouses. Nonetheless, these studies have regarded warehouses merely as black-box systems, failing to take into account their internal processes when determining efficiency. In addition, the literature reveals a lack of research focused on assessing the efficiency of cross-docking systems utilizing KPIs, and there has been no effort to apply DEA models. This concern is amplified by the fact that cross-docking systems involve more intricate internal processes and components compared to standard warehouses. Consequently, evaluating the efficiency of these facilities solely based on external metrics, without taking into account the internal processes, may lead to a flawed assessment. Thus, relying on traditional DEA models is inadequate for addressing this issue, making it imperative to employ NDEA models for a comprehensive evaluation of the efficiency of cross-docking systems. Furthermore, across all studies that have applied standard DEA models to assess warehouse efficiency, the notion of undesirable factors has been overlooked. Nevertheless, many of these studies have indicated the presence of undesirable elements such as pollution and waste, raising potential concerns about the validity of their findings. Additionally, another issue that has received insufficient focus in this context is the aspect of uncertainty. In practical terms, several parameters such as costs, energy consumption, and travel times, which are integral to the computation of KPIs, are characterized by uncertainty. Consequently, both the input and output parameters of DEA models are subject to this uncertainty. Additionally, many factors cannot be quantitatively assessed which makes it essential to adopt effective qualitative evaluation strategies.

III. PROBLEM DEFINITION AND FORMULATION

In contemporary warehouses and distribution centers, the predominant forms of cross-docking systems are pre-distribution and post-distribution (Torballi and Alpan, 2023). Pre-distribution cross-docking refers to the process of assigning products to customers prior to their departure from suppliers. So, upon arrival, products are unloaded, unpacked, sorted into kits, and repackaged following predetermined distribution standards prior to their distribution. In contrast, post-distribution cross-docking postpones the sorting process until the most suitable destination and customers are determined. Consequently, this may result in products lingering in the cross-docking system for a longer duration (Liu and Li, 2023). However, by allowing additional time to analyze market trends, sales forecasts, and inventory data, suppliers and retailers are better equipped to make informed decisions regarding shipping. It is important to highlight that certain cross-docking systems integrate these strategies to enhance efficiency and profitability. Nevertheless, they usually assign a substantial share of their activities to one of the parties involved. As a result, a generalized design of a cross-docking system, based on our case study, is suggested to integrate both strategies.

Regarding operational strategy, warehouses today implement three distinct cross-docking techniques: continuous, consolidation arrangements, and de-consolidation arrangements (Vis and Roodbergen, 2011; Liu and Li, 2023). It is crucial to highlight that the selection of these techniques may depend on the types of shipments received, allowing for the possibility of using one or more techniques concurrently. It is essential to recognize that the simultaneous application of various techniques necessitates the use of efficient material handling equipment, a thoughtfully designed layout, and exceptional coordination among the different operational components of the system. On this matter, the continuous operation of cross-docking facilitates the seamless transfer of products from incoming shipments directly onto outbound trucks. Consequently, each product is processed a single time, starting from its entry into the cross-docking facility and finishing with its shipment. Perishable items and fresh vegetables serve as prevalent examples of shipments that necessitate prompt processing. Conversely, consolidation entails the accumulation of incoming shipments prior to their sorting and loading onto outgoing trucks. This method is often utilized when items from multiple suppliers, arriving from different locations and at varying times, reach the cross-docking facility. Hence, for cross-docking operators, the operations of sorting and loading products are considerably more straightforward. In contrast to consolidation, de-consolidation involves the division of incoming products into smaller units prior to their loading onto outbound vehicles. This method is commonly utilized when products need to be dispatched to various destinations or at different times, allowing warehouse operators to more efficiently sort and load items onto outbound vehicles by breaking down inbound shipments.

In light of the aforementioned description, it is essential to take into account various factors when assessing the efficiency of cross-docking systems, ensuring that the impact of each factor is incorporated into the efficiency calculations. To facilitate this analysis, Fig. 1 presents the cross-docking layout under examination. The configuration is characterized by a U-shape, and the arrangement of the inbound and outbound doors does not follow a straight line. This design is a result of spatial limitations and is intended to optimize the use of the road infrastructure. The system employs a range of transportation equipment, including conveyors, jack pallets, and forklifts, to facilitate the movement of products within this system. Unlike standard warehouses, the efficiency of cross-docking systems is significantly influenced by the design and functionality of their inbound and outbound doors. In addition, three other sectors, specifically storage/retrieval, kitting, and staging, have a significant influence on the operational efficiency of cross-docking systems. Staging areas are specifically designed for the short-term storage of products within these systems and are located in proximity to inbound or outbound doors to accommodate a range of purposes. Furthermore, the kitting area serves as the location where new stock-keeping units (SKUs) are generated by combining various SKUs obtained from different inbound shipments. Another critical aspect of cross-docking systems, which offers a more vibrant function than traditional warehouses, is the inspection area. Here, operators assess various factors, including the visual condition of incoming products, as well as their quantity, volume, and type in accordance with the accompanying remittance.

It is evident that the various components of the cross-docking system undertake interrelated tasks, working in concert with one another. The performance of each individual component has a significant impact on the overall efficiency of the system. To assess the efficiency of the system in question, the network configuration of the cross-dock is shown in Fig. 2. The depiction reveals five inbound doors (IDs), three outbound doors (ODs), two conveyors (Cs), three jack pallets (JPs), four forklifts (Fs), a storage and retrieval (S/R) area, two unpacked and inspection (U&I) sections, two kitting (K) areas, and two staging (S) areas. Every component is viewed as a part of a cross-docking system referred to as a decision-making unit (DMU), which consists of a range of inputs and outputs. These inputs and outputs can be described as the values of KPIs or criteria that characterize the mentioned divisions. It is evident that the inputs and outputs associated with divisions can exhibit both quantitative and qualitative traits. Thus, it is necessary to convert qualitative characteristics into quantitative forms to facilitate a proper evaluation of performance.

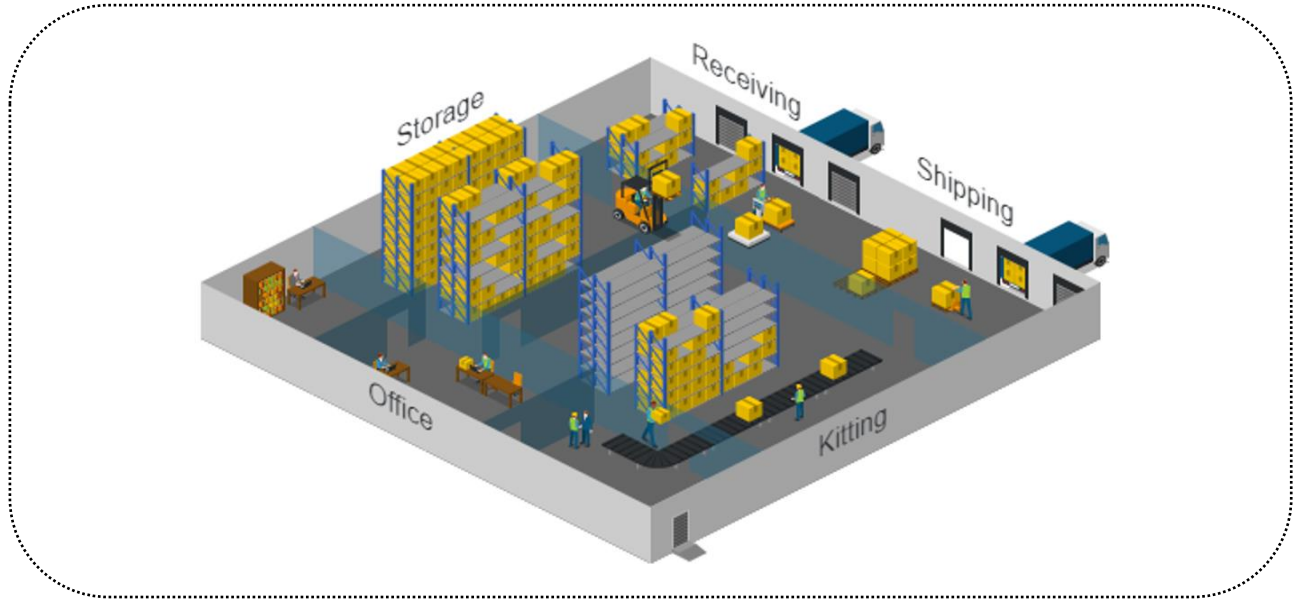


Fig. 1. The U-shape layout of the investigated U-shape cross-docking system

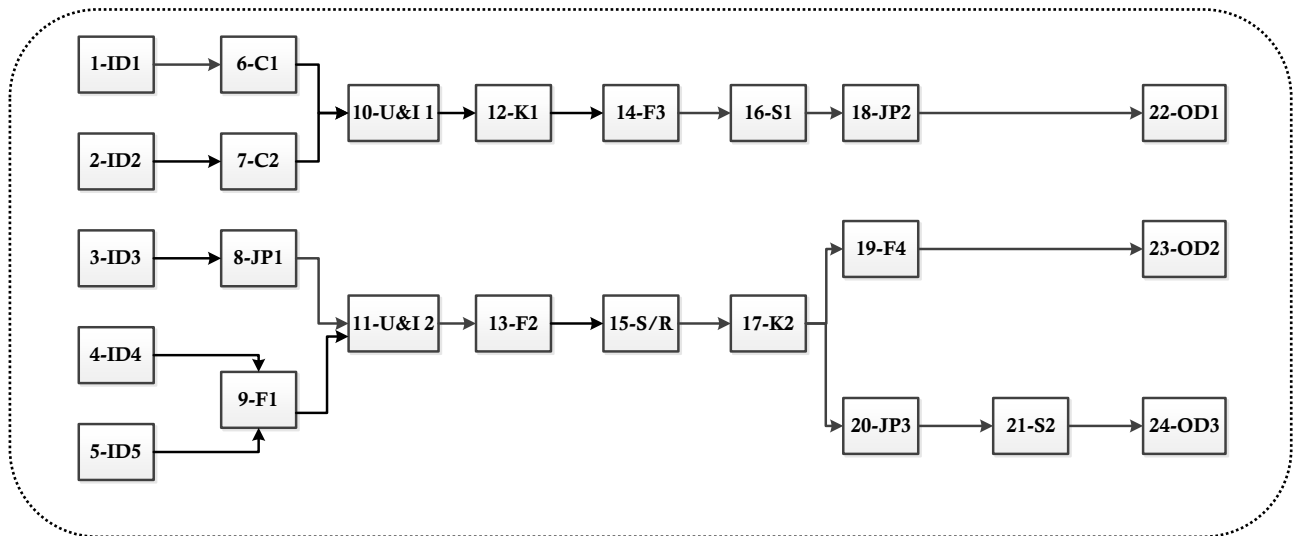


Fig. 2. The network structure of the investigated U-shape cross-docking system

A. Sets and indices

n Number of DMUs ($j = 1, 2, \dots, n$)

K Number of divisions ($k = 1, 2, \dots, K$), $K = K_1 \cup K_2 \cup K_3 \cup K_4 \cup K_5 \cup K_6 \cup K_7 \cup K_8$

K_1 Number of conveyors

K_2 Number of jack pallets

K_3 Number of forklifts

K_4 Number of unpacking and inspection sections

K_5	Number of storage/retrieval area
K_6	Number of kitting area
K_7	Number of staging area
K_8	Number of outbound doors
q^k	Number of exogenous inputs utilized by division k ($i = 1, 2, \dots, q^k$)
s^k	Number of final outputs made by division k ($r = 1, 2, \dots, s^k$)
h^k	Number of intermediate outputs made by division k ($g = 1, 2, \dots, h^k$)

