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Sustainable supplier selection and order allocation based on stochastic programming and dynamic programming

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Abstract *–In recent years, supplier selection (SS) has been one of the fundamental issues in the supply chain (SC). Sustainable suppliers are selected based on the quantitative and qualitative evaluation of economic, environmental, and social studies. This paper examines sustainable SS and order allocation (OA) for a single product in a multistage setting. To model the problem, in the first step, the best-worst method (BWM) is used to determine the weights of the sustainability criteria. Then, given the uncertainty of personal judgment, evidential reasoning (ER) is used to evaluate suppliers. Suppliers are also compared based on the minimum and maximum utility function in case of lack of information. In the next step, a model is designed to optimize suppliers' total purchase value (TVP) and optimize total purchase cost (TCP). Demand is assumed to be random. This assumption leads to a set of scenarios based on the time horizon. To solve the model, a dynamic programming (DP) is presented. Finally, a case study is given to detail the methodology.*

Keywords– Sustainable Supplier selection (SS), Order Allocation (OA), Best Worst Method, Evidential Reasoning, dynamic programming (DP), Stochastic Programming (SP).

I. INTRODUCTION

Supplier evaluation and SS has a vital role in improving the efficiency and effectiveness of the organization(Ghadimi et al., 2017).Before 1980, selecting suppliers was mainly based on costs. However, years later, factors like quality and delivery time also became important. Recently, sustainable SS has become essential, requiring the consideration of environmental and social factors in addition to economic ones. Due to increased environmental pollution and decreasing natural resources, considering environmental criteria is crucial (Cheraghalipour and Farsad, 2018; Zailani et al., 2012).Thus, SS involves multiple criteria, both qualitative and quantitative. Sustainable SS is helpful to design a sustainable SC (Shen et al., 2013; Govindan et al., 2013) and is a key strategic decision in SC management (Amindoust et al., 2012).

On the other hand, uncertainty in SS and OA is a common challenge in SC management. Uncertainty can arise from various sources such as demand variability, supplier performance changes, price fluctuations, quality issues, delivery delays, and also the performance and efficiency of SC. For example, demand uncertainty in SS may vary depending on factors such as market conditions, customer preferences, weather, etc. Demand uncertainty can make planning SC

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challenging for a business, as it must balance maintaining sufficient inventory. Meeting customers' needs and avoiding excess inventory may result in additional costs or waste. Also, the uncertain personal judgment in the SS means that the decision-maker must rely on his subjective evaluation of suppliers rather than the objective and measurable criteria. This can happen when the decision-maker is faced with a complex and uncertain environment, where information about suppliers is incomplete, inconsistent, or contradictory. Uncertainty of personal judgment can affect the ranking of suppliers, because different decision-makers may have different preferences, opinions, and perceptions of suppliers' performance.

This research presents a decision-making framework to address sustainable SS and OA in a multi-supplier, single-product, and multi-stage environment. Initially, sustainable criteria and sub-criteria are identified to evaluate and select the suppliers within the environmental SC, drawing on company strategies, expert opinions, and a thorough literature review. The BWM is then used to determine the weights of these criteria and sub-criteria. BWM is preferred over the AHP method as it requires fewer pairwise comparisons and produces more consistent and reliable results (Rezaei, 2015). Finally, the Evidential Reasoning (ER) method is applied to evaluate and rank the suppliers. ER is chosen due to the qualitative nature of most criteria and sub-criteria, and the high uncertainty evaluators face due to incomplete information. This method effectively models the uncertainties of personal judgments and the gaps in information regarding certain sub-criteria. In the second phase, a model is designed to optimize supplier value and optimize total cost. The demand is treated as stochastic, resulting in various scenarios across different stages. A new algorithm is introduced, integrating SP and DP. This approach effectively handles uncertainty and dependencies among sub-problems. Unlike traditional methods that inefficiently handle repetitive sub-problems, DP solves each sub-problem for the next stages, thereby reducing the computational burden. In summary, this study proposes a novel approach to managing uncertainty by using DP for sustainable SS and OA, offering a more efficient and reliable framework for decision-making in SC management.

II. LITERATURE REVIEW

This section presents an overview of prior research endeavors. The scope of the literature review encompasses sustainable SS and OA, supplier evaluation, selection methodologies, and OA solution strategies. The subsequent part of this section delineates the existing research gaps and outlines the distinctive contributions of this paper.

A. SUSTAINABLE SS AND OA

Over recent decades, the matter of sustainable SS and OA has occupied a prominent position in SC management. These studies have delved into various criteria to discern optimal supplier choices.

Ho et al. (2010) and Chai et al. (2013) presented two basic methods for SS and OA. Initially, the focus was predominantly on cost-centric factors, with Degraeve and Roodhoft (1999) underscoring the importance of cost in SS. However, organizations progressively recognized that assessing suppliers solely on cost criteria might not yield optimal outcomes. Dickson (1966) meticulously documented 23 criteria for supplier evaluation and selection. Weber et al. (1991) reviewed SS criteria from research dating back to 1966, and three criteria for SS were obtained as quality, cost, and delivery time. Alidaee and Kochenberger (2005) introduced a DP approach to solve the single-sink fixed charge transportation (SSFCT) problem to show its applicability in optimizing order quantities. Li et al. (2009) considered SS based on price and demand criteria. Razmi and Rafiei (2010) selected a two-stage approach, based on qualitative traits using the analytic network process (ANP), and mixed-integer nonlinear programming (MINLP).

Mendoza and Ventura (2010) proposed a mathematical model to optimize inventory policies based on the transfer of goods between SC tiers during supplier OA. Mafakheri et al. (2011) analyzed the SS and OA problem through DP. Kannan et al. (2013) considered green SS, addressing economic and environmental criteria, and employed fuzzy multi-attribute utility to rank suppliers. Ware et al. (2014) investigated dynamic SS (DSSP) within a SC to determine the parameter variations across different stages. Singh (2014) proposed a heuristic algorithm for supplier evaluation and

OA. Scott et al. (2014) advocated an integrated approach amalgamating the AHP with QFD for SS and OA. Lee et al. (2014) applied the dynamic SS method to identify better suppliers based on quality. Jadidi et al. (2014) incorporated three objective functions (OFs) and proposed two solution approaches. Guo and Li (2015) considered a multi-tier SC scenario, encompassing wholesalers, numerous retailers, and a consortium of suppliers. Kuo et al. (2015) proposed a method to select suppliers and developed an ANN approach for OA. Moghaddam (2015) presented a model to identify and rank the premier suppliers within a SC, based on stochastic demand through fuzzy methods. Pazhani et al. (2016) developed an MINLP model to optimize OA across SC stages. Amorim et al. (2016) devised an MIP model based on the stochastic SS, which is particularly suited for the food industry. Sodenkamp et al. (2016) suggested a model to optimize risk based on the cooperation among SC stages through knowledge sharing. PrasannaVenkatesan and Goh (2016) adopted a hybrid approach, combining the fuzzy AHP for SS with PSO to handle SS and OA. Çebi and Otay (2016) executed a two-stage fuzzy methodology for SS and OA. Ghorabaee et al. (2017) extended their method to incorporate environmental criteria, particularly environmental pollution, in supplier evaluation under conditions of uncertainty. Noori-Daryan et al. (2017) developed a multi-national decision model considering capacity with stochastic demand. Ghadimi et al. (2017) developed their evaluation of environmental, social, and economic criteria for SS, considering multi-agent systems (MAS) to handle SS and OA. Hamdan and Cheaitou (2017a) probed into the intricacies of SS and green OA based on the fluctuating availability of suppliers. Hamdan and Cheaitou (2017b) revisited the SS and green OA problem while assuming quantity discounts. Vahidi et al. (2018) focused on sustainable SS, mobilizing a two-stage SP framework complemented by a hybrid SWOT-QFD methodology for SS and OA. Cheraghalipour and Farsad (2018) investigated sustainable SS and OA, adopting the (BWM) to ascertain supplier weights. Esmaeili-Najafabadi et al. (2019) assumed integrated SS and OA within a centralized SC fraught with disruption risks. Gören (2018) proposed a decision framework for sustainable SS and OA. Foroozesh and Tavakkoli-Moghaddam (2018) proposed a method under uncertainty to assess green supplier development programs. Hosseini and Fallah Nezhad (2019) introduced a two-level SC model for green SS and OA across multiple stages and a single product. The method employs an innovative integrated method that combines SP and DP. Nasr et al. (2020) introduced an innovative two-stage approach for fuzzy SS and OA within a closed-loop SC. They utilized the fuzzy BWM for SS and employed a fuzzy goal programming approach. Li et al. (2020) proposed an original two-stage mathematical model that dynamically selects suppliers and determines order quantities, addressing green considerations and supplier risks. Foroozesh et al. (2020) suggested a new decision-making method for SS. Jahangirzadeh et al. (2020) investigated a new extended grey relational analysis based on the complex proportional evaluation and this combined methodology was applied to SS problems. Meanwhile, Kaur and Singh (2021) developed a model that integrated supplier segmentation, SS, and OA using DEA. Cui et al. (2023) introduced a model based on fuzzy theory and Bayesian networks to assess critical criteria in the SS process. Sontake et al. (2021) formulated an MILP framework for SS and OA, emphasizing the selection of transportation methods. Finally, Beiki et al. (2021) proposed the entropy method to address sustainability concerns in SS and OA. Islam et al. (2022) proposed a three-stage framework to address SS and OA planning challenges. Keramati et al. (2022) developed a multi-product Economic Production Quantity (EPQ) model. Ali and Zhang (2023) introduced an approach by integrating economic, environmental, and foreign transportation risk factors to create a comprehensive model for global green SS and OA. Nazari-Shirkouhi et al. (2023) presented a framework for addressing the SS and OA problem within the context of multiple items, multiple suppliers, multiple price levels, and various time stages. Nayeri et al. (2023) introduced a novel decision-making methodology termed the Stochastic Fuzzy Best–Worst Method. In the second phase, they proposed a multi-objective model to address decisions related to SS and OA. Islam et al. (2024) introduced a novel three-stage framework for addressing SS and OA challenges. It considers a modified deep-learning forecasting method.

