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Development of a simulation-based optimization approach to integrate condition-based maintenance, production control and control chart design in deteriorating production processes

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Abstract –A statistical model is proposed to schedule maintenance actions, control the inventory, and design a proper control chart in unreliable manufacturing systems. Process deterioration can lead to quality degradation and affect maintenance scheduling and production control. Integrated control of three aspects can enhance the productivity of production processes. A deteriorating manufacturing system is considered that has two states of in-control and out-of-control. Quality control of the products and process monitoring are implemented by employing a control chart. Maintenance actions are scheduled based on the machine's condition. The duration of maintenance is considered a continuous random variable that follows a general distribution. The purpose is to schedule the maintenance tasks and control the safety stock with respect to the data collected from the control chart related to the machine condition to minimize the total cost per time unit. A Genetic algorithm (GA) is used as a solution method. Then, to reduce the solution time of the proposed GA, a Monte Carlo simulation method is presented. These two methods are combinted, and as a result a simulation-optimization technique is proposed. The performance of the model is validated by a simulation-based optimization technique. Finally, sensitivity analysis of the model is performed.

***Keywords**– Control chart, Genetic algorithm, Imperfect manufacturing system, Monte Carlo simulation, Preventive maintenance.*

I. INTRODUCTION

The main goal of managers and engineers is to improve the productivity of production processes. There are many factors that can affect the performance of the production processes. Maintenance, production/inventory and quality have the main impacts on the efficiency of processes (Pandey et al., 2011). The availability of machines can be increased and the deterioration level of the process can be reduced by maintenance scheduling. Therefore, the quality of the produced items can be improved and the total quality cost can be decreased (Nourelfath et al., 2016). In imperfect manufacturing systems, the machines and the equipment may deteriorate during the production process, and the machine's deterioration can lead to quality degradation and defective items being produced. Producing non-conforming items has

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an impact on the production level and quality control can affect the production plan and decrease the cost. Therefore, there are interactions and interrelationships among maintenance scheduling, quality control, and production/inventory control and planning of these aspects simultaneously, which can enhance productivity and decrease cost (Hadian et al., 2023).

Quality degradation can be derived from process variation when an assignable cause occurs. Different statistical tools can be used to monitor the process and detect the assignable cause with the purpose of decreasing the process variation (Atashgar and Adelian, 2023). The stability of the process can be improved, and the assignable cause can be detected by employing a control chart, which is the most important method of SPC[1] and a widely used process monitoring tool (Mohammadipour et al., 2022). By applying this effective method, the process can be monitored and the quality of products will be enhanced (Rasay et al., 2022).

In deteriorating systems, the machines may break down, the process deteriorates during the production, and the state of the process transitions from an in-control (State 0) to an out-of-control (State 1) condition. With respect to the process state, maintenance actions are scheduled and implemented to restore the machine to an operational condition. During maintenance, the process is stopped, and the demand can be met by buffer stock. During production, some inventory can be maintained as safety stock to satisfy customer’s demand on time. Therefore, buffer stocks can be helpful to improve the flexibility of manufacturing systems (Alfieri et al., 2020; Shi et al., 2024).

The models of production and maintenance that consider these two aspects jointly have been developed by researchers (Kang & Subramaniam, 2018; Polotski et al., 2019; Yan et al., 2020; Boufellouh & Belkaid, 2020; Sharifzadegan et al., 2021; Hadian et al., 2023). Integrating the decisions of maintenance scheduling and quality control has been noticed by researchers in different studies. Process monitoring and quality control can be implemented by different policies.

Ardakan et al. (2016) described a model for joint planning of maintenance and quality control. They designed a multi-variate EWMA (MEWMA) chart to control the process variability and the failure mechanism following a Weibull distribution. Rasay et al. (2018) integrated the decisions of maintenance and control chart design for a two-stage process. Chi-square chart was employed for process monitoring. Gouiaa-Mtibaa et al. (2018) considered a deteriorating manufacturing system that produced conforming and non-conforming items and provided a model to plan PM and quality control. They assumed that some imperfect PM actions are conducted before a perfect PM and a 100% inspection policy was employed for quality control. The rework process was considered for substandard quality items and defective items. They considered the Weibull distribution for failure mechanism. Bahria et al. (2018) planned maintenance and quality control, simultaneously. An $\bar{x}$ chart was used to determine the machine’s condition. Corrective maintenance (CM) and PM were considered as maintenance actions that were conducted with constant time duration. Rasay et al. (2019) provided a model for maintenance and SPC for a two-stage process. In this paper, $\bar{x}$ chart and cause-selecting control (CSC) chart were applied to monitor the process state. They considered a general distribution for failure mechanism and developed the model according to the process state in an inspection interval. He et al. (2019) presented a model to schedule PM and designed a time-between-events (TBE) chart. The model was developed based on the Markov decision process. Rasay et al. (2022) proposed a scenario-based integrated model for the maintenance and design of a chart. They considered a general distribution for failure mechanism and a stochastic geometric process to indicate the impact of imperfect maintenance tasks. Finally, a numerical algorithm was employed as a solution method. Rasay et al. (2024) integrated condition-based maintenance and attributed it to the Bayesian chart. Markov decision process was applied for the process deterioration and Bayesian chart design. Shojaee et al. (2024a) proposed a statistical model to schedule maintenance actions and design an SVSSI-np control chart and the Genetic Algorithm (GA) was applied. In this study, it was indicated that the SVSSI scheme reduced costs and performed better in detecting failures.

1 . Statistical process control

According to the interrelationship among maintenance, production and quality, interactions between these aspects have been investigated in different studies by researchers with different approaches for each aspect. Pandey et al. (2011) planned these three aspects simultaneously. At first, a model was proposed to determine the PM interval and control chart parameters. Then, the production was scheduled with respect to the optimal PM interval to optimize the batch sequence and minimize the cost. Weibull distribution was considered for the transition time and it was assumed that the maintenance duration had constant values. Nourelfath et al. (2016) provided a model for the capacitated lot-sizing problem in an imperfect system. They considered constant values for the duration of maintenance. Product inspection was employed for quality control and they proposed a solution algorithm. Bouslah et al. (2018) considered a system with multiple stages. CM and age-based PM were implemented on the machine. It was considered a variable for the maintenance duration. With regard to the level of quality degradation, 0% or 100% inspection was employed. Finally, a discrete/continuous simulation approach was developed. Salmasnia et al. (2017) provided a model for maintenance, SPC and inventory and they assumed that multiple assignable causes lead to quality degradation and process deterioration. A Weibull distribution was considered for the failure mechanism and they neglected the maintenance duration. They designed an $\bar{x}$ chart and used a particle swarm optimization (PSO) algorithm. Cheng et al. (2018) described a model to control maintenance, production, and quality jointly. 100% inspection was applied for the quality monitoring. It was assumed that the failure mechanism had a gamma stochastic process. Finally, a simulation technique and the Response Surface methodology were applied as a simulation-optimization method. Salmasnia et al. (2018) described a model to schedule maintenance actions, control the production, and design a VP-T^2 Hotelling chart. Failure time of the process followed an exponential distribution. Finally, the PSO algorithm was used for cost minimization. Wang et al. (2019) considered an imperfect process. PM interval and batch size were optimized in the model. 100% inspection was applied for each produced batch to control the quality. They developed a mixed-integer non-linear programming model for the problem. Finally, the GA method was employed. Duffuaa et al. (2020) presented a binary model for PM planning, production scheduling, inventory control and an $\bar{x}$ chart design. They considered exponential distribution for shift time and a constant value for maintenance duration. A model was developed to schedule PM and production and control the inventory. Wang et al. (2020) considered a serial manufacturing system for the integrated planning. An inspection policy was employed for process monitoring. They considered gamma distribution for the machine deterioration and used a simulation method. Rivera-Gómez et al. (2020) considered a continuous system to provide an integrated model with a dynamic acceptance sampling plan, and a non-linear stochastic model was presented. Simulation techniques and response surface methodology were used for optimization. Cadi et al. (2021) presented an analytical model. They used a make-to-stock strategy for production and a 100% inspection policy. They assumed that the failure rate and the proportion of non-conforming items depend on the machine's age. They proposed an analytical model for the problem and used recursive equations to compute the objective function. A simulation model was applied to investigate the performance of the model. Tasias (2022) developed an integrated model of the three aspects, considering a Bayesian approach, and Tambe and Kulkarni (2022) presented a model for each of them. Fallahi et al. (2023) provided a convex model for PM scheduling, quality and production/inventory control. They considered transportation policies and environmental issues and provided an integrated model. A numerical algorithm was proposed. Wan et al. (2023) provided a joint mathematical model for continuous production systems with considering multiple assignable causes and compared the presented model with a model considering one assignable cause. They developed GA for optimization. Shojaee et al. (2024b) integrated the three aspects and proposed a statistical model. T^2 Hotelling control chart was used for process monitoring and the duration of maintenance had a constant value. They applied the PSO algorithm. Salmasnia et al. (2024) proposed a statistical model of three aspects for two-stage systems. They considered the possibility of occurring multiple assignable causes.

The integrated models are complex and the exact methods cannot solve such models in a reasonable time. Therefore, meta-heuristic algorithms are applied to different problems. GA was applied by Charongrattanasakul and Pongpullponsak (2011) to plan maintenance tasks and design the EWMA chart, jointly. GA was employed by Hadian et al. (2021) for maintenance and inventory. Wang et al. (2019), Hafidi et al. (2020), Sharifzadegan et al. (2021), Wan et al. (2023) and Shojaee et al. (2024a) used GA to solve the integrated models. Salmasnia et al. (2017), Salmasnia et al. (2018) and Shojaee et al. (2024b) applied the PSO algorithm. Also, the simulation approach has been employed by some researchers in joint planning of three aspects (Bouslah et al., 2016; Bouslah et al., 2018; Cheng et al., 2018; Cheng and Li, 2020; Wang et al., 2020; Rivera-Gómez et al., 2020; Cadi et al., 2021). Table I summarizes the main characteristics of the explained studies.

This study proposes a statistical scenario-based model to schedule condition-based maintenance, determine the optimal level of buffer stock, and develop an economic-statistical design for a proper control chart for quality monitoring. A scenario-based model is presented according to the different process states, and different scenarios in process state transition. The main purpose of this scenario-based model is to determine the optimal values of the quality control chart parameters, schedule the maintenance actions and determine the value of the stock level simultaneously. In the proposed model, different types of charts can be applied with respect to the production process condition when applying the model in practice. It is assumed that the machine has two states including State 0 and 1 and a general distribution is considered for the state's transition time. Then, the proposed stochastic model is compared with a model without quality control. Finally, Monte-Carlo simulation and GA are combined, and a simulation-based optimization technique is developed.

Table I. Main features of different studies

	Integrated approach					
	Maintenance and quality	**Maintenance, quality and inventory**	**Failure mechanism**	**Maintenance time duration**	**Quality control policy**	**Solution method**
Ardakan et al. (2016)	✓		Weibull distribution	Constant	MEWMA chart	Hooke and Jeeves algorithm
Rasay et al. (2018)	✓		A general distribution	Constant	Chi-square chart	Grid search algorithm
Gouiaa-Mtibaa et al. (2018)	✓		Weibull distribution	Constant	100% inspection policy	MATHEMATICA software
Bahria et al.(2018)	✓		A general distribution	Constant	$\bar{x}$ chart	A proposed numerical optimization method
Rasay et al. (2019)	✓		A general distribution	Constant	$\bar{x}$ chart and CSC chart	Grid search algorithm
He et al. (2019)	✓		Markov model	Constant	TBE chart	Dynamic search algorithm
Rasay et al. (2022)	✓		A general distribution	Constant	A proper control chart	A numerical algorithm
Rasay et al. (2024)	✓		Markov decision process	Constant	Attribute Bayesian chart	Markov decision process
Shojaee et al. (2024a)	✓		Markov chain	Constant	SVSSI-np chart	Genetic algorithm
Pandey et al. (2011)		✓	Weibull distribution	Constant	$\bar{x}$ chart	global optimization tool box of Maple
Nourelfath et al. (2016)		✓	A general distribution	Constant	Products inspection	A proposed solution algorithm
Bouslah et al. (2018)		✓	Stochastic process	Random variable	100% inspection	Simulation

Continue Table I. Main features of different studies

	Integrated approach					
	Maintenance and quality	**Maintenance, quality and inventory**	**Failure mechanism**	**Maintenance time duration**	**Quality control policy**	
Salmasnia et al. (2017)		✓	Weibull distribution	Neglected	$\bar{x}$ chart	PSO algorithm
Cheng et al. (2018)		✓	Gamma distribution	Random variable	100% inspection	Monte Carlo simulation and Response Surface methodology
Salmasnia et al. (2018)		✓	Exponential distribution	Neglected	VP-T^2 hotelling chart	PSO algorithm
Wang et al. (2019)		✓	General distribution	Random variable	100% inspection	Genetic algorithm
Duffuaa et al. (2020)		✓	Exponential distribution	Constant	$\bar{x}$ chart	A proposed solution algorithm
Wang et al. (2020)		✓	Gamma distribution	Constant	Periodic inspection	Simulation-based optimization
Rivera-Gómez et al. (2020)		✓	General distribution	Random variable	Dynamic sampling plan	Simulation and Response Surface methodology
Cadi et al. (2021)		✓	Machine age	Constant	100% inspection	Simulation model (signal-based approach)
Fallahi et al. (2023)		✓	Number of defective items	Constant	100 % inspection	Proposed a numerical algorithm
Wan et al. (2023)		✓	General distribution	Constant	$\bar{x}$ chart	Genetic algorithm
Shojaee et al. (2024b)		✓	General distribution	Constant	T^2 Hotelling chart	PSO algorithm
Salmasnia et al. (2024)		✓	General distribution	Constant	Cause-selecting control chart	PSO algorithm
This paper		✓	General distribution	Random variable with a general distribution	A proper chart	Monte Carlo Simulation-based optimization and GA

The structure of the research is organized as follows. In section II, the problem statement is presented. In section III, the scenarios in a cycle are elaborated. Section IV presents the proposed model. Numerical experiments and the optimization method are explained in Section V. Two models are compared in Section VI, and finally, the conclusions of the research are discussed in Section VII.

