Analyzing the Hybrid Approach of PCA-DEA in Two Different Modes

Kiyana Salehi [1] , Ahmad Mehrabian [2,*] , Hossein Amoozad Khalili[3] , Mehrzad Navabakhsh [4]

** Corresponding Author, a.mehrabian@aliabadiau.ac.ir*
1-Department of Industrial Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran
2-Department of Industrial Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran
3-Department of Industrial Engineering, Sari Branch, Islamic Azad University, Sari, Iran
4-Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran

Abstract—The present work compared input- and output-based integrated principal component analysis-data envelopment analysis (PCA-DEA) approaches. The approach minimizing the number of decision-making units (DMUs) identified as efficient would be the superior one as it facilitates DMU ranking. This would somewhat handle a major drawback of DEA– i.e., the emergence of an excessively high number of DMUs. The input and output-based approaches were independently implemented in MATLAB and were compared to identify the superior one. A number of numerical examples were carried out to demonstrate the performance of the superior approach. The results show that The second approach (the output-based approach) is superior to the first approach (the input-based approach). Therefore, it is better to divide the outputs by the inputs to create the PCA-DEA indices. In order to achieve better results in this way. And this point(divide the outputs by the inputs) is not specific to this research alone and can be used in other research(In case study research).

Keywords— DEA, Efficient DMUs, Performance Evaluation, PCA.

I. Introduction

According to Neely (2005), performance measurement is a frequently discussed concept with inadequate definitions. Therefore, Neely described performance measurement as an activity quantification process(Neely, 2005).Performance evaluation is defined as the process of measuring the performance of a business thoroughly in terms of efficiency, effectiveness, empowerment, and accountability within the framework of scientific principles and concepts of management for organizational tasks and purposes in executive plans(Timothy & Gerald, 1993).Performance evaluation dates back to a very long time ago. In fact, humans have somehow considered evaluation since they started living together and practicing division of labor in the very early form. However, the official use of evaluation systems dates from the 19thcentury, in which very primitive tools were employed to evaluate the output quality levels of organizations. However, performance evaluation has

Manuscript Received day-Month-year and revised day-Month-year *ISSN: ****-*****
Accepted on day- month-year

now become much more evolved than ever before. In other words, it has been developed in line with the management notion transformation model, its nature, and its functions(Gibbons & Murphy, 1990).The performance evaluation of a company has always been considered a challenge in management areas. Efficiency measurement has drawn a great deal of attention due to its importance, especially in the past two decades. Since 1957 when Farrell proposed a method for efficiency measurement, comprehensive reviews have been conducted on this subject so far. Moreover, parametric and non-parametric views are used extensively in efficiency evaluation(Battess & Coelli, 1995).

Today, in light of technological advances and the contributions of product and service organizations to human life, different and newer decision-making units (DMUs) are emerging in urban and rural areas. For an organization with numerous DMUs, it is essential for senior managers to identify the DMUs of higher efficiency– i.e., evaluating each DMU relative to the others(Ghalayini et al, 1997). The continuous evaluation of DMUs motivates employees and encourages DMUs to create added value in products and services. This reflects the necessity of deploying mechanisms to tackle the shortcomings of DMUs of poor efficiency and avoid resource waste(Vittorio et al, 2008).

Performance management refers to the deployment of a system to utilize information on organizational performance measurement using performance evaluation results to define objectives, allocate resources, and improve managers' awareness in order to maintain or change the current policy and achieve the objectives(Li, 2001). Data envelopment analysis (DEA) is an efficient tool for measuring and evaluating productivity. It is used as a nonparametric method for calculating the efficiency of decision-making units (DMUs) (Soofizadeh & Fallahnejad, 2022). The use of DEA is now rapidly growing. In fact, this technique is employed to evaluate different organizations and industries such as banking, post services, hospitals, education centers, power plants, and refineries(Omrani et al, 2022). Abbreviated to DEA, data envelopment analysis is considered a novel method of operations research and economics intended to measure or estimate the performance efficiency of production units. However, a production unit may refer to a factory and also a service-providing company(Rakhshan & Alirezaee, 2019). Hence, DEA can be adopted in any economic activities to determine the efficiency of relevant DMUs(Moazeni et al, 2022).

In typical DEA methods, index dependency is less discussed(Dong et al, 2015) and the researchers often do not consider the assumption of data independency(Xiao et al, 2022), which leads to alteration in the results of implementation of this method(Razavi Hajiagha et al, 2022). Therefore, weaknesses and strengths of DEA method should be considered. In other words, we should find the way to obtain better results by eliminating weaknesses of the DEA method. We will elaborate this further in the following.

II. Problem Statement

Efficiency represents the extent to which an organization (or an organizational DMU) exploits its resources to maximize production quality(Golany & Roll, 1989). The efficiency levels of DMUs provide a clear representation of their statuses. DMU efficiency is dependent on several parameters (or criteria). This confuses managers and decision-makers in the organization(Choudhuri, 2014).In other words, a given DMU may be satisfactory in a criterion and poor in another one. This challenge can be somewhat handled using the importance (weights) of the criteria and multi-criteria decision-making (MCDM) techniques. However, the importance (weights) of decision criteria may be biased for some DMUs. To cope with the challenge, non-parametric methodologies, e.g., principal component analysis (PCA), are employed(Charnes et al, 1978). However, PCA has also drawbacks, e.g., the emergence of an excessively high number of efficient DMUs(Andersen, & Petersen, 1993). The integration of PCA and data envelopment analysis (DEA) is a popular technique to minimize efficient DMUs(Kardıyen & Örkcü,, 2010).

The average of the absolute values of the correlation of indices(components) in the first approach, before and after implementation of PCA, was 0.59 and 0.11, respectively, and in the second approach, 0.56 and 0.11, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices(components) were reduced, which confirms efficiency of PCA in reducing dependency of indices.

A hybrid PCA-DEA approach converts all the criteria into either inputs only or outputs only. An output-based approach converts all the criteria into profit indices, while an input-based one converts all the criteria into cost indices. It remains a challenge to realize whether the input-based or output-based approach is the superior one. To this end, both approaches are implemented to identify the outperforming one. The superior approach might not certainly be identified through one or two numerical examples. Hence, different and new numerical examples will be simulated in one of the final cases to draw a general conclusion and identify the superior approach by comparing the results.

III. Necessity

There may be several approaches to solve a given problem, while they yield different results. Therefore, it is necessary to identify the approach that works best for the problem. The present study compared input- and output-based PCA-DEA approaches.

A. *Input-based PCA-DEA*

The inputs were divided by the outputs, obtaining a large number of new input indices($z_{m(r-1)+i,j} = \frac{x_{i,j}}{y_{r,j}}$; $i = 1,2,\dots,m$ and $r = 1,2,\dots,s$ and $j = 1,2,\dots,n$). To exclude the insignificant inputs, PCA can be employed. The efficiency of DMUs can be calculated using DEA(During this research we call this approach the first approach).