B. RESEARCH GAP

Table 1 is a comprehensive compendium of studies scrutinized in the preceding section. A discernible pattern emerges from the table. Most of the studies have not addressed the holistic spectrum of sustainability criteria, spanning economic, social, and environmental dimensions. Notably, the exigent issue of stochastic demand, a ubiquitous challenge in real-world scenarios, has been an important subject in past research. Furthermore, this paper introduces innovative elements, primarily centered on the integration of stochastic demand considerations and the application of DP techniques to address these research gaps.

- This study distinguishes itself through the formulation of a bi-objective mathematical model. It considers the scenarios involving multiple suppliers, a singular product, and varying stages, all within sustainable SS and OA. The primary objective aims to optimize the cumulative score of all suppliers, considering three vital sustainability facets, denoted as the Total Value of Purchase (TVP). The secondary objective seeks to optimize the total procurement cost (TCP). Notably, the model accounts for stochastic demand quantities during each stage because of inherent uncertainties.
- To determine the optimal supplier ranking, the study employs the (BWM) and the Evidential Reasoning (ER) method. Of particular note, the ER approach is applied because of its capacity to factor in uncertainties stemming from subjective judgments and incomplete information.
- In addition to these innovations, the paper introduces a pioneering integrated solution approach based on the methods of SP and DP to confront the challenges of sustainable SS and OA under stochastic conditions.

Table 1. A brief literature of the sustainable SS problem

Reference	Aspects			Example	Item	OA	SD	EOCUU	Solution approach
	Economic	Environmental	Social						
Lu et al. (2007)		*		NuEx	SI				AHP, Fuzzy logic
Mendoza and Ventura (2010)	*			NuEx	SI	*			Power-of-Two (POT) approach
Mafakheri et al. (2011)	*			NuEx	SI	*			AHP
Kannan et al. (2013)	*	*		CaSt	SI	*			Fuzzy AHP, fuzzy TOPSIS
Nazari-Shirkouhi et al. (2013)	*			NuEx	MI	*			QFD and DEA
Ware et al. (2014)	*			NuEx	MI	*			A mixed-integer non-linear program (MINLP,dynamic SS problem
Jadidi et al. (2014)	*			NuEx	SI	*			TOPSIS
Guo and Li (2015)	*			NuEx	SI	*	*		MINLP
Moghaddam (2015)	*			NuEx	MI				Fuzzy goal programming
Kuo et al. (2015)	*	*		CaSt	SI				DANP; VIKOR
Torabi et al. (2015)	*			NuEx	MI	*			Two-stage SP
Pazhani et al. (2016)	*			NuEx	SI	*			MILP
Amorim et al. (2016)	*			CaSt	MI		*		Bender's decomposition algorithm
Hamdan and Cheaitou (2016)	*	*		NuEx	SI	*			Fuzzy TOPSIS, AHP
Çebi and Otay (2016)	*			CaSt	SI	*			Two-stage fuzzy approach, multi-objective linear programming
Noori-Daryan et al. (2017)	*			NuEx	SI	*			Game-theoretic approaches
Ghadimi et al. (2017)	*	*	*	CaSt	MI	*			Multi-agent system approach

Continue Table 1. A brief literature of the sustainable SS problem

Reference	Aspects			Example	Item	OA	SD	EOCUU	Solution approach
	Economic	Environmental	Social						
Hamdan and Cheaitou (2017a)	*	*		NuEx	SI	*			FTOPSIS, AHP, ILP
Hamdan and Cheaitou (2017b)	*	*		NuEx	MI	*			ε-constraint method and metaheuristics
Vahidi et al. (2018)	*	*		NuEx	MI	*			ε-constraint method and metaheuristics
Kaur and Singh (2021)	*			CaSt	SI			*	DEA and Evidential Reasoning
Luthra et al. (2017)	*	*	*	CaSt	MI	*			AHP+ VIKOR
Babbar and Amin (2018)	*	*		CaSt	MI	*	*		QFD
Cheraghalipour and Farsad (2018)	*	*	*	CaSt	MI	*			BWM-RMCGP
Dickson (1966)	*			NuEx	MI	*			Nonlinear mixed integer programming
Gören (2018)	*	*	*	CaSt	SI	*			DEMATEL
Park et al. (2018)	*	*	*	CaSt	MI	*			Multi-attribute utility theory (MAUT)
Lo et al. (2018)	*	*		NuEx	MI	*			BWM, Modified fuzzy TOPSIS, FMOLP
Jia et al. (2015)	*	*	*	CaSt	SI	*	*		Distributionally robust goal programming model, Tractable approximation
Cui et al. (2023)	*	*	*	CaSt	SI				Bayesian network
This research	*	*	*	CaSt	SI	*	*	*	BWM and Evidential Reasoning,

Notes:
CaSt: Case study NuEx: Numerical example S/M-I: Single/Multi-Item OA: OA SD: stochastic demand EOCUU: evaluation of criteria under uncertainty

III. METHODOLOGY

In this research, multi-stage, multi-suppliers, and single-product are introduced for sustainable SS and OA decisions. The model is developed under time-varying prices, limited capacity, and stochastic demand assumptions. In most realistic optimization problems, the data is stochastic; therefore, SP should be utilized.

In this study, we applied the scenario-based technique. This method treats random quantities as stochastic variables. A scenario represents a hypothesis about the future, detailing the interaction between different factors under specific conditions. Scenarios combine stochastic parameters that are summarized with different data states in a few simple cases. Compared to sensitivity analysis, scenarios change several parameters simultaneously. In SP, stochastic parameters are defined by a discrete distribution and have a finite number of states. Therefore, all random parameters depend on a finite set of scenarios. In a multi-stage, SP problem, uncertainty can be expressed as a multi-level scenario tree, which represents the possible states of a sequence of events (Schildbach and Morari, 2016).

In this model, the demand quantities are considered the primary source of randomness. Sales and income fluctuations cause the producer to have no accurate estimation of demand value for the required raw material. We assume three states are pessimistic, probabilistic, and optimistic about demand. It is also assumed that each value can occur with the same probability. A discrete uniform distribution is used. This assumption leads to scenarios during the time horizon (T). The number of potential demands in every stage is 3. As a result, the total number of scenarios is denoted by 3^T; thus, the S= {1... 3^T} can be assumed as the set of potential scenarios with equal probabilities $\frac{1}{S}$.

There are different economic, social, and environmental criteria for SS and OA problems. Some of them are qualitative and their exact score is impossible. Accurate scoring methods have been used in the relevant literature. Therefore, exceptional results may be subjective and inaccurate. To address this issue, we used the BWM to determine the weights of criteria and sub-criteria. Next, the suppliers' weights were obtained using the ER approach, where a higher weight corresponds to a higher-ranked supplier. In the second step, which links the multi-criteria evaluation to the OA method, these weights are included in the initial model as an OF to optimize the suppliers' total purchase value (TVP). We present an MILP model to determine the OA for each supplier at each stage under different scenarios. The two objectives of OF are TVP optimization and Total procurement Cost (TCP) optimization. The model is solved using a combination of SP and DP. Figure 1 shows the methodology.

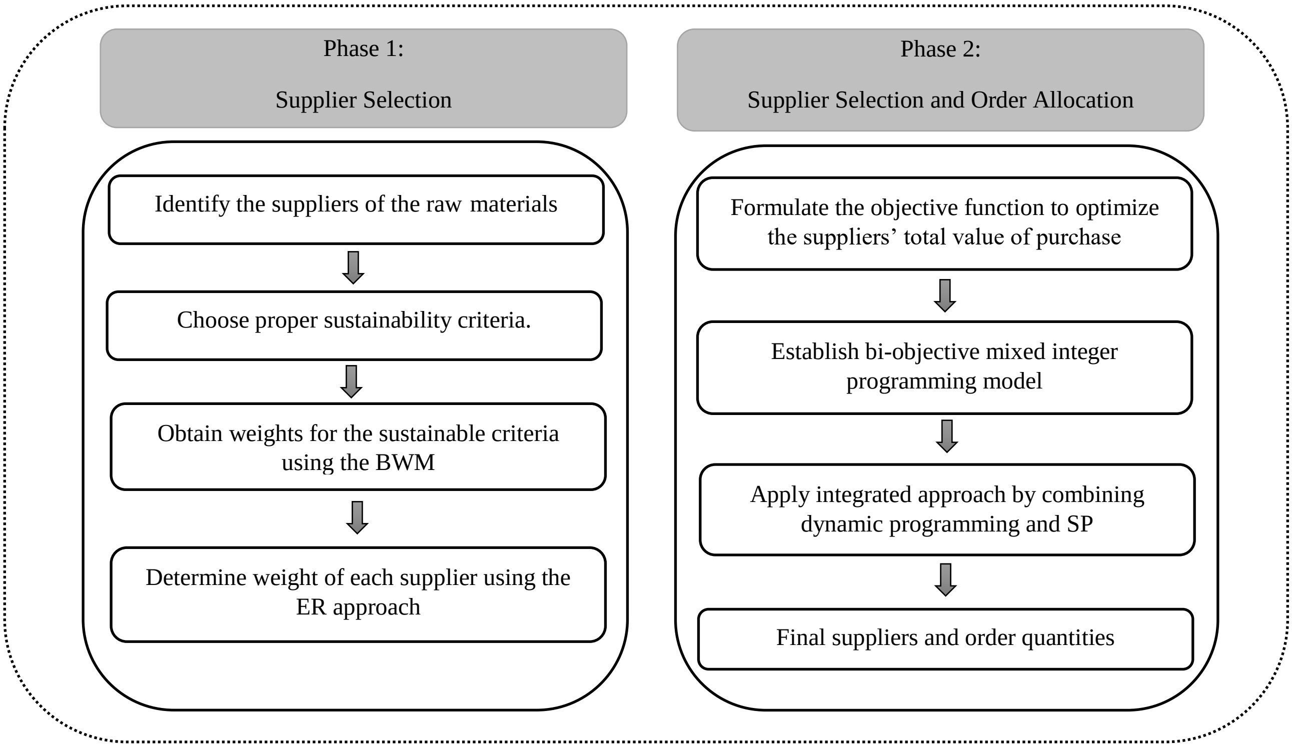

Fig. 1. Two-phased approach for sustainable supplier section and order allocation

The critical assumptions of the model are as follows:

- The model is considered multi-stages, multi-suppliers, and single-product.
- Shortages are allowed at the SC level.
- The demand in each stage is stochastic.
- The stages are limited.
- The prices from each supplier in all stages are fixed.