II. PROBLEM STATEMENT

A. Notation

Notation	Description
Decision variables	
h	Inspection interval
k	Maximum number of inspections in a cycle
l	The width of the control chart limits

n	Sample size
S	Buffer stock level
T	Maximum production run time ($T = (k+1)h$)
Parameters	
ARL_{in}	Average run length in State 0
ARL_{out}	Average run length in State 1
C_f	Fixed cost of sampling
C_{fa}	The unit cost of searching for a false alarm
C_h	The cost of inventory holding per unit item per unit time
C_{PM}	PM cost per time unit
$C_{CM}(C_{CM} > C_{PM})$	CM cost per time unit
C_{sh}	The shortage cost per unit item
C_v	Variable cost of sampling per item
C_0	Production cost in State 0 per time unit
$C_1(C_1 \geq C_0)$	Production cost in State 1 per time unit
P_1	Production rate
$P_2(P_2 \geq P_1)$	Maximum production rate
t_{ac}	Time duration to detect the assignable cause
t_{fa}	Time duration for searching a false alarm
t_{sa}	Time duration for one sampling and plot the statistic on chart
γ	Equal to 1 if the process is not stopped during searches for a false alarm, otherwise is equal to 0
X	A random variable denoting the transition time from State 0 (in-control) to State 1 (out-of-control)
$f(x)$	Probability density function (p.d.f) of X
$F(x)$	Cumulative distribution function (c.d.f) of X
$\bar{F}(x)$	$1 - F(x)$
Z_1	A random variable of the time duration of the PM
$g_1(z_1)$	p.d.f of Z_1
$G_1(z_1)$	c.d.f of Z_1
Z_2	A random variable of the time duration of the CM
$g_2(z_2)$	p.d.f of Z_2
$G_2(z_2)$	c.d.f of Z_2

B. Problem definition

An unreliable manufacturing system is considered with quality degradation that produces the products at a known rate. The deterioration of the process can lead to quality degradation. An assignable cause occurs at a random time during the production and the machine state shifts from 0 to 1. The shift time has a general distribution. The quality of the products is decreased, and non-conforming items may be produced in State 1. Therefore, the production cost of the system is increased.

The process is monitored by sampling inspection and applying a proper control chart. The data are collected from the chart and analyzed. Then, the process condition, the state transition, and the defective items can be detected by the chart. The process is inspected at time points h, $2h$, $3h$, …, kh. n products are randomly selected in each sampling and the computed statistic is plotted on a proper chart. If the statistic falls between the limits, the chart does not signal anything. Therefore, the process is in State 0 and the production is continued until T. Then, the process is stopped at T, the condition of the machine is controlled and a PM is performed. If the statistic falls outside the limits, the chart signals before the k^{th} sampling and the machine may be in State 1. The process is investigated to detect the assignable cause. If the investigation indicates that the process is in State 0, the production is continued. However, if it indicates that the process is in State 1, the production is stopped, a CM is performed and the machine is returned to the as-good-as-new condition.

The production is stopped during conducting maintenance. Since random variables (z_1,z_2) were considered for maintenance, the demand is met from the buffer stock during the maintenance. Therefore, the amount of stock is reduced with rate P_1. Then, the machine produces the parts with rate P_2 to reach the stock to S and then, the rate is set at P_1. It is assumed that after conducting the maintenance, the machine operates in State 0 until the amount of stock reaches S.

If $z_1 or\ z_2 \leq \frac{S}{P_1}$, no shortage happens, but, if $z_1 or\ z_2 > \frac{S}{P_1}$, shortage happens. The unsatisfied demands during the maintenance are lost.

The objective is to minimize the average total cost per time unit, including maintenance cost (PM, CM), quality cost (sampling, false alarms, operational costs in States 0 and 1) and inventory cost (inventory holding and shortage).

The assumptions considered in this study are presented as follows.

1. The production rate and demand rate are equal.
2. The machine is in State 0 at the beginning of the process and the products are initially produced in State 0
3. PM and CM’s actions are perfect
4. The duration of maintenance are random variables
5. After conducting maintenance, the machine is in State 0 until the stock level reaches S again
6. The unsatisfied demands are lost

III. POSSIBLE SCENARIOS DURING A CYCLE

There are three possible scenarios during each cycle depending on the process state and the performance of the control chart in detecting the states.

Scenario 1: The production is performed in State 0 until T. The machine is stopped at T, the condition of the machine is controlled and the PM is implemented on the machine. Then, the machine produces the parts with rate P_2 to reach the stock of S. Then, the items are produced with rate P_1. Figures 1(a) and 1(b) illustrate this scenario with shortage and without shortage, respectively. Equation (1) is the probability of occurrence.

$$P(S_1) = \bar{F}\big((k+1)h\big), \tag{1}$$

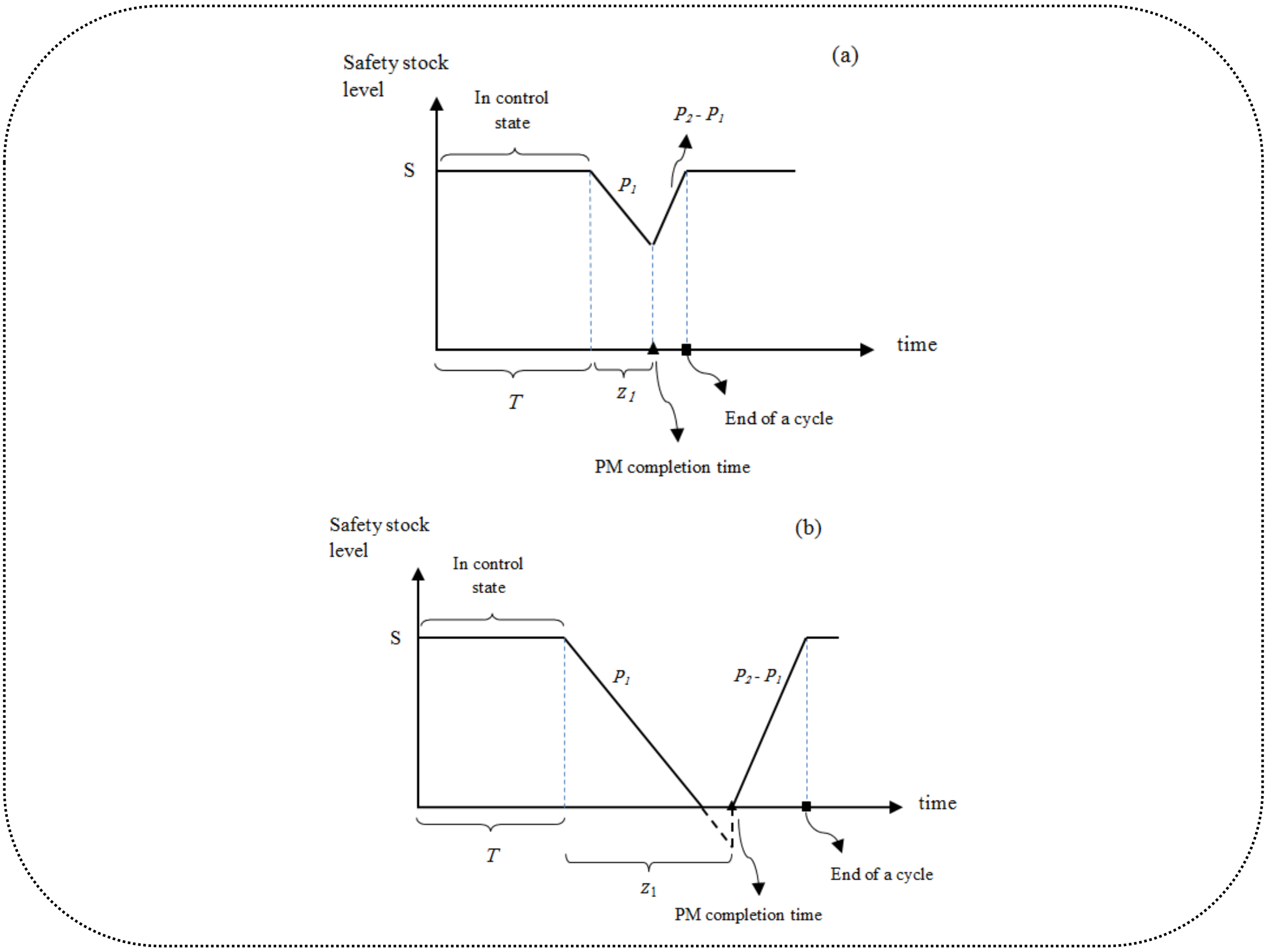

Fig. 1. A cycle in Scenario 1, a) without shortage, b) with shortage

Scenario 2: At first, the products are produced in State 0. At a random time (*x*) between the *j*th and (*j+1*)th inspection, the machine shifts to State 1 when an assignable cause occurs and the items are produced in State 1. The statistics of the samples are plotted on the chart, and it detects State 1 during the first *k* sampling. Therefore, the production is stopped and the CM is implemented. After that, the production is performed in State 0 with rate P_2 to reach the stock of *S*. Then, the machine produces the parts with rate P_1. Figures 2(a) and 2(b) illustrate this scenario with shortage and without shortage, respectively. Equation (2) is the probability of occurrence.

$$P(S_2) = P(control\ chart\ signals|out-of-control).F(kh), \tag{2}$$

$P(control\ chart\ signals|out-of-control)$ computes the probability of detecting the state shift while the machine is in State 1.

$$P(control\ chart\ signals|out-of-control) = 1 - \prod_{j=1}^{k-u} \beta, \tag{3}$$

Where *u* is the number of samples in State 0.

$$u = \left\lceil \frac{\int_0^{kh} xf(x)dx}{h} \right\rceil, \tag{4}$$

$\lceil . \rceil$ is the next largest integer value.

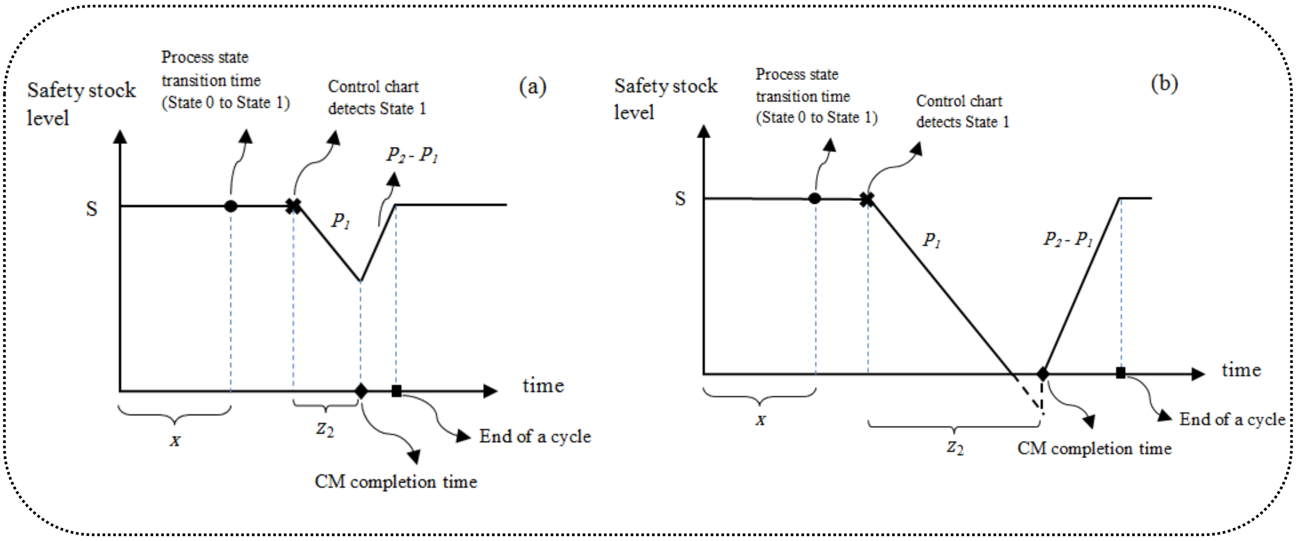

Fig2. A cycle in Scenario 2, a) without shortage, b) with shortage

Scenario 3: The products are initially produced in State 0. At a random time (x) between the j^{th} and $(j+1)^{th}$ inspection, the machine state shifts from 0 to 1 due to the occurrence of an assignable cause. However, State 1 cannot be detected by the chart due to the existence of Type II error (β) and the products are produced in State 1 until T. The machine is stopped at T, the condition of the machine is controlled and CM is implemented on the machine. Then, the machine produces the items in State 0 with rate P_2 to reach the stock to S. Then, the machine produces the parts with rate P_1. Figures 3(a) and 3(b) illustrate this scenario with shortage and without shortage, respectively. Equation (5) is the probability of occurrence.

$$P(S_3) = F\big((k+1)h\big) - P(control\ chart\ signals|out-of-control).F(kh), \tag{5}$$