B. Parameters

x_{ij}^k	i th fuzzy input provided from outside to k th division of j th DMU
$x_{ij}^{k,D}$	i th fuzzy desirable input provided from outside to k th division of j th DMU
$x_{ij}^{k,UD}$	i th fuzzy undesirable input provided from outside to k th division of j th DMU
y_{rj}^k	r th fuzzy final output made from k th division of j th DMU
$y_{rj}^{k,D}$	r th fuzzy final desirable output made from k th division of j th DMU
$y_{rj}^{k,UD}$	r th fuzzy final undesirable output made from k th division of j th DMU
$z_{gj}^{(a,b)}$	g th intermediate output made by division a for division b in j th DMU

C. Decision variables

s_i^-, s_g^-	Surplus values
s_r^+, s_g^+	Slack values
λ_g^k	Auxiliary variable to guarantee convexity

In alignment with the specified parameters, Table I presents the inputs, final outputs, and intermediate outputs pertinent to each segment of the cross-docking component. These inputs and outputs have been meticulously offered by analyzing a comprehensive selection of KPIs from existing literature, alongside the perspectives of cross-docking specialists. As can be seen, the inputs and final outputs are rendered by fuzzy numbers (Allaei et al. 2024). In this regard, an effective fuzzy logic approach is used to quantify the parameters with the nature of risk. Also, a Neutrosophic optimization approach is used for the other parameters to handle their uncertainty. In addition, certain inputs and final outputs are identified as undesirable factors, including waiting times and CO2 emissions. In this context, on the input side, a reduced quantity of consumed input may be viewed as less favorable, while on the output side, an increased volume of generated output may also be deemed undesirable. In this regard, several strategies have been suggested in the literature to manage these situations, including input-output exchange, data transformation, weak disposability, and the slacks-based approach. However, it remains unproven which of these strategies is the most effective, as their implementation is influenced by the particular application and the perspectives of the users (Kao, 2020). The present

study employs an input-output exchange methodology to address undesirable factors. This involves treating an undesirable input as an output and the opposite as well. Additionally, the sustainability dimensions of the cross-docking system are incorporated into the analysis of the examined problem. In pursuit of this aim, the defined KPIs encompass three key pillars of sustainability: environmental, economic, and social factors. Illustratively, aspects such as CO2 emissions, the number of personnel, and costs associated with packaging materials reflect these important considerations.

Table I. The KPIs employed in cross-docking efficiency assessment

Divisions	Notations	The definition of KPIs
Inbound Door (ID)	$x_{1j}^{1-5,UD}$	Number of inbound products
	$x_{2j}^{1-5,D}$	Number of employees
	$x_{3j}^{1-5,UD}$	Proper assignment of inbound doors
	$x_{4j}^{1-5,UD}$	On-time arrival of inbound trucks
	$x_{5j}^{1-5,UD}$	Flexibility in inbound trucks' scheduling
	$x_{6j}^{1-5,UD}$	Information sharing with suppliers and their fleet
	$x_{7j}^{1-5,UD}$	Readiness of inbound doors' equipment
	$x_{8j}^{1-5,D}$	Risk of information system shutdown
	$x_{9j}^{1-5,D}$	Risk of damage to products
	$y_{1j}^{1-5,D}$	Timely and fast unloading capability
Conveyor (C)	$x_{1j}^{6,7,D}$	Maintenance cost
	$x_{2j}^{6,7,D}$	Number of employees
	$x_{3j}^{6,7,D}$	Energy cost
	$x_{4j}^{6,7,D}$	Risk of conveyor breakdown
	$x_{5j}^{6,7,D}$	Risk of damage to products
	$y_{1j}^{6,7,D}$	Timely transfer
Jack Pallet (JP)	$x_{1j}^{8,18,20,D}$	Maintenance cost
	$x_{2j}^{8,18,20,D}$	Number of employees
	$x_{3j}^{8,18,20,D}$	Risk of jack pallet breakdown
	$x_{4j}^{8,18,20,D}$	Risk of damage to products
	$x_{5j}^{8,18,20,D}$	Risk of manual handling injuries
	$y_{1j}^{8,18,20,D}$	Timely transfer
Forklift (F)	$x_{1j}^{9,13,14,19,D}$	Maintenance cost
	$x_{2j}^{9,13,14,19,D}$	Number of employees
	$x_{3j}^{9,13,14,19,D}$	Energy cost

Continue Table I. The KPIs employed in cross-docking efficiency assessment

Divisions	Notations	The definition of KPIs
Forklift (F)	$x_{4j}^{9,13,14,19,D}$	Training cost
	$x_{5j}^{9,13,14,19,UD}$	Neatness and cleanliness of cross-docking floor
	$x_{6j}^{9,13,14,19,D}$	Risk of forklift breakdown
	$x_{7j}^{9,13,14,19,D}$	Risk of damage to products
	$x_{8j}^{9,13,14,19,D}$	Risk of injuries during transferring
	$y_{1j}^{9,13,14,19,UD}$	Safe and quick maneuvering power
	$y_{2j}^{9,13,14,19,D}$	Timely transfer
	$y_{3j}^{9,13,14,19,UD}$	CO2 emission
Unpacking & Inspection (U&I)	$x_{1j}^{10,11,D}$	Number of employees
	$x_{2j}^{10,11,D}$	Training cost
	$x_{3j}^{10,11,D}$	Maintenance cost of inspection tools
	$x_{4j}^{10,11,UD}$	Information sharing level
	$x_{5j}^{10,11,UD}$	Technology level
	$x_{6j}^{10,11,UD}$	Barcode reliability
	$x_{7j}^{10,11,D}$	Risk of damage to products
	$x_{8j}^{10,11,D}$	Risk of manual handling injuries
	$y_{1j}^{10,11,D}$	Speed and accuracy of unpacking process
	$y_{2j}^{10,11,D}$	Speed and accuracy of inspection
Kitting (K)	$x_{1j}^{12,17,D}$	Number of employees
	$x_{2j}^{12,17,D}$	Training cost
	$x_{3j}^{12,17,D}$	Packaging material cost
	$x_{4j}^{12,17,UD}$	Information sharing level
	$x_{5j}^{12,17,UD}$	Technology level
	$x_{6j}^{12,17,UD}$	Barcode reliability
	$x_{7j}^{12,17,D}$	Risk of damage to products
	$x_{8j}^{12,17,D}$	Risk of manual handling injuries
	$y_{1j}^{12,17,D}$	Speed and accuracy of packing
Storage/Retrieval (S/R) area	$x_{1j}^{15,D}$	Number of employees
	$x_{2j}^{15,D}$	Training cost
	$x_{3j}^{15,D}$	Energy cost

Continue Table I. The KPIs employed in cross-docking efficiency assessment

Divisions	Notations	The definition of KPIs
Storage/Retrieval (S/R) area	$x_{4j}^{15,D}$	Maintenance cost
	$x_{5j}^{15,UD}$	Information sharing level
	$x_{6j}^{15,UD}$	Proper assignment of products to storage locations
	$x_{7j}^{15,UD}$	Flexibility in storage
	$x_{8j}^{15,D}$	Risk of damage to products
	$x_{9j}^{15,D}$	Risk of machinery and equipment breakdown
	$x_{10j}^{15,D}$	Risk of information system shutdown
	$x_{11j}^{15,D}$	Risk of information security
	$y_{1j}^{15,D}$	Inventory turnover rate
	$y_{2j}^{15,D}$	Real-time visibility into inventory level
	$y_{3j}^{15,UD}$	Timely storage
	$y_{4j}^{15,D}$	Timely retrieval and transfer
Staging (S)	$x_{1j}^{16,21,D}$	Number of employees
	$x_{2j}^{16,21,UD}$	Availability of temporary storage space
	$x_{3j}^{16,21,UD}$	Neatness and cleanliness of cross-docking floor
	$x_{4j}^{16,21,D}$	Risk of unexpected order change
	$x_{5j}^{16,21,D}$	Risk of damage to products
	$x_{6j}^{16,21,UD}$	Information sharing level
	$y_{1j}^{16,21,D}$	Non-interference and disturbance with other activities
	$y_{2j}^{16,21,UD}$	Waiting time
Outbound Door (OD)	$x_{1j}^{22-24,D}$	Number of employees
	$x_{2j}^{22-24,UD}$	Proper assignment of outbound doors
	$x_{3j}^{22-24,UD}$	Flexibility in outbound trucks' scheduling
	$x_{4j}^{22-24,UD}$	Information sharing with customers and their fleet
	$x_{5j}^{22-24,UD}$	Readiness of outbound doors' equipment
	$x_{6j}^{22-24,D}$	Risk of unexpected order change
	$x_{7j}^{22-24,D}$	Risk of damage to products
	$x_{8j}^{22-24,D}$	Risk of information system shutdown
	$y_{1j}^{22-24,D}$	Customer satisfaction
	$y_{2j}^{22-24,D}$	On-time departure of outbound trucks
	$y_{3j}^{22-24,D}$	Timely and fast loading capability
	$y_{4j}^{22-24,D}$	Number of intact products delivered from inbound doors to outbound trucks

Continue Table I. The KPIs employed in cross-docking efficiency assessment

Divisions	Notations	The definition of KPIs
Intermediate outputs	$z_{1j}^{(1,6)}$ $z_{1j}^{(2,7)}$	Number of intact, unloaded products moved from IDs on Cs
	$z_{1j}^{(3,8)}$	Number of intact, unloaded products moved from IDs on JPs
	$z_{1j}^{(4,9)}$ $z_{1j}^{(5,9)}$	Number of intact, unloaded products moved IDs on Fs
	$z_{1j}^{(6,10)}$ $z_{1j}^{(7,10)}$	Number of intact products moved from Cs to U&I workstations
	$z_{1j}^{(8,11)}$	Number of intact products moved from JPs to U&I workstations
	$z_{1j}^{(9,11)}$	Number of intact products moved from Fs to U&I workstations
	$z_{1j}^{(10,12)}$	Number of intact products moved from U&I workstations to Ks
	$z_{1j}^{(11,13)}$	Number of intact products moved from U&I workstations on Fs
	$z_{1j}^{(12,14)}$ $z_{1j}^{(17,19)}$	Number of intact products moved from Ks on Fs
	$z_{1j}^{(13,15)}$	Number of intact products moved from Fs to the S/R
	$z_{1j}^{(15,17)}$	Number of intact products moved from the S/R to Ks
	$z_{1j}^{(17,20)}$	Number of intact products moved from Ks on JPs
	$z_{1j}^{(14,16)}$	Number of intact products moved by Fs to the staging area
	$z_{1j}^{(16,18)}$	Number of intact products moved from the staging area on JPs
	$z_{1j}^{(20,21)}$	Number of intact products moved by JPs to the staging area
	$z_{1j}^{(18,22)}$	Number of intact products moved by JPs to ODs
	$z_{1j}^{(19,23)}$	Number of intact products moved by Fs to ODs
	$z_{1j}^{(21,24)}$	Number of intact products moved from the staging area to ODs by workforces

D. The proposed model

In light of the aforementioned description, the SBM-NDEA model is developed to evaluate the efficiency of the cross-docking system under examination. It is essential to note that this model applies to cases where all KPIs are viewed as desirable. To account for undesirable KPIs, as outlined, undesirable inputs are regarded as outputs, and the reverse is also true.