ER approach and BWM, the MILP model for the problem of sustainable SS and OA, and the combined DP, and SP approach are detailed in the sections that follow.

A. The ER Approach

Decision-making often involves both qualitative and quantitative factors, which must be considered to accurately address complex problems. The presence of uncertain information and qualitative factors further complicates these decisions. Thus, a powerful method is needed to tackle multi-criteria decision analysis under uncertainty. Over the past two decades, significant research in artificial intelligence and operations research has focused on analyzing uncertain information. ER has been developed to address multi-criteria decision-making involving uncertainty (Huynh et al., 2006).

The ER approach, rooted in evidential theory (Yang and Singh, 1994), integrates multi-criteria decision-making. This method is widely used in various fields, including engineering design, risk and safety assessment, design selection, and marine safety system analysis (Yang and Sen, 1994; Sen and Yang, 1995; Wang et al., 1996). The ER approach uses a multi-criteria evaluation matrix and the Dempster-Shafer (D-S) evidence combination rule. It effectively combines the factors of multi-level structures and accounts for incomplete evaluations caused by information gaps, misjudgments, or errors in group decision-making. Useful intervals are then provided to assess the degree of incompleteness of the original data (Huynh et al., 2006; Wang and Elhag, 2008).

A.1. The Supplier Assessment Method

Based on previous research (Zimmer et al., 2016; Luthra et al., 2017; Cheraghalipour and Farsad, 2018) and input from company experts, several sustainability criteria and sub-criteria were selected to assess suppliers on sustainability aspects. These chosen criteria, along with brief descriptions, are presented in Table 2.

Table 2. Selected criteria and sub-criteria (Zimmer et al., 2016; Luthra et al., 2017; Cheraghalipour and Farsad, 2018)

S. NO.	Criteria	Sub-criteria	Brief description
1	Economic (EC)	Quality (ECQ)	Providing products at a significant quality level
2		Cost (ECC)	Capability of supplying products at reasonable prices
3		Delivery and Service of product (ECDS)	Ensuring correct delivery and service of products
4		Technological & financial capacity (ECTF)	Handling technological and financial aspects within the supplier domain
5		Long-term relationship – continuity (ECLR)	Establishing a long-term relationship between manufacturer and supplier for raw material procurement
6		Flexibility (ECF)	Flexibility to handle market variations
7	Environmental (EN)	Environmental management Systems (ENEM)	Structuring, planning, and implementing environmental protection policies
8		Green design and purchasing (END)	Incorporating eco-friendly practices during design and purchasing stages
9		Green packing and labeling (ENGL)	Considering environmental factors for packaging and labeling
10		Environmental Pollution & Waste management (ENPW)	Minimizing wastage and pollution during production
11		Energy consumption management (ENEC)	Managing energy consumption effectively

Continue Table 2. Selected criteria and sub-criteria (Zimmer et al., 2016; Luthra et al., 2017; Cheraghalipour and Farsad, 2018)

S. NO.	Criteria	Sub-criteria	Brief description
12	Social (SO)	Occupational health and safety (SOOHS)	Ensuring safety, health, and welfare of employees at the supplier's workplace
13		Social management commitment (SOSMC)	Planning and implementing commitments for social management
14		The interests & rights of employees (SOIE)	Planning and implementing commitments for social management
15		Wages & working hours (SOWW)	Ensuring fair wages and working hours for employees

A.2. The Weights Of Criteria

The BWM is used to determine the weight of each criterion. BWM, which was introduced by Rezaei (2015), stands out as one of the most efficient multi-criteria decision making techniques. Compared to AHP, BWM requires fewer pairwise comparisons and ensures efficiency. Moreover, pairwise comparisons in BWM lead to reliable results. The main steps in determining the weight of the criteria are as follows:

Step 1. A set of decision criteria is determined.

Step 2. The best and worst criteria are selected.

Step 3. Pairwise comparisons are made between the best criterion and other criteria. Therefore, the preference of the best criterion over other criteria is evaluated with a number in the range of 1 to 9, and these numbers are assigned according to the opinions of experts and company policies. First, the best criterion is assigned to the number 1, and then a comparison is made between the best and other criteria. In this comparison, if the importance of the criterion is closer to the best criterion, a number close to 1 is assigned to it. Also, the criterion is less important than the best criterion, and the assigned number will be closer to 9. The vector of "best-to-others" is obtained as follows:

$$A_B = (a_{B1}, a_{B2}, \dots, a_{Bn})$$

a_{Bj} denotes the preference of the criterion B to the criterion j and $a_{BB} = 1$.

Step 4. Pairwise comparisons are performed between the worst criterion and all the other criteria. In this step, the preferences of criteria over the worst one are obtained with a number in the range mentioned above, and the vector of "others-to-worst" is obtained as follows:

$$A_w = (a_{1w}, a_{2w}, \dots, a_{nw})^T$$

a_{jw} represents the preference of the criterion j to the worst criterion W, and $a_{ww} = 1$

Step 5. The weights of the criteria are determined. In this step, using the following optimization model, the weights of the criteria are obtained. This model can be implemented using commonly available mathematical programming software like Lingo.

$$\min \max \left\{ \left| \frac{w_B}{w_j} - a_{Bj} \right|, \left| \frac{w_j}{w_w} - a_{jw} \right| \right\} \tag{1}$$

$$s.t.$$

$$\sum_j w_j = 1$$

$$w_j \geq 0 \qquad for\ all\ j$$

Problem (1) can be rewritten as the following problem:

$$\min \xi \tag{2}$$

$$s.t.$$

$$\left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi \qquad for\ all\ j$$

$$\left|\frac{w_j}{w_w} - a_{jw}\right| \leq \xi \qquad for\ all\ j$$

$$\sum_j w_j = 1$$

$$w_j \geq 0 \qquad for\ all\ j$$

After solving the optimization problem (2), the optimal weights $(w_1^*, w_2^*, \dots, w_n^*)$ and ξ^*can be determined.

Step 6. When the optimal weights are determined, their consistency index must be obtained. The consistency index is obtained using Equation (3) and Table (3) (Reazei, 2015). The closer this index reaches zero, the more consistent the results are.

$$\text{Consistency ratio} = \frac{\xi^*}{\text{Consistency index}} \tag{3}$$

Table 3. Consistency index proposed by Rezaei (Rezaei, 2015)

a_{Bj}	1	2	3	4	5	6	7	8	9
Consistency index	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

A.3. Constructing The Model Of ER

Once the criteria weights are established, the distribution model of ER method needs to be constructed. To achieve this, assessors and experts must assign scores to each sub-criterion for each supplier and product. These scores serve as inputs for the ER model. Using the ER approach, the scores of the main criteria are then computed based on the scores of the sub-criteria. The ER approach is iterated at the main criteria to determine the overall score of each supplier for each product. This process ensures a comprehensive evaluation of suppliers' sustainability performance across various criteria and sub-criteria.

As mentioned before, the advantage of the ER approach compared with other existing methods of supplier evaluation is that the uncertainty that lies in the experts' subjective judgments can be considered in the model. This information is considered in the inputs of the model. As an example, the experts can state their judgments about a sub-criterion of environmental management systems for a supplier as {(3, 0.7), (4, 0.3)}, meaning that the expert's degree of belief in assigning the score, 3 to this sub-criterion is 70%, and with a 30% degree of belief assigns the score of 4. The distributed evaluation can be shown as {(1,0),(2,0),(3,0.7),(4,0.3),(5,0),(6,0),(7,0)}. Also, the sub-criterion social management commitment can be shown as {(2, 0.5), (3, 0.2)}, meaning that the supplier, in this criterion, gets the score 2 and 3 with a belief degree, 50%, and 20%, respectively. Because the sum of the degrees is less than 100%, it can be said that the evaluation is incomplete. This case may be the result of the inability or carelessness of the expert to perform a precise evaluation or lack of information.