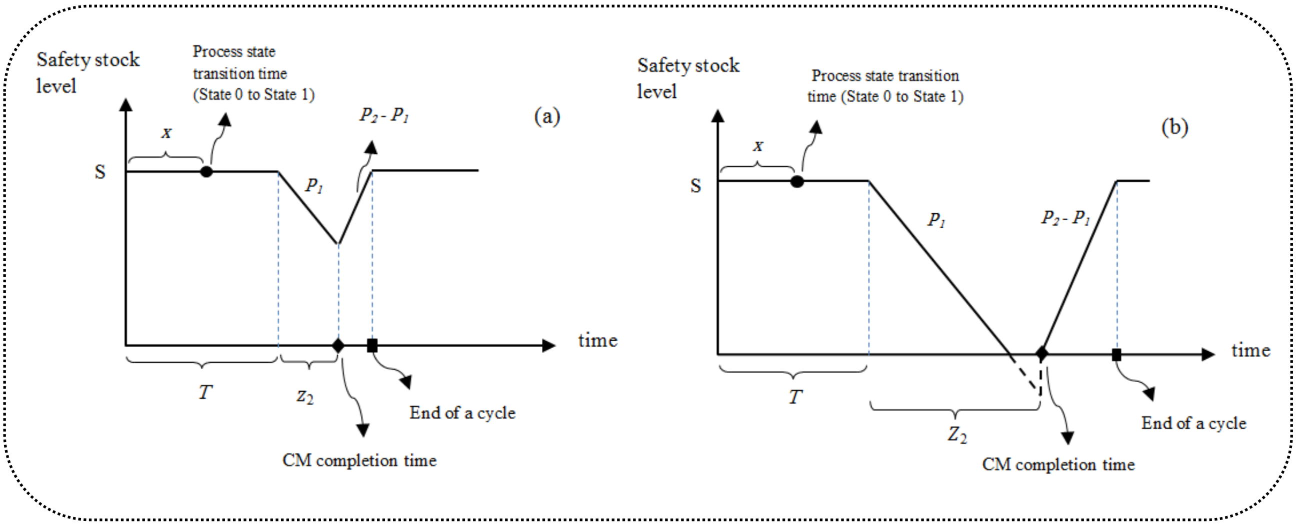

Fig3. A cycle in Scenario 3, a) without shortage, b) with shortage

IV. STATISTICAL MODEL OF MAINTENANCE, PRODUCTION AND QUALITY

The expected total cost per time unit (ETC) is obtained as follows:

$$ETC = \frac{E[TC]}{E[CL]}, \tag{6}$$

$E[CL]$ is the average time length of a cycle and is computed as follows:

$$E[CL] = P(S_1)E[CL|S_1] + P(S_2)E[CL|S_2] + P(S_3)E[CL|S_3], \tag{7}$$

Which $E[CL|S_i]$ is the expected duration of each cycle if the i^{th} scenario occurs and scenario i occurs with the probability $P(S_i), i = 1,2,3$.

$E[TC]$ is the average total cost in a cycle and is computed as follows:

$$E[TC] = E[M] + E[HC] + E[SH] + E[C_{in}] + E[C_{out}] + E[C_{inspection}] + E[FA], \tag{8}$$

$E[M]$ is the average cost of maintenance (costs of PM and CM). This cost is computed as follows:

$$E[M] = E[PM] + E[CM], \tag{9}$$

$E[HC]$ is the expected cost of inventory held in one cycle.

$$E[HC] = P(S_1)E[HC|S_1] + P(S_2)E[HC|S_2] + P(S_3)E[HC|S_3], \tag{10}$$

Which $E[HC|S_i], i = 1,2,3$ is the holding cost if the Scenario i occurs.

$E[SH]$ is the expected cost of shortage that happened in one cycle.

$$E[SH] = P(S_1)E[SH|S_1] + P(S_2)E[SH|S_2] + P(S_3)E[SH|S_3], \tag{11}$$

Which $E[SH|S_i], i = 1,2,3$ is the cost of shortage if the Scenario i occurs.

$E[C_{in}]$ is the average cost of production in State 0 and is obtained as follows:

$$E[C_{in}] = P(S_1)\, E[C_{in\text{-}control} \mid S_1] + P(S_2)\, E[C_{in\text{-}control} \mid S_2] + P(S_3)\, E[C_{in\text{-}control} \mid S_3], \tag{12}$$

Which $E[C_{in\text{-}control} \mid S_i]$ is the cost of production in State 0 if the i^{th} scenario occurs.

$E[C_{out}]$ is the average cost of production in State 1 and is computed as follows:

$$E[C_{out}] = P(S_1)\, E[C_{out\text{-}of\text{-}control} \mid S_1] + P(S_2)\, E[C_{out\text{-}of\text{-}control} \mid S_2] + P(S_3)\, E[C_{out\text{-}of\text{-}control} \mid S_3], \tag{13}$$

Which $E[C_{out\text{-}of\text{-}control} \mid S_i]$ is the cost of production in State 1 if the i^{th} scenario occurs.

$E[C_{inspection}]$ is the expected sampling cost and is obtained as follows:

$$E[C_{inspection}] = P(S_1)E[C_{inspection}|S_1] + P(S_2)E[C_{inspection}|S_2] + P(S_3)E[C_{inspection}|S_3], \tag{14}$$

Which $E\left[C_{inspection}\middle|S_i\right], i = 1,2,3$ is the average sampling cost if the i^{th} scenario occurs.

$E[FA]$ is the expected false alarms cost and is obtained as follows:

$$E[FA] = P(S_1)E[FA|S_1] + P(S_2)E[FA|S_2] + P(S_3)E[FA|S_3], \tag{15}$$

Which $E[FA|S_i], i = 1,2,3$ is the expected false alarm cost if the i^{th} scenario occurs.

A. Expected cost of maintenance

The PM is performed in Scenario 1 and the CM is performed in Scenario 2 or 3. Therefore, $E[PM]$ and $E[CM]$ are computed as follows:

$$E[PM] = P(S_1) \times C_{PM} \int_0^{\infty} z_1 g_1(z_1) dz_1 = P(S_1).C_{PM} \times E(z_1), \tag{16}$$

$$E[CM] = [P(S_2) + P(S_3)] \times C_{CM} \int_0^{\infty} z_2 g_2(z_2) dz_2 = [P(S_2) + P(S_3)] \times C_{CM} \times E(z_2), \tag{17}$$

The equations of each scenario are presented in the following:

B. Scenario 1

$E[CL|S_1]$ is obtained using the following formula:

$$E[CL|S_1] = E[T_{in\&m}|S_1] + E[T_{out}|S_1], \tag{18}$$

$E[T_{in\&m}|S_1]$ is the expected time length of State 0 and the PM if Scenario 1 happens and is calculated as follows:

$$[T_{in\&m}|S_1] = \frac{\left[\int_0^{\frac{S}{P_1}} \{ (k+1)h + (1-\gamma)\frac{k.t_{fa}}{ARL_{in}} + z_1 + \frac{P_1 z_1}{P_2 - P_1}\} g_1(z_1) dz_1\right]}{P(S_1)} + \frac{\left[\int_{\frac{S}{P_1}}^{\infty} \{(k+1)h + (1-\gamma)\frac{k.t_{fa}}{ARL_{in}} + z_1 + \frac{S}{P_2 - P_1}\} g_1(z_1) dz_1\right]}{P(S_1)}, \tag{19}$$

In the first statement, the time lengths of production in State 0 is computed before the PM, searching for false alarms, conducting the PM and the time length of State 0 after the PM is computed without shortage (PM duration is less than $\frac{S}{P_1}$). The second statement calculates this time length with shortage (PM duration is more than $\frac{S}{P_1}$). The production process in Scenario 1 is indicated in Figures 1(a) and 1(b). In Figure 1(a), the production is performed in State 0. The PM action is implemented at T in z_1 time unit so that $0 < z_1 < \frac{S}{P_1}$. After PM, the machine produces the parts with maximum rate to reach the stock of S which takes $\frac{P_1 z_1}{P_2 - P_1}$ time unit. In Figure 1(b), after PM, the machine produces the parts with maximum rate to reach the stock of S which takes $\frac{S}{P_2 - P_1}$ time unit.

$E[T_{out}|S_1]$ is the time length of State 1 (equal to 0 in Scenario 1).

$E[HC|S_1]$ is computed as the following:

$$E[HC|S_1] = \frac{C_h\left[\int_0^{\frac{S}{P_1}} \left\{s\big((k+1)h\big) + \left(z_1 + \frac{P_1 z_1}{P_2 - P_1}\right)s - \frac{1}{2}\left(z_1 + \frac{P_1 z_1}{P_2 - P_1}\right)P_1 z_1\right\} g_1(z_1) dz_1\right]}{P(S_1)} + \frac{C_h\left[\left\{s((k+1)h) + \frac{s^2}{2P_1} + \frac{s^2}{2(P_2 - P_1)}\right\}\overline{G_1}\left(\frac{S}{P_1}\right)\right]}{P(S_1)}, \tag{20}$$

In this equation, the expected holding cost is computed without shortage (PM duration is less than $\frac{S}{P_1}$) in the first statement. In the second statement, this cost is calculated with shortage (PM duration is more than $\frac{S}{P_1}$). In this equation, the surface area of the chart in Figures 1(a) and 1(b) are calculated.

$E[SH|S_1]$ is computed as follows:

$$E[SH|S_1] = \frac{C_{sh}\left[\int_{(k+1)h}^{\infty}\int_{\frac{S}{P_1}}^{\infty}(P_1 z_1 - s)g_1(z_1)f(x)dz_1 dx\right]}{P(S_1)}, \tag{21}$$

This equation calculates the shortage cost (PM duration is more than $\frac{S}{P_1}$).

$E[C_{in-control}|S_1]$ is computed as follows:

$$E[C_{in-control}|S_1] = \frac{C_0\left[\int_0^{\frac{S}{P_1}}\left\{(k+1)h+\frac{P_1 z_1}{P_2-P_1}\right\}g_1(z_1)dz_1\right]}{P(S_1)} + \frac{C_0\left[\int_{\frac{S}{P_1}}^{\infty}\left\{(k+1)h+\frac{S}{P_2-P_1}\right\}g_1(z_1)dz_1\right]}{P(S_1)}, \tag{22}$$

In the first statement, the operational cost in State 0 is calculated without shortage (PM duration is less than $\frac{S}{P_1}$). In the second statement, this cost is computed with shortage (PM duration is more than $\frac{S}{P_1}$). With respect to Figure 1(a), the production is performed in State 0 until *T*. PM action is implemented at *T*. After PM, the machine produces the parts with the maximum rate to reach the stock to *S* which takes $\frac{P_1 z_1}{P_2 - q_1}$ time unit. In Figure 1(b), after PM, the machine produces the parts with maximum rate to reach the stock of *S* which takes $\frac{s}{P_2-P_1}$ time unit.

$E[C_{out\text{-}of\text{-}control} \mid S_1]$ is equal to 0 in this scenario.

In Scenario 1, k samples are inspected during a cycle. Therefore, the sampling cost is computed as follows:

$$E[C_{inspection}|S_1] = \frac{(C_f + nC_v)k}{P(S_1)}, \tag{23}$$

$E[FA|S_1]$ is computed as follows:

$$E[FA|S_1] = \frac{\frac{k.C_{fa}}{ARL_{in}}}{P(S_1)}, \tag{24}$$

C. Scenario 2

$E[CL|S_2]$ is obtained using the following formula:

$$E[CL|S_2] = E[T_{in\&m}|S_2] + E[T_{out}|S_2], \tag{25}$$

$E[T_{in\&m}|S_2]$ is the expected time length of State 0 and the CM if Scenario 2 happens and is calculated as follows:

$$E[T_{in\&m}|S_2] = \frac{\int_0^{kh}\int_0^{\frac{S}{P_1}}\{x+(1-\gamma)\frac{m.t_{fa}}{ARL_{in}}+z_2+\frac{P_1 z_2}{P_2-P_1}\}f(x)g_2(z_2)dz_2dx}{P(S_2)} + \frac{\int_0^{kh}\int_{\frac{S}{P_1}}^{\infty}\{x+(1-\gamma)\frac{m.t_{fa}}{ARL_{in}}+z_2+\frac{S}{P_2-P_1}\}f(x)g_2(z_2)dz_2\,dx}{P(S_2)}, \quad (26)$$

In the first statement, the time lengths of State 0 before CM, searching for false alarms, conducting the CM and the time length of State 0 after CM are computed without shortage (CM duration is less than $\frac{S}{P_1}$). The second statement calculates this time length with shortage (CM duration is more than $\frac{S}{P_1}$). In Figure 2(a), the state shift occurs at *x*, and the CM is performed which takes z_2 time unit so that $0 < z_2 < \frac{S}{P_1}$. After the CM, the machine produces the parts with the maximum rate to reach the stock of S which takes $\frac{P_1 z_2}{P_2-P_1}$ time unit. The second term of this equation can be explained similarly.

In this equation, *m* is the expected number of samples in State 0 and can be obtained as follows:

$$m = E[number\ of\ samples\ taken\ from\ the\ process\ in\ an\ in-control\ state|scenario\ 2] = \sum_{j=0}^{k} jP(jh < x < (j+1)h) = \sum_{j=1}^{k} j[F((j+1)h) - F(jh)], \quad (27)$$

$E[T_{out}|S_2]$ is the expected time length of State 1 and is obtained as follows:

$$E[T_{out}|S_2] = \frac{h.ARL_{out} - \omega + n.t_{sa} + t_{ac}}{P(S_2)}, \quad (28)$$

Equation 28 includes the time duration between an assignable cause and the next sampling, the time of State 1 detection, the duration to plot a sample on the chart and the expected time to detect the assignable cause.

In equation 28, ω is the expected time length between the last inspection before the assignable cause and the occurrence of the assignable cause. The equation for this parameter is proved in Appendix A.