$$\text{Min } Z = \sum_{k=1}^{24} \rho_0^{(k)} \quad (1)$$

S. t.

Divisions 1-5:

$$\rho_0^{(k)} = \frac{1 - \left(\frac{1}{q^{(k)}} \right) \sum_{i=1}^q \frac{S_i^-}{x_{io}^k}}{1 + \left(\frac{1}{s^{(k)} + h^{(k)}} \right) \left(\sum_{r=1}^{s^{(k)}} \frac{S_r^+}{y_{ro}^k} + \sum_{g=1}^{h^{(k)}} \frac{S_g^+}{z_{go}^k} \right)} \quad \forall k = 1, 2, \dots, 5 \quad (2)$$

Division 6:

$$\rho_0^{(6)} = \frac{1 - \left(\frac{1}{q^{(6)} + h^{(1)}} \right) \left(\sum_{i=1}^{q^{(6)}} \frac{S_i^-}{x_{io}^6} + \sum_{g=1}^{h^{(1)}} \frac{S_g^-}{z_{go}^6} \right)}{1 + \left(\frac{1}{s^{(6)} + h^{(6)}} \right) \left(\sum_{r=1}^{s^{(6)}} \frac{S_r^+}{y_{ro}^6} + \sum_{g=1}^{h^{(6)}} \frac{S_g^+}{z_{go}^6} \right)} \quad (3)$$

Division 7:

$$\rho_0^{(7)} = \frac{1 - \left(\frac{1}{q^{(7)} + h^{(2)}} \right) \left(\sum_{i=1}^{q^{(7)}} \frac{S_i^-}{x_{io}^7} + \sum_{g=1}^{h^{(2)}} \frac{S_g^-}{z_{go}^7} \right)}{1 + \left(\frac{1}{s^{(7)} + h^{(7)}} \right) \left(\sum_{r=1}^{s^{(7)}} \frac{S_r^+}{y_{ro}^7} + \sum_{g=1}^{h^{(7)}} \frac{S_g^+}{z_{go}^7} \right)} \quad (4)$$

Division 8:

$$\rho_0^{(8)} = \frac{1 - \left(\frac{1}{q^{(8)} + h^{(3)}} \right) \left(\sum_{i=1}^{q^{(8)}} \frac{S_i^-}{x_{io}^8} + \sum_{g=1}^{h^{(3)}} \frac{S_g^-}{z_{go}^8} \right)}{1 + \left(\frac{1}{s^{(8)} + h^{(8)}} \right) \left(\sum_{r=1}^{s^{(8)}} \frac{S_r^+}{y_{ro}^8} + \sum_{g=1}^{h^{(8)}} \frac{S_g^+}{z_{go}^8} \right)} \quad (5)$$

Division 9:

$$\rho_0^{(9)} = \frac{1 - \left(\frac{1}{q^{(9)} + h^{(4)} + h^{(5)}} \right) \left(\sum_{i=1}^{q^{(9)}} \frac{S_i^-}{x_{io}^9} + \sum_{g=1}^{h^{(4)}} \frac{S_g^-}{z_{go}^4} + \sum_{g=1}^{h^{(5)}} \frac{S_g^-}{z_{go}^5} \right)}{1 + \left(\frac{1}{s^{(9)} + h^{(9)}} \right) \left(\sum_{r=1}^{s^{(9)}} \frac{S_r^+}{y_{ro}^9} + \sum_{g=1}^{h^{(9)}} \frac{S_g^+}{z_{go}^9} \right)} \quad (6)$$

Division 10:

$$\rho_0^{(10)} = \frac{1 - \left(\frac{1}{q^{(10)} + h^{(6)} + h^{(7)}} \right) \left(\sum_{i=1}^{q^{(10)}} \frac{S_i^-}{x_{io}^{10}} + \sum_{g=1}^{h^{(6)}} \frac{S_g^-}{z_{go}^{10}} + \sum_{g=1}^{h^{(7)}} \frac{S_g^-}{z_{go}^{10}} \right)}{1 + \left(\frac{1}{s^{(10)} + h^{(10)}} \right) \left(\sum_{r=1}^{s^{(10)}} \frac{S_r^+}{y_{ro}^{10}} + \sum_{g=1}^{h^{(10)}} \frac{S_g^+}{z_{go}^{10}} \right)} \quad (7)$$

Division 11:

$$\rho_0^{(11)} = \frac{1 - \left(\frac{1}{q^{(11)} + h^{(8)} + h^{(9)}} \right) \left(\sum_{i=1}^{q^{(11)}} \frac{S_i^-}{x_{io}^{11}} + \sum_{g=1}^{h^{(8)}} \frac{S_g^-}{z_{go}^{11}} + \sum_{g=1}^{h^{(9)}} \frac{S_g^-}{z_{go}^{11}} \right)}{1 + \left(\frac{1}{s^{(11)} + h^{(11)}} \right) \left(\sum_{r=1}^{s^{(11)}} \frac{S_r^+}{y_{ro}^{11}} + \sum_{g=1}^{h^{(11)}} \frac{S_g^+}{z_{go}^{11}} \right)} \quad (8)$$

Division 12:

$$\rho_0^{(12)} = \frac{1 - \left(\frac{1}{q^{(12)} + h^{(10)}} \right) \left(\sum_{i=1}^{q^{(12)}} \frac{S_i^-}{x_{io}^{12}} + \sum_{g=1}^{h^{(10)}} \frac{S_g^-}{z_{go}^{12}} \right)}{1 + \left(\frac{1}{s^{(12)} + h^{(12)}} \right) \left(\sum_{r=1}^{s^{(12)}} \frac{S_r^+}{y_{ro}^{12}} + \sum_{g=1}^{h^{(12)}} \frac{S_g^+}{z_{go}^{12}} \right)} \quad (9)$$

Division 13:

$$\rho_0^{(13)} = \frac{1 - \left(\frac{1}{q^{(13)} + h^{(11)}} \right) \left(\sum_{i=1}^{q^{(13)}} \frac{S_i^-}{x_{io}^{13}} + \sum_{g=1}^{h^{(11)}} \frac{S_g^-}{z_{go}^{13}} \right)}{1 + \left(\frac{1}{s^{(13)} + h^{(13)}} \right) \left(\sum_{r=1}^{s^{(13)}} \frac{S_r^+}{y_{ro}^{13}} + \sum_{g=1}^{h^{(13)}} \frac{S_g^+}{z_{go}^{13}} \right)} \quad (10)$$

Division 14:

$$\rho_0^{(14)} = \frac{1 - \left(\frac{1}{q^{(14)} + h^{(12)}} \right) \left(\sum_{i=1}^{q^{(14)}} \frac{S_i^-}{x_{io}^{14}} + \sum_{g=1}^{h^{(12)}} \frac{S_g^-}{z_{go}^{14}} \right)}{1 + \left(\frac{1}{s^{(14)} + h^{(14)}} \right) \left(\sum_{r=1}^{s^{(14)}} \frac{S_r^+}{y_{ro}^{14}} + \sum_{g=1}^{h^{(14)}} \frac{S_g^+}{z_{go}^{14}} \right)} \quad (11)$$

Division 15:

$$\rho_0^{(15)} = \frac{1 - \left(\frac{1}{q^{(15)} + h^{(13)}} \right) \left(\sum_{i=1}^{q^{(15)}} \frac{S_i^-}{x_{io}^{15}} + \sum_{g=1}^{h^{(13)}} \frac{S_g^-}{z_{go}^{15}} \right)}{1 + \left(\frac{1}{s^{(15)} + h^{(15)}} \right) \left(\sum_{r=1}^{s^{(15)}} \frac{S_r^+}{y_{ro}^{15}} + \sum_{g=1}^{h^{(15)}} \frac{S_g^+}{z_{go}^{15}} \right)} \quad (12)$$

Division 16:

$$\rho_0^{(16)} = \frac{1 - \left(\frac{1}{q^{(16)} + h^{(14)}} \right) \left(\sum_{i=1}^{q^{(16)}} \frac{S_i^-}{x_{io}^{16}} + \sum_{g=1}^{h^{(14)}} \frac{S_g^-}{z_{go}^{16}} \right)}{1 + \left(\frac{1}{s^{(16)} + h^{(16)}} \right) \left(\sum_{r=1}^{s^{(16)}} \frac{S_r^+}{y_{ro}^{16}} + \sum_{g=1}^{h^{(16)}} \frac{S_g^+}{z_{go}^{16}} \right)} \quad (13)$$

Division 17:

$$\rho_0^{(17)} = \frac{1 - \left(\frac{1}{q^{(17)} + h^{(15)}} \right) \left(\sum_{i=1}^{q^{(17)}} \frac{S_i^-}{x_{io}^{17}} + \sum_{g=1}^{h^{(15)}} \frac{S_g^-}{z_{go}^{17}} \right)}{1 + \left(\frac{1}{s^{(17)} + h^{(17,19)} + h^{(17,20)}} \right) \left(\sum_{r=1}^{s^{(17)}} \frac{S_r^+}{y_{ro}^{17}} + \sum_{g=1}^{h^{(17,19)}} \frac{S_g^+}{z_{go}^{17}} + \sum_{g=1}^{h^{(17,20)}} \frac{S_g^+}{z_{go}^{17}} \right)} \quad (14)$$

Division 18:

$$\rho_0^{(18)} = \frac{1 - \left(\frac{1}{q^{(18)} + h^{(16)}} \right) \left(\sum_{i=1}^{q^{(18)}} \frac{s_i^-}{x_{io}^{18}} + \sum_{g=1}^{h^{(16)}} \frac{s_g^-}{z_{go}^{18}} \right)}{1 + \left(\frac{1}{s^{(18)} + h^{(18)}} \right) \left(\sum_{r=1}^{s^{(18)}} \frac{s_r^+}{y_{ro}^{18}} + \sum_{g=1}^{h^{(18)}} \frac{s_g^+}{z_{go}^{18}} \right)} \quad (15)$$

Division 19:

$$\rho_0^{(19)} = \frac{1 - \left(\frac{1}{q^{(19)} + h^{(17)}} \right) \left(\sum_{i=1}^{q^{(19)}} \frac{s_i^-}{x_{io}^{19}} + \sum_{g=1}^{h^{(17)}} \frac{s_g^-}{z_{go}^{19}} \right)}{1 + \left(\frac{1}{s^{(19)} + h^{(19)}} \right) \left(\sum_{r=1}^{s^{(19)}} \frac{s_r^+}{y_{ro}^{19}} + \sum_{g=1}^{h^{(19)}} \frac{s_g^+}{z_{go}^{19}} \right)} \quad (16)$$

Division 20:

$$\rho_0^{(20)} = \frac{1 - \left(\frac{1}{q^{(20)} + h^{(17)}} \right) \left(\sum_{i=1}^{q^{(20)}} \frac{s_i^-}{x_{io}^{20}} + \sum_{g=1}^{h^{(17)}} \frac{s_g^-}{z_{go}^{20}} \right)}{1 + \left(\frac{1}{s^{(20)} + h^{(20)}} \right) \left(\sum_{r=1}^{s^{(20)}} \frac{s_r^+}{y_{ro}^{20}} + \sum_{g=1}^{h^{(20)}} \frac{s_g^+}{z_{go}^{20}} \right)} \quad (17)$$

Division 21:

$$\rho_0^{(21)} = \frac{1 - \left(\frac{1}{q^{(21)} + h^{(20)}} \right) \left(\sum_{i=1}^{q^{(21)}} \frac{s_i^-}{x_{io}^{21}} + \sum_{g=1}^{h^{(20)}} \frac{s_g^-}{z_{go}^{21}} \right)}{1 + \left(\frac{1}{s^{(21)} + h^{(21)}} \right) \left(\sum_{r=1}^{s^{(21)}} \frac{s_r^+}{y_{ro}^{21}} + \sum_{g=1}^{h^{(21)}} \frac{s_g^+}{z_{go}^{21}} \right)} \quad (18)$$