B. *Output-based PCA-DEA*

The outputs were divided by the inputs, leading to numerous new output indices($z_{m(r-1)+i,j} = \frac{y_{r,j}}{x_{i,j}}$; $i = 1,2, \dots, m$ and $r = 1,2, \dots, s$ and $j = 1,2, \dots, n$). Likewise, PCA could be utilized to exclude the insignificant outputs. DMU efficiency may be measured using DEA(During this research we call this approach the second approach).

IV. LITERATURE REVIEW

Wu (2009) employed an integrated model of a decision tree (DT), DEA, and an artificial neural network (ANN) to evaluate suppliers. They classified suppliers into efficient and inefficient and trained the DT and ANN using a dataset. The trained DT was applied to new suppliers.

Ahmadvand *et al.* (2011) evaluated the road safety level performance in different provinces of Iran. They employed the data envelopment analysis (DEA) to calculate the efficiency score of each province. They also adopted the principal component analysis (PCA) to improve the DEA distinguish ability, create independent variables, and prevent the information overlap in decision-making units (DMUs). An innovation of their paper was to develop the PCA when some inputs or outputs were undesirable.

Hosseinzadeh Lotfi *et al.* (2012) evaluated the performance of banks. They aimed to propose a framework for evaluating the general performance of bank branches in terms of profitability effectiveness and efficiency by using the bi-level DEA model. In this model, all outputs of the first level are used as the inputs of the second level to help evaluate DMUs in the best way possible.

Mohaghar *et al*. (2013) studied the supplier selection problem. They employed DEA and the VIKOR method to calculate the efficiency of suppliers.

Sadraei-Javaheri and Ostadzad (2014) estimated the efficiency of Iranian fossil-fueled and renewable power plants using DEA. The DEA network was developed with several inputs (generation inputs) and an output (power generation). They used the fuel cost (zero for renewable power plants, labor cost, and operation costs as the inputs, whereas annual power generation was treated to be the output. The Iranian power plants were classified based on their efficiency, proposing solutions to improve power plant efficiency under different scenarios.

Shokrollahpour *et al*. (2016) investigated the relative efficiency calculation and pattern determination of Tejarat Bank branches in Iran through a hybrid DEA-ANN model. They sought to tackle some drawbacks of DEA. The hybrid model was demonstrated to improve DEA.

Rostamy-Malkhalifeh *et al.* (2018) analyzed uncertainty in the calculation of efficiency weight at DMUs. They used fuzzy numbers for modeling to handle uncertainty. They also employed rank functions to solve the mathematical models. In other words, a fuzzy problem can be converted into a certain (crisp) problem through the rank functions of fuzzy numbers. It is then solved through classical methods. In this paper, some numerical examples were presented to better perceive the proposed method.

Heidary *et al*. (2018) developed a hybrid model for efficiency evaluation. The model consisted of two stages: (1) evaluating DMUs using DEA and (2) identifying DMUs with an efficiency weight of 1 using an ANN. It was found that the hybrid model could introduce only one efficient DMU.

Jafari and Ehsanifar (2020) studied a widely-used technique in multi-attribute decision-making (MADM) problems. They developed the VIKOR method under non-crisp (grey) conditions. Their proposed method can evaluate decision alternatives under crisp (interval) conditions. The potential application of the proposed method was illustrated by a numerical example.

Firoozishahmirzadi (2020) introduced a new DEA approach that could rank efficient DMUs. It was found to outperform some earlier methodologies.

Tsolas *et al*. (2020) proposed an integrated artificial intelligence (AI)-DEA approach to calculate the efficiency of bank branches. DEA was used for preprocessing. Then, the branches were classified into efficient and inefficient groups using an ANN. The integrated approach was found to be significantly helpful in future decision-making.

Rahimpour *et al*. (2020) evaluated the efficiency of organizational DMUs. They aimed to develop an evaluation model for organizational DMUs based on intellectual capital (IC) and employee loyalty using PCA and DEA. The operation, design and manufacturing, production planning, internal, quality control, and security units were identified to be efficient. The operation, internal, and quality control units had the first, second, and third ranks, respectively, while the human resource unit had the last rank.

Jafarigorzin and Asadi Talooki (2021) analyzed the ranking problem of efficient units through the DEA. Their proposed approach was to use the Malmquist index (MI) based on common weights in addition to calculating the final efficiency of DMUs. According to numerical examples, their proposed approach performed a complete ranking process on DMUs.

Salehi *et al*. (2022) assessed DEA in specific conditions. By specific conditions, they refer the conditions that assessment of decision-making units is done without any input index so that all the indices are of output type. Through proving a mathematical theorem, the authors showed that a simple mathematical model can solve the problem.

Moazeni *et al.* (2022) used the DEA technique to evaluate industrial units. In fact, they aimed to propose a network mode for calculating the efficiency of partial and total DMUs. They also employed the principal component analysis (PCA) technique to improve the distinguish ability of network DEA results. The proposed model was executed in 26 stone factories. The results indicated acceptable differences between industrial units.

A. Research Innovation

According to the literature review, each of the previous studies has somehow tried to fully rank DMUs (*i.e.*, to reduce the number of efficient units)(Amirteimoori et al, 2014) and (Soltani et al, 2022) and (Zanboori et al, 2014).In some studies, the PCA–DEA method was employed to reduce the quantity of efficient units(Mohammadnazari et al, 2022). However, none of those studies compared different execution modes of the PCA–DEA method, a problem which is partially addressed in this study. In fact, this study aims to compare two different approaches to the execution of PCA–DEA.

V. METHODOLOGY

This paper adopted a descriptive-analytical methodology. In fact, mathematical-statistical analyses and descriptions were used to compare the input- and output-based approaches. In simpler terms, two numerical examples are first presented to compare the two approaches. Further numerical examples will then be presented to improve the reliability of comparison results (through a simple numerical simulation). These numerical examples will be created randomly. Finally, their results will be discussed. For this purpose, the two approaches are first introduced and then compared.

A. Data Envelopment Analysis

DEA is employed to measure the relative technical efficiency of organizational DMUs. It was developed in 1976 and introduced by Charnes et al. (1978) as the CCR model in the paper "Measuring the Efficiency of DMUs." The DEA model allocates importance weights to the criteria to maximize the performance of DMUs(Xiuli et al, 2010). Several mathematical models can be used for efficiency calculation, including the DEA-CCR method. It is a strict method in the calculation of efficiency. Therefore, the present work adopted DEA-CCR.

B. DEA-CCR Method

CCR seeks to allocate optimal weights to the inputs and outputs to maximize the efficiency fraction of a DMU such that the efficiency of the other DMUs would not exceed 1. Constant returns to scale mean a portion of inputs producing the same portion of the output. The CCR model assumes constant returns to scale. Hence, large and small DMUs are compared. For a total of *n* DMUs with *m* inputs and *s* outputs, the CCR model is written as(Abbasi, 2022):

$$Max\ E_o = \sum_{r=1}^{s} u_r y_{ro} \tag{1.1}$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1 \tag{1.2}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \ \ ; \ \ j = 1, \dots, n \tag{1.3}$$

$$v_i, u_r \geq \varepsilon \, ; i = 1,2, \dots, m \ and \ r = 1,2, \dots, s \tag{1.4}$$

As can be seen, it is an input-based model that keeps constant inputs and seeks to maximize the output(Premachandra, 2001).