$E[HC|S_2]$ is computed as follows.

$$E[HC|S_2] = \frac{C_h\left[\int_0^{kh}\int_0^{\frac{S}{P_1}}\left\{s(x+h.ARL_{out}-\omega+n.t_{sa}+t_{ac})+\left(z_2+\frac{P_1z_2}{P_2-P_1}\right)s-\frac{1}{2}\left(z_2+\frac{P_1z_2}{P_2-P_1}\right)P_1z_2\right\}f(x)g_2(z_2)dz_2dx\right]}{P(S_2)} + \frac{C_h\left[\int_0^{kh}\left\{s(x+h.ARL_{out}-\omega+n.t_{sa}+t_{ac})+\frac{s^2}{2P_1}+\frac{s^2}{2(P_2-P_1)}\right\}\overline{G_2}\left(\frac{S}{P_1}\right)f(x)dx\right]}{P(S_2)}, \quad (29)$$

In this equation, the expected holding cost is calculated without shortage (CM duration is less than $\frac{S}{P_1}$), in the first statement. In the second statement this cost is computed with shortage (CM duration is more than $\frac{S}{P_1}$). In this equation, the surface area of the chart in Figures 2(a) and 2(b) are calculated.

$E[SH|S_2]$ is computed as follows.

$$E[SH|S_2] = \frac{C_{sh}\left[\int_0^{kh}\int_{\frac{S}{P_1}}^{\infty}(P_1 z_2 - s)g_2(z_2)f(x)dz_2dx\right]}{P(S_2)}, \tag{30}$$

This equation calculates the shortage cost (CM duration is more than $\frac{S}{P_1}$).

$E[C_{in-control}|S_2]$ is computed as follows.

$$E[C_{in-control}|S_2] = \frac{C_0\left[\int_0^{kh}\int_0^{\frac{S}{P_1}}\left\{x + \frac{P_1 z_2}{P_2 - P_1}\right\}f(x)g_2(z_2)dz_2dx\right]}{P(S_2)} + \frac{C_0\left[\int_0^{kh}\int_{\frac{S}{P_1}}^{\infty}\left\{x + \frac{S}{P_2 - P_1}\right\}f(x)g_2(z_2)dz_2dx\right]}{P(S_2)}, \tag{31}$$

In the first statement, the expected cost of production in State 0 is calculated without shortage (CM duration is less than $\frac{S}{P_1}$). In the second statement, this cost is computed with shortage (CM duration is more than $\frac{S}{P_1}$). With respect to Figure 2(a), the shift of the process occurs at x, the shift is detected by the chart and CM action is implemented. After the CM, the machine produces the parts with maximum rate to reach the stock of S which takes $\frac{P_1 z_2}{P_2 - P_1}$ time unit. The second term of this equation can be explained similarly with respect to Figure 2(b).

$E\left[C_{out-of-control}\middle|S_2\right]$ is computed as follows:

$$E\left[C_{out-of-control}\middle|S_2\right] = \frac{C_1(h.ARL_{out} - \tau + n.t_{sa} + \gamma t_{ac})}{P(S_2)}, \tag{32}$$

The sampling cost is computed as follows:

$$E[C_{inspection}|S_2] = \frac{(C_f + nC_v)\frac{\int_0^{kh} xf(x)dx + h.ARL_{out} - \omega + n.t_{sa} + \gamma t_{ac}}{h}}{P(S_2)}, \tag{33}$$

$E[FA|S_2]$ is computed as follows:

$$E[FA|S_2] = \frac{\frac{m.C_{fa}}{ARL_{in}}}{P(S_2)}, \tag{34}$$

D. Scenario 3

$E[CL|S_3]$ is obtained from the following formula:

$$E[CL|S_3] = E[T_{in\&m}|S_3] + E[T_{out}|S_3], \tag{35}$$

$E[T_{in\&m}|S_3]$ is the expected time length of State 0 and the CM if Scenario 3 happens.

$$E[T_{in\&m}|S_3] = \frac{\int_0^{(k+1)h}\int_0^{\frac{S}{P_1}}\left\{x + (1-\gamma)\frac{m.t_{fa}}{ARL_{in}} + z_2 + \frac{P_1 z_2}{P_2 - P_1}\right\}f(x)g_2(z_2)dz_2dx}{P(S_3)} + \frac{\int_0^{(k+1)h}\int_{\frac{S}{P_1}}^{\infty}\{x + (1-\gamma)\frac{m.t_{fa}}{ARL_{in}} + z_2 + \frac{S}{P_2 - P_1}\}f(x)g_2(z_2)dz_2dx}{P(S_3)}, \tag{36}$$

In the first statement, the time lengths of State 0 before CM, searching for false alarms, conducting CM and the time length of State 0 after CM are computed without shortage (CM duration is less than $\frac{S}{P_1}$). The second statement calculates this time length with shortage (CM duration is more than $\frac{S}{P_1}$). With respect to Figure 3(a), the process state transits at *x*, and CM action is performed. After CM, the machine produces the parts with the maximum rate to reach the stock to *S* which takes $\frac{P_1 z_2}{P_2 - P_1}$ time unit. The second term of this equation can be explained similarly with respect to Figure 3(b).

$E[T_{out}|S_3]$ is the expected time length of State 1 and is obtained as follows:

$$E[T_{out}|S_3] = \frac{(k+1)h - \int_0^{(k+1)h} x f(x) dx}{P(S_3)}, \tag{37}$$

In this scenario, the process shift occurs at *x* but the chart cannot detect State 1 until *T* and the CM is conducted at this time. This equation computes the difference between the production run time, (*k+1*)*h*, and the average time length of State 0.

$E[HC|S_3]$ is computed as follows:

$$E[HC|S_3] = \frac{C_h \left[\int_0^{(k+1)h} \int_0^{\frac{S}{P_1}} \left\{ s\big((k+1)h\big) + \left(z_2 + \frac{P_1 z_2}{P_2 - P_1}\right) s - \frac{1}{2}\left(z_2 + \frac{P_1 z_2}{P_2 - P_1}\right) P_1 z_2 \right\} g_2(z_2) f(x) dz_2 dx\right]}{P(S_3)} + \frac{C_h \left[\int_0^{(k+1)h} \left\{ s\big((k+1)h\big) + \frac{s^2}{2P_1} + \frac{s^2}{2(P_2 - P_1)} \right\} \overline{G_2}\left(\frac{s}{P_1}\right) f(x) dx\right]}{P(S_3)}, \tag{38}$$

In this equation, the expected holding cost is calculated without shortage (CM duration is less than $\frac{S}{P_1}$), in the first statement. In the second statement this cost is computed with shortage (CM duration is more than $\frac{S}{P_1}$). In this equation, the surface area of the chart in Figures 3(a) and 3(b) are calculated.

$E[SH|S_3]$ is computed as follows:

$$E[SH|S_3] = \frac{C_{sh} \left[\int_0^{(k+1)h} \int_{\frac{S}{P_1}}^{\infty} (P_1 z_2 - s) g_2(z_2) f(x) dz_2 dx\right]}{P(S_3)}, \tag{39}$$

This equation calculates the shortage cost (CM duration is more than $\frac{S}{P_1}$).

$E[C_{in-control}|S_3]$ is computed as follows:

$$E[C_{in-control}|S_3] = \frac{C_0 \left[\int_0^{(k+1)h} \int_0^{\frac{S}{P_1}} \left\{ x + \frac{P_1 z_2}{P_2 - P_1} \right\} f(x) g_2(z_2) dz_2 dx\right]}{P(S_3)} + \frac{C_0 \left[\int_0^{(k+1)h} \int_{\frac{S}{P_1}}^{\infty} \left\{ x + \frac{S}{P_2 - P_1} \right\} f(x) g_2(z_2) dz_2 dx\right]}{P(S_3)}, \tag{40}$$

The expected production cost in State 0 is calculated without shortage (CM duration is less than $\frac{S}{P_1}$), in the first statement. In the second statement, this cost is computed with shortage (CM duration is more than $\frac{S}{P_1}$). With respect to

Figure 3(a), the machine state transits to State 1 at x and CM is performed. After CM, the machine produces the parts with maximum rate to reach the stock to S which takes $\frac{P_1 z_2}{P_2 - P_1}$ time unit. The second term of this equation can be explained similarly with respect to Figure 3(b).

$E\left[C_{out-of-control}\middle|S_3\right]$ is obtained as follows:

$$E\left[C_{out-of-control}\middle|S_3\right] = \frac{C_1\left[(k+1)h - \int_0^{(k+1)h} xf(x)dx\right]}{P(S_3)}, \tag{41}$$

In this scenario, the chart cannot detect State 1 during the production and *k* samples are inspected during a cycle. Therefore, the sampling cost is computed as follows:

$$E[C_{inspection}|S_3] = \frac{\left(C_f + nC_v\right)k}{P(S_3)}, \tag{42}$$

$E[FA|S_3]$ is computed as follows.

$$E[FA|S_3] = \frac{\frac{m.C_{fa}}{ARL_{in}}}{P(S_3)}, \tag{43}$$

E. Integration of maintenance and production

In this subsection, the integration of maintenance and production is investigated, but quality control is not considered. Two scenarios may occur. In Scenario 1, the machine is in State 0 during the cycle. Then, the process is stopped at time *T* and the PM is performed on the machine. After that, the parts are produced with maximum rate to reach the stock of *S*. In Scenario 2, the shift of the machine occurs at *x*, and the machine produces the products in State 1 until *T*. Then, the machine is stopped, the CM is performed on the machine, and the machine is returned to State 0. Then, the machine produces the parts at the maximum rate. There are two decision variables: the time to conduct the PM (*T*) and the amount of safety stock (*S*). These scenarios occurr with the following probabilities:

$$P(S_1) = \bar{F}(T) \quad, P(S_2) = F(T), \tag{44}$$

$$ETC' = \frac{E[TC']}{E[CL']}, \tag{45}$$

$$\text{E}[TC'] = \text{E}[M'] + \text{E}[HC'] + \text{E}[SH'] + \text{E}[C_{in}{}'] + \text{E}[C_{out}{}'], \tag{46}$$

$E[M']$, $E[HC']$, $E[SH']$, $E[C_{in}{}']$ and $E[C_{out}{}']$ are the expected costs of maintenance, inventory, shortage, the operational cost in State 0 and 1, respectively.

The equations of each cost are presented in the following:

$$E[M'] = E[PM] + E[CM] = \left[P(S_1) \times C_{PM} \int_0^\infty z_1 g_1(z_1)dz_1\right] + \left[P(S_2) \times C_{CM} \int_0^\infty z_2 g_2(z_2)dz_2\right], \tag{47}$$

$$E[HC'] = P(S_1)\left[\frac{C_h\left[\int_0^{\frac{S}{P_1}}\left\{sT+\left(z_1+\frac{P_1z_1}{P_2-P_1}\right)s-\frac{1}{2}\left(z_1+\frac{P_1z_1}{P_2-P_1}\right)P_1z_1\right\}g_1(z_1)dz_1\right]}{P(S_1)}+\frac{C_h\left[\left\{sT+\frac{s^2}{2P_1}+\frac{s^2}{2(P_2-P_1)}\right\}\overline{G_1}\left(\frac{s}{P_1}\right)\right]}{P(S_1)}\right]+ P(S_2)\left[\frac{C_h\left[\int_0^{\frac{S}{P_1}}\left\{sT+\left(z_2+\frac{P_1z_2}{P_2-P_1}\right)s-\frac{1}{2}\left(z_2+\frac{P_1z_2}{P_2-P_1}\right)P_1z_2\right\}g_2(z_2)dz_2\right]}{P(S_2)}+\frac{C_h\left[\left\{sT+\frac{s^2}{2P_1}+\frac{s^2}{2(P_2-P_1)}\right\}\overline{G_2}\left(\frac{s}{P_1}\right)\right]}{P(S_2)}\right], \tag{48}$$

$$E[SH'] = P(S_1)\left[\frac{C_{sh}\int_{\frac{S}{P_1}}^{\infty}(P_1z_1-s)g_1(z_1)dz_1}{P(S_1)}\right]+P(S_2)\left[\frac{C_{sh}\int_{\frac{S}{P_1}}^{\infty}(P_1z_2-s)g_2(z_2)dz_2}{P(S_2)}\right], \tag{49}$$

$$E[C_{in}'] = P(S_1)\left[\frac{C_0\int_0^{\frac{S}{P_1}}(T+\frac{P_1z_1}{P_2-P_1})g_1(z_1)dz_1}{P(S_1)}+\frac{C_0\int_{\frac{S}{P_1}}^{\infty}(T+\frac{S}{P_2-P_1})g_1(z_1)dz_1}{P(S_1)}\right]+ P(S_2)\left[\frac{C_0\int_0^T\int_0^{\frac{S}{P_1}}\left(x+\frac{P_1z_2}{P_2-P_1}\right)f(x)g_2(z_2)dz_2dx}{P(S_2)}+\frac{C_0\int_0^T\int_{\frac{S}{P_1}}^{\infty}\left(x+\frac{S}{P_2-P_1}\right)f(x)g_2(z_2)dz_2dx}{P(S_2)}\right], \tag{50}$$

$$E[C_{out}'] = P(S_2)\left[\frac{C_1\int_0^T(T-x)f(x)dx}{P(S_2)}\right], \tag{51}$$

$$E[CL'] = P(S_1)\left[\frac{\int_0^{\frac{S}{P_1}}\left(T+z_1+\frac{P_1z_1}{P_2-P_1}\right)g_1(z_1)dz_1+\int_{\frac{S}{P_1}}^{\infty}\left(T+z_1+\frac{S}{P_2-P_1}\right)g_1(z_1)dz_1}{P(S_1)}\right]+ P(S_2)\left[\frac{\int_0^{\frac{S}{P_1}}\left(T+z_2+\frac{P_1z_2}{P_2-P_1}\right)g_2(z_2)dz_2+\int_{\frac{S}{P_1}}^{\infty}\left(T+z_2+\frac{S}{P_2-P_1}\right)g_2(z_2)dz_2}{P(S_2)}\right], \tag{52}$$

V. OPTIMIZATION AND NUMERICAL EXPERIMENTS

The presented model is complex, and also, due to the existence of continuous and discrete decision variables within the limits of integrals in some equations, the exact methods cannot reach an optimal solution in a reasonable time. Therefore, at the first step, the GA method is employed for optimization by the MATLAB software. In subsection 2, to reduce the GA solution time and to validate the model, a Monte Carlo (MC) simulation method is applied, the GA and the simulation methods are combined, and a simulation-based optimization technique is proposed. In this approach, the value of *ETC* for each chromosome in a solution of GA is calculated using MC. In the last subsection, a comprehensive sensitivity analysis is provided.