Division 22:

$$\rho_0^{(22)} = \frac{1 - \left(\frac{1}{q^{(22)} + h^{(18)}} \right) \left(\sum_{i=1}^{q^{(22)}} \frac{s_i^-}{x_{io}^{22}} + \sum_{g=1}^{h^{(18)}} \frac{s_g^-}{z_{go}^{22}} \right)}{1 + \left(\frac{1}{s^{(22)} + h^{(22)}} \right) \left(\sum_{r=1}^{s^{(22)}} \frac{s_r^+}{y_{ro}^{22}} + \sum_{g=1}^{h^{(22)}} \frac{s_g^+}{z_{go}^{22}} \right)} \quad (19)$$

Division 23:

$$\rho_0^{(23)} = \frac{1 - \left(\frac{1}{q^{(23)} + h^{(19)}} \right) \left(\sum_{i=1}^{q^{(23)}} \frac{s_i^-}{x_{io}^{23}} + \sum_{g=1}^{h^{(19)}} \frac{s_g^-}{z_{go}^{23}} \right)}{1 + \left(\frac{1}{s^{(23)} + h^{(23)}} \right) \left(\sum_{r=1}^{s^{(23)}} \frac{s_r^+}{y_{ro}^{23}} + \sum_{g=1}^{h^{(23)}} \frac{s_g^+}{z_{go}^{23}} \right)} \quad (20)$$

Division 24:

$$\rho_0^{(24)} = \frac{1 - \left(\frac{1}{q^{(24)} + h^{(21)}} \right) \left(\sum_{i=1}^{q^{(24)}} \frac{s_i^-}{x_{io}^{24}} + \sum_{g=1}^{h^{(21)}} \frac{s_g^-}{z_{go}^{24}} \right)}{1 + \left(\frac{1}{s^{(24)} + h^{(24)}} \right) \left(\sum_{r=1}^{s^{(24)}} \frac{s_r^+}{y_{ro}^{24}} + \sum_{g=1}^{h^{(24)}} \frac{s_g^+}{z_{go}^{24}} \right)} \quad (21)$$

Division 1:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^1 x_{ij}^1 + s_i^- &= x_{io}^1 \quad \forall i = 1, 2, \dots, q^{(1)} \\
 \sum_{j=1}^n \lambda_j^1 y_{rj}^1 - s_r^+ &= y_{ro}^1 \quad \forall r = 1, 2, \dots, s^{(1)} \\
 \sum_{j=1}^n \lambda_j^1 z_{gj}^{(1,6)} - s_g^+ &= z_{go}^1 \quad \forall g = 1, 2, \dots, h^{(1)}
 \end{aligned} \tag{22}$$

Division 2:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^2 x_{ij}^2 + s_i^- &= x_{io}^2 \quad \forall i = 1, 2, \dots, q^{(2)} \\
 \sum_{j=1}^n \lambda_j^2 y_{rj}^2 - s_r^+ &= y_{ro}^2 \quad \forall r = 1, 2, \dots, s^{(2)} \\
 \sum_{j=1}^n \lambda_j^2 z_{gj}^{(2,7)} - s_g^+ &= z_{go}^2 \quad \forall g = 1, 2, \dots, h^{(2)}
 \end{aligned} \tag{23}$$

Division 3:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^3 x_{ij}^3 + s_i^- &= x_{io}^3 \quad \forall i = 1, 2, \dots, q^{(3)} \\
 \sum_{j=1}^n \lambda_j^3 y_{rj}^3 - s_r^+ &= y_{ro}^3 \quad \forall r = 1, 2, \dots, s^{(3)} \\
 \sum_{j=1}^n \lambda_j^3 z_{gj}^{(3,8)} - s_g^+ &= z_{go}^3 \quad \forall g = 1, 2, \dots, h^{(3)}
 \end{aligned} \tag{24}$$

Division 4:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^4 x_{ij}^4 + s_i^- &= x_{io}^4 \quad \forall i = 1, 2, \dots, q^{(4)} \\
 \sum_{j=1}^n \lambda_j^4 y_{rj}^4 - s_r^+ &= y_{ro}^4 \quad \forall r = 1, 2, \dots, s^{(4)} \\
 \sum_{j=1}^n \lambda_j^4 z_{gj}^{(4,9)} - s_g^+ &= z_{go}^4 \quad \forall g = 1, 2, \dots, h^{(4)}
 \end{aligned} \tag{25}$$

Division 5:

$$\sum_{j=1}^n \lambda_j^5 x_{ij}^5 + s_i^- = x_{io}^5 \quad \forall i = 1, 2, \dots, q^{(5)} \tag{26}$$

$$\sum_{j=1}^n \lambda_j^5 y_{rj}^5 - s_r^+ = y_{ro}^5 \quad \forall r = 1, 2, \dots, s^{(5)}$$

$$\sum_{j=1}^n \lambda_j^5 z_{gj}^{(5,9)} - s_g^+ = z_{go}^5 \quad \forall g = 1, 2, \dots, h^{(5)}$$

Division 6:

$$\sum_{j=1}^n \lambda_j^6 x_{ij}^6 + s_i^- = x_{io}^6 \quad \forall i = 1, 2, \dots, q^{(6)}$$

$$\sum_{j=1}^n \lambda_j^6 z_{gj}^{(1,6)} + s_g^- = z_{go}^6 \quad \forall g = 1, 2, \dots, h^{(6)}$$

$$\sum_{j=1}^n \lambda_j^6 z_{gj}^{(6,10)} - s_g^+ = z_{go}^6 \quad \forall g = 1, 2, \dots, h^{(6)}$$

$$\sum_{j=1}^n \lambda_j^6 y_{rj}^6 - s_r^+ = y_{ro}^6 \quad \forall r = 1, 2, \dots, s^{(6)}$$

(27)

Division 7:

$$\sum_{j=1}^n \lambda_j^7 x_{ij}^7 + s_i^- = x_{io}^7 \quad \forall i = 1, 2, \dots, q^{(7)}$$

$$\sum_{j=1}^n \lambda_j^7 z_{gj}^{(2,7)} + s_g^- = z_{go}^7 \quad \forall g = 1, 2, \dots, h^{(7)}$$

$$\sum_{j=1}^n \lambda_j^7 z_{gj}^{(7,10)} - s_g^+ = z_{go}^7 \quad \forall g = 1, 2, \dots, h^{(7)}$$

$$\sum_{j=1}^n \lambda_j^7 y_{rj}^7 - s_r^+ = y_{ro}^7 \quad \forall r = 1, 2, \dots, s^{(7)}$$

(28)

Division 8:

$$\sum_{j=1}^n \lambda_j^8 x_{ij}^8 + s_i^- = x_{io}^8 \quad \forall i = 1, 2, \dots, q^{(8)}$$

$$\sum_{j=1}^n \lambda_j^8 z_{gj}^{(3,8)} + s_g^- = z_{go}^8 \quad \forall g = 1, 2, \dots, h^{(8)}$$

$$\sum_{j=1}^n \lambda_j^8 z_{gj}^{(8,11)} - s_g^+ = z_{go}^8 \quad \forall g = 1, 2, \dots, h^{(8)}$$

$$\sum_{j=1}^n \lambda_j^8 y_{rj}^8 - s_r^+ = y_{ro}^8 \quad \forall r = 1, 2, \dots, s^{(8)}$$

(29)

Division 9:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^9 x_{ij}^9 + s_i^- &= x_{io}^9 \quad \forall i = 1, 2, \dots, q^{(9)} \\
 \sum_{j=1}^n \lambda_j^9 (z_{gj}^{(4,9)} + z_{gj}^{(5,9)}) + s_g^- &= z_{go}^9 \quad \forall g = 1, 2, \dots, h^{(9)} \\
 \sum_{j=1}^n \lambda_j^9 z_{gj}^{(9,11)} - s_g^+ &= z_{go}^9 \quad \forall g = 1, 2, \dots, h^{(9)} \\
 \sum_{j=1}^n \lambda_j^9 y_{rj}^9 - s_r^+ &= y_{ro}^9 \quad \forall r = 1, 2, \dots, s^{(9)}
 \end{aligned} \tag{30}$$

Division 10:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{10} x_{ij}^{10} + s_i^- &= x_{io}^{10} \quad \forall i = 1, 2, \dots, q^{(10)} \\
 \sum_{j=1}^n \lambda_j^{10} (z_{gj}^{(6,10)} + z_{gj}^{(7,10)}) + s_g^- &= z_{go}^{10} \quad \forall g = 1, 2, \dots, h^{(10)} \\
 \sum_{j=1}^n \lambda_j^{10} z_{gj}^{(10,12)} - s_g^+ &= z_{go}^{10} \quad \forall g = 1, 2, \dots, h^{(10)} \\
 \sum_{j=1}^n \lambda_j^{10} y_{rj}^{10} - s_r^+ &= y_{ro}^{10} \quad \forall r = 1, 2, \dots, s^{(10)}
 \end{aligned} \tag{31}$$

Division 11:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{11} x_{ij}^{11} + s_i^- &= x_{io}^{11} \quad \forall i = 1, 2, \dots, q^{(11)} \\
 \sum_{j=1}^n \lambda_j^{11} (z_{gj}^{(8,11)} + z_{gj}^{(9,11)}) + s_g^- &= z_{go}^{11} \quad \forall g = 1, 2, \dots, h^{(11)} \\
 \sum_{j=1}^n \lambda_j^{11} z_{gj}^{(11,12)} - s_g^+ &= z_{go}^{11} \quad \forall g = 1, 2, \dots, h^{(11)} \\
 \sum_{j=1}^n \lambda_j^{11} y_{rj}^{11} - s_r^+ &= y_{ro}^{11} \quad \forall r = 1, 2, \dots, s^{(11)}
 \end{aligned} \tag{32}$$

Division 12:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{12} x_{ij}^{12} + s_i^- &= x_{io}^{12} \quad \forall i = 1, 2, \dots, q^{(12)} \\
 \sum_{j=1}^n \lambda_j^{12} z_{gj}^{(10,12)} + s_g^- &= z_{go}^{12} \quad \forall g = 1, 2, \dots, h^{(12)} \\
 \sum_{j=1}^n \lambda_j^{12} z_{gj}^{(12,14)} - s_g^+ &= z_{go}^{12} \quad \forall g = 1, 2, \dots, h^{(12)}
 \end{aligned} \tag{33}$$

$$\sum_{j=1}^n \lambda_j^{12} y_{rj}^{12} - s_r^+ = y_{ro}^{12} \quad \forall r = 1, 2, \dots, s^{(12)}$$

Division 13:

$$\sum_{j=1}^n \lambda_j^{13} x_{ij}^{13} + s_i^- = x_{io}^{13} \quad \forall i = 1, 2, \dots, q^{(13)}$$

$$\sum_{j=1}^n \lambda_j^{13} z_{gj}^{(11,13)} + s_g^- = z_{go}^{13} \quad \forall g = 1, 2, \dots, h^{(13)}$$

$$\sum_{j=1}^n \lambda_j^{13} z_{gj}^{(13,15)} - s_g^+ = z_{go}^{13} \quad \forall g = 1, 2, \dots, h^{(13)}$$

$$\sum_{j=1}^n \lambda_j^{13} y_{rj}^{13} - s_r^+ = y_{ro}^{13} \quad \forall r = 1, 2, \dots, s^{(13)}$$

(34)