C. Principal Component Analysis

PCA is a transformation in the vector space. It is often employed to downsize datasets. PCA was introduced by Pearson (1901) and analyzes the eigenvalues of the covariance matrix(Sarkar, 2015). It is an orthogonal linear transformation that maps data into a new coordinate system in which, for example, the largest variance lies on the first coordinate axis, and the second-largest covariance rests on the second coordinate axis. PCA can be employed to downsize a dataset. Thus, the dataset components with the highest contribution to the variance are maintained(Han et al, 2020).

D. Principal Component Analysis algorithm

The implementation of PCA on a data matrix $Z = [Z_1, Z_2, \dots, Z_n]_{m\times n}$ includes a number of steps(Omrani et al, 2015) and (MendonçaPeixoto et al, 2020) and (Saqafi et al, 2018):

1) Calculate the normalized data matrix $D = [D_1, D_2, \dots, D_n]_{m\times n}$;

Matrix Z is normalized using $D_j = \frac{Z_j - \bar{Z}_j}{\sqrt{Var(Z_j)}}$; $j = 1,2,\dots,n$ as:

$$\begin{cases} \bar{Z}_j = \frac{1}{m}\sum_{i=1}^{m} z_{i,j} \\ Var(Z_j) = \frac{1}{m-1}\sum_{i=1}^{m}\left(z_{i,j} - \bar{Z}_j\right)^2 \end{cases} \tag{2}$$

2) Calculate the variance-covariance matrix as:

$$\begin{cases} c_{i,j} = Cov(D_i, D_j) \\ Cov(D_i, D_j) = \frac{1}{m-1}\sum_{k=1}^{m}\left(d_{k,i} - \bar{D}_i\right)\left(d_{k,j} - \bar{D}_j\right) \end{cases}; i = 1,2,\dots,n \;\; and \;\; j = 1,2,\dots,n \tag{3}$$

3) Calculation of the matrix of correlation coefficients ($R = \left(r_{i,j}\right)_{n\times n}$) as follows:

$$r_{i,j} = \frac{c_{i,j}}{\sqrt{c_{i,i}\times c_{j,j}}}; i = 1,2,\dots,n \;\; and \;\; j = 1,2,\dots,n \tag{4}$$

4) Calculate the eigenvalues of matrix R;

To calculate the eigenvalues of C, it is required to find the roots of the next determinants:

$$\det(R - \lambda \times I_{n\times n}) = 0 \tag{5}$$

Let $\lambda_1, \lambda_1, \dots, \lambda_n$ be the roots of Eq. (4).

5) Calculate the eigenvector matrix V;

To calculate the eigenvectors, a system of linear equations is solved as:

$$R \times V_j = \lambda_j \times V_j \;\; ; j = 1,2,\dots,n \tag{6}$$

6) Exclude the insignificant eigenvectors;

The eigenvectors whose eigenvalues are smaller than 1 are excluded from matrix V (a total of p eigenvectors remain in matrix V, $p \le n$).

7) Calculate the principal components $PC = \left[PC_1, PC_2, \dots, PC_p\right]_{m\times p}$;

The principal components are calculated as:

$$PC = D \times V \tag{7}$$

7) End.

DEA assumes all the data to be higher than zero. Therefore, when matrix PC contains a non-positive element, $-\min\{pc_{i,j}\}+1$ is added to matrix PC so that the negative and zero elements could become positive(Rahimpour et al, 2020).

VI. Mathematical model for efficiency calculation in the first approach(PCA-DEA(1))

It is recommended that the following mathematical model with a virtual output of 1 be employed in the first approach(Because in the first approach we do not have any indicators of the output type):

$$Max\ E_o = u_1 \tag{8.1}$$

Subject to:

$$\sum_{i=1}^{m} v_i pc_{i,o} = 1 \tag{8.2}$$
$$u_1 - \sum_{i=1}^{m} v_i pc_{i,j} \le 0 \ \ ; \ \ j = 1, \dots, n \tag{8.3}$$
$$u_1, v_i \ge \varepsilon \ ; \ i = 1,2, \dots, m \tag{8.4}$$

VII. Mathematical model for efficiency calculation in the second approach(PCA-DEA(2))

It is recommended that the following mathematical model with a virtual input of 1 be employed in the second approach(Because in the second approach we do not have any input type index):

$$Max\ E_o = \sum_{r=1}^{s} u_r pc_{r,o} \tag{9.1}$$

Subject to:

$$v_1 = 1 \tag{9.2}$$
$$\sum_{r=1}^{s} u_r pc_{r,j} - v_1 \le 0 \ \ ; \ \ j = 1, \dots, n \tag{9.3}$$
$$u_r, v_1 \ge \varepsilon \ ; \ r = 1,2, \dots, s \tag{9.4}$$

Numerical Example(1)

Table 1 provides the data of twenty DMUs.

Table 1. Data and outputs of twenty DMUs

	Inputs			Outputs		
	x_1	x_2	x_3	y_1	y_2	y_3
DMU_1	43	36	27	141	133	122
DMU_2	47	11	25	146	101	119
DMU_3	15	44	41	106	143	139
DMU_4	47	48	42	146	147	140
DMU_5	35	37	17	132	134	109
DMU_6	13	41	30	104	138	124
DMU_7	21	40	28	114	137	122
DMU_8	32	26	36	127	120	132
DMU_9	49	36	39	148	133	136
DMU_{10}	49	17	40	149	108	138
DMU_{11}	16	38	21	108	136	114
DMU_{12}	49	11	37	149	101	134
DMU_{13}	49	21	36	148	114	133

DMU_{14}	29	11	16	124	102	108
DMU_{15}	42	13	14	140	104	106
DMU_{16}	15	43	30	107	141	125
DMU_{17}	27	38	49	121	135	148
DMU_{18}	47	23	23	146	116	117
DMU_{19}	42	48	33	140	148	129
DMU_{20}	49	11	19	148	101	111

The PCA algorithm was implemented, calculating the principal components through the two approaches, as shown in Table 2.