A. Optimization with Genetic Algorithm

GA is a meta-heuristic method that has been applied to optimize the problems related to the joint models. This algorithm is applied in different scientific fields, such as engineering, operations research and computer science (Wang et al. 2019). GA can be applied for optimization in different types of complex problems, it uses crossover and mutation to reach a near-optimal solution and is easy to perform (Charongrattanasakul and Pongpullponsak, 2011). Therefore, GA is employed in this study.

In GA, a chromosome is considered for each solution, and each chromosome includes six genes. The values of the decision variables are assigned to the first five genes, and the objective function is assigned to the sixth gene. At first, the initial population is generated randomly. A reasonable numerical range is considered for each decision variable, and a random value is selected from this interval for each decision variable to generate the initial population. The random values for all the decision variables are generated based on a uniform distribution in the considered numerical intervals and integer random values are generated for the decision variables *n* and *k*. Then, the Roulette wheel method is used to select some chromosomes as parents to perform crossover and mutation on them. The crossover operation is performed by a uniform single-point method. After these two operations, the new solutions are obtained and by using the roulette wheel method, the new solution is replaced. Studies of Yin et al. (2015) and Charongrattanasakul and Pongpullponsak (2011) are used to determine the GA parameters. Therefore, 80 is considered for generation number, 20 for population size and 0.4 and 0.1 are considered for crossover rate and mutation rate, respectively.

As mentioned before, the random variables related to the transition time and maintenance are considered with general distributions. For numerical analysis, the variables x, z_1 and z_2 are considered with the Weibull distribution. The Weibull distribution can used to indicate the failure of industrial machines. This distribution is simple to interpret and can be used as the distribution of the maintenance actions time duration and the distribution of the time duration between failures (Ardakan et al., 2016). The importance of this distribution in different types of industrial machines was studied by McWilliams (1989), Dodson (1994) ,and Hopp and Spearman (1996). Then, it was employed in maintenance and production planning (Fakher et al., 2018, Bouslah et al., 2018, Salmasnia et al., 2017, Salmasnia et al., 2020, Cadi et al., 2021, Rasay et al., 2022). The P.D.F of distributions are as follows (Rasay et al., 2022).

$$f(x) = \lambda v(\lambda x)^{v-1} e^{-(\lambda x)^v} \quad ; x \geq 0 \,, \lambda \geq 0 \,, v \geq 1 \tag{53}$$

$$g_1(z_1) = \lambda_1 v_1 (\lambda_1 z_1)^{v_1 - 1} e^{-(\lambda_1 z_1)^{v_1}} \quad ; z_1 \geq 0 \,, \lambda_1 \geq 0 \,, v_1 \geq 1 \tag{54}$$

$$g_2(z_2) = \lambda_2 v_2 (\lambda_2 z_2)^{v_2 - 1} e^{-(\lambda_2 z_2)^{v_2}} \quad ; z_2 \geq 0 \,, \lambda_2 \geq 0 \,, v_2 \geq 1 \tag{55}$$

v, v_1 and v_2 are the shape parameters and λ, λ_1 and λ_2 are scale parameters of the Weibull distributions.

For the numerical experiment, an $\bar{x}$ chart is applied and the chart limits are as follows (Montgomery, 2009):

$$\mu_0 \pm l \frac{\sigma}{\sqrt{n}} \ , \tag{56}$$

l is the width of the control limits. The quality characteristic follows a normal distribution with mean μ_0 and standard deviation σ in State 0. The occurrence of an assignable cause leads to the transition of the process mean from μ_0 to $\mu_1 = \mu_0 \pm \delta\sigma$. In this chart, α (the probability of Type I error) and β (The probability of Type II error) are computed as follows (Montgomery, 2009):

$$\alpha = 2\emptyset(-l), \tag{57}$$

$$\beta = \emptyset\left(l - \delta\sqrt{n}\right) - \emptyset(-l - \delta\sqrt{n}), \tag{58}$$

Φ is the C.D.F of the normal distribution. δ is the magnitude of the shift and is constant.

Two constraints are considered in the modeling. The constraint $ARL_{\text{in}} \geq 100$ can help to decrease the false alarms and the constraint $ARL_{\text{out}} \leq 10$ can help earlier detection of State 1. ARL_{in} and ARL_{out} are obtained as follows (Montgomery, 2009):

$$ARL_{in} = \frac{1}{\alpha} \ , \tag{59}$$

$$ARL_{out} = \frac{1}{1 - \beta} \ , \tag{60}$$

Data from the studies of Linderman et al. (2005) and Chakraborty and Giri (2012) are used in the numerical experiments. Table II shows the data and the parameters of the Weibull distribution.

Table II. Data used for the optimization

δ	γ	t_{fa}	t_{sa}	t_{ac}	C_f	C_v	C_0	C_1	C_{fa}	C_h
1	1	1	0.01	1	1	0.2	100	300	200	0.5
C_{sh}	C_{PM}	C_{CM}	P_1	P_2	v	v_1	v_2	λ	λ_1	λ_2
3	2400	5000	90	160	2	2	2	0.3	0.4	0.4

An example is optimized with GA using the data presented in Table II. Table III illustrates the optimization results. Every 2.2604 time unit, the process is inspected with n = 26 and the statistics are plotted on $\bar{x}$ chart which l is 3.1616. At most, 27 inspections are implemented in a cycle and 101.0751 units of products are maintained as the buffer stock. The minimum value of cost is 315.9644.

Table III. The optimization results

n	h	k	l	S	T	ETC
26	2.2604	27	3.1616	101.0751	63.2912	315.9644

B. Development of a simulation-based optimization approach

During the run of the proposed GA, it is observed that due to the complexities of the equations, the GA is slow to find the optimal solution. Hence, in the following, we proceed to develop a simulation approach. An MC simulation is proposed (using MATLAB software), the GA is combined with the simulation method and a simulation-optimization technique is proposed. Then, the value of the objective function for each chromosome is calculated using the simulation method.

MC simulation can be employed for stochastic problems. In this method, random numbers are generated randomly for the random variables of the model to simulate the stochastic events. In our study, the shift time of the states (x) and the time length of maintenance (z_1, z_2) are stochastic events.

The optimal values of *n*, *h*, *k*, *l*, *S* and *ETC* are obtained by optimizing the model with GA. Then, for the given values of the decision variables, the simulation method runs 10,000 times and the expected values of *ETC* are considered as the fitness function of the chromosome. For this purpose, the model is solved with data presented in Table IV and the optimization results are illustrated in Table V. Figure 4 shows the stages of the simulation method.

Table IV. Data used for simulation

δ	γ	t_{fa}	t_{sa}	t_{ac}	C_f	C_v	C_0	C_1	C_{fa}	C_h
1	1	1	0.01	1	1	0.2	100	300	200	0.2
C_{sh}	C_{PM}	C_{CM}	P_1	P_2	v	v_1	v_2	λ	λ_1	λ_2
0.8	5	50	90	160	2	2	2	0.3	0.4	0.4

Table V. The optimal values obtained in simulation-optimization approach

n	h	k	l	S	ETC (GA)	ETC(simulation)
26	1.2702	17	3.5390	170.7902	195.9067	172.5832

To validate the efficiency of MC, the simulation and GA results are compared. The comparison results are shown in Table VI and Figure 5. The difference percentage between the *ETC* values in GA and simulation is on average 4.79% and this difference is negligible. These results indicate the effectiveness of the MC simulation method.

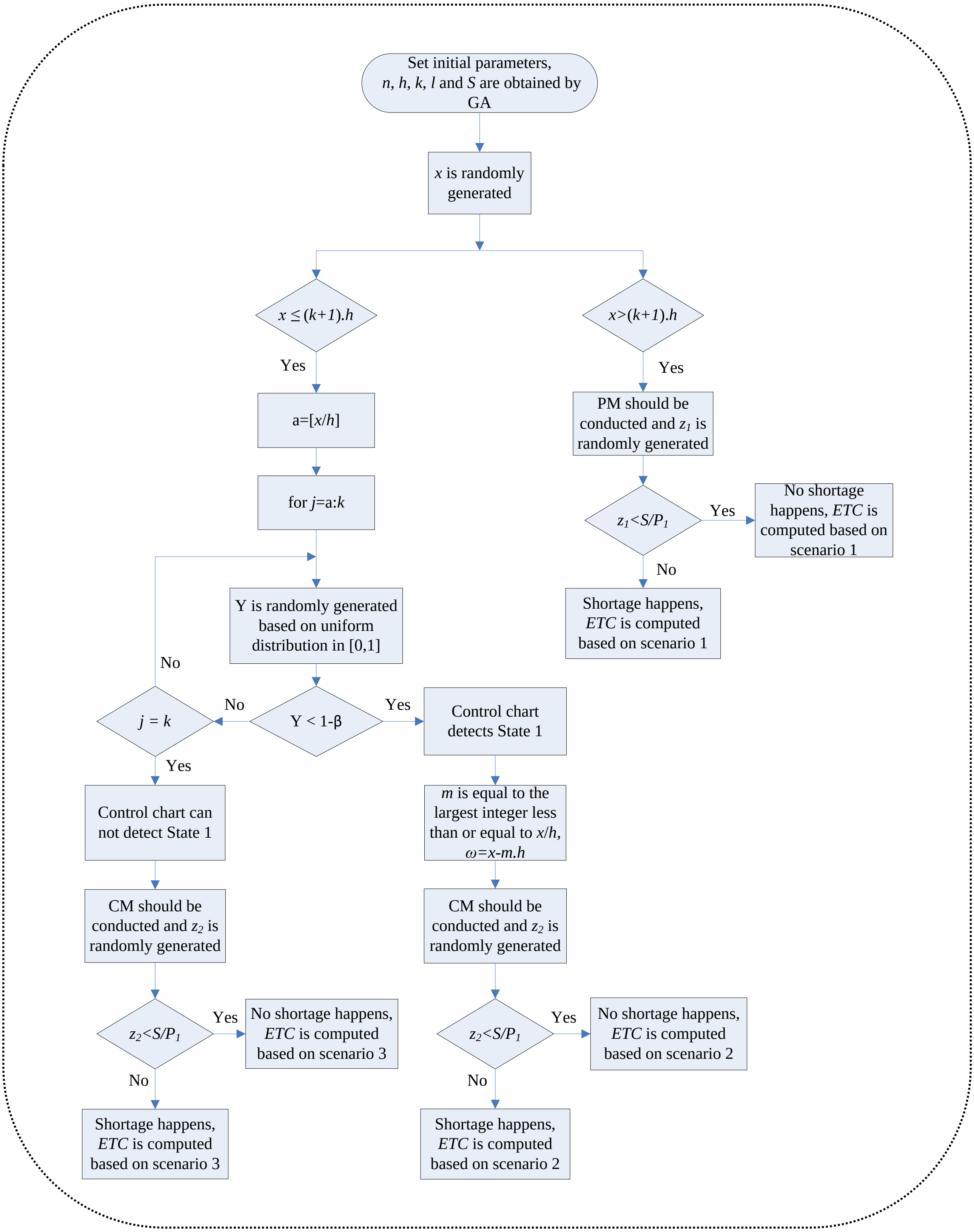

Fig4. The procedure of Monte Carlo simulation

Table VI. The comparison results of GA and simulation

Problem number			*n*	*h*	*k*	*l*	*S*	*ETC* (GA)	*ETC* (simulation)	Percentage of difference
1	λ	0.2	18	1.4671	2	2.7619	274.3534	172.0280	162.0288	6.17
2		0.5	37	1.8688	27	2.7803	238.2210	231.2160	219.9691	5.11
3	λ_1	0.2	20	1.1311	14	2.8703	187.4542	179.7848	171.8688	4.61
4		0.6	31	2.3051	29	2.9727	188.3503	217.7064	218.6562	0.44
5	λ_2	0.2	13	1.1235	2	3.7344	257.9689	159.3250	165.9091	4.13
6		0.6	32	1.2730	14	2.5763	107.4559	186.4484	173.7539	7.31
7	C_{PM}	3	15	1.1805	29	3.5130	276.9449	225.6008	216.3884	4.26
8		10	26	2.3817	8	3.4348	207.3098	198.9180	186.0150	6.94
9	C_{CM}	30	10	1.8150	24	2.5951	122.6047	204.3017	200.8812	1.70
10		70	31	2.3504	13	3.6359	292.0408	226.7909	202.7507	11.86
11	C_h	0.4	34	2.6290	3	3.3761	121.0147	184.2490	195.8167	6.28
12		0.6	9	1.2985	3	3.2007	121.7420	187.2603	193.6289	3.40
13	C_s	2	20	2.6116	10	2.6496	117.2465	196.8881	192.0686	2.51
14		5	11	2.8272	15	3.1619	130.5050	213.4961	215.9577	1.15
15	C_v	0.5	10	2.2457	11	2.7922	179.8844	203.3491	196.0001	3.75
16		1	3	2.3311	28	2.8616	223.5236	238.8739	255.0282	6.76
17	C_1	200	38	1.5537	17	2.9329	197.7159	170.4753	155.4500	9.67
18		500	29	1.3114	12	3.7178	209.3591	284.2095	284.6878	0.17