Generally, it is assumed that in the supplier evaluation hierarchy, there are L main criteria represented by F_i ‘$i = 1, \dots, L$, Also, the *i*th main criterion has L_i sub-criteria and is denoted as $F_{ij}, i = 1, \dots, L,\ j = 1, \dots, L_i$. The company’s experts must state their judgments about each sub-criterion for each supplier as a distributed evaluation model. The distributed evaluation model is shown as follows:

$$S(F_{i-j}) = \{(g_k, \beta_{k,i-j}),\ k = 1, \dots, N\}, \qquad i = 1, \dots, L, j = 1, \dots, L_i$$

In the above equation, $S(F_{i-j})$ is the distributed evaluation concerning the criterion F_{ij}, and N is the number of evaluation grades and is shown as $g = \{g_1, \dots, g_N\}$. $\beta_{k,ij}$ is the expert's degree of belief in grade g_k. Note that $\beta_{k,ij} \geq 0$ and $\sum_{K=1}^{N} \beta_{k,i-j} \leq 1$. If $\sum_{K=1}^{N} \beta_{k,i-j} = 1$, then the evaluation is complete, otherwise it is incomplete. Moreover, if it equals zero, it implies lack of information.

The evaluation result of each main criterion is shown in a column matrix called the distributed evaluation matrix, denoted as follows.

$$\mathrm{D_i} = \left[\mathrm{S}(\mathrm{F_{i-j}})\right]_{\mathrm{L_i} \times 1}; \mathrm{i} = 1, \dots, \mathrm{L} \tag{4}$$

This matrix is not a typical one because each element is a distribution instead of a specific value.

A.4. The ER Approach For Combining Criteria Of Supplier Selection

It is assumed that for the *i*th main criterion, the distributed evaluation matrix (D_i) is given. Here, the ER approach is derived to combine the elements of this matrix and assign the score of the main criterion i. The output of the algorithm, which is the *i*th main criterion score, and we call it the distributed score (DS), is expressed by the evaluation grades $g = \{g_1, \dots, g_N\}$, in which degrees of belief are presented for each score. Once the DSs for all core criteria are determined, the ER approach is repeated at the core criteria level to obtain DSs for suppliers. The ER approach merges elements within the distributed matrix (Wang and Elhag, 2008).

The recursive ER approach combines evidence sequentially, offering clarity in concept and progression. On the other hand, the analytic evidential algorithm presents a more flexible approach to combining a multitude of supplier evaluation criteria. Its non-linear characteristics are easily comprehensible, allowing for straightforward sensitivity analysis on ER parameters such as weights and belief degrees. Additionally, it facilitates the approximation and optimization of these parameters. In this paper, we employed the analytic ER approach.

In the following, the application of analytical ER for calculating the DS of the *i*th main criterion based on the distributed matrix D_i is described. First, the belief degrees must be converted to probability values. For this purpose, the relative weights combined with the belief degrees are used according to the following equations:

$$m_{k,i-j} = w_{i-j}\beta_{k,i-j}, \qquad k = 1, \dots, N, \quad j = 1, \dots, L_i \tag{5}$$

$$m_{g,i-j} = 1 - \sum_{k=1}^{N} m_{k,i-j} = 1 - w_{i-j} \sum_{k=1}^{N} \beta_{k,i-j}, \quad j = 1, \dots, L_i \tag{6}$$

$$\bar{m}_{g,i-j} = 1 - w_{i-j}, \quad j = 1, \dots, L_i \tag{7}$$

$$\tilde{m}_{g,i-j} = w_{i-j}(1 - \sum_{k=1}^{N} \beta_{k,i-j}), \quad j = 1, \dots, L_i \tag{8}$$

$$m_{g,i-j} = \bar{m}_{g,i-j} + \tilde{m}_{g,i-g} \tag{9}$$

Here, $m_{k,i-j}$ is the probability mass distribution function of assigning the grade $g_k$for the sub-criteria F_{i-j}. $m_{g,i-j}$ represents the probability value that is not assigned to the set g. This value can be separated into two parts, $\bar{m}_{g,i-j}$ and $\tilde{m}_{g,i-j}$, in this $\bar{m}_{g,i-j}$ is caused by the relative importance of the criterion j and $\tilde{m}_{g,i-j}$ is caused by the incompleteness of the evaluation of the sub-criterion j. $\bar{m}_{g,i-j}$ shows to what extent other factors can affect the evaluation of the overall objective.

Then, the probability values are combined based on the following analytic evidential algorithm:

$$m_k = p\left[\prod_{j=1}^{L_i}\left(m_{k,i-j} + \bar{m}_{g,i-j} + \tilde{m}_{g,i-j}\right) - \prod_{j=1}^{L_i}(\bar{m}_{\mathrm{g},i-j} + \tilde{m}_{g,i-j})\right], \quad k = 1, \dots, N \tag{10}$$

$$\tilde{m}_g = p\left[\prod_{j=1}^{L_i}\left(\bar{m}_{g,i-j} + \tilde{m}_{g,i-j}\right) - \prod_{j=1}^{L_i}(\bar{m}_{g,i-j})\right] \tag{11}$$

$$\bar{m}_g = p\left[\prod_{j=1}^{L_i}\bar{m}_{g,i-j}\right] \tag{12}$$

$$p = \left[\sum_{k=1}^{N}\prod_{j=1}^{L_i}\left(m_{k,i-j} + \bar{m}_{g,i-j} + \tilde{m}_{g,i-j}\right) - (N-1)\prod_{j=1}^{L_i}\left(\bar{m}_{g,i-j} + \tilde{m}_{g,i-j}\right)\right]^{-1} \tag{13}$$

$$\beta_k = \frac{m_k}{1 - \bar{m}_g}, \quad k = 1, \dots, N \tag{14}$$

$$\beta_g = \frac{\tilde{m}_g}{1 - \bar{m}_g} \tag{15}$$

Therefore, the *i*th main factor takes the grade g_k with the degree of belief g_k. In other words, the DS of the *i*th factor is as $S(F_i) = \{(g_k, \beta_k), k = 1, \dots, N\}$. β_g is the degree of belief caused by the incompleteness of the evaluation which is assigned to the evaluation grades set (g). The equation $\sum_{k=1}^{N}\beta_k + \beta_g = 1$ should be satisfied to verify the accuracy of the calculation (Yang and Xu, 2002).

A.5. Ranking The Suppliers

Assuming M suppliers, each of which assessed by the ER approach and their DSs are as $S(R_h) = \{(g_k, \beta_k(R_h)), k = 1, \ldots\ldots, N\}$, $h = 1, \ldots, M$, in which $\beta_k(R_h)$ is the degree of belief in grade g_k on the h_{th} supplier. The distribution score gives an approximate evaluation of suppliers, but cannot be used directly for comparison and ranking.

We consider the mean value for each supplier to rank M suppliers based on sustainability criteria. Thus a utility value for each of the evaluation grades(g_k; $k = 1, \ldots, N$) must be determined. The utility value for (g_k) is denoted by $Du(g_k)$. Thus, the expected value for the h_{th} supplier is obtained as follows:

$$E(S(R_h)) = \sum_{k=1}^{N} \beta_k(R_h)u(g_k) \tag{16}$$

$\beta_k(R_h)$ is a lower bound for the degree of belief in the grade g_k for supplier R_h, and its upper limit is determined by $(\beta_k(R_h) + \beta_g(R_h))$. Consequently, a belief degree interval is assigned to the grade $g_k$in cases of an incomplete evaluation. In the evaluation grade set g, it is assumed that $g_1$is the lowest rank, which has the min utility and g_N as the highest rank with the max utility. The max, min, and average of the expected value of the supplier $R_h$can be obtained as follows:

$$E_{max}(R_h) = (\beta_N(R_h) + \beta_g(R_h))Du(g_N) + \sum_{k=1}^{N-1} \beta_k(R_h)Du(g_k) \tag{17}$$

$$E_{min}(R_h) = \sum_{k=2}^{N} \beta_k(R_h)Du(g_k) + (\beta_1(R_h) + \beta_g(R_h))Du(g_1) \tag{18}$$

$$E_{avg}(R_h) = \frac{E_{max}(R_h) + E_{min}(R_h)}{2} \tag{19}$$

Obviously, if $Du(g_1) = 0$, then $E(S(R_h)) = E_{min}(R_h)$. If the distributed evaluations on all sub-criteria are complete, then we have $(\beta_g(R_h) = 0$ and $E(S(R_h)) = E_{min}(R_h) = E_{max}(R_h) = E_{avg}(R_h)$.

If the distributed evaluations on all sub-criteria are complete, then the value of the supplier h is more than that of the supplier q if and only if $E(S(R_h)) > E(S(R_q))$. However, if the distributed evaluations on all sub-criteria of the suppliers are incomplete, then the comparison between two suppliers is done based on their max and min expected value as follows.

1. If $E_{min}(R_h) \geq E_{max}(R_q)$ $\rightarrow$ then the value of the supplier R_h is more than that of the supplier R_q.
2. If $E_{min}(R_h) = E_{min}(R_q)$ and $E_{max}(R_h) = E_{max}(R_q)$ $\rightarrow$ then the value of the supplier R_h is not different from that of the supplier R_q.
3. In other cases, the comparison of R_h and R_q can be done based on the following formula (Wang and Elhag, 2008).

$$P(R_h > R_q) = \frac{max[0, E_{max}(R_h) - E_{min}(R_q)] - max[0, E_{min}(R_h) - E_{max}(R_q)]}{[E_{max}(R_h) - E_{min}(R_h)] + [E_{max}(R_q) - E_{min}(R_q)]} \tag{20}$$

If $P(R_h > R_q) > 0.5$, the value of the supplier R_h is more than that of the supplier R_q to the degree of $P(R_h > R_q)$. If $P(R_h > R_q) = 0.5$, then the value of the supplier R_h is equal to that of the supplier R_q. If $P(R_h > R_q) < 0.5$, the value of the supplier $R_h$is less than that of the supplier $R_q$with a $1 - P(R_h > R_q)$degree.

B. Bi-Objective Milp Model

we develop a bi-objective model. The first objective is to optimize the TVP based on the weights in ER approach considering economic, environmental, and social criteria. The second OF aims to optimize the TCP.