Table 2. Principal components in the input- and output-based approaches

	Input-based		Output-based		
	PC_1	PC_2	PC_1	PC_2	PC_3
DMU_1	3.99	4.35	6.07	4.48	4.56
DMU_2	4.70	1.53	3.33	3.84	7.50
DMU_3	5.44	8.31	3.21	4.73	1.00
DMU_4	5.90	5.62	5.98	3.10	3.72
DMU_5	2.11	4.64	6.58	7.11	5.21
DMU_6	3.75	7.83	3.05	5.93	1.01
DMU_7	3.61	6.78	4.79	5.13	2.83
DMU_8	5.62	4.87	4.76	3.39	4.03
DMU_9	5.94	4.40	5.77	3.08	4.20
DMU_{10}	6.94	2.75	4.20	2.53	5.57
DMU_{11}	2.30	6.79	4.35	6.92	2.61
DMU_{12}	6.77	2.00	2.75	2.45	7.07
DMU_{13}	6.10	2.95	4.88	2.97	5.20
DMU_{14}	2.53	2.61	3.54	6.44	7.46
DMU_{15}	2.44	1.53	4.91	7.35	8.12
DMU_{16}	3.70	7.75	3.65	5.65	1.52
DMU_{17}	7.03	6.92	4.61	3.06	2.75
DMU_{18}	3.86	2.64	5.65	4.78	5.68
DMU_{19}	4.52	5.66	6.12	3.96	3.88
DMU_{20}	3.68	1.00	3.84	5.15	8.11

Table 3 shows the efficiency weights under three scenarios: (1) DEA approach, (2) input-based PCA-DEA approach, and (3) output-based PCA-DEA approach.

Table 3. Efficiency weights under the three scenarios

	Efficiecy		
	DEA	PCA-DEA(1)	PCA-DEA(2)
DMU_1	0.737	0.585	0.922
DMU_2	1.000	0.727	0.924

DMU_3	1.000	0.411	0.645
DMU_4	0.608	0.400	0.909
DMU_5	1.000	1.000	1.000
DMU_6	1.000	0.567	0.807
DMU_7	0.885	0.601	0.727
DMU_8	0.884	0.424	0.739
DMU_9	0.635	0.406	0.876
DMU_{10}	0.801	0.449	0.773
DMU_{11}	1.000	0.916	0.941
DMU_{12}	1.000	0.525	0.871
DMU_{13}	0.708	0.462	0.822
DMU_{14}	1.000	0.926	0.919
DMU_{15}	1.000	1.000	1.000
DMU_{16}	0.954	0.574	0.769
DMU_{17}	0.920	0.335	0.700
DMU_{18}	0.775	0.628	0.931
DMU_{19}	0.680	0.508	0.929
DMU_{20}	1.000	1.000	0.999

According to Table 3, a total of nine efficient DMUs were identified using DEA. The integrated input-based PCA-DEA approach identified three efficient DMUs. The output-based PCA-DEA approach identified two efficient DMUs. Although both integrated PCA-DEA approaches reduced the number of efficient DMUs, the output-based approach introduced fewer efficient DMUs and, therefore, outperformed the input-based one.

The average of the absolute values of the correlation of indices(components) in the first approach, before and after implementation of PCA, was 0.59 and 0.11, respectively, and in the second approach, 0.56 and 0.11, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices(components) were reduced, which confirms efficiency of PCA in reducing dependency of indices. More details are included in Table 10 to Table 13.

Numerical Example(2)

Table 4 reports the data of thirty DMUs.

Table 4. Inputs and outputs of thirty DMUs

	Inputs			Outputs			
	x_1	x_2	x_3	y_1	y_2	y_3	y_4
DMU_1	17	27	11	259	210	264	131
DMU_2	39	53	59	146	275	138	165
DMU_3	13	22	59	220	108	124	163
DMU_4	51	39	16	122	281	265	279
DMU_5	46	57	33	203	126	228	149

DMU_6	57	52	43	268	267	203	162
DMU_7	35	12	24	285	260	280	182
DMU_8	43	11	48	200	284	203	242
DMU_9	55	44	38	155	127	209	128
DMU_{10}	37	40	31	231	201	221	275
DMU_{11}	24	15	23	284	181	252	116
DMU_{12}	59	50	58	102	134	171	192
DMU_{13}	11	41	55	295	215	176	106
DMU_{14}	26	13	47	139	221	117	251
DMU_{15}	59	13	30	122	143	247	240
DMU_{16}	28	16	57	159	204	166	143
DMU_{17}	25	50	23	179	298	268	236
DMU_{18}	16	14	37	184	198	174	212
DMU_{19}	56	22	58	162	239	266	270
DMU_{20}	16	22	23	239	182	135	212
DMU_{21}	26	15	22	118	106	126	281
DMU_{22}	55	53	57	180	158	276	184
DMU_{23}	35	45	13	159	261	108	171
DMU_{24}	41	47	25	161	169	238	198
DMU_{25}	39	43	40	121	116	247	151
DMU_{26}	45	36	20	219	202	187	286
DMU_{27}	11	26	42	156	173	176	193
DMU_{28}	36	43	50	131	248	296	151
DMU_{29}	11	15	35	100	205	180	186
DMU_{30}	52	17	43	157	261	188	241

The principal components were found through the input- and output-based approaches, as shown in Table 5.

Table 5. Principal components under the two approaches

	Input-based			Output-based		
	PC_1	PC_2	PC_3	PC_1	PC_2	PC_3
DMU_1	5.54	2.74	1.20	5.84	10.43	9.83
DMU_2	5.19	5.68	7.19	4.82	5.36	3.13
DMU_3	4.56	8.37	4.07	3.23	4.39	5.82
DMU_4	3.63	1.61	3.76	7.52	9.03	5.83
DMU_5	5.80	2.86	7.12	5.40	6.30	3.32
DMU_6	4.98	3.31	5.75	5.56	5.99	3.60
DMU_7	3.64	3.18	1.21	8.71	4.47	9.30
DMU_8	2.48	4.57	2.49	8.40	2.29	7.49
DMU_9	4.11	3.12	8.14	5.60	5.76	2.87
DMU_{10}	4.60	3.24	3.18	5.58	6.42	4.94
DMU_{11}	4.39	3.63	1.64	6.56	5.47	7.96

DMU_{12}	2.51	4.79	10.40	5.36	5.29	2.33
DMU_{13}	7.15	6.96	3.98	1.00	5.32	7.31
DMU_{14}	2.65	5.92	2.94	6.33	3.30	6.11
DMU_{15}	1.00	2.69	4.40	8.18	4.22	5.62
DMU_{16}	3.21	6.69	3.83	5.77	3.86	5.08
DMU_{17}	5.72	2.94	2.63	4.79	7.74	6.41
DMU_{18}	3.97	5.07	1.40	4.91	3.87	7.86
DMU_{19}	2.05	4.60	4.16	6.63	4.20	4.72
DMU_{20}	4.87	3.89	1.51	4.48	6.03	7.36
DMU_{21}	3.01	3.58	2.91	6.63	5.21	5.66
DMU_{22}	4.24	4.42	7.20	5.30	5.49	3.04
DMU_{23}	5.78	1.62	4.83	6.58	8.96	5.29
DMU_{24}	4.93	2.51	4.79	5.77	6.90	4.18
DMU_{25}	4.44	4.10	7.02	5.23	5.68	3.24
DMU_{26}	4.11	2.23	3.29	6.74	7.66	5.33
DMU_{27}	4.94	5.72	2.44	2.11	4.92	7.28
DMU_{28}	4.64	4.78	5.26	4.92	5.51	4.02
DMU_{29}	3.96	5.44	1.76	3.28	4.20	8.22
DMU_{30}	2.05	3.92	3.71	7.15	4.12	5.22

Table 6 represents the efficiency weights under the three scenarios.