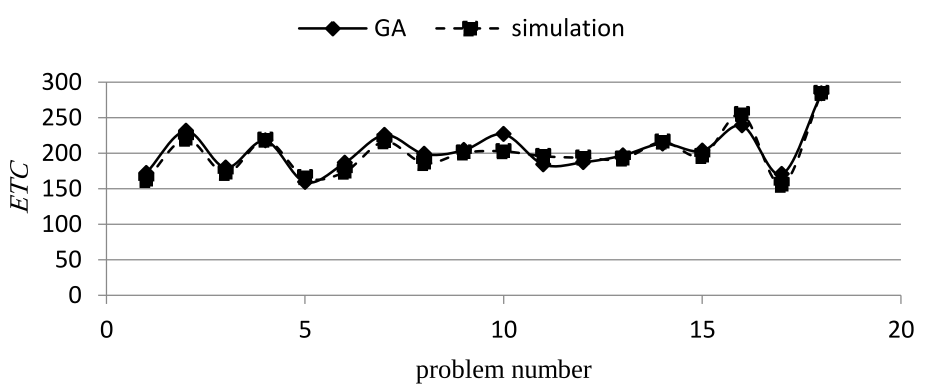

Fig5. Comparing the *ETC* values in GA and simulation

Table VII. The results obtained for decision variables changes in simulation

h	k	l	s	n	ETC (simulation)
1.2702	17	3.5390	170.7902	5	262.9787
1.2702	17	3.5390	170.7902	10	202.1584
1.2702	17	3.5390	170.7902	15	181.1411
1.2702	17	3.5390	170.7902	18	176.0451
1.2702	17	3.5390	170.7902	20	174.0175
1.2702	**17**	**3.5390**	**170.7902**	**26**	**172.5832**
1.2702	17	3.5390	170.7902	30	172.8886
1.2702	17	3.5390	170.7902	35	173.6646
1.2702	17	3.5390	170.7902	38	174.5488
1.2702	17	3.5390	170.7902	40	175.0774

n	k	l	s	h	ETC (simulation)
26	17	3.5390	170.7902	1	184.1571
26	17	3.5390	170.7902	1.1	179.2076
26	17	3.5390	170.7902	1.2	172.7548
26	**17**	**3.5390**	**170.7902**	**1.2702**	**172.5832**
26	17	3.5390	170.7902	1.5	174.0422
26	17	3.5390	170.7902	1.7	176.1765
26	17	3.5390	170.7902	2	178.5680
26	17	3.5390	170.7902	2.3	180.9000
26	17	3.5390	170.7902	2.6	183.2305
26	17	3.5390	170.7902	3	186.3770

n	h	l	s	k	ETC (simulation)
26	1.2702	3.5390	170.7902	5	183.1296
26	1.2702	3.5390	170.7902	7	182.6937
26	1.2702	3.5390	170.7902	10	172.7395
26	1.2702	3.5390	170.7902	15	172.7811
26	**1.2702**	**3.5390**	**170.7902**	**17**	**172.5832**
26	1.2702	3.5390	170.7902	20	172.6897
26	1.2702	3.5390	170.7902	23	172.6783
26	1.2702	3.5390	170.7902	25	172.7980
26	1.2702	3.5390	170.7902	28	172.7465
26	1.2702	3.5390	170.7902	30	172.9931

n	h	k	s	l	ETC (simulation)
26	1.2702	17	170.7902	1	182.6856
26	1.2702	17	170.7902	1.2	179.7748
26	1.2702	17	170.7902	1.5	176.2002
26	1.2702	17	170.7902	2	172.9312
26	1.2702	17	170.7902	2.5	172.8206
26	1.2702	17	170.7902	3	172.8744
26	**1.2702**	**17**	**170.7902**	**3.5390**	**172.5832**
26	1.2702	17	170.7902	3.8	173.5938
26	1.2702	17	170.7902	4	174.7521

n	h	k	l	S	ETC (simulation)
26	1.2702	17	3.5390	50	182.3855
26	1.2702	17	3.5390	100	179.0169
26	1.2702	17	3.5390	150	175.5619
26	**1.2702**	**17**	**3.5390**	**170.7902**	**172.5832**
26	1.2702	17	3.5390	200	175.6926
26	1.2702	17	3.5390	230	180.4499
26	1.2702	17	3.5390	250	183.8136
26	1.2702	17	3.5390	280	188.0143
26	1.2702	17	3.5390	300	191.3319

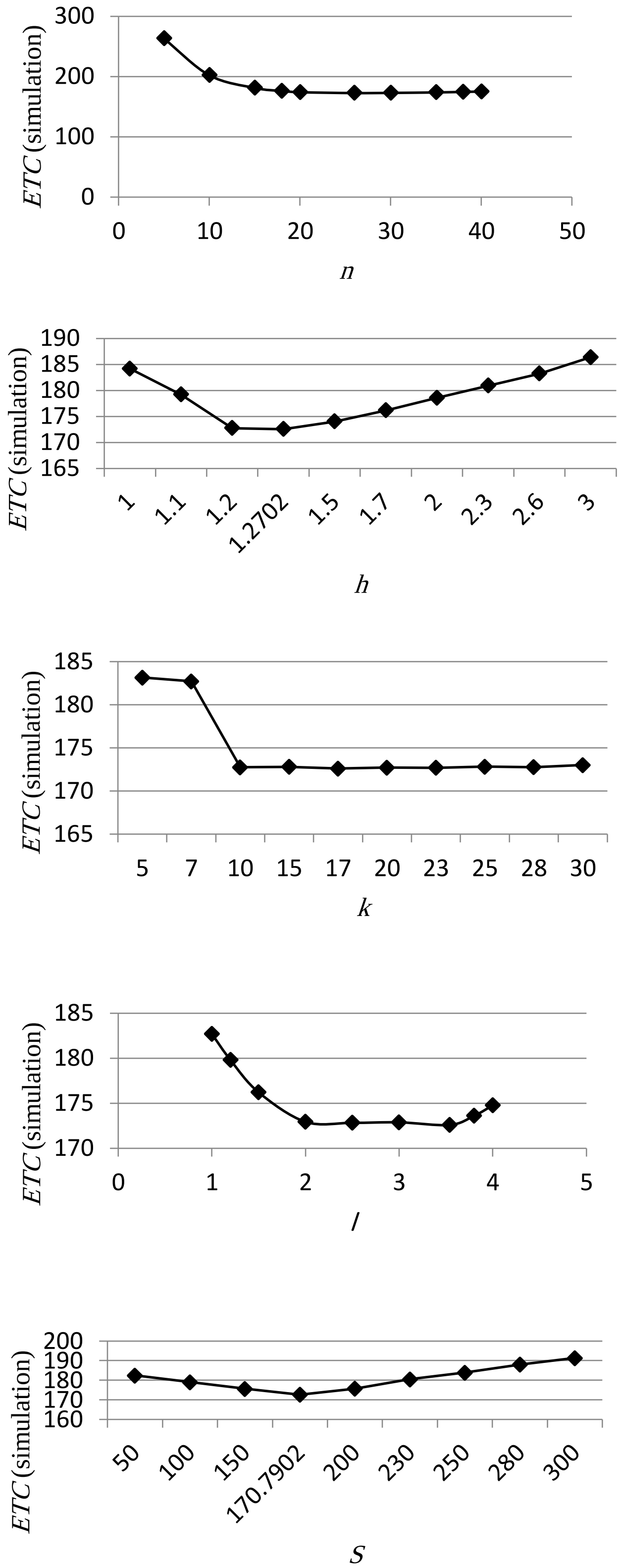

Fig6. The impact of decision variables changes on *ETC* in simulation

With respect to the results, the values of *ETC* are more sensitive to *n* and *h* changes. Therefore, the impact of changes in these two variables on *ETC* is investigated. The results are illustrated in Table VIII and Figure 7. For example, if the value 26 is considered for *n* and 1.2702 for *h*, *ETC* is 172.5832, and this is the minimum value.

Table VIII. The objective function values obtained for different combinations of *n* and *h*

		n									
		5	10	15	18	20	**26**	30	35	38	40
h	1	252.2119	194.5003	186.7402	180.7506	180.8745	180.2301	181.5170	184.4751	188.2989	197.3235
	1.1	256.6828	197.3878	178.5656	179.5076	178.6453	179.2076	182.7175	183.4870	187.1380	193.1718
	1.2702	262.9787	202.1584	181.1411	176.0451	174.0175	**172.5832**	172.8886	173.6646	174.5488	175.0774
	1.5	269.3755	207.5589	184.9126	179.2721	176.1870	174.0422	174.3932	175.3675	176.1735	176.4957
	1.7	274.5918	212.6746	187.6887	181.4641	178.3821	176.1765	176.0948	176.6505	177.1005	177.6003
	2	280.7584	219.3777	191.9596	184.6149	182.0169	178.5680	178.0621	178.7726	179.2300	179.6924
	2.3	285.4703	225.0157	196.0252	188.1043	184.9428	180.9000	180.1090	180.2251	181.2868	181.6438
	2.6	289.7821	230.0868	200.1282	191.8619	187.9426	183.2305	182.5890	182.9782	183.3667	183.5081
	3	294.3370	236.1598	204.5956	195.5068	191.2300	186.3770	185.6485	185.8882	186.2371	186.3884

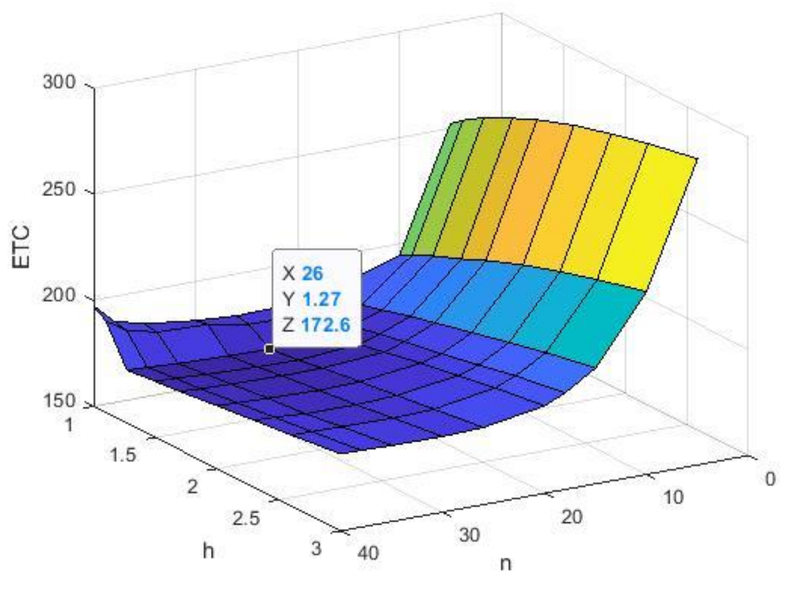

Fig7. The impact of changes of *n* and *h* on *ETC* in simulation

C. Sensitivity analysis

In this section, for further investigation and to indicate the effect of parameters changes on the optimal values, a comprehensive sensitivity analysis is conducted.

Larger values of δ decrease n but increase l. Because, by increasing δ, the shift can be detected more easily by the chart. Therefore, the width of the chart limits (l) should be increased, and the sample size (n) can be decreased in product quality inspections to reduce the quality inspection cost. Figure 8 indicates the results.

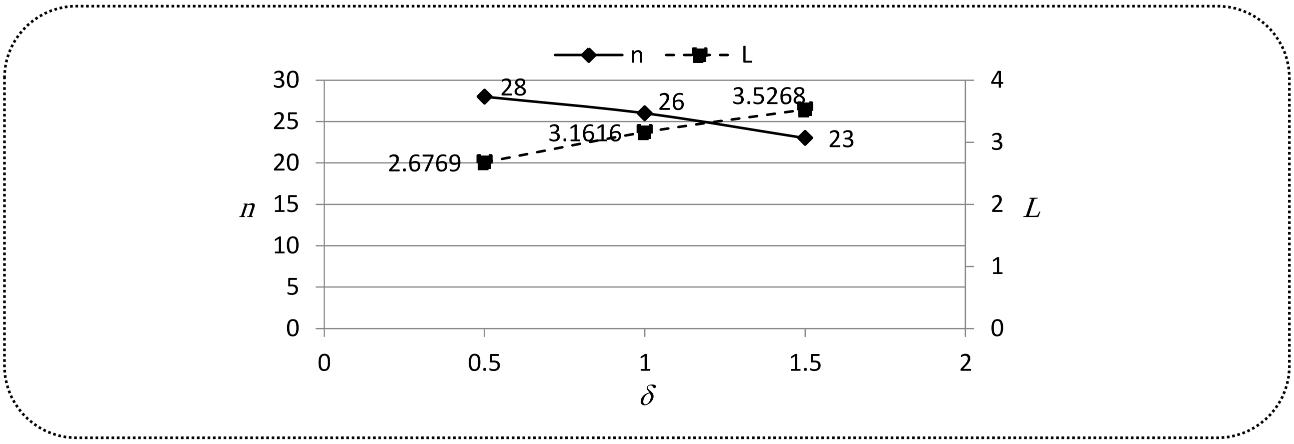

Fig8. The impact of δ on n and l

The optimal values of the decision variables and *ETC* are decreased by increasing the value of v. With an increase in v, the variance of the distribution is decreased, and the chart can predict the shift easier. Therefore, n and l can be decreased in quality inspections. Also, h and k can be decreased and finally, the total cost (*ETC*) is decreased. Figures 9-13 indicate the results.