Division 14:

$$\sum_{j=1}^n \lambda_j^{14} x_{ij}^{14} + s_i^- = x_{io}^{14} \quad \forall i = 1, 2, \dots, q^{(14)}$$

$$\sum_{j=1}^n \lambda_j^{14} z_{gj}^{(12,14)} + s_g^- = z_{go}^{14} \quad \forall g = 1, 2, \dots, h^{(14)}$$

$$\sum_{j=1}^n \lambda_j^{14} z_{gj}^{(14,16)} - s_g^+ = z_{go}^{14} \quad \forall g = 1, 2, \dots, h^{(14)}$$

$$\sum_{j=1}^n \lambda_j^{14} y_{rj}^{14} - s_r^+ = y_{ro}^{14} \quad \forall r = 1, 2, \dots, s^{(14)}$$

(35)

Division 15:

$$\sum_{j=1}^n \lambda_j^{15} x_{ij}^{15} + s_i^- = x_{io}^{15} \quad \forall i = 1, 2, \dots, q^{(15)}$$

$$\sum_{j=1}^n \lambda_j^{15} z_{gj}^{(13,15)} + s_g^- = z_{go}^{15} \quad \forall g = 1, 2, \dots, h^{(15)}$$

$$\sum_{j=1}^n \lambda_j^{15} z_{gj}^{(15,17)} - s_g^+ = z_{go}^{15} \quad \forall g = 1, 2, \dots, h^{(15)}$$

$$\sum_{j=1}^n \lambda_j^{15} y_{rj}^{15} - s_r^+ = y_{ro}^{15} \quad \forall r = 1, 2, \dots, s^{(15)}$$

(36)

Division 16:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{16} x_{ij}^{16} + s_i^- &= x_{io}^{16} \quad \forall i = 1, 2, \dots, q^{(16)} \\
 \sum_{j=1}^n \lambda_j^{16} z_{gj}^{(14,16)} + s_g^- &= z_{go}^{16} \quad \forall g = 1, 2, \dots, h^{(16)} \\
 \sum_{j=1}^n \lambda_j^{16} z_{gj}^{(16,18)} - s_g^+ &= z_{go}^{16} \quad \forall g = 1, 2, \dots, h^{(16)} \\
 \sum_{j=1}^n \lambda_j^{16} y_{rj}^{16} - s_r^+ &= y_{ro}^{16} \quad \forall r = 1, 2, \dots, s^{(16)}
 \end{aligned} \tag{37}$$

Division 17:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{17} x_{ij}^{17} + s_i^- &= x_{io}^{17} \quad \forall i = 1, 2, \dots, q^{(17)} \\
 \sum_{j=1}^n \lambda_j^{17} z_{gj}^{(15,17)} + s_g^- &= z_{go}^{17} \quad \forall g = 1, 2, \dots, h^{(17)} \\
 \sum_{j=1}^n \lambda_j^{17} (z_{gj}^{(17,19)} + z_{gj}^{(17,20)}) - s_g^+ &= z_{go}^{17} \quad \forall g = 1, 2, \dots, h^{(17)} \\
 \sum_{j=1}^n \lambda_j^{17} y_{rj}^{17} - s_r^+ &= y_{ro}^{17} \quad \forall r = 1, 2, \dots, s^{(17)}
 \end{aligned} \tag{38}$$

Division 18:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{18} x_{ij}^{18} + s_i^- &= x_{io}^{18} \quad \forall i = 1, 2, \dots, q^{(18)} \\
 \sum_{j=1}^n \lambda_j^{18} z_{gj}^{(16,18)} + s_g^- &= z_{go}^{18} \quad \forall g = 1, 2, \dots, h^{(18)} \\
 \sum_{j=1}^n \lambda_j^{18} z_{gj}^{(18,22)} - s_g^+ &= z_{go}^{18} \quad \forall g = 1, 2, \dots, h^{(18)} \\
 \sum_{j=1}^n \lambda_j^{18} y_{rj}^{18} - s_r^+ &= y_{ro}^{18} \quad \forall r = 1, 2, \dots, s^{(18)}
 \end{aligned} \tag{39}$$

Division 19:

$$\sum_{j=1}^n \lambda_j^{19} x_{ij}^{19} + s_i^- = x_{io}^{19} \quad \forall i = 1, 2, \dots, q^{(19)} \tag{40}$$

$$\sum_{j=1}^n \lambda_j^{19} z_{gj}^{(17,19)} + s_g^- = z_{go}^{19} \quad \forall g = 1, 2, \dots, h^{(19)}$$

$$\sum_{j=1}^n \lambda_j^{19} z_{gj}^{(19,23)} - s_g^+ = z_{go}^{19} \quad \forall g = 1, 2, \dots, h^{(19)}$$

$$\sum_{j=1}^n \lambda_j^{19} y_{rj}^{19} - s_r^+ = y_{ro}^{19} \quad \forall r = 1, 2, \dots, s^{(19)}$$

Division 20:

$$\sum_{j=1}^n \lambda_j^{20} x_{ij}^{20} + s_i^- = x_{io}^{20} \quad \forall i = 1, 2, \dots, q^{(20)}$$

$$\sum_{j=1}^n \lambda_j^{20} z_{gj}^{(17,20)} + s_g^- = z_{go}^{20} \quad \forall g = 1, 2, \dots, h^{(20)}$$

$$\sum_{j=1}^n \lambda_j^{20} z_{gj}^{(20,21)} - s_g^+ = z_{go}^{20} \quad \forall g = 1, 2, \dots, h^{(20)}$$

$$\sum_{j=1}^n \lambda_j^{20} y_{rj}^{20} - s_r^+ = y_{ro}^{20} \quad \forall r = 1, 2, \dots, s^{(20)}$$

(41)

Division 21:

$$\sum_{j=1}^n \lambda_j^{21} x_{ij}^{21} + s_i^- = x_{io}^{21} \quad \forall i = 1, 2, \dots, q^{(21)}$$

$$\sum_{j=1}^n \lambda_j^{21} z_{gj}^{(20,21)} + s_g^- = z_{go}^{21} \quad \forall g = 1, 2, \dots, h^{(21)}$$

$$\sum_{j=1}^n \lambda_j^{21} z_{gj}^{(21,24)} - s_g^+ = z_{go}^{21} \quad \forall g = 1, 2, \dots, h^{(21)}$$

$$\sum_{j=1}^n \lambda_j^{21} y_{rj}^{21} - s_r^+ = y_{ro}^{21} \quad \forall r = 1, 2, \dots, s^{(21)}$$

(42)

Division 22:

$$\sum_{j=1}^n \lambda_j^{22} x_{ij}^{22} + s_i^- = x_{io}^{22} \quad \forall i = 1, 2, \dots, q^{(22)}$$

$$\sum_{j=1}^n \lambda_j^{22} z_{gj}^{(18,22)} + s_g^- = z_{go}^{22} \quad \forall g = 1, 2, \dots, h^{(22)}$$

$$\sum_{j=1}^n \lambda_j^{22} y_{rj}^{22} - s_r^+ = y_{ro}^{22} \quad \forall r = 1, 2, \dots, s^{(22)}$$

(43)

Division 23:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{23} x_{ij}^{23} + s_i^- &= x_{io}^{23} \quad \forall i = 1, 2, \dots, q^{(23)} \\
 \sum_{j=1}^n \lambda_j^{23} z_{gj}^{(19,23)} + s_g^- &= z_{go}^{22} \quad \forall g = 1, 2, \dots, h^{(23)} \\
 \sum_{j=1}^n \lambda_j^{23} y_{rj}^{23} - s_r^+ &= y_{ro}^{23} \quad \forall r = 1, 2, \dots, s^{(23)}
 \end{aligned} \tag{44}$$

Division 24:

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j^{24} x_{ij}^{24} + s_i^- &= x_{io}^{24} \quad \forall i = 1, 2, \dots, q^{(24)} \\
 \sum_{j=1}^n \lambda_j^{24} z_{gj}^{(21,24)} + s_g^- &= z_{go}^{24} \quad \forall g = 1, 2, \dots, h^{(24)} \\
 \sum_{j=1}^n \lambda_j^{24} y_{rj}^{24} - s_r^+ &= y_{ro}^{24} \quad \forall r = 1, 2, \dots, s^{(24)}
 \end{aligned} \tag{45}$$

$$\sum_{k=1}^{24} \lambda_j^k = 1 \tag{46}$$

$$\lambda_j^k, s_i^-, s_g^-, s_r^+, s_r^+ \geq 0 \tag{47}$$

IV. PROPOSED HYBRID UNCERTAINTY APPROACH

The developed SBM-NDEA model incorporates uncertain parameters, and a hybrid methodology that combines fuzzy reasoning with Neutrosophic fuzzy programming is offered to manage these uncertainties. In this framework, the uncertain parameters are categorized into two groups; the initial group concerns risk parameters, which are tackled through a proficient fuzzy reasoning approach. The other parameters may be expressed through Neutrosophic fuzzy numbers, and a robust Neutrosophic programming method is utilized to manage these parameters (Mohammadi et al. 2020). Notably, several linguistic variables are employed to assess certain qualitative KPIs, which can subsequently be transformed into Neutrosophic fuzzy numbers. It is noteworthy that all elements of a Neutrosophic fuzzy number associated with KPIs that exhibit no uncertainty are regarded as identical.

A. Risk evaluation stage

This section presents several fundamental definitions of trapezoidal fuzzy numbers (TFNs), which are utilized within the fuzzy reasoning framework.

Definition 1: The notation for a trapezoidal fuzzy number (TFN) is $\tilde{A} = (a, b, c, d)$, and its membership function (MF) $\mu_{\tilde{A}}$ is established as follows, as described by Hwang and Masud (2012).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x > d \end{cases} \quad (48)$$

Definition 2: If $a > 0$ and a, b, c , and d are not identical, \tilde{A} is a positive TFN.

Definition 3: Consider $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ two positive TFNs and k will be a positive scalar, the following arithmetic operators can be established as described by Hwang and Masud (2012):

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \quad (49)$$

$$k\tilde{A} = (ka_1, ka_2, ka_3, ka_4) \quad (50)$$

To quantify risk through a metric termed risk magnitude (RM), three primary risk parameters are analyzed: the frequency of failure (FOF), the severity of consequence (SOC), and the probability of consequence (POC). FOF indicates how often an event occurs within a specified timeframe. FOF can be assessed using linguistic variables such as “Very unlikely,” “Unlikely,” “Fairly unlikely,” “Likely,” and “Very likely,” as illustrated in Table II. SOC denotes the extent of the damaging impact that a phenomenon may have on the system. The assessment of SOC can be categorized as “Negligible,” “Minor,” “Moderate,” “Major,” and “Catastrophic,” as illustrated in Table II. Additionally, POC pertains to the probability of a phenomenon occurring within the system. POC can be classified as “Highly unlikely,” “Unlikely,” “Reasonably unlikely,” “Likely,” “Reasonably likely,” and “Highly likely,” as shown in Table III. Additionally, RM is characterized as linguistic variables that encompass five distinct levels: “Low,” “Acceptable,” “Average,” “High,” and “Unacceptable,” as illustrated in Table II. The TFNs associated with these linguistic variables, which are utilized to quantify the risk parameters, are detailed in Tables II and III.