B.1. Subscripts

t	The stage index
i	The supplier index
s	The disruption scenarios (s= 1, 2, ... , S)

B.2. Parameters

x	Available inventory
X	Maximum allowed inventory level
Y	Maximum allowed shortage level
b_i	The capacity of supplier *i*
w_i	Importance weight of supplier *i*
D_t^s	Demand in stage t under scenario *s*
h_t	Holding cost per each item in stage *t*
A_t	Shortage cost per each item in stage *t*
p_{it}	Buying cost from the supplier i per each item in stage *t*

B.3. Decision Variables

x_t^s	Inventory level at the end of stage *t*
y_t^s	Shortage level at the end of stage *t*
q_{it}^s	Order quantity from the supplier i in stage *t*
k_t^s	$\begin{cases} 1 & \text{if there is inventory} \\ 0 & \text{if there is shortage} \end{cases}$

All these variables depend on the scenario considered, and the here-and-now variable in our stochastic model is the order quantity in each stage under each scenario.

B.4. Objective Function and Constraints

To address multiple demand scenarios in the SC, we formulate a SC model as follows:

$$\max \mathrm{TVP} = \sum_{s=1}^{S} \frac{1}{S} \sum_{t=1}^{T} \sum_{i=1}^{n} w_i q_{it}^s \tag{21}$$

$$min\,\mathrm{TCP} = \sum_{s=1}^{S}\frac{1}{S}(\sum_{t=1}^{T}\mathrm{h_t\,x_t^s} + \sum_{t=1}^{T}\mathrm{A_t\,y_t^s} + \sum_{t=1}^{T}\sum_{i=1}^{n}\mathrm{p_{it}q_{it}^s}) \tag{22}$$

s. t.

$$0 \le \mathrm{q_{it}^s} \le \mathrm{b_i} \qquad \forall \mathrm{i,t,s} \tag{23}$$

$$0 \le \mathrm{x_t^s} \le \mathrm{X}\mathrm{k}_t^s \qquad \forall \mathrm{t,s} \tag{24}$$

$$0 \le \mathrm{y_t^s} \le \mathrm{Y}(1 - \mathrm{k}_t^s\ ) \qquad \forall \mathrm{t,s} \tag{25}$$

$$\mathrm{x_{t-1}^s} + \sum_{i=1}^{n}\mathrm{q_{it}^s} = \mathrm{x_t^s} + \mathrm{D_t^s} - \mathrm{y_t^s} \qquad \forall \mathrm{t,s} \tag{26}$$

$$\mathrm{x_t^s} \ge 0,\ \mathrm{y_t^s} \ge 0, \mathrm{q_{it}^s} \ge 0,\ \mathrm{k}_t^s\ = 0{,}1 \tag{27}$$

In this model, equation (21) represents the first OF, which aims to maximize TVP along with the weight obtained from the ER approach, incorporating the total economic, environmental, and social score of the suppliers. Since order of the supplier *i* in stage *t* under scenario *s* is different, this equation is multiplied by $\frac{1}{S}$. This optimization ensures that the average value of OF is optimized. Equation (22) shows the second OF, which aims to optimize the total purchase cost (TCP). This equation calculates the total cost, including holding cost, shortage cost and purchase cost. Similar to equation (21), the scenario-based technique is used and the total equation is multiplied by $\frac{1}{S}$, which represents the possible scenarios *s*. Constraint (23) specifies the maximum capacity of suppliers. Constraint (24) guarantees that the amount of raw materials supplied does not exceed the predetermined limit. Constraint (25) implies that the raw material shortage amount is less than or equal to the predetermined amount at each stage under each scenario. Constraint (26) shows inventory changes based on demand in each scenario. Constraint (27) forces the decision variables to be non-negativity. The aim of this model is to optimize both TVP and TCP simultaneously.

C. Integrated SP and DP Approach

Since inventory holding cost, inventory shortage cost, purchasing prices, and demand are time-varying in the developed model, the cost OF (and its related constraints (23), (24), (25), (26), and (27) are defined using a DP with a recursive formula (Bellman and Kalaba, 1957).

$$V_{1t}\left(x^s\right) = \min\left\{\left(h_t x^s + A_t y^s + \sum_{i=1}^{n} p_{it} q_{it}^s\right) + \sum_{s=1}^{S}\frac{1}{S}\left(V_{1,t+1}\left(x^s + \sum_{i=1}^{n} q_{it}^s - D_t^s + y^s\right)\right)\right\} \quad \forall \mathrm{i,t,s} \tag{28}$$

$$0 \le q_{it}^s \le b_i$$

$$0 \le x^s \le X k_t^s$$

$$0 \le y^s \le Y(1 - k_t^s\ )$$

The backward approach is used to solve DP where stage is the decision dates in stages, *t*= 1, 2 ..., T. State variable is the inventory level in stages of decision under scenario *s*, x^s= 1, 2... X. The decision variable is the order value from the supplier *i* in stage *t* under scenario s, $q_{it}^{s} = 0, \dots, b_i$. $V_{1t}(x^s)$ is the minimum total purchase cost if the inventory level is *x* under scenario s.

To optimize both the TVP and TCP OFs simultaneously, we used a distance-to-ideal method (Collette et al., 2004). This method integrates the TVP and TCP functions by considering the optimal values of individual objectives and their related constraints (23, 24, 25, 26, and 27) (Mafakheri et al., 2011).

Finally, we formulate a DP model as follows:

$$V_{2t}(x^s) = min\left\{(TVP_{max}^{t}(x^s) - \sum_{i=1}^{n} w_i q_{it}^{s}(x^s))/(TVP_{max} - TVP_{min}) + ((h_t x^s + A_t y^s + \sum_{i=1}^{n} p_{it} q_{it}^{s}\,(x^s)) - TCP_{min}^{t}(x^s))/(TCP_{max} - TCP_{min}) + \sum_{s=1}^{S} \frac{1}{S} V_{2,t+1}(x^s + \sum_{i=1}^{n} q_{it}^{s} - D_t^s + y^s\right\} \qquad \forall i,t,s \; (29)$$

$0 \le q_{it}^{s} \le b_i$

$0 \le x^s \le X k_t^s$

$0 \le y^s \le Y(1 - k_t^s\,)$

Our goal was to optimize the total normalized deviation of each OF from its optimal value based on the inventory level (x^s) in stage *t* in scenario s . $TVP_{max}^{t}(x^s)$ is the optimal value of the TVP function in stage *t* in scenario *s* when the inventory level is (x^s). $TCP_{min}^{t}(x^s)$ is the optimal value of the TCP function in stage *t* in scenario *s* when the inventory level is (x^s). We analyzed the problem with TVP as the only objective. Once TVP_{max} is obtained, we use these optimized order values in the TCP function to obtain its worst value (TCP_{max}). Similarly, the function TCP_{min} can be determined.

The optimal solution is obtained using the DP and SP approach. We used the backward method to solve the decision tree with DP. OF is obtained by adding the OF at the value of that stage to the average expected optimal value of OF. The flow chart of the DP method is shown in Figure 2.

IV. NUMERICAL RESULTS

This section uses a real-world numerical example to demonstrate the model's performance. First, we introduce the case, and then utilize the model to generate the solutions.

A. Case study

In this research, we applied the problem of sustainable SS and OA in the Composite Products Company. The manager intends to improve supplier evaluation and allocation by including sustainability aspects and prioritizing suppliers. For this purpose, managers selected criteria based on sustainability criteria.

Table 4 outlines the criteria, the company's list of related products, identified suppliers, and the raw materials required.