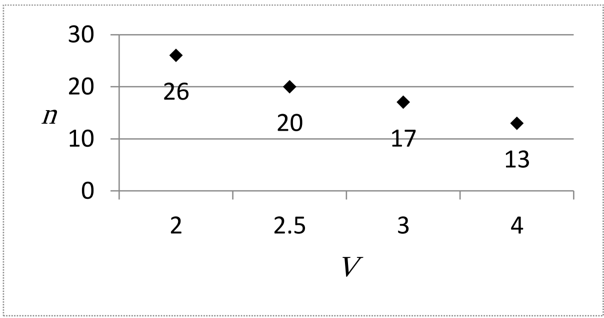

Fig9. The effect of v on n

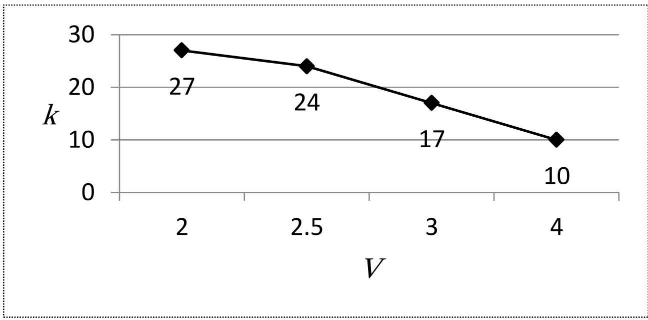

Fig10. The effect of v on k

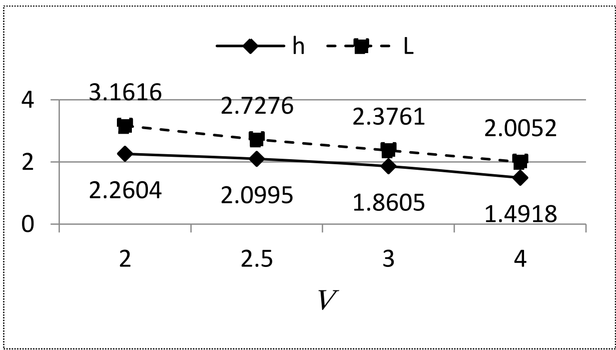

Fig11. The impact of v on h and l

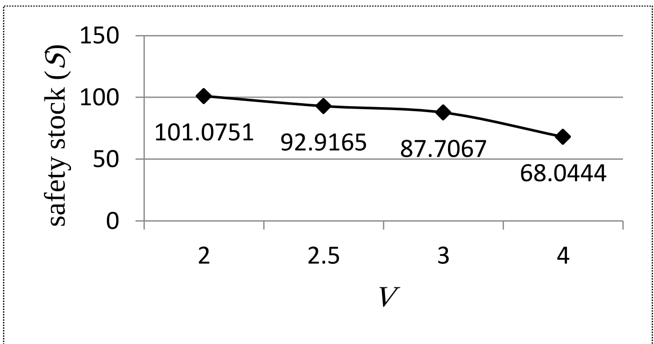

Fig12. The effect of v on S

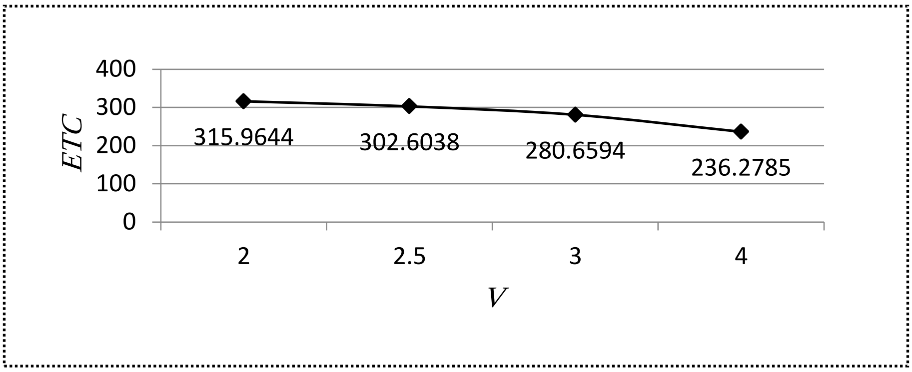

Fig13. The impact of *v* on *ETC*

The values of *n*, *S* and *ETC* are increased when *λ* increases, but *h* and *k* are decreased. The probability of the state transition is increased with an increase in *λ*. Therefore, *n* is increased, and *h* is decreased to detect the shift earlier. Also, *S* increases to reduce the shortage level. By increasing *n* and *S* and by decreasing *h*, *ETC* increases too. Figures 14, 15 and 16 indicate the results.

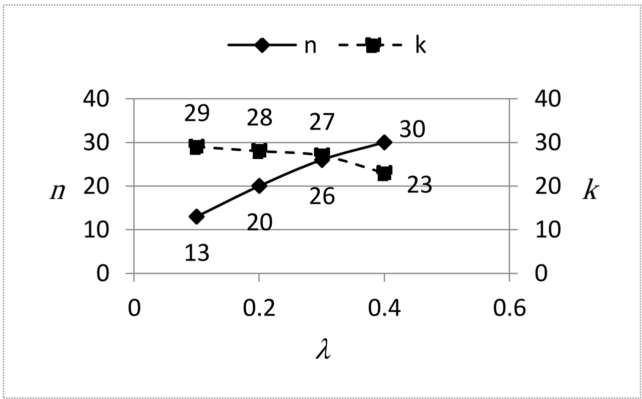

Fig14. The effect of *λ* on *n* and *k*

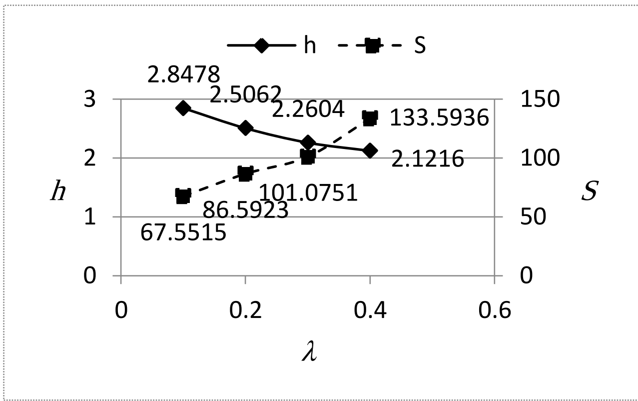

Fig15. The effect of *λ* on *h* and *S*

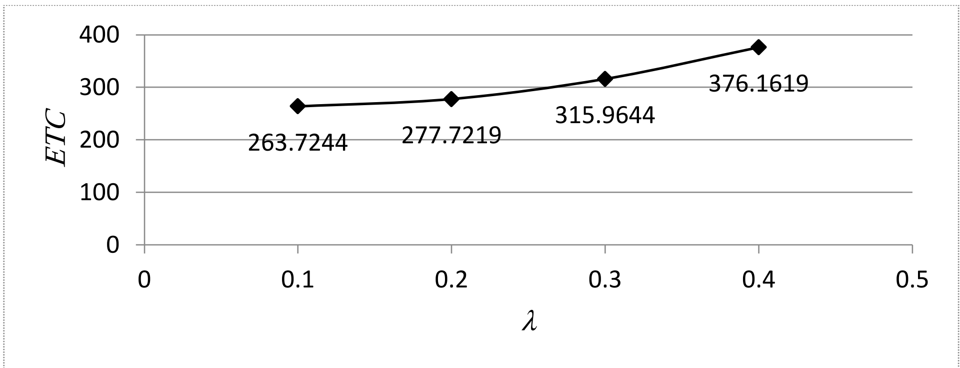

Fig16. The effect of *λ* on *ETC*

Larger values of λ_1 and λ_2, increase S to reduce the cost and the level of shortage during the maintenance actions. By increasing the scale parameters of the Weibull distribution in maintenance duration, λ_1 and λ_2, the duration of maintenance tasks is increased and the safety stock level (S) is increased to satisfy the demands while conducting the maintenance and reduce the shortage cost. Figures 17 and 18 indicate the results.

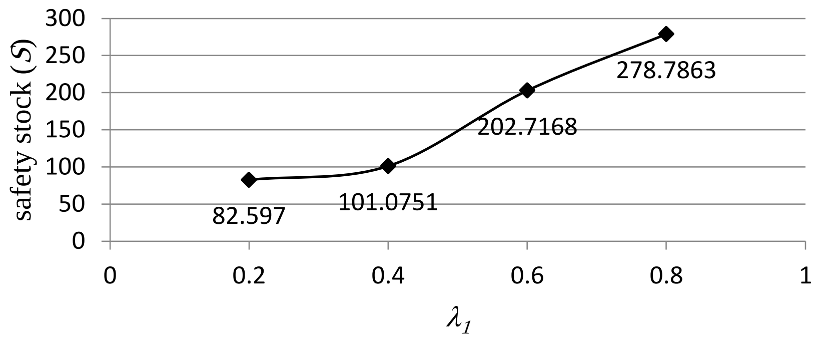

Fig17. The impact of λ_1 on S

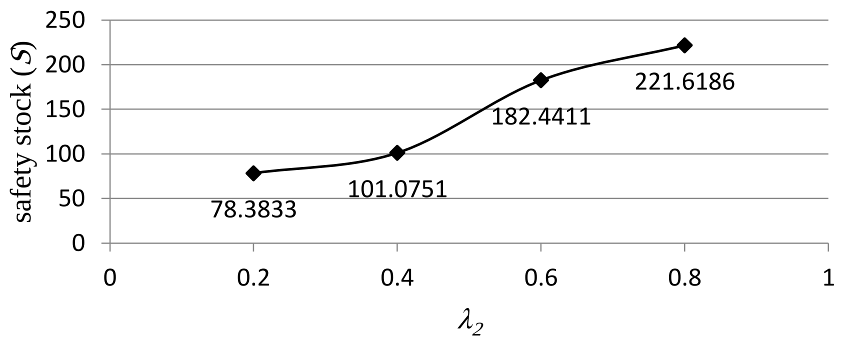

Fig18. The impact of λ_2 on S

By increasing the value of C_v, n is decreased but h is increased to reduce the total sampling inspection cost. Also, l is decreased to compensate the effect of n reduction. Figures 19, 20 and 21 indicate the results.

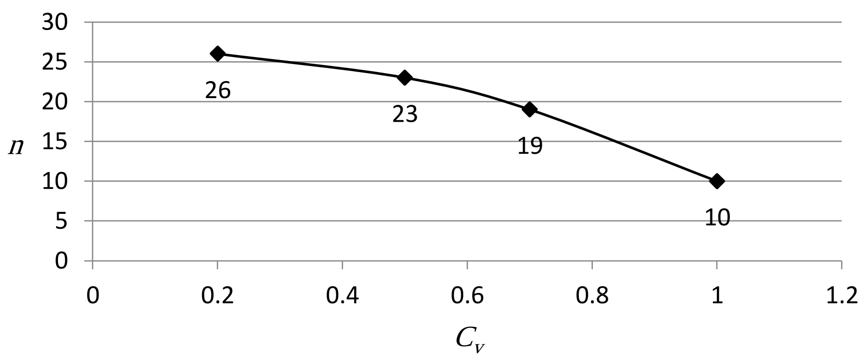

Fig19. The effect of C_V on n

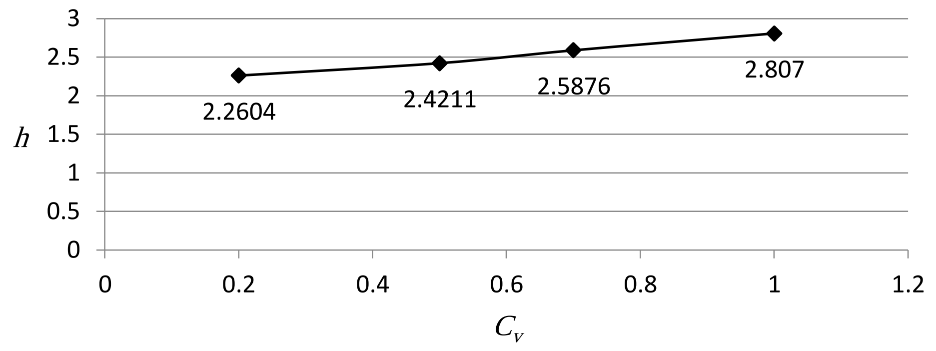

Fig20. The effect of C_V on *h*

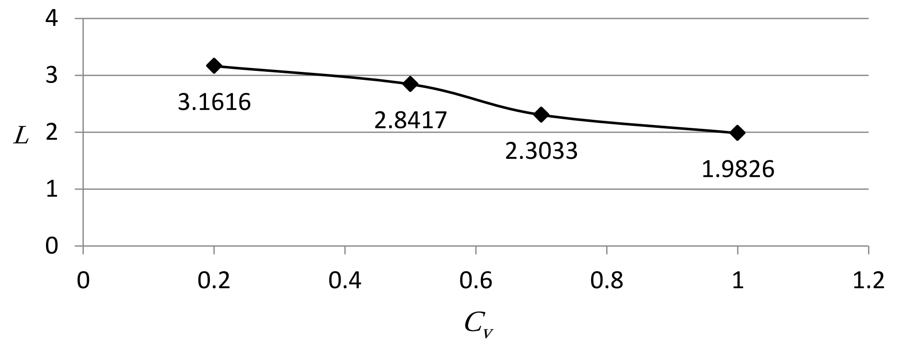

Fig21. The impact of C_V on *l*

By increasing the value of C_{PM} as the cost of planned maintenance action per time unit, the time duration of PM action should be decreased and therefore, *S* is decreased to reduce the shortage cost and finally reduce the total cost. But increasing C_{CM} as the cost of maintenance action for unexpected failures increases *S*. Figures 22 and 23 indicate the results.