Table II. Qualitative descriptors of FOF, SOC and RM and corresponding TFNs

linguistic variables for FOF	linguistic variables for SOC	linguistic variables for RM	TFNs
Very Unlikely (VU)	Negligible (NL)	Low (L)	(0,0,0.5,1)
Unlikely (U)	Minor (MN)	Acceptable (AC)	(0.5,1,1.5,2)
Fairly Unlikely (FU)	Moderate (MD)	Average (AV)	(1.5,2,3,3.5)
Likely (L)	Major (MJ)	High (H)	(3,3.5,4,4.5)
Very Likely (VL)	Catastrophic (CT)	Unacceptable (UAC)	(4,4.5,5,5)

Table III. Qualitative descriptors of POC and corresponding fuzzy numbers

linguistic variables for POC	TFNs
Highly Unlikely (HU)	(0,0,0.5,1)
Unlikely (U)	(0.5,1,1.5,2)
Reasonably Unlikely (RU)	(1.5,2,2.5,3)
Likely (L)	(2.5,3,4,4.5)
Reasonably Likely (RL)	(4,4.5,5,5.5)
Highly Likely (HL)	(5,5.5,6,6)

A.1. Aggregated of TFNs

Given that the risk assessment team is composed of multiple experts whose views on PO and SC may vary, it is essential to aggregate these perspectives in order to derive a singular score. Therefore, Eq. (51) is employed for this aggregation.

$$\text{Total score} = w_1 \times TFN_{1i} + w_2 \times TFN_{2i} + \dots + w_n \times TFN_{ni} \quad (51)$$

where the importance of expert j is denoted by w_j and TFN_{ji} is the score of risk parameter i assessed by expert j , wherein $\sum_{j=1}^n w_j = 1$.

A.2. Compute fuzzy values of total score

Assume that A_{FOF}^i , A_{SOC}^i , and A_{POC}^i denote total scores for FOF, SOC, and POC related to i th risk phenomenon, respectively. Their corresponding fuzzy sets, denoted by \tilde{A}_{FOF}^i , \tilde{A}_{SOC}^i , and \tilde{A}_{POC}^i are defined in the following manner (An et al. 2011):

$$\tilde{A}_{FOF}^i = \left\{ \left(u, \mu_{A_{FOF}^i}(u) \right) \mid u \in U = [0, u], \mu_{A_{FOF}^i}(u) \in [0, 1] \right\} \quad (52)$$

$$\tilde{A}_{SOC}^i = \left\{ \left(v, \mu_{A_{SOC}^i}(v) \right) \mid v \in V = [0, v], \mu_{A_{SOC}^i}(v) \in [0, 1] \right\} \quad (53)$$

$$\tilde{A}_{POC}^i = \left\{ \left(w, \mu_{A_{POC}^i}(w) \right) \mid w \in W = [0, w], \mu_{A_{POC}^i}(w) \in [0, 1] \right\} \quad (54)$$

where trapezoidal MFs of A_{FOF}^i , A_{SOC}^i , and A_{POC}^i are denoted by $\mu_{A_{FOF}^i}$, $\mu_{A_{SOC}^i}$, and $\mu_{A_{POC}^i}$, and u , v , and w are input variables within the universe of discourse (UD) U , V , and W of FOF, SOC, and POC, respectively.

A.3. Fuzzy reasoning approach

A fuzzy reasoning methodology based on the Mamdani technique is utilized to identify the appropriate rules for a specific situation in order to compute fuzzy output. This involves considering a set of if-then rules that connect the input risk parameters, including FOF, SOC, and POC, to the resulting output, as described by An et al. (2011).

$$R_i: \text{if } u \text{ is } \tilde{B}_{FOF}^i \text{ and } v \text{ is } \tilde{B}_{SOC}^i \text{ and } w \text{ is } \tilde{B}_{POC}^i \text{ then } x \text{ is } \tilde{B}_{RL}^i, i = 1, 2, \dots, n$$

where the qualitative descriptors (QDs) of FOF, SOC, POC and fuzzy output are denoted by \tilde{B}_{FOF}^i , \tilde{B}_{SOC}^i , \tilde{B}_{POC}^i , and \tilde{B}_{RL}^i , respectively. In what follows, the following fuzzy intersection operator is used to calculate the fire strength of α_i of i th rule (An et al. 2011).

$$\alpha_i = \min \left[\max \left(\mu_{A_{FOF}^i}(u) \bigwedge \mu_{B_{FOF}^i}(u) \right), \max \left(\mu_{A_{SOC}^i}(v) \bigwedge \mu_{B_{SOC}^i}(v) \right), \max \left(\mu_{A_{POC}^i}(w) \bigwedge \mu_{B_{POC}^i}(w) \right) \right] \quad (55)$$

where MFs of fuzzy sets \tilde{B}_{FOF}^i , \tilde{B}_{SOC}^i , and \tilde{B}_{POC}^i of QDs in rule R_i are denoted by $\mu_{B_{FOF}^i}(u)$, $\mu_{B_{SOC}^i}(v)$, and $\mu_{B_{POC}^i}(w)$. Next, the truncated MF $\mu'_{B_{RL}^i}$ of the inferred outcome fuzzy set of rule R_i is computed as follows (An et al. 2011):

$$\mu'_{B_{RL}^i} = \alpha_i \bigwedge \mu_{B_{RL}^i}(x) \quad (56)$$

where $\mu_{B_{RL}}^i$ is the MF of the QDs \tilde{B}_{RL}^i and x is an input variable in the UD X . Also, $\mu'_{B_{RL}}$ of outcome fuzzy set is calculated as follows (An et al. 2011):

$$\mu'_{B_{RL}} = \bigvee_{i=1}^n \mu_{B_{RL}}^i(x) \quad (57)$$

where n is the total number of rules in the rule base.

A.4. Defuzzification

The final result of the fuzzy reasoning approach is derived through the application of a defuzzification method referred to as the centroid of area method. For this analysis, we define the outcome fuzzy set from the fuzzy reasoning approach as $\mu'_{B_{RL}} = \{(x, \mu'_{B_{RL}}(x)) | x \in X, \mu'_{B_{RL}}(x) \in [0,1]\}$, the aggregated outcome MF RL_i is computed as follows (An et al. 2011):

$$RL_i = \frac{\sum_{j=1}^m \mu'_{B_{RL}}(x_j) \cdot x_j}{\sum_{j=1}^m \mu'_{B_{RL}}(x_j)} \quad (58)$$

where $x_j \in X$ and x_j indicates the center of $\mu'_{B_{RL}}$ in the outcome expression, and m is the number of quantization level of X .

B. Neutrosophic fuzzy programming

In order to overcome the challenges posed by ambiguity and uncertainty, Zadeh proposed fuzzy set theory in 1965. Since then, numerous extensions of fuzzy sets have been introduced in the academic literature, including type 2, multi-sets, hesitant, intuitionistic, Neutrosophic, and Pythagorean fuzzy sets (Vahdani and Zandieh, 2010; Vahdani et al., 2012; Mousavi et al., 2013; Mousavi et al., 2014; Mohagheghi et al., 2015; Otay, Oztaysi, Mohagheghi et al., 2016; Mohagheghi et al., 2017; Moradi et al., 2017; Gitinavard et al., 2017; Davoudabadi et al., 2019). These advancements aim to enhance the interpretation of imprecise and ambiguous information. The intuitionistic fuzzy set (IFS) is widely recognized for its incorporation of membership, non-membership, and hesitancy functions, which effectively address issues of vagueness and imprecision. Nevertheless, it falls short of accurately representing the human decision-making process. To address the limitations of IFS and to manage inconsistent, imprecise, and vague information, the Neutrosophic set (NS) was introduced by Broumi et al. (2016). Consequently, NS theory is capable of modeling the human decision-making process by encompassing all aspects of this complex procedure. In fact, NS serves as an advancement of fuzzy logic and IFS, wherein each component of the set possesses membership functions for truth, indeterminacy, and falsity. This allows NS to adeptly and efficiently handle ambiguous, imprecise, and conflicting information (Deli and Şubaş, 2017).

Hence, so as to address the uncertainty associated with the other parameters, a commonly adopted Neutrosophic fuzzy programming method is utilized (Abdelfattah, 2021). This involves using linguistic variables, presented in Table IV, to represent the relevant parameters, which can subsequently be transformed into Neutrosophic fuzzy numbers. Subsequently, a defuzzification process is applied to transform the model into an equivalent deterministic representation. In this regard, this section presents several preliminary concepts, including NS, the single-valued NS (SVNS), the single-valued triangular Neutrosophic number (SVTNN), and the various operations performed on SVTNNs.

Table IV. Linguistics variables and corresponding Neutrosophic fuzzy numbers

linguistic variables	SVTNNs
Very Low (VL)	$< [0,0.5,1.5]; [0.7,0.4,0.3] >$
Low (L)	$< [1,2,3]; [0.6,0.5,0.2] >$
Fairy Low (FL)	$< [2,3.5,5]; [0.6,0.4,0.1] >$
Medium (M)	$< [3,5,7]; [0.4,0.3,0.2] >$
Fairly High (FH)	$< [5,6.5,8]; [0.4,0.3,0.2] >$
High (H)	$< [7,8,9]; [0.7,0.4,0.3] >$
Very High (VH)	$< [8.5,9.5,10]; [0.6,0.5,0.2] >$

B.1. Theoretical preliminaries

Definition 4: Assume that E be a UD, a NS A over E can be defined by $A = \{< x, (T_A(x), I_A(x), F_A(x)) > | x \in E\}$, wherein $T_A(x)$, $I_A(x)$ and $F_A(x)$ are truth-MF, indeterminacy-MF, and falsity-MF, respectively. In this regard, a SVNS over E is a NS, where $T_A(x): E \rightarrow [0,1]$, $I_A(x): E \rightarrow [0,1]$, and $F_A(x): E \rightarrow [0,1]$ and $0 \leq T_V(x) + I_V(x) + F_V(x) \leq 3$ (Deli and Şubaş, 2017).

Definition 5: A SVTNN $\tilde{a} = < [a_1, a_2, a_3]; (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) >$ is a special NS on the real number set R , where the MFs of truth ($\mu_{\tilde{a}}$), indeterminacy ($v_{\tilde{a}}$), and falsity ($\lambda_{\tilde{a}}$) can be defined as follows (Deli and Şubaş, 2017):

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x - a_1)T_{\tilde{a}}}{(a_2 - a_1)} & a_1 \leq x \leq a_2 \\ \frac{(a_3 - x)T_{\tilde{a}}}{(a_3 - a_2)} & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (59)$$

$$v_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + I_{\tilde{a}}(x - a_1))}{(a_2 - a_1)} & a_1 \leq x \leq a_2 \\ \frac{(x - a_2 + I_{\tilde{a}}(a_3 - x))}{(a_3 - a_2)} & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (60)$$

$$\lambda_{\tilde{a}} = \begin{cases} \frac{(a_2 - x + F_{\tilde{a}}(x - a_1))}{(a_2 - a_1)} & a_1 \leq x \leq a_2 \\ \frac{(x - a_2 + F_{\tilde{a}}(a_3 - x))}{(a_3 - a_2)} & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (61)$$

Definition 6: Let $\tilde{a} = < [a_1, a_2, a_3]; (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) >$ and $\tilde{b} = < [b_1, b_2, b_3]; (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) >$ be two SVTNNs and $\rho \neq 0$ be any real number (Deli and Şubaş, 2017). So,

- $\tilde{a} + \tilde{b} = < [a_1 + b_1, a_2 + b_2, a_3 + b_3]; (T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \wedge I_{\tilde{b}}, F_{\tilde{a}} \wedge F_{\tilde{b}}) >$
- $\rho \tilde{a} = \begin{cases} \tilde{a} = < [\rho a_1, \rho a_2, \rho a_3]; (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) > & \rho > 0 \\ \tilde{a} = < [\rho a_3, \rho a_2, \rho a_1]; (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) > & \rho < 0 \end{cases}$

Definition 7: Let $\tilde{a} = < [a_1, a_2, a_3]; (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) >$ be a SVTNN, the α, β , and γ cuts of SVTNN \tilde{a} for truth, indeterminacy, and falsity MFs are defined as follows (Deli and Şubaş, 2017):