Table 6.Efficiency weights under three scenarios

	Efficiecy		
	DEA	PCA-DEA(1)	PCA-DEA(2)
DMU_{1}	1.000	1.000	1.000
DMU_{2}	0.508	0.458	0.630
DMU_{3}	1.000	0.575	0.592
DMU_{4}	1.000	1.000	1.000
DMU_{5}	0.397	0.591	0.713
DMU_{6}	0.472	0.618	0.721
DMU_{7}	1.000	1.000	1.000
DMU_{8}	1.000	1.000	0.963
DMU_{9}	0.361	0.645	0.719
DMU_{10}	0.739	0.767	0.745
DMU_{11}	1.000	0.843	0.845
DMU_{12}	0.331	0.532	0.682
DMU_{13}	1.000	0.445	0.744
DMU_{14}	1.000	0.896	0.727
DMU_{15}	1.000	1.000	0.939
DMU_{16}	0.731	0.714	0.686
DMU_{17}	0.987	0.809	0.767
DMU_{18}	1.000	0.905	0.804

DMU_{19}	0.705	0.836	0.782
DMU_{20}	1.000	0.801	0.750
DMU_{21}	1.000	0.857	0.809
DMU_{22}	0.420	0.530	0.682
DMU_{23}	1.000	0.991	0.945
DMU_{24}	0.607	0.725	0.766
DMU_{25}	0.489	0.546	0.680
DMU_{26}	1.000	0.915	0.885
DMU_{27}	1.000	0.662	0.741
DMU_{28}	0.588	0.561	0.645
DMU_{29}	1.000	0.853	0.836
DMU_{30}	0.792	0.900	0.832

According to Table 4, DEA identified sixteen efficient DMUs. The input- and output-based integrated approaches yielded five and three efficient DMUs. Hence, the output-based PCA-DEA approach showed higher performance.

The average of the absolute values of the correlation of indices(components) in the first approach, before and after implementation of PCA, was 0.43 and 0.08, respectively, and in the second approach, 0.35 and 0.08, before and after implementation of PCA, respectively. Therefore, it can be seen that in both studied approaches, the correlation of indices(components) were reduced, which confirms efficiency of PCA in reducing dependency of indices.

Numerical Example(3)

Table 7 provides twenty random examples solved by the two integrated approaches. As can be seen, no approach outperformed the other one in all the cases; however, the output-based PCA-DEA approach showed higher performance in most cases.

Table 7. Twenty random examples

				Number of Efficient DMUs		
Example	Number of Inputs	Number of Outputs	Number of DMUs	DEA	PCA-DEA(1)	PCA-DEA(2)
1	3	3	21	9	8	6
2	2	4	19	4	4	2
3	2	5	30	9	9	3
4	2	5	27	7	6	1
5	2	4	19	9	5	3
6	3	4	26	11	6	5
7	2	4	19	5	3	2
8	5	2	26	8	3	3
9	2	3	16	6	6	2
10	3	4	29	7	7	4
11	6	3	35	25	10	9

12	4	4	29	18	10	5
13	3	3	25	8	8	5
14	6	5	35	22	21	5
15	3	2	24	6	2	2
16	6	3	30	20	18	17
17	5	5	39	25	13	12
18	5	5	40	30	13	10
19	6	5	35	22	21	5
20	2	3	22	6	4	2
Sum	**72**	**76**	**546**	**257**	**177**	**103**

According to Table 7, it can be said that the output-based PCA-DEA approach identified fewer total efficient DMUs than the input-based one ($103 < 177$). As a result, the output-based PCA-DEA approach outperformed the input-based PCA-DEA approach in general.For further clarity, Figure (1) indicates the number of efficient units. Evidently, the second approach is superior to the first approach. In fact, it has fewer efficient units than the first approach and the classical DEA.

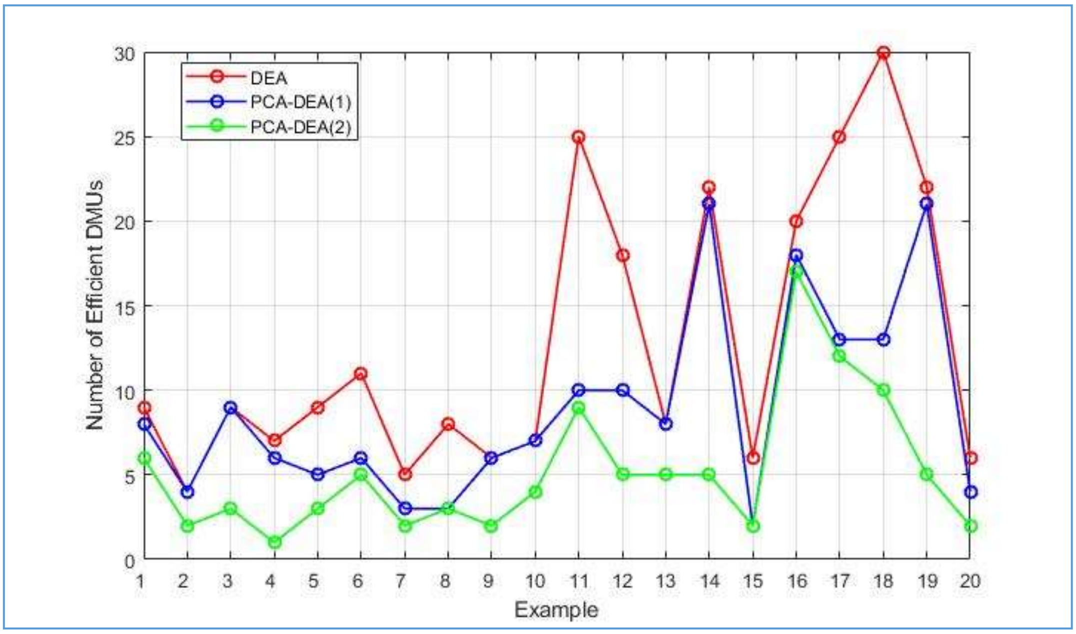

Figure 1. The number of efficient units in the simulated examples

In plain language, the performance distinguish ability of DMUs is higher in the second approach than the other two approaches. As a result, the second approach is better than the first. It is better to divide the outputs by the inputs to create the PCA indices.

In previous studies, an important goal of integrating PCA with DEA was to decrease the number of efficient units. According to Figure (1), the number of efficient units in the second approach (*i.e.*, the greed diagram) is smaller than or equal to the number of efficient units in the first approach (*i.e.*, the blue approach) in all simulation examples. In other words, the second approach (*i.e.*, PCA–DEA (2)) could further reduce the number of efficient DMUs. For instance, the first approach (*i.e.*, PCA–DEA

(1)) had six efficient DMUs in this simulation example; however, the second approach (*i.e.*, PCA–DEA (2)) had two. Other examples are interpreted similarly.