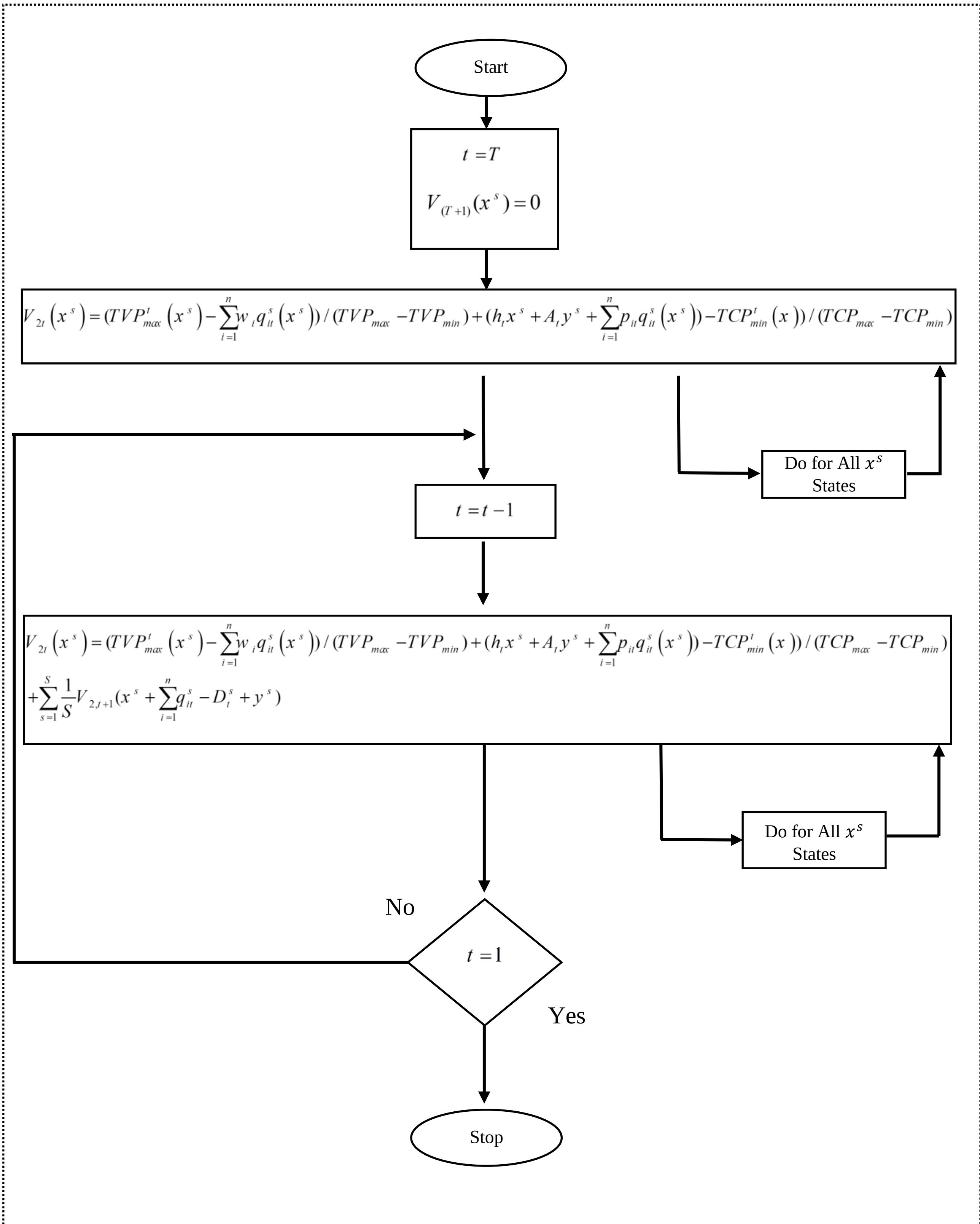

Fig. 2. Flow chart of dynamic programming algorithm for solving decision tree

Table 4. The detail of case study

List of firm's products	List of raw materials	List of suppliers' names
Composite Pipes and reservoir Epoxy adhesive Different composite structures and coverings	Resin – Unsaturated Poly Ester	Supplier from Tehran (TES) Supplier from Ghazvin (GHS) Supplier from Esfahan (ESS) Taiwanese Supplier (TAS) European supplier (EUS)

B. Solution method

In this section, supplier evaluation method, prioritization and OA are described. In the first step, sustainable criteria are weighted using BWM. In the second step, supplier prioritization is determined using the ER approach. Because the criteria are qualitative and the experts may lack complete experience, uncertainty is high. This method allows evaluators to express scores as probability distributions or even scores for specific criteria. Finally, the suppliers are prioritized based on this, and the weight of each supplier is determined. In the second step, a two-objective mixed integer programming model is developed. Combining DP and SP is an integrated approach to solve this model.

B.1. Implementing the BWM

First, experts determine the best and worst criteria. Then, pairwise comparisons are shown in Tables 5 and 6.

Table 5. Best-to-Others vectors of main criteria

Criteria	Economic (EC)	Environmental (EN)	Social (SO)
Economic (EC)	1	2	4

Table 6. Others -to-Worst vectors of main criteria

Criteria	Social (SO)
Economic (EC)	4
Environmental (EN)	2
Social (SO)	1

Steps 5 and 6 of the BWM are executed to find the optimal weight for each criterion and calculate the consistency ratio. The comparison results are presented in Table 7. It's worth noting that Lingo 17 software is used for step 5 of the BWM.

Table 7. Results of BWM for main criteria

Criteria	weight
Economic (EC)	0.5714286
Environmental (EN)	0.2857143
Social (SO)	0.1428571
ξ^*	0.0001285
Consistency ratio	0.0001285/1.63= 0.00007883

Similarly, the same steps are carried out for all sub-criteria. Pairwise comparisons for economic criteria are displayed in Tables 8 and 9. The optimal weight and consistency ratio are provided in Table 10.

Table 8. Best-to-Others vectors for Economic criteria

Criteria	ECQ	ECC	ECDS	ECTF	ECLR	ECF
Cost (ECC)	2	1	4	5	4	8

Table 9. Others -to-Worst vectors for economic criteria

Criteria	ECF
ECQ	7
ECC	8
ECDS	4
ECTF	2
ECLR	3
ECF	1

Table 10. Results of BWM for economic criteria

Criteria	Weight
ECQ	0.3129128
ECC	0.3520269
ECDS	0.1173423
ECTF	0.0756199
ECLR	0.1029839
ECF	0.0391141
ξ^*	1.000000
Consistency ratio	1.000000/4.47= 0.2237136

The pairwise comparisons for environmental criteria are illustrated in Tables 11 and 12. Table 13 presents their optimal weights and consistency ratios.

Table 11. Best-to-Others vectors for environmental criteria

Criteria	ENEM	END	ENGL	ENPW	ENEC
ENEM	1	3	5	2	2

Table 12. Others -to-Worst vectors for environmental criteria

Criteria	ENGL
ENEM	5
END	2
ENGL	1
ENPW	3
ENEC	3

Table 13. Results of BWM for environmental criteria

Criteria	Weight
ENEM	0.3786797
END	0.1338835
ENGL	0.07322331
ENPW	0.2071068
ENEC	0.2071068
ξ^*	0.1715736
Consistency ratio	0.1715736/2.3= 0.07459217

The pairwise comparisons for social criteria are displayed in Tables 14 and 15. Table 16 presents their optimal weight and consistency ratio.

Table 14. Best-to-Others vectors for social criteria

Criteria	SOOHS	SOSMC	SOIE	SOWW
SOSMC	2	1	2	4

Table 15. Others -to-Worst vectors for social criteria

Criteria	SOWW
SOOHS	3
SOSMC	4
SOIE	3
SOWW	1

Table 16. Results of BWM for social criteria

Criteria	Weight
SOOHS	0.2742919
SOSMC	0.4514162
SOIE	0.2742919
SOWW	0.1036726
ξ^*	0.3542487
Consistency ratio	0.3542487/ 1.63= 0.2173304

B.2. Implementing the ER

In this step, the suppliers are ranked based on evidence reasoning. Table 17 shows the results of the ER approach for supplier evaluation (Wang and Elhag, 2008).

Table 17. Suppliers' criteria evaluation standards defined for Suppliers

Grade	Meaning
7	Excellent condition
6	Very good condition
5	Good condition
4	Fair condition
3	Poor condition
2	Very poor condition
1	Critical condition

The distributed evaluations of five suppliers are presented in Table 18. The score {(5, 1.0)} is assigned to the sub-criterion “quality” for the second supplier which means there is a 100% probability that the materials produced by this supplier are suitable. The score {(4, 0.7), (5, 0.3)} is obtained from the sub-criterion “environmental management” for the first supplier, meaning there's a 30% possibility that this supplier is in good state and a 70% probability that it is in an average condition. Other scores in this table can be interpreted in a similar manner.

Table 18. Distributed evaluation information for the five suppliers

Suppliers criteria	Supplier 1 (TES)	Supplier 2 (GHS)	Supplier 3 (ESS)	Supplier 4 (TAS)	Supplier 5 (EUS)
Economic (0.5714286)					
ECQ (0.3129128)	{(6,1.0)}	{(5,1.0)}	{(4,1.0)}	{(6,1.0)}	{(7,1.0)}
ECC (0.3520269)	{(4, 1.0)}	{(6,1.0)}	{(6,1.0)}	{(3,1.0)}	{(1,1.0)}
ECDS (0.1173423)	{(6, 0.85),(7,0.15)}	{(5,0.9),(6,0.1)}	{(3, 0.8),(4, 0.2)}	{(4, 0.7),(3,0.3)}	{(2, 0.85),(1,0.15)}
ECTF (0.0756199)	{(6, 0.9),(7,0.1)}	{(3, 0.95),(2,0.05)}	{(2, 0.6),(3, 0.4)}	{(5, 0.95),(4,0.05)}	{(7,1.0)}
ECLR (0.1029839)	{(5, 1.0)}	{(2, 0.8),(3, 0.2)}	{(3, 0.95),(4,0.05)}	{(5,1.0)}	{(1, 0.6),(2,0.4)}
ECF (0.0391141)	{(6, 0.7),(5,0.3)}	{(4, 0.85),(3,0.15)}	{(3, 0.9),(2,0.1)}	{(6,0.8),(5,0.2)}	{(6,1.0)}
Environmental (0.2857143)					
ENEM (0.3786797)	{(4, 0.7),(5,0.3)}	{(3, 0.5),(4,0.5)}	{(3, 0.6),(4, 0.4)}	{(5, 0.9),(4, 0.1)}	{(7, 0.95),(6,0.05)}
END (0.1338835)	{(2, 0.85),(3,0.15)}	{(1, 0.75),(2,0.25)}	{(1, 1.0)}	{(2, 0.5),(3, 0.5)}	{(7, 0.9),(6,0.1)}
ENGL (0.0732233)	{(5, 0.6),(4,0.4)}	{(5, 0.9),(4,0.1)}	{(5, 0.7),(4, 0.3)}	{(4, 0.6),(3, 0.4)}	{(4, 0.95),(3,0.0.5)}
ENPW (0.2071068)	-	{(6, 1.0)}	{(3, 0.6),(2, 0.4)}	{(6, 0.7),(5, 0.3)}	{(7, 0.9),(6,0.1)}
ENEC (0.2071068)	-	-	{(1, 0.7),(2, 0.3)}	-	-
Social (0.1428571)					
SOOHS (0.274291)	{(6, 0.99),(7,0.01)}	{(4, 0.75),(3,0.25)}	{(5, 0.5),(6,0.5)}	{(6, 0.85),(5,0.15)}	{(7,1.0)}
SOSMC (0.451416)	{(5, 0.75),(4,0.25)}	{(3, 0.5),(2,0.5)}	{(4, 0.8),(5,0.2)}	{(6, 0.9),(7,0.1)}	{(6,1.0)}
SOIE (0.2742919)	{(5, 1.0)}	{(4, 0.85),(3,0.15)}	{(5, 1.0)}	-	{(6, 0.7),(7,0.3)}
SOWW (0.103672)	{(6, 1.0)}	{(4, 0.9),(3,0.1)}	{(5, 0.9),(4,0.1)}	{(6, 0.5),(5,0.5)}	{(6, 0.75),(5,0.25)}

To enhance the applicability of this algorithm, Microsoft Excel is utilized. Computations commence from the lowest level in a stepwise manner. Information from each level is combined and utilized as input for the subsequent level. Eventually, a distribution score is obtained for each supplier. It's noteworthy that some suppliers have incomplete DSs. Table 19 displays the final DSs for all suppliers. Then, a DS is obtained for each criterion using equations (5) to (9). These steps are reiterated at the primary criteria level to ascertain the distribution score of each supplier. For instance, in Table 19, the first supplier has the DS {(1,0), (2,0), (3,0), (4,0.36494), (5,0.08626), (6,0.5298), (7,0.019)}. Thus, this supplier has an excellent economic status with about 52% probability, an average status with about 36% probability, a good status with about 8% probability, and an excellent status with about a 1.9% probability. When the information is incomplete, the $\beta_g$column takes a value that indicates the probability of incomplete information. This method allows us to evaluate the DSs in this table.