- $\tilde{a}(\alpha) = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)] = \left[\frac{(T_{\tilde{a}} - \alpha)a_1 + \alpha a_2}{T_{\tilde{a}}}, \frac{(T_{\tilde{a}} - \alpha)a_3 + \alpha a_2}{T_{\tilde{a}}} \right]; \quad \alpha \in [0, T_{\tilde{a}}]$
- $\tilde{a}(\beta) = [L_{\tilde{a}}(\beta), R_{\tilde{a}}(\beta)] = \left[\frac{(1-\beta)a_2 + (\beta - I_{\tilde{a}})a_1}{1 - I_{\tilde{a}}}, \frac{(1-\beta)a_2 + (\beta - I_{\tilde{a}})a_3}{1 - I_{\tilde{a}}} \right]; \quad \beta \in [I_{\tilde{a}}, 1]$
- $\tilde{a}(\gamma) = [L_{\tilde{a}}(\gamma), R_{\tilde{a}}(\gamma)] = \left[\frac{(1-\gamma)a_2 + (\gamma - F_{\tilde{a}})a_1}{1 - F_{\tilde{a}}}, \frac{(1-\gamma)a_2 + (\gamma - F_{\tilde{a}})a_3}{1 - F_{\tilde{a}}} \right]; \quad \gamma \in [F_{\tilde{a}}, 1]$

Definition 8: The aggregate coefficient $\tilde{a}(\alpha, \beta, \gamma)$ which is based on the ordered normalized sum of lower bounds of $\tilde{a}(\alpha)$, $\tilde{a}(\beta)$, and $\tilde{a}(\gamma)$ can be calculated as follows (Abdelfattah, 2021):

$$\tilde{a}(\alpha, \beta, \gamma) = [a_1 + (a_2 - a_1)\theta_{\tilde{a}(\alpha, \beta, \gamma)}, a_3 - (a_3 - a_2)\theta_{\tilde{a}(\alpha, \beta, \gamma)}] = [a^l(\alpha, \beta, \gamma), a^u(\alpha, \beta, \gamma)]$$

where $\theta_{\tilde{a}(\alpha, \beta, \gamma)}$ is a variation degree of SVTNN \tilde{a} and can be calculated as follows:

$$\theta_{\tilde{a}(\alpha, \beta, \gamma)} = \frac{1}{4} \left[\frac{\alpha}{T_{\tilde{a}}} + 2 \frac{(1-\beta)}{1 - I_{\tilde{a}}} + \frac{(1-\gamma)}{1 - F_{\tilde{a}}} \right]; \quad \alpha \in [0, T_{\tilde{a}}], \beta \in [I_{\tilde{a}}, 1], \text{ and } \gamma \in [F_{\tilde{a}}, 1]$$

As can be seen, the obtained aggregate coefficient is an interval.

B.2. Neutrosophic programming model

Consider the following mathematical programming model, wherein parameters are SVNNS (Abdelfattah, 2021):

$$\begin{aligned}
 & \text{Max/Min } Z = \sum_{j=1}^n \tilde{c}_j x_j \\
 & \text{S. t.} \\
 & \sum_{j=1}^n \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad \forall i = 1, 2, \dots, l \\
 & \sum_{j=1}^n \tilde{a}_{ij} x_j \geq \tilde{b}_i \quad \forall i = l + 1, \dots, k \\
 & \sum_{j=1}^n \tilde{a}_{ij} x_j = \tilde{b}_i \quad \forall i = k + 1, \dots, m \\
 & x_j \geq 0 \quad \forall j = 1, 2, \dots, n
 \end{aligned} \tag{62}$$

where $\tilde{c}_j = \langle [c_{j1}, c_{j2}, c_{j3}], (T_{\tilde{c}_j}, I_{\tilde{c}_j}, F_{\tilde{c}_j}) \rangle$, $\tilde{a}_{ij} = \langle [a_{ij1}, a_{ij2}, a_{ij3}], (T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}}) \rangle$ and $\tilde{b}_i = \langle [b_{i1}, b_{i2}, b_{i3}], (T_{\tilde{b}_i}, I_{\tilde{b}_i}, F_{\tilde{b}_i}) \rangle$ are SVNNS. By exploiting the aggregate coefficient $\tilde{a}(\alpha, \beta, \gamma)$, the equivalent model of the model (62) is provided as follows (Abdelfattah, 2021):

$$\begin{aligned}
 & \text{Max/Min } Z_{(\alpha, \beta, \gamma)} = \left[\sum_{j=1}^n c_j^l(\alpha, \beta, \gamma), \sum_{j=1}^n c_j^u(\alpha, \beta, \gamma) \right] x_j \\
 & \text{S. t.}
 \end{aligned}$$

$$\begin{aligned}
\left[\sum_{j=1}^n a_{ij}^l(\alpha, \beta, \gamma), \sum_{j=1}^n a_{ij}^u(\alpha, \beta, \gamma) \right] x_j &\leq [b_i^l(\alpha, \beta, \gamma), b_i^u(\alpha, \beta, \gamma)] \quad \forall i = 1, 2, \dots, l \\
\left[\sum_{j=1}^n a_{ij}^l(\alpha, \beta, \gamma), \sum_{j=1}^n a_{ij}^u(\alpha, \beta, \gamma) \right] x_j &\geq [b_i^l(\alpha, \beta, \gamma), b_i^u(\alpha, \beta, \gamma)] \quad \forall i = 1, 2, \dots, l \\
\left[\sum_{j=1}^n a_{ij}^l(\alpha, \beta, \gamma), \sum_{j=1}^n a_{ij}^u(\alpha, \beta, \gamma) \right] x_j &= [b_i^l(\alpha, \beta, \gamma), b_i^u(\alpha, \beta, \gamma)] \quad \forall i = 1, 2, \dots, l
\end{aligned} \tag{63}$$

$$x_j \geq 0 \quad \forall j = 1, 2, \dots, n$$

$$\alpha \in [0, \min\{T_{\bar{c}_j}, T_{\bar{a}_{ij}}, T_{\bar{b}_i}\}], \beta \in [\max\{I_{\bar{c}_j}, I_{\bar{a}_{ij}}, I_{\bar{b}_i}\}, 1], \gamma \in [\max\{F_{\bar{c}_j}, F_{\bar{a}_{ij}}, F_{\bar{b}_i}\}, 1].$$

It is essential to note that model (63) is classified as an interval programming model, thereby necessitating the application of the interval planning technique for its solution.

V. CASE STUDY AND NUMERICAL RESULTS

The developed mathematical model is based on the cross-docking configuration utilized by an Iranian retail company. Furthermore, an analysis of five additional cross-docking systems with analogous operations is conducted to measure the effectiveness of different DMUs, each corresponding to a unique cross-docking system. It is important to highlight that disparities exist among these cross-docking systems in terms of their structures and procedures, resulting in certain parameters being recorded as zero. On this matter, in the case of cross-docking No. 2 (DMU2), products are required to be temporarily stored before they can be unloaded and subsequently loaded. Additionally, the risk assessment team is composed of three specialists, with their respective importance values being 0.25, 0.4, and 0.35. Notably, as indicated by the assessed levels for the three risk parameters—FOF, SOC, and POC—in Tables II and III, this research can formulate 105 rules, which are outlined as follows:

Rule #1: IF FOF is Very Unlikely, SOC is Negligible, and POC is Highly Unlikely **THEN** RM is Low

Rule #2: IF FOF is Very Unlikely, SOC is Negligible, and POC is Unlikely **THEN** RM is Low

⋮

Rule #150: IF FOF is Very likely, SOC is Catastrophic, and POC is Highly Likely **THEN** RM is Unacceptable

Tables V to VII present various input and output data associated with cross-docking 1. It is evident that certain parameters, which decision-makers qualitatively express using linguistic variables, are transformed into corresponding fuzzy numbers. Notably, as seen in Table VII, the values of the parameters that have certainty are presented in the form of an SVTNN, wherein all of its components are equal to each other, and the MF truth is equal to one, and the rest are equal to zero. The developed model is executed using GAMS software on a laptop equipped with Core i5 CPUs operating at 1.6 GHz and 8 GB of RAM to determine the efficiencies of DMUs and their respective rankings.

Table V. A number of risk parameters related to the case study

Parameters	Expert 1			Expert 2			Expert 3		
	FOF	SOC	POC	FOF	SOC	POC	FOF	SOC	POC
$\chi_{8,1}^{1,D}$	FU	MJ	L	L	MD	RU	L	MD	RL
$\chi_{9,1}^{1,D}$	VL	CT	RL	L	MJ	RL	L	CT	HL
$\chi_{4,1}^{8,D}$	VU	CT	U	U	MJ	RU	VU	MJ	U
$\chi_{5,1}^{8,D}$	L	MJ	RL	VL	MJ	L	VL	MJ	HL
$\chi_{7,1}^{10,D}$	FU	MN	RU	L	MD	L	FU	MN	RU
$\chi_{8,1}^{10,D}$	VL	MJ	L	L	MJ	L	L	MJ	L
$\chi_{10,1}^{15,D}$	U	MJ	L	FU	MJ	U	U	MD	L

The outcome fuzzy set for $\chi_{81}^{1,D}$ can be determined by adhering to the steps outlined in the fuzzy reasoning approach previously described.

Step 1: Aggregate experts' opinions in the following manner:

$$Total\ Score_{\tilde{\chi}_{81-FOF}^{1,D}} = 0.25 \times (1.5, 2, 3, 3.5) + 0.4 \times (3, 3.5, 4, 4.5) + 0.35 \times (3, 3.5, 4, 4.5) = (2.625, 3.125, 3.75, 4.25)$$

$$Total\ Score_{\tilde{\chi}_{81-SOC}^{1,D}} = 0.25 \times (3, 3.5, 4, 4.5) + 0.4 \times (1.5, 2, 3, 3.5) + 0.35 \times (1.5, 2, 3, 3.5) = (1.875, 2.375, 3.25, 3.75)$$

$$Total\ Score_{\tilde{\chi}_{81-POC}^{1,D}} = 0.25 \times (2.5, 3, 4, 4.5) + 0.4 \times (1.5, 2, 2.5, 3) + 0.35 \times (4, 4.5, 5, 5.5) = (2.625, 3.125, 3.75, 4.25)$$

Step 2: The aggregated fuzzy numbers of FOF, SOC, and POC are transformed into their respective fuzzy sets for the purpose of fuzzy inference in the following manner.

$$FOF = \{(Fairly\ unlikely, 0.875), (Likely, 1), (Very\ likely, 0.25)\}$$

$$SOC = \{(Minor, 0.125), (Moderate, 1), (Major, 0.75)\}$$

$$POC = \{(Reasonably\ unlikely, 0.375), (Likely, 1), (Reasonably\ likely, 0.25)\}$$

The results obtained for FOF, SOC, and POC indicate that a total of 27 rules can be utilized from a possible 105 rules. Some of these rules are as follows:

Rule #1: IF FOF is Fairly Unlikely, SOC is Minor, and POC is Reasonably Unlikely **THEN** RM is Low

:

Rule #27: IF FOF is Very Likely, SOC is Major, and POC is Reasonably Likely **THEN** RM is High

Step 3: Compute the fire strength i th rule in the following manner:

$$= \min(0.875, 0.125, 0.375) = 0.125$$

:

$$\alpha_{27} = \min(0.25, 0.75, 0.25) = 0.25$$

Step 4: Calculate the truncated MF of the inferred outcome fuzzy set of rule R_i in the following manner:

$$\mu'_{B_{RL}} = \alpha_1 \wedge \mu_{RM}^{Low} = \min(0.125, \mu_{RM}^{Low})$$

⋮

$$\mu'_{B_{RL}} = \alpha_{27} \wedge \mu_{RM}^{High} = \min(0.25, \mu_{RM}^{High})$$