Moreover, Table (8) presents the mean absolute values of correlations(MAC) between indices(components) before and after the execution of PCA.

Table 8. The mean absolute values of correlations between indices(components)

	PCA-DEA(1)		PCA-DEA(2)	
Example	Before	After	Before	After
1	0.42	0.11	0.35	0.11
2	0.37	0.13	0.49	0.13
3	0.35	0.10	0.46	0.10
4	0.32	0.10	0.50	0.10
5	0.42	0.13	0.39	0.13
6	0.28	0.08	0.32	0.08
7	0.27	0.13	0.42	0.13
8	0.44	0.10	0.40	0.10
9	0.35	0.17	0.45	0.17
10	0.28	0.08	0.31	0.08
11	0.30	0.06	0.27	0.06
12	0.27	0.06	0.33	0.06
13	0.38	0.11	0.34	0.11
14	0.21	0.03	0.22	0.03
15	0.53	0.17	0.38	0.17
16	0.31	0.06	0.22	0.06
17	0.25	0.04	0.23	0.04
18	0.24	0.04	0.27	0.04
19	0.21	0.03	0.22	0.03
20	0.44	0.17	0.45	0.17

For further clarity, Figure (2) demonstrates these data.

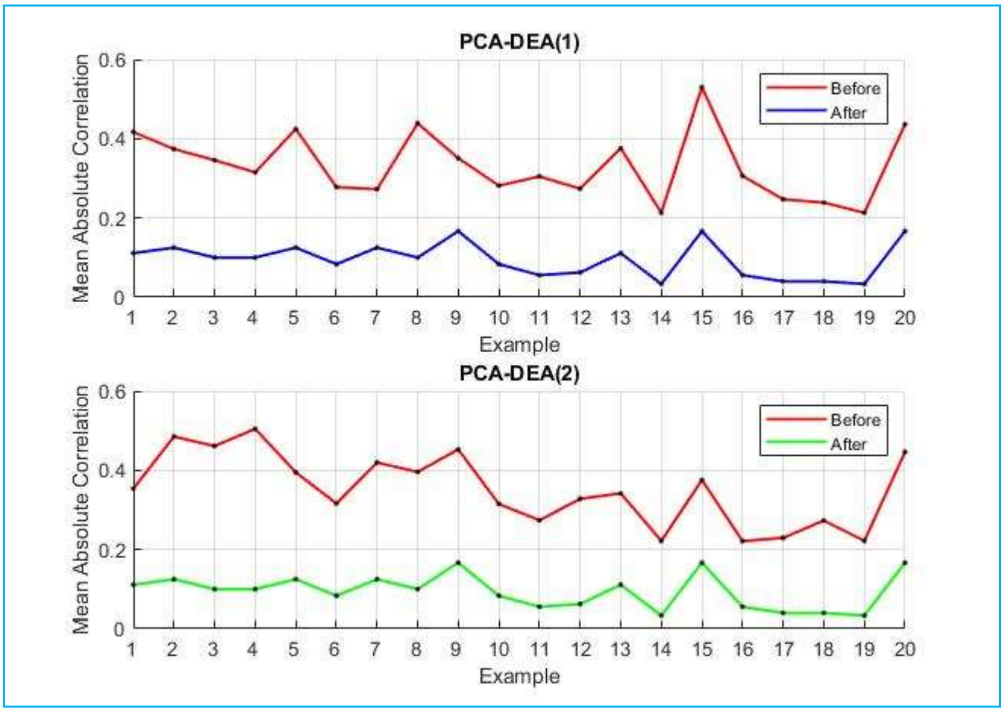

Figure 2. The mean absolute values of correlations between indices(components)

According to Figure (2), both approaches analyzed in this study reduced the correlations between criteria (components). Therefore, PCA is confirmed to be efficient in decreasing the correlations between indices(components).

Table (9) presents the mean squared errors (MSEs) of each of the twenty random examples($MSE = \frac{1}{n}\sum_{j=1}^{n}\left(Efficiency_j^{DEA} - Efficiency_j^{PCA-DEA}\right)^2$).

Table 9. The MSEs of each of the twenty random examples

	MSE	
Example	PCA-DEA(1)	PCA-DEA(2)
1	0.16	0.20
2	0.25	0.01
3	0.26	0.16
4	0.19	0.10
5	0.07	0.04
6	0.18	0.14
7	0.15	0.18
8	0.09	0.04
9	0.08	0.04
10	0.13	0.06
11	0.02	0.02
12	0.03	0.05
13	0.26	0.20
14	0.04	0.03
15	0.14	0.13

16	0.04	0.05
17	0.04	0.04
18	0.03	0.04
19	0.04	0.03
20	0.27	0.06
Sum	**2.48**	**1.64**

According to Table (9), the second approach had smaller MSEs than the first approach in most of the random examples. In the cases where the second approach had greater MSEs, the two approaches were slightly different in terms of MSE. In total, the first and second approaches had 2.48 and 1.64 MSEs, respectively, in the entire simulation. Therefore, this comparison indicates that the efficiency scores of the second approach were closer to the DEA efficiency scores. It is now possible to conclude that the second approach outperformed the first approach by further separating DMUs and yielding higher accuracy in efficiency calculation.

VIII. Conclusion

DEA is a mathematical programming-based technique to evaluate the efficiency of a congruent set of DMUs. A DMU receives inputs and produces a set of outputs. However, problems mostly involve an excessively high number of DMUs and cannot be solved using basic methods to identify the efficient DMUs. In such cases, PCA can be exploited to substantially handle the challenge. This paper implemented input- and output-based integrated PCA-DEA approaches. The approaches were compared in the number of efficient DMUs to identify the superior approach through a number of examples. It was found that both integrated approaches improved DEA; however, the output-based PCA-DEA approach had higher performance. Therefore, it is suggested that future works focus on output-based approaches. Researchers are recommended to analyze the problem addressed by this study in the conditions with data uncertainty (*e.g.*, Fuzzy data or Grey data).And also the investigation of this challenge for DEA-BCC models

References

Abbasi, M., Ghomashi, A., & Shahghobadi, S. (2022). Finding common weights in DEA using a compromise solution approach. *International Journal of Data Envelopment Analysis*,10(2),63-72.

Ahmadvand, A., Abtahy, Z., & Bashiri, M. (2011). Considering undesirable variables in PCA-DEA method: a case of road safety evaluation in Iran.*Journal of Industrial Engineering International*,7(15),43-50.

Amirteimoori, A.R., Despotis, D., & Kordrostami, S.(2014).Variables reduction in data envelopment analysis.*Optimization*,63(5),735-745.

Andersen, P., & Petersen, N.C. (1993). A Procedure for Ranking Efficient Units in Data Envelopment Analysis.*Management Science*,39(10),1261-1264.

Battess, G.E., & Coelli, T.J.(1995).A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data.*Empirical Economics*,20,325-332.

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision making units.*European Journal of Operational Research*,2(6),429-444.