Table 19. The aggregated distributed evaluations for the five suppliers

Supplier	Suppliers criteria	Degrees of belief assessed to each grade							
		1	2	3	4	5	6	7	β_g
Supplier 1 (TES)	Economic (EC)	0	0	0	0.36494	0.08626	0.5298	0.019	0
	Environmental (EN)	0	0.10664	0.17066	0.38286	0.03847	0	0	0.30137
	Social (SO)	0	0	0	0.10437	0.62323	0.27048	0.00192	0
	The whole Supplier	0	0.01920	0.03073	0.36276	0.12496	0.39513	0.01294	0.054266
Supplier 2 (GHS)	Economic (EC)	0	0.06719	0.07617	0.02414	0.43919	0.39331	0	0
	Environmental (EN)	0	0.10892	0.33369	0.29600	0.06681	0	0	0.19458
	Social (SO)	0	0.21262	0.33656	0.45081	0	0	0	0
	The whole Supplier	0	0.08985	0.15892	0.11975	0.32305	0.27110	0	0.037331
Supplier 3 (ESS)	Economic (EC)	0	0.02637	0.15501	0.24847	0.30162	0.26853	0	0
	Environmental (EN)	0.24961	0.13039	0.39571	0.18653	0.03775	0	0	0
	Social (SO)	0	0	0	0.34336	0.56086	0.09578	0	0
	The whole Supplier	0.04906	0.04380	0.19539	0.25181	0.27337	0.18657	0	0
Supplier 4 (TAS)	Economic (EC)	0	0	0.33676	0.0543	0.32898	0.27996	0	0
	Environmental (EN)	0	0.05684	0.08188	0.08181	0.49267	0.13447	0	0.15232
	Social (SO)	0	0	0	0	0.06177	0.75126	0.04315	0.143822
	The whole Supplier	0	0.01046	0.24091	0.05146	0.35277	0.30219	0.00343	0.03879
Supplier 5 (EUS)	Economic (EC)	0.50019	0.04894	0	0	0	0.02983	0.42104	0
	Environmental (EN)	0	0.10469	0.01465	0.08114	0.46676	0.17691	0	0.15585
	Social (SO)	0	0	0	0	0.01398	0.72773	0.25829	0
	The whole Supplier	0.34744	0.05587	0.00287	0.01592	0.09298	0.12688	0.32747	0.03057

To rank the suppliers, we need a single numerical value instead of a distribution. For this purpose, equations (17), (18), and (19) are used, and the maximum, minimum, and average values of degrees for each supplier are obtained. The utility value for each degree, as defined by Wang and Elhag (2008), are as follows:

The results are presented in Table 20. In particular, when complete information is available from one supplier, the max and min values for that supplier are equal, as seen with the third supplier. Conversely, when information is incomplete, there is a difference between the max and min value. For example, the min and max values of the first supplier are 0.55 and 0.61, respectively. Equation (20) is used to evaluate relative superiority based on their minimum and maximum values. Table 21 shows the preferences between suppliers. For example, the first supplier is better than the second, third, and fifth suppliers with 100% probability. The probability that the first supplier is preferred over the fourth supplier is 0.91. Since this probability is greater than 0.5, it indicates that the first supplier is also better than the fourth supplier. Likewise, the fourth supplier is better than the third supplier with 100% probability. Furthermore, it has an 86.784% probability of being better than the second supplier and a 98% probability of being better than the fifth supplier. The second supplier outperforms the third supplier with 100% certainty and has a 58% chance of being better than the fifth supplier. Finally, the fifth supplier is definitely better than the third supplier with 100% probability. The last column of the table shows the normalized weight of the suppliers, which is obtained based on the average values.

Table 20. The expected utilities of the five suppliers

Supplier	$E_{max}(R_h)$	$E_{min}(R_h)$	$E_{avg}(R_h)$
Supplier 1	0.61147	0.55720	0.58433
Supplier 2	0.53671	0.49938	0.51804
Supplier 3	0.45746	0.45746	0.45746
Supplier 4	0.56544	0.52665	0.54605
Supplier 5	0.52786	0.49729	0.51258

Table 21. Priority of suppliers based on sustainability criteria Using the ER

Supplier	Supplier 1	Supplier 2	Supplier 3	Supplier 4	Supplier 5	Ranking order	Normalized weights
Supplier 1	-	0	0	0.088545	0	1	0.22316
Supplier 2	1	-	0	0.86784	0.41944	3	0.19784
Supplier 3	1	1	-	1	1	5	0.17471
Supplier 4	0.911455	0.13216	0	-	0.017445	2	0.20854
Supplier 5	1	0.58056	0	0.982555	-	4	0.19576

B.3. Validation of the ER

To validate our model, we employed the AHP and fuzzy AHP methods. Initially, the criteria were assessed using the AHP approach, and the supplier rankings were obtained, as shown in Table 22. Then, the criteria were ranked using fuzzy AHP, and the results are described in Table 23. It can be observed that the results obtained from the AHP method differ from the results of the fuzzy AHP and ER methods. This difference is due to the uncertainty in evaluating the criteria and sub-criteria, which must be considered definitively in the AHP method. However, the results of the ER and fuzzy AHP methods are similar, both providing a ranking for SS. Taleghani et al. (2012), in their study focusing on the household appliance industry, conducted a comparison between the AHP and fuzzy AHP methods. They concluded that the results obtained from the fuzzy AHP method were closer to reality and more reliable. Therefore, it can be inferred that the results obtained from the ER approach are also valid. One of the advantages of this approach over fuzzy AHP is that, in fuzzy AHP, all information about sub-criteria must be available. At the same time, in the ER approach, it can provide an acceptable evaluation despite missing data. Additionally, the simplicity of implementing this method is another advantage. Instead of pairwise comparisons between suppliers for each criterion, it is only necessary to assign a distribution score to each supplier based on the criteria. Then, the best ranking is determined based on the available steps in this method, reducing the need for extensive questioning and, consequently, lowering the likelihood of errors.

Table 22. Priority of suppliers based on sustainability criteria Using the AHP

Supplier	Economic (EC)	Environmental (EN)	Social (SO)	AHP	Ranking order	Normalized weights
Supplier 1	0.268762	0.231087	0.237651	0.253566	1	0.253552
Supplier 2	0.187597	0.13102	0.149814	0.166042	4	0.166032
Supplier 3	0.175687	0.120987	0.072939	0.145387	5	0.145378
Supplier 4	0.240313	0.201549	0.220251	0.226383	2	0.226370
Supplier 5	0.127641	0.315357	0.319345	0.208678	3	0.208666

Table 23. Priority of suppliers based on sustainability criteria Using the Fuzzy-AHP

Supplier	Economic (EC)	Environmental (EN)	Social (SO)	Fuzzy-AHP	Ranking order	Normalized weights
Supplier 1	0.273683	0.254055	0.294792	0.271105	1	0.270833
Supplier 2	0.248896	0.175221	0.130619	0.210959	3	0.210748
Supplier 3	0.159612	0.048903	0.003597	0.105696	5	0.10559
Supplier 4	0.193895	0.223463	0.25693	0.211361	2	0.211149
Supplier 5	0.123914	0.298358	0.320691	0.201883	4	0.201681

B.4. Implementing the integrated DP and Sp approach

The input data for the case study are shown in Tables 24, and 25, and these parameters are obtained by the company's experts. These data are related to the three stages of five suppliers. It should be noted that demand in each stage is selected from possible values of 320, 420, and 500 with equal probability, and as a result, $3^{3} = 27$ scenarios are created in three stages. The holding cost per unit in all stages is 1000 ($h_t = 1000$). The cost of one shortage is a 12,000 for all stages ($A_t = 12000$). The maximum allowed inventory level is 200 (X = 200) and the maximum allowed shortage level is 1000 (Y = 1000).