Step 5: Calculate the defuzzified aggregated outcome MF as follows:

$$RL_1 = \frac{0.875 \times 1.5 + 1 \times 4 + 0.75 \times 5 + 0.125 \times 3}{0.875 + 1 + 0.75 + 0.125} = 3.43$$

Table VI. A number of SVTNNs related to the case study

Parameters	Linguistic variables	SVTNNs	Parameters	Linguistic variables	SVTNNs
$x_{3j}^{1,UD}$	H	$< [7,8,9]; [0.7,0.4,0.3] >$	$y_{1j}^{1,D}$	FH	$< [5,6.5,8]; [0.4,0.3,0.2] >$
$x_{4j}^{1,UD}$	FH	$< [5,6.5,8]; [0.4,0.3,0.2] >$	$y_{1j}^{8,D}$	H	$< [7,8,9]; [0.7,0.4,0.3] >$
$x_{5j}^{1,UD}$	M	$< [3,5,7]; [0.4,0.3,0.2] >$	$y_{1j}^{10,D}$	H	$< [7,8,9]; [0.7,0.4,0.3] >$
$x_{6j}^{1,UD}$	FH	$< [5,6.5,8]; [0.4,0.3,0.2] >$	$y_{1j}^{13,D}$	VH	$< [8.5,9.5,10]; [0.6,0.5,0.2] >$
$x_{7j}^{1,UD}$	VH	$< [8.5,9.5,10]; [0.6,0.5,0.2] >$	$y_{2j}^{13,D}$	H	$< [7,8,9]; [0.7,0.4,0.3] >$
$x_{5j}^{11,UD}$	M	$< [3,5,7]; [0.4,0.3,0.2] >$	$y_{2j}^{15,D}$	FL	$< [2,3.5,5]; [0.6,0.4,0.1] >$

Table VII. A number of certain parameters related to the case study

Parameters	Nominal values	Parameters	Nominal values
$x_{1j}^{1,UD}$	$< [1450,1450,1450]; [1,0,0] >$	$x_{3j}^{15,D}$	$< [17,17,17]; [1,0,0] >$
$x_{2j}^{1,D}$	$< [10,10,10]; [1,0,0] >$	$x_{3j}^{17,D}$	$< [1402,1402,1402]; [1,0,0] >$
$x_{1j}^{6,D}$	$< [750,750,750]; [1,0,0] >$	$x_{1j}^{15,D}$	$< [5,5,5]; [1,0,0] >$
$x_{3j}^{6,D}$	$< [43,43,43]; [1,0,0] >$	$x_{3j}^{14,UD}$	$< [23,23,23]; [1,0,0] >$
$x_{4j}^{9,D}$	$< [148,148,148]; [1,0,0] >$	$x_{1j}^{15,D}$	$< [24,24,24]; [1,0,0] >$
$x_{3j}^{10,D}$	$< [361,361,361]; [1,0,0] >$	$x_{2j}^{21,UD}$	$< [19,19,19]; [1,0,0] >$

In light of the parameter values and after applying the proposed hybrid uncertainty approach, the interval programming is established and subsequently solved using GAMS software, where $\alpha \in [0,0.4]$, $\beta \in [0.5,1]$, $\gamma \in [0.3,1]$ and to be set in $\alpha = 0.25$, $\beta = 0.65$, $\gamma = 0.45$. The efficiency metrics for the five assessed cross-docking systems, along with their rankings, are illustrated in Table IX. The results indicate that only cross-docking system 2 is efficient, whereas the other systems are deemed inefficient. It should be noted that the average of the upper and lower bounds of the obtained efficiency range is used to determine the ranking of the systems.

Table VIII. Efficiency scores and ranking of the cross-docking systems (DMUs)

DMUs	Lower Efficiency	Upper Efficiency	Rank
Cross-docking 1	0.681	0.863	3
Cross-docking 2	1.000	1.000	1
Cross-docking 3	0.405	0.571	5
Cross-docking 4	0.589	0.704	4
Cross-docking 5	0.774	0.824	2

A. Sensitivity analysis

To evaluate the proposed model's accuracy and validity, a range of sensitivity analyses is performed, targeting the upper bounds of system efficiency. This evaluation encompasses four defined scenarios. In the first scenario, there is a 10% increase in the values of intermediate outputs, which correspond to the number of intact products that are transferred between various divisions of cross-docking systems. Fig. 3 indicates that this increase has the potential to enhance the efficiency scores of inefficient DMUs, with a significant portion of the improvement attributed to cross-docking 3. In the subsequent scenario, a 10% increase in the risk of product damage is observed. As illustrated in Fig. 3, this alteration may lead to a reduction in the efficiency scores of inefficient DMUs, with the most pronounced decline associated with cross-docking 5. The third scenario indicates a 10% enhancement in the accuracy and timeliness of operations. This improvement is evident in the efficiency scores of the inefficient DMUs, particularly for cross-docking 3, which experiences the most substantial increase. In the fourth scenario, a similar 10% rise in the level of information sharing is implemented. This modification is expected to further boost the efficiency scores of the inefficient DMUs, with cross-docking 3 again showing the highest level of improvement. The findings depicted in Fig. 3 reveal that the proposed model is more responsive to scenario 2 than to the other scenarios. Consequently, it is imperative to consider corrective actions aimed at mitigating the risks associated with product damage. Furthermore, it can be inferred that even minor adjustments can have a substantial effect on inefficient DMUs, especially in cases where inefficiency is considerable.

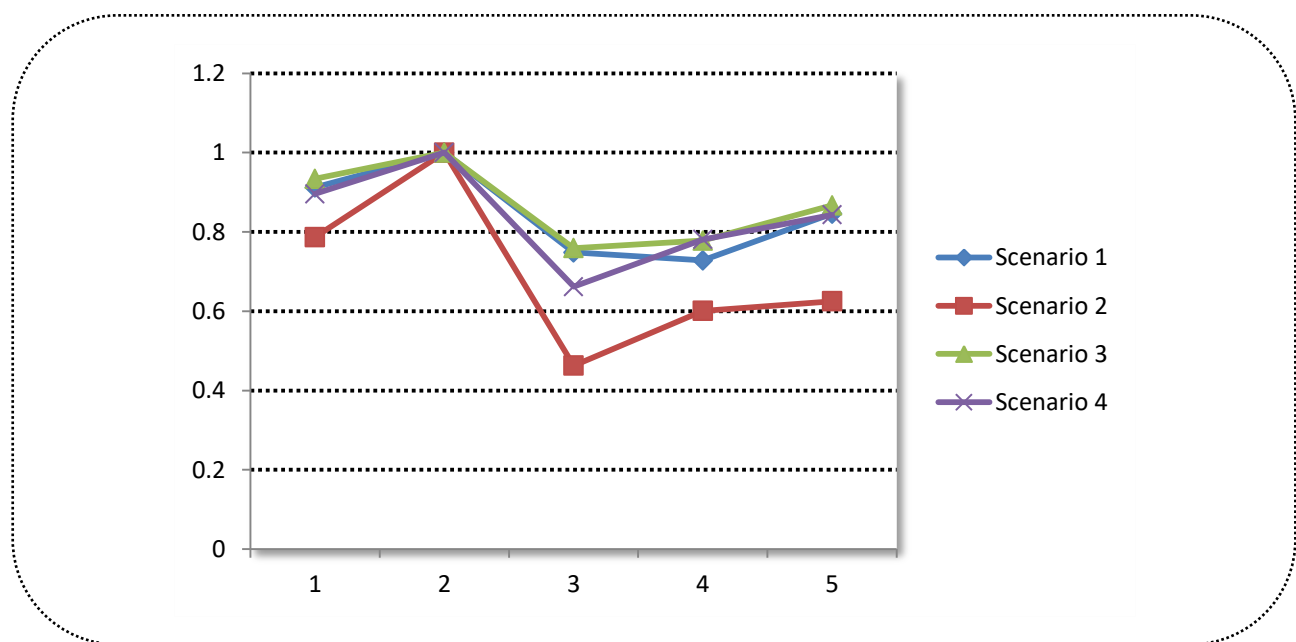


Fig. 3. Sensitivity analyses on the upper bound of systems' efficiency regarding 4 scenarios

B. Managerial insights

To offer managerial insights derived from the maximum potential of system efficiency, an in-depth examination is conducted on cross-docking systems 3 and 4, which are identified as the most inefficient. In order to gain insight into the structure of these systems, it is essential to highlight that cross-docking 3 first stores the received products. After an order is placed, it conducts the kitting process on these products and subsequently sends them to customers. So, in this cross-docking system, products are not able to be kitted immediately upon receipt and they are not transferred directly to outbound trucks for delivery to customers. Conversely, in cross-docking 4, there is no storage phase involved, as products are sorted right after they are received and then dispatched to outbound trucks.

Insight 1: The investigations into the parameters of cross-docking 3 reveal that a substantial proportion of products experience damage throughout the storage and recovery stages. Additionally, the inventory turnover rate in this system is alarmingly low, which casts doubt on the fundamental principles of a cross-docking system and, to some extent, reverts it to the characteristics of a traditional warehouse. To enhance this system, a direct transmission line that connects the inbound doors to the outbound doors is incorporated, as represented in the upper part of Fig. 2. The reassessment reveals that this adjustment yields a 33% improvement in the overall efficiency of the system. Consequently, enhancing operational capabilities within such a system has had a profound impact on its overall efficiency. It is important to clarify that while this enhancement can be readily attained from a mathematical perspective, managers must engage in meticulous decision-making and strategic planning to realize this improvement. Among the most significant measures are the augmentation of information sharing, the alteration of order types, the refinement of vehicle scheduling accuracy, and the preparation of relevant equipment.

Insight 2: A thorough examination of the parameters related to cross-docking 4 has indicated that the implementation of two distinct groups of parameters, along with a modification in layout, can significantly enhance the efficiency of such a system. Concerning the modification in layout, it is essential to highlight that the system includes a kitting division, where 14 operators work simultaneously. The configuration of this kitting division is straightforward and located at a significant distance from the outbound doors. Therefore, products that have been kitted must be transported to the front of the outbound doors with the assistance of a forklift or pallet jack. In an effort to boost the efficiency of this system, modifications have been made to the layout of the process to eliminate unnecessary movements. The final workstation in this process is now positioned close to the outbound doors. Consequently, this adjustment has led to a 21% improvement in system efficiency. Moreover, advancements in two factors associated with the flexibility of truck scheduling and the risk of unexpected changes in orders can further elevate system efficiency by 7% and 11%, respectively.

VI. CONCLUSION

This study presented an extensive framework for cross-docking systems aimed at assessing their efficiency, highlighting various operational elements that impacted their performance. These elements encompassed inbound and outbound doors, diverse transportation methods, inspection processes, kitting, storage, retrieval, and staging activities. A novel data envelopment analysis model was developed to assess such a system, incorporating a wide array of practical key performance indicators as both inputs and outputs within the model. The key performance indicators addressed a diverse set of issues, including automation, digitization, resilience, sustainability, and lean methodologies, to establish a comprehensive evaluation framework. Recognizing that many key performance indicators were linked to uncertainty and necessitated qualitative evaluations, a hybrid uncertainty approach was suggested, which integrated fuzzy reasoning techniques with Neutrosophic fuzzy programming. The findings indicated that enhancing operational capabilities could have a substantial impact on the efficiency of cross-docking systems. Additionally, a modest positive adjustment in such systems that are currently inefficient could result in a significant enhancement of their performance. The current study can be extended in various ways, with one primary option being the application of machine learning methods to estimate risk factors instead of relying on the fuzzy reasoning approach. Furthermore, investigating different types of data envelopment analysis models offers another promising avenue for research.

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