Choudhuri, P. K.(2014).Application of Multi-Criteria Decision Making (MCDM) Technique for Gradation of Jute Fibres.Journal *of The Institution of Engineers (India): Series E*,2(95),63-68.

Dong, F., Mitchell P.D., & Colquhoun, J.(2015).Measuring farm sustainability using data envelope analysis with principal components: The case of Wisconsin cranberry.*Journal of Environmental Management*,147(1),175-183.

Elhaik, E.(2022).Principal Component Analyses (PCA)-based findings in population genetic studies are highly biased and must be reevaluated.*Sci Rep*,12,146-152.

Firoozishahmirzadi, P.(2020).Ranking Efficient Decision Making Units in Data Envelopment Analysis based on Changing Reference Set.*International Journal of Data Envelopment Analysis*,8(1),21-26.

Ghalayini, A.M., Noble, J.S., & Crowe, T.J.(1997).An Integrated Dynamic performance Measurement system for Improving Manufacturing competitiveness.*Intermational Journal of Production Economics*,(48),111-123.

Gibbons, R., & Murphy, K.(1990).Relative Performance Evaluation for Chief Executive Officers.*ILR Review*,43(3),30-51.

Golany, B., & Roll, Y. (1989). An application procedure for DEA.*Omega – The International Journal of Management Science*,17(3),237-250.

Han, X., Peng, J., Cui, A., & Zhao, F.(2020).Sparse Principal Component Analysis via Fractional Function Regularity.*Mathematical Problems in Engineering*,Special Issue,1-10.

Heidary, S., Zanburi, E., & Parvin, H. (2018).A Hybrid model based on neural network and Data Envelopment Analysis model for Evaluation of unit Performance.*Iranian Journal of Optimization*,10(2),101-112.

Hosseinzadeh Lotfi, F., Toloie Eshlaghy, A., Saleh, H., Nikoomaram, H., & Seyedhoseini, S.M.(2012).A new two-stage data envelopment analysis (DEA) model for evaluating the branch performance of banks).*African Journal of Business Management*,6(24),7230-7241.

Jafari, H., & Ehsanifar, M.(2020).Using interval arithmetic for providing a MADM approach.*Journal of Fuzzy Extension and Applications*,1(1),60-68.

Jafarigorzin, S., & Asadi Talooki, I. (2021). Malmquist Productivity Index Based on Means of Weights for Ranking of Decision Making Units in Data Envelopment Analysis. *International Journal of Data Envelopment Analysis*,9(2),1-8.

Kardıyen, F., & Örkcü, H. (2010). The Comparison of Principal Component Analysis and Data Envelopment Analysis in Ranking of Decision Making Units .*Gazi University Journal of Science*,19(2),127-133.

Li, p.(2001).Design of Performance Measurement Systems: a Stakeholder Analysis Framework The Academy of Management Review.*Mississippi State April*.

MendonçaPeixoto, M.G., AndreottiMusetti, M., & Mendonça, M.C.A.(2020). Performance management in hospital organizations from the perspective of Principal Component Analysis and Data Envelopment Analysis: the case of Federal University Hospitals in Brazil.*Computers & Industrial Engineering*,150,1-14.

Moazeni, H., Arbabshirani, B., & Hejazi, S. R. (2022). An integrated model of network Data Envelopment Analysis and principal component analysis approach to calculate the efficiency of industrial units (Case study: Stone Industry). *Journal of Industrial and Systems Engineering*,14(2),172-192.

Mohaghar, A., Fathi, M.R., & Jafarzadeh, A.H.(2013).A Supplier Selection Method Using AR- DEA and Fuzzy VIKOR.*International Journal of Industrial Engineering: Theory, Applications and Practice*,20(5),387-400.

Mohammadnazari, Z., Aghsami, A., & Rabbani, M.(2022).A hybrid novel approach for evaluation of resiliency and sustainability in construction environment using data envelopment analysis, principal component analysis, and mathematical formulation.*Environ Dev Sustain*,11,231-239.

Neely, A.D.(2005).Defining performance measurement: adding to the debate.*Perspectives on Performance*,4(2),14-15.

Omrani, H., Fahimi, P., & Emrouznejad, A.(2022). A common weight credibility data envelopment analysis model for evaluating decision making units with an application in airline performance.*RAIRO-Oper.Res*,56(2),911-930.

Omrani, H., Gharizadeh Beiragh, R., & Shafiei Kaleibari, S.(2015). Performance assessment of Iranian electricity distribution companies by an integrated cooperative game data envelopment analysis principal component analysis approach.*International Journal of Electrical Power & Energy Systems*,(64),617-625.

Premachandra, I.M. (2001).A note on DEA vs principal component analysis: An improvement to Joe Zhu's approach.*European Journal of Operational Research*,3(132), 553–560.

Rahimpour, K., Shirouyehzad, H., Asadpour, M., & Karbasian, M.(2020). A PCA-DEA method for organizational performance evaluation based on intellectual capital and employee loyalty: A case study.*Journal of Modelling in Management*,15(4),1479-1513.

Rakhshan, F., & Alirezaee, M. R. (2019). An ethics-based decomposition of Malmquist productivity index using data envelopment analysis. *Journal of Industrial and Systems Engineering*,12(4),1-17.

Razavi Hajiagha, S.H., Amoozad Mahdiraji, H., Hashemi, S.S., Garza-Reyes, J.A., & Joshi, R.(2022).Public Hospitals Performance Measurement through a Three-Staged Data Envelopment Analysis Approach: Evidence from an Emerging Economy.*Cybernetics and Systems*,53(8),1-27.

Rostamy-Malkhalifeh, M., Poudineh, E., & Payan, A. (2018). A Fully Fuzzy Method of Network Data Envelopment Analysis for Assessing Revenue Efficiency Based on Ranking Functions. *Control and Optimization in Applied Mathematics*,3(2),77-96.

Sadraei Javaheri, A., & Ostadzad, A.(2014). Estimating Efficiency of Thermal and Hydroelectric Power Plants in Iranian Provinces.*Iranian Journal of Economic Studies*,3(2),19-42.

Salehi, K., Mehrabian, A., Amoozad Khalili, H., & Navabakhsh, M. (2022). ANALYSIS OF SPECIFIC STATES IN NONPARAMETRIC DECISION-MAKING METHODS. *International Journal of Industrial Engineering: Theory, Applications, and Practice*,29(2),192-205.

Sarkar, S. (2015). Assessment of Cost Effectiveness of a Firm Using Multiple Cost Oriented DEA and Validation with MPSS based DEA.*International Journal of Data Envelopment Analysis*,3(1),593-607.

Saqafi, A., Osta, S., Amiri, M., & Barzideh, F. (2018). A Model for Performance Assessment of the Investment Companies with Data Envelopment Analysis Approach and Principal Component Segregation Method. *Journal of Financial Accounting Research*,10(1),75-94.

Skokrollahpour, E., Hosseinzadeh Lotfi, F., & Zandieh, M.(2016). An integrated data envelopment analysis-artificial neural network approach for benchmarking of bank branches. *International Journal of Industrial Engineering*,12,137-143.