Table 24. Demand information

Scenario	Stage 1 (t=1)	Stage 2 (t=2)	Stage 3 (t=3)
Scenario 1	320	320	320
Scenario 2	320	320	420
Scenario 3	320	320	500
Scenario 4	320	420	320
Scenario 5	320	420	420
Scenario 6	320	420	500
Scenario 7	320	500	320
Scenario 8	320	500	420
Scenario 9	320	500	500
Scenario 10	420	320	320
Scenario 11	420	320	420
Scenario 12	420	320	500

Continue Table 24. Demand information

Scenario	Stage 1 (t=1)	Stage 2 (t=2)	Stage 3 (t=3)
Scenario 13	420	420	320
Scenario 14	420	420	420
Scenario 15	420	420	500
Scenario 16	420	500	320
Scenario 17	420	500	420
Scenario 18	420	500	500
Scenario 19	500	320	320
Scenario 20	500	320	420
Scenario 21	500	320	500
Scenario 22	500	420	320
Scenario 23	500	420	420
Scenario 24	500	420	500
Scenario 25	500	500	320
Scenario 26	500	500	420
Scenario 27	500	500	500

Table 25. Price and capacity information

Suppliers	Ordering price (per unit)			Capacity
	Stage 1	Stage 2	Stage 3	
Supplier 1	91000	94000	98000	300
Supplier 2	56000	57000	62000	300
Supplier 3	55000	53000	51000	400
Supplier 4	112000	110000	109000	450
Supplier 5	150000	165000	168000	600

To analyze the performance of the proposed model, we initially considered the model with a single OF. The model was solved to optimize the Total Value of Production (TVP), achieving its optimal value. Subsequently, the optimal order values from this scenario were used in the Total Cost of Production (TCP) OF to determine its worst value. The values of optimal order quantity and TVP value in different scenarios are shown in Table 26. The subsequent step involves the same procedure for TCP. Initially, the TCP OF is included in the model, and its best value is determined, specifying the optimal value for each scenario. Subsequently, akin to the preceding step, these optimal values are inserted into the TVP OF to ascertain its worst value. The outcomes of these computations are displayed in Table 27.

Table 26. Optimal order quantities with respect to maximizing TVP

Total TCP=TCP_{max}				**1733800.586**							
Total TVP=TVP_{max}				**286.9914**							
Scenario1				**Scenario2**				**Scenario3**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	20	20	20	q_4	20	20	120	q_4	20	20	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario4				**Scenario5**				**Scenario6**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	20	120	20	q_4	20	120	120	q_4	20	120	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario7				**Scenario8**				**Scenario9**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	20	200	20	q_4	20	200	120	q_4	20	200	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario10				**Scenario11**				**Scenario12**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	140	0	20	q_4	140	0	120	q_4	140	0	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario13				**Scenario14**				**Scenario15**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	120	120	20	q_4	120	120	120	q_4	120	120	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario16				**Scenario17**				**Scenario18**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	320	0	20	q_4	320	0	120	q_4	320	0	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

Continue Table 26. Optimal order quantities with respect to maximizing TVP

Scenario19				Scenario20				Scenario21			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	200	20	20	q_4	200	20	120	q_4	200	20	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario22				**Scenario23**				**Scenario24**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	200	120	20	q_4	200	120	120	q_4	200	120	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario25				**Scenario26**				**Scenario27**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	300	q_1	300	300	300	q_1	300	300	300
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	200	200	20	q_4	200	200	120	q_4	200	200	200
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

Table 27. Optimal order quantities with respect to minimizing TCP

Total TCP=TCP_{min}				943800.428							
Total TVP=TVP_{min}				**178.54537**							
Scenario1				**Scenario2**				**Scenario3**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	0	0	0	q_2	0	0	0	q_2	0	0	0
q_3	320	320	320	q_3	320	340	400	q_3	340	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario4				**Scenario5**				**Scenario6**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	0	0	0	q_2	0	0	0	q_2	0	40	0
q_3	340	400	320	q_3	360	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario7				**Scenario8**				**Scenario9**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	0	20	0	q_2	0	40	0	q_2	120	0	40
q_3	400	400	320	q_3	400	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

Continue Table 27. Optimal order quantities with respect to minimizing TCP

Scenario10				**Scenario11**				**Scenario12**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	60	300	0	q_2	60	300	0	q_2	60	300	80
q_3	400	0	300	q_3	400	0	400	q_3	400	0	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario13				**Scenario14**				**Scenario15**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	40	0	0	q_2	60	0	0	q_2	20	120	0
q_3	400	400	320	q_3	400	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario16				**Scenario17**				**Scenario18**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	20	100	0	q_2	20	120	0	q_2	20	200	0
q_3	400	400	320	q_3	400	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario19				**Scenario20**				**Scenario21**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	100	0	0	q_2	100	0	0	q_2	120	0	0
q_3	400	320	320	q_3	400	340	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario22				**Scenario23**				**Scenario24**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	120	0	0	q_2	140	0	0	q_2	220	0	0
q_3	400	400	320	q_3	400	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario25				**Scenario26**				**Scenario27**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	0	0	0	q_1	0	0	0	q_1	0	0	0
q_2	200	0	0	q_2	220	0	0	q_2	300	0	0
q_3	400	400	320	q_3	400	400	400	q_3	400	400	400
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

In the third step, we calculate the optimal values of both OFs using equation (29), to obtain the optimal order. The results reveal that the outcomes for both objectives, when optimized together using this method, are very near to those obtained when the OFs are optimized separately. The results are presented in Table 28.

Table 28. Optimal order quantities obtained in DP model

Total TCP				**996750.428**							
Total TVP				**246.8212**							
Scenario1				**Scenario2**				**Scenario3**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	100	300	q_1	300	100	300	q_1	300	100	300
q_2	240	0	20	q_2	240	0	120	q_2	240	0	200
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario4				**Scenario5**				**Scenario6**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	100	q_1	300	300	200	q_1	300	300	280
q_2	240	120	0	q_2	240	120	0	q_2	240	120	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario7				**Scenario8**				**Scenario9**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	100	q_1	300	300	200	q_1	300	300	280
q_2	240	200	0	q_2	240	200	0	q_2	240	200	0
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario10				**Scenario11**				**Scenario12**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	140	300	q_1	300	140	300	q_1	300	140	300
q_2	300	0	20	q_2	300	0	120	q_2	300	0	200
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario13				**Scenario14**				**Scenario15**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	140	q_1	300	300	240	q_1	300	300	300
q_2	300	0	0	q_2	300	0	0	q_2	300	0	20
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	120	0	q_4	0	120	0	q_4	0	120	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

Continue Table 28. Optimal order quantities obtained in DP model

Scenario16				Scenario17				Scenario18			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	140	q_1	300	300	240	q_1	300	300	300
q_2	300	0	0	q_2	300	0	0	q_2	300	0	20
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	200	0	q_4	0	200	0	q_4	0	200	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario19				**Scenario20**				**Scenario21**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	220	300	q_1	300	220	300	q_1	300	220	300
q_2	300	0	20	q_2	300	0	120	q_2	300	0	200
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario22				**Scenario23**				**Scenario24**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	220	q_1	300	300	300	q_1	300	300	300
q_2	300	120	0	q_2	300	120	20	q_2	300	120	100
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0
Scenario25				**Scenario26**				**Scenario27**			
Stage	1	2	3	Stage	1	2	3	Stage	1	2	3
q_1	300	300	220	q_1	300	300	300	q_1	300	300	300
q_2	300	200	0	q_2	300	200	20	q_2	300	200	100
q_3	0	0	0	q_3	0	0	0	q_3	0	0	0
q_4	0	0	0	q_4	0	0	0	q_4	0	0	0
q_5	0	0	0	q_5	0	0	0	q_5	0	0	0

Based on the method proposed by Mafakheri et al. (2011) and an analysis of the percentage change in the OF before and after applying the model, it was found that the TVP value decreased by 13% from its maximum value, while the TCP value increases by 5.6% from its minimum value. Conversely, if we only focus on maximizing the TVP function, the TCP value increases by 83% from its minimum value. Similarly, if we only focus on minimizing the TCP function, the TVP value decreases by 37% from its maximum value. Therefore, minimizing the deviation of OFs from their optimal values using a DP has better overall solutions.

The complexity of the proposed method in the first phase (ranking suppliers) depends on the number of suppliers and evaluation criteria. With increasing supplier numbers, complexity will increase. In the second phase, the size of the solution space to search for the optimal policy in the DP method depends on the number of stages (T), the maximum allowable inventory level (X), the maximum allowable shortage level (Y), the maximum amount of products purchased from suppliers (Q), the number of possible scenarios for demand D_t^s. As each of the mentioned parameters increases, finding the optimal policy in the DP method may be challenging.

The distinctive advantage of our proposed solution is in its utilization of DP, which sets it apart from commonly employed methods. This approach streamlines the primary problem, resulting in reduced computation time. Unlike other methods for our specific challenge, which grapple with an extensive array of scenarios demanding substantial memory capacity when using GAMS software, our approach harnesses the power of DP. This technique excels in situations where sub-problems are interconnected. Unlike traditional approaches that redundantly solve sub-problems, DP tackles them just once, storing their solutions for subsequent analysis. This systematic algorithm alleviates the computational load, making it compatible with various computer systems and obviating the need for extensive memory resources.

V. CONCLUSION

In this research, we focus on sustainable SS, and OA in a multi-supplier, multi-stage, single-product context and emphasize the importance of sustainability in SS in recent decades. This study addresses all three dimensions of sustainability: economic, environmental and social aspects. The weights are obtained using BWM and the ER approach is used to rank suppliers. Subsequently, considering stochastic demand, a model for sustainable SS and OA is developed. Finally, a new integrated approach based on DP and SP is introduced to solve the model. One of the key advantages of this solution method is its ability to provide optimal solutions for multi-stage and multi-objective problems.

To further enhance the presented research, the following areas are suggested for exploration:

- Extend the model to incorporate additional stochastic parameters.
- Expand the problem to a multi-product scenario, where each product has varying prices across different stages.
- Other multi-objective solution methods can be considered for the problem.

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