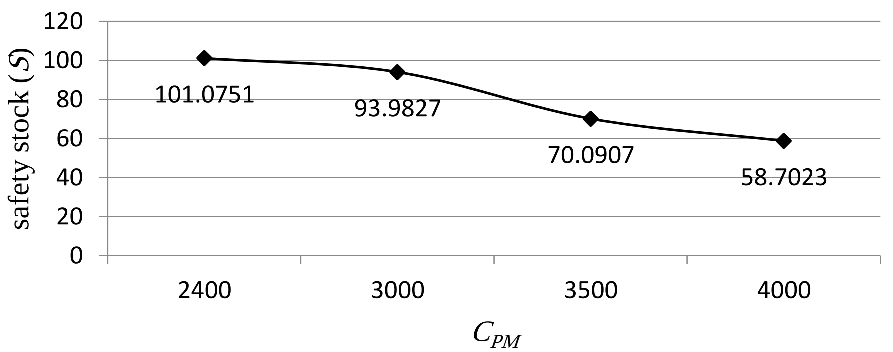

Fig22. The impact of C_{PM} on *S*

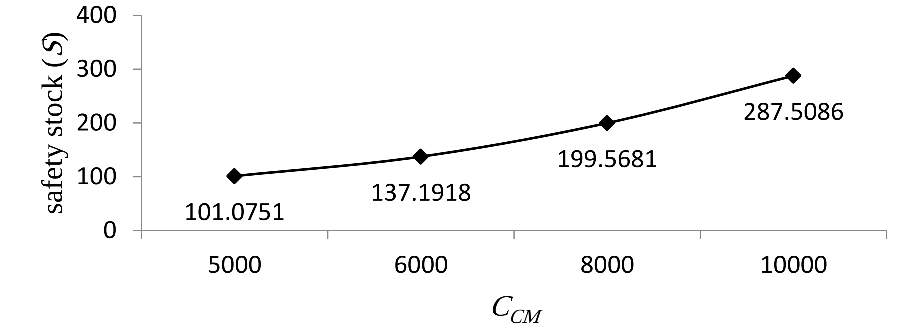

Fig23. The impact of C_{CM} on S

By increasing the value of C_h, S is decreased to reduce the total holding cost, but increasing C_{sh}, increases S to reduce the total shortage cost. Figures 24 and 25 indicate the results.

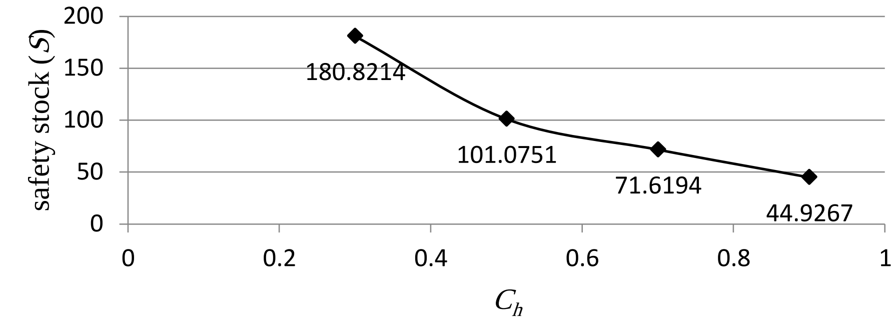

Fig24. The impact of C_h on S

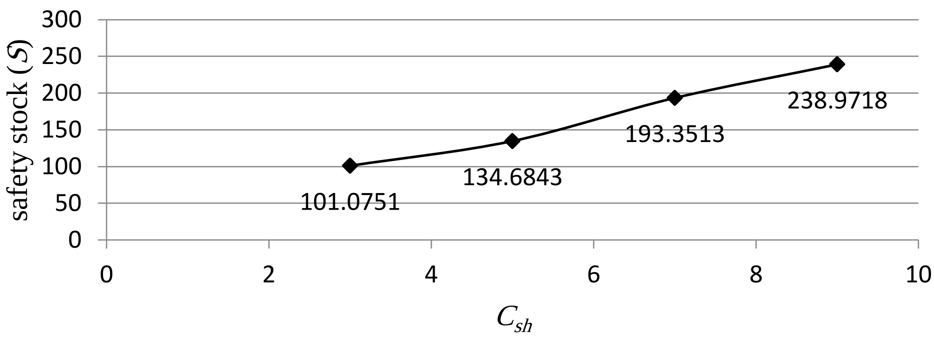

Fig25. The impact of C_{sh} on S

By increasing the value of C_1, h is decreased to detect the state transition from State 0 to 1 earlier and reduce the cost of production in State 1. Figure 26 shows the results.

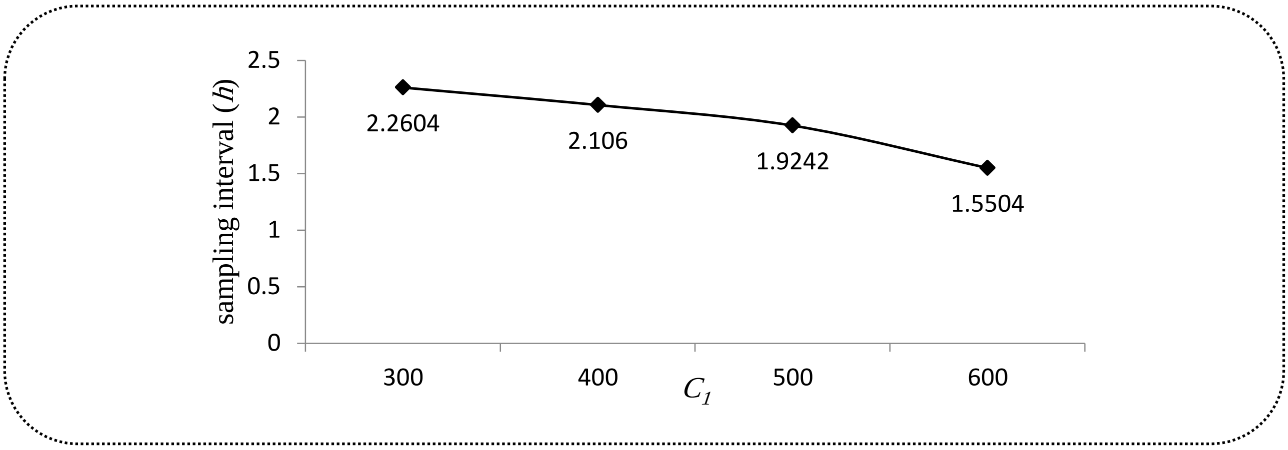

Fig26. The impact of C_1 on h

VI. COMPARING THE MODELS

The integration of the three aspects and the integration of maintenance and production are compared to investigate the effect of considering quality control in planning. Table IX and Figure 27 show the results obtained by GA. The values of ETC' is on average 67.7% more than the values of ETC. The results indicate the effect of quality control in integrated planning. Therefore, the integration of the three aspects can be effective in the total cost reduction of a process, and integrated planning is an efficient policy in the management of the production system.

Table IX. The results of comparison

v	ETC	ETC'	Percentage of difference
2	315.9644	612.0208	93.7
2.5	302.6038	524.8575	73.4
3	280.6594	436.5097	55.5
3.5	255.7488	399.2942	56.1
4	236.2785	377.4956	59.8

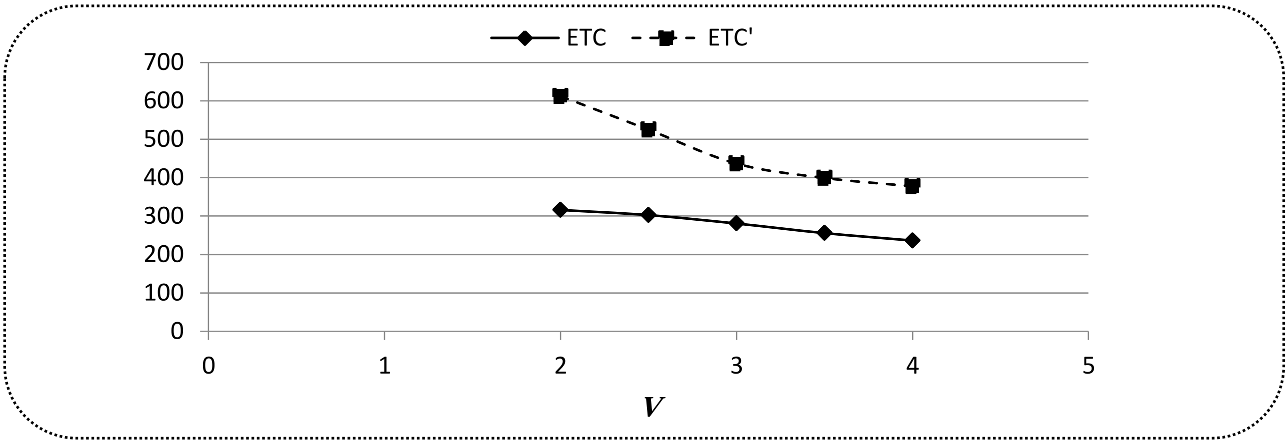

Fig27. The comparison of the total costs

VII. CONCLUSION

An unreliable manufacturing system was considered with quality degradation and the decisions of the three aspects were integrated. When an assignable cause occurs, this may transit the process state at a random time. The process is inspected at time points h, $2h$, $3h$,…, kh and the calculated statistics are plotted on a chart. Then, it is decided for maintenance to perform the PM or CM. The machine returns to State 0 after the maintenance. Random variables with general distributions were considered for the time length of maintenance. A safety stock is maintained to reduce the shortage effects during the maintenance. The purpose is to jointly design a control chart, schedule the maintenance and determine the buffer stock for cost minimization. A stochastic joint model of maintenance, quality and production was developed with respect to the different scenarios that occurred in each cycle based on the process state and the machine condition. The integration of three aspects was compared to the integration of two aspects, and the results indicated that the total cost was decreased by considering quality in joint planning. To solve the proposed complex integrated model, at first, the GA method was applied. During the run of the proposed GA, it was observed that due to the complexities of the equations, the GA was slow to optimize the problem. Hence, the GA was combined with the MC and a simulation-optimization technique was proposed to optimize the value of *ETC*. The values of the decision variables are specified using the GA, while the corresponding fitness function, i.e., *ETC*, is computed using the proposed simulation method. It was observed that the proposed simulation-optimization approach was faster than GA to reach the optimal solutions. To validate the efficiency of the MC method, the results obtained by MC and GA were compared. The difference percentage between the *ETC* values in GA and simulation was on average 4.79% and this difference is negligible. These results indicate the efficiency of the MC method. In another step to investigate the efficiency of MC and validate the model, *ETC* was computed by simulation-optimization method for different values of the decision variables. The results indicated that for the values of the decision variables larger or smaller than the optimal values, *ETC* increases. These results illustrated that the model and the presented simulation-optimization method for MC and GA have appropriate performance in achieving the optimal solutions. Finally, a comprehensive sensitivity analysis was conducted to investigate the efficiency of the model and the effect of parameter changes on the optimal values of the decision variables and objective. The results indicated the effectiveness of the model.

Different manufacturing systems can apply these integrated models. The integrated models can be helpful for the JIT (Just In Time) systems. In JIT strategy, it is tried to receive customers' orders and then the items are manufactured with respect to the customers' demand. By using the integrated model, the managers can achieve their purpose. Also, applying the integrated models can be beneficial in continuous production processes and serial production processes that; the purpose of such systems is to produce the products without interruptions. Therefore, such manufacturing systems can be more productive if the three aspects are planned. The technologies in manufacturing systems became more complex, and the intelligence level increased in different manufacturing systems in industry 4. The machines and the equipment conditions, production planning policy and the quality monitoring policy are the most important aspects in the management of such systems. Simultaneous planning of three aspects can be employed in intelligent systems to increase the process's efficiency. Due to the physical and logical connections between the machines and operations in intelligent manufacturing systems, joint planning of three aspects can be more efficient in process's management than separate planning.

Some limitations in this study can be considered in future work. We assumed that a single quality characteristic in products is inspected. Considering multi-quality characteristics and applying multi-variate charts can be an idea for future research. In this study, it was assumed that the process has one machine, and the system is single-stage and a single assignable cause affects the performance of the machine. Development of the model for the multi-machines processes and multi-stage systems and considering multiple assignable causes leading to quality degradation can be other assumptions for future studies.

Appendix A

$$\omega = E\left[x - h\left\lfloor\frac{x}{h}\right\rfloor \middle| x < (k+1)h\right] = \int_0^{(k+1)h}\left(x - h\left\lfloor\frac{x}{h}\right\rfloor\right) f(x)dx =$$

$$\int_0^{(k+1)h} x f(x)dx - h\sum_{j=0}^{k}\int_{jh}^{(j+1)h}\left(\left\lfloor\frac{x}{h}\right\rfloor\right) f(x)dx,$$

Where, $\left\lfloor\frac{x}{h}\right\rfloor$ is the largest integer less than or equal to $\frac{x}{h}$. If $jh \leq x < (j+1)h$, therefore, $\left\lfloor\frac{x}{h}\right\rfloor = j$.

$$\omega = \int_0^{(k+1)h} x f(x)dx - h\sum_{j=0}^{k}\int_{jh}^{(j+1)h} j f(x)dx =$$

$$\int_0^{(k+1)h} x f(x)dx - h\sum_{j=0}^{k} j\left[F\big((j+1)h\big) - F(jh)\right],$$

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