Soltani, N., Yang, Z., & Lozano, S.(2022).Ranking decision making units based on the multi-directional efficiency measure.*Journal of the Operational Research Society*,73(9),1996-2008.

Soofizadeh, S., & Fallahnejad, R. (2022). A bargaining game model for performance evaluation in network DEA considering shared inputs in the presence of undesirable outputs. *Journal of Mathematical Modeling*,10(2),227-245.

Timothy, A.J., & Gerald R.F.(1993).Social Context of Performance Evaluation Decisions. *The Academy of Management Journal*,36(1),80-105.

Tsolas, L.E.,Charles, V., & Gherman, T.(2020).Supporting better practice benchmarking: A DEA-ANN approach to bank branch performance assessment.*Expert Systems with Applications*,160,1-26.

Vittorio, C., Federico, F., Valentina, L., & Manzini, R.(2008).Designing a Performance Measurement System for the Research Activities: A Reference Framework and an Empirical Study.*Journal of Engineering and Technology Management*,(25),213-225.

Wu, D.(2009).Supplier selection: A hybrid model using DEA, decision tree and neural network.*Expert Systems with Applications*,36 (5),9105–9112.

Xiao, Q.W., Tian, Z., & Ren, F.R.(2022). Efficiency assessment of electricity generation in China using meta-frontier data envelopment analysis: Cross-regional comparison based on different electricity generation energy sources.*Energy Strategy Reviews*,39,1-12.

Xiuli, G., Xuening, C., Deyi, X., & Zaifang, Z.(2010). An integrated approach for rating engineering characteristics' final importance in product-service system development .*Journal of Computers & Industrial Engineering*,59,585-594.

Zanboori, E., Rostamy-Malkhalifeh, M., Jahanshahloo, G.R., Shoja, N.(2014).Calculating super efficiency of DMUs for ranking units in data envelopment analysis based on SBM model.*ScientificWorldJournal*,1-7.

Appendix

Table 10. Correlation value of indicators before implementing the PCA method in the first approach

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Z_1	1	0.94	0.96	-0.73	-0.51	-0.55	-0.39	0.24	0.00
Z_2	0.94	1	0.95	-0.89	-0.75	-0.77	-0.46	0.24	-0.11
Z_3	0.96	0.95	1	-0.79	-0.61	-0.60	-0.59	0.02	-0.24
Z_4	-0.73	-0.89	-0.79	1	0.96	0.95	0.58	-0.08	0.34
Z_5	-0.51	-0.75	-0.61	0.96	1	0.98	0.55	-0.02	0.41
Z_6	-0.55	-0.77	-0.60	0.95	0.98	1	0.39	-0.21	0.23
Z_7	-0.39	-0.46	-0.59	0.58	0.55	0.39	1	0.73	0.91
Z_8	0.24	0.24	0.02	-0.08	-0.02	-0.21	0.73	1	0.90
Z_9	0.00	-0.11	-0.24	0.34	0.41	0.23	0.91	0.90	1

Table 11. Correlation value of the components after the implementation of the PCA method in the first approach

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	1.18E-12	5.78E-13	2.03E-13	-6.2E-15	3.69E-14	3.32E-14	3.85E-15	-5.1E-15
C_2	1.18E-12	1	1.75E-13	-4.2E-13	1.81E-14	-5.9E-14	-5.1E-15	-1.4E-14	9.26E-15
C_3	5.78E-13	1.75E-13	1	3.45E-14	1.08E-14	-1.1E-15	4.09E-15	1.93E-15	-2.4E-15
C_4	2.03E-13	-4.2E-13	3.45E-14	1	1.5E-14	2.64E-14	1.28E-15	-1.2E-15	-8.6E-16
C_5	-6.2E-15	1.81E-14	1.08E-14	1.5E-14	1	-5.5E-15	1.93E-16	-7E-16	2.23E-15
C_6	3.69E-14	-5.9E-14	-1.1E-15	2.64E-14	-5.5E-15	1	-1.1E-16	9.48E-16	-8E-16
C_7	3.32E-14	-5.1E-15	4.09E-15	1.28E-15	1.93E-16	-1.1E-16	1	-5.3E-17	-1.7E-16
C_8	3.85E-15	-1.4E-14	1.93E-15	-1.2E-15	-7E-16	9.48E-16	-5.3E-17	1	-1.8E-16
C_9	-5.1E-15	9.26E-15	-2.4E-15	-8.6E-16	2.23E-15	-8E-16	-1.7E-16	-1.8E-16	1

Table 12. Correlation value of indicators before implementing the PCA method in the second approach

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Z_1	1	-0.53	-0.34	0.99	-0.45	0.13	0.98	-0.48	-0.08
Z_2	-0.53	1	0.54	-0.62	0.99	0.03	-0.57	0.99	0.40
Z_3	-0.34	0.54	1	-0.41	0.57	0.81	-0.47	0.43	0.96
Z_4	0.99	-0.62	-0.41	1	-0.55	0.09	0.99	-0.57	-0.16
Z_5	-0.45	0.99	0.57	-0.55	1	0.10	-0.50	0.98	0.46
Z_6	0.13	0.03	0.81	0.09	0.10	1	-0.02	-0.07	0.92
Z_7	0.98	-0.57	-0.47	0.99	-0.50	-0.02	1	-0.50	-0.23
Z_8	-0.48	0.99	0.43	-0.57	0.98	-0.07	-0.50	1	0.29
Z_9	-0.08	0.40	0.96	-0.16	0.46	0.92	-0.23	0.29	1

Table 13. Correlation value of the components after the implementation of the PCA method in the second approach

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1	-4.6E-13	-1E-13	-2.1E-13	-9E-14	6.28E-14	7.59E-15	-1.8E-15	9.72E-15

C_2	-4.6E-13	1	3.84E-14	-1.2E-13	-1.6E-13	-2.1E-14	5.18E-15	2.67E-15	9.99E-15
C_3	-1E-13	3.84E-14	1	-1E-14	-1E-14	-2.1E-16	2.74E-15	9.4E-16	4.85E-15
C_4	-2.1E-13	-1.2E-13	-1E-14	1	-3E-14	1.55E-14	-2.6E-15	-2.5E-15	7.13E-15
C_5	-9E-14	-1.6E-13	-1E-14	-3E-14	1	-1.1E-15	1.4E-15	-1E-16	2.73E-15
C_6	6.28E-14	-2.1E-14	-2.1E-16	1.55E-14	-1.1E-15	1	-2.2E-15	5.75E-16	5.15E-16
C_7	7.59E-15	5.18E-15	2.74E-15	-2.6E-15	1.4E-15	-2.2E-15	1	1.8E-16	2.24E-17
C_8	-1.8E-15	2.67E-15	9.4E-16	-2.5E-15	-1E-16	5.75E-16	1.8E-16	1	3.34E-16
C_9	9.72E-15	9.99E-15	4.85E-15	7.13E-15	2.73E-15	5.15E-16	2.24E-17	3.34E-16	1