# Manufacturing Process Optimization to Increase Products Reliability by Control Charts

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*Abstract* — **The need to produce products with high reliability is a requirement for survival for many industries in today's world, so one of the main and important quality characteristics of products is lifetime or reliability. The production process must be monitored to produce highly reliable products, and in this regard, control charts are one of the most important and widely used process monitoring tools. In this study, it is assumed that the quality characteristic that is monitored is the lifetime of the products, which follows the distribution of Weibull with a variable scale parameter and a fixed shape parameter. The purpose is to monitor the average lifetime of the products, which is conducted by monitoring the Weibull distribution scale parameter. To design a control chart, first, the control limits of the chart for different parameters are obtained and presented. Relationships and values regarding control limits show that the control limit of a one-sided chart is always larger than the control limit of a two-sided chart. To evaluate the performance of the proposed control chart, the average run length in the out-of-control mode for various parameters is presented and sensitivity analysis is performed. The computational results show that the one-sided control chart has better performance in detecting reduced lifetime compared to the two-sided control chart. One of the most important parameters that increase the performance of the control chart is the number of failures in the failure censored life test, as the number of failures increases, the test continues for a longer time and therefore it increases the chances of detecting the changes. Finally, a practical example is provided to show the performance of the proposed control chart.**

***Keywords*:** Statistical Process Control, Control chart, lifetime, failure censoring, Weibull distribution

## 1. INTRODUCTION

With the start of the production era, customers began to compare different goods, which intensified competition between manufacturers. As one of the issues always facing human beings, quality control is a process or set of processes to check that a product or service meets a specific set of standards and responds to customer needs (Kale, 2014). There are three statistical methods for quality control and

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improvement, one of the most important and powerful of which is the statistical process control, which includes seven tools. The most important tools for statistical process control are control charts and the Average Run Length (ARL) is one of the most important criteria for evaluating the control charts performance (Montgomery, 2020).

Today, in the industry, products with high reliability are produced and the lifetime or reliability of products is considered a main and important quality characteristic. Time and cost are important factors in the inspection. Due to the duration and cost of inspection, it is impossible to observe the lifetime of all products. Therefore, the concept of lifetime tests has been introduced to minimize inspection costs as well as save time. A lifetime test usually involves testing a set of products to see their lifetime (Klyatis & Klyatis, 2010).

Lifetime data have two important features that make it difficult to design a Control chart for lifetime monitoring. First, the lifetime data are often followed by non-normal distributions, second, considering that the components are designed and produced for a long time, obtaining the lifetime of a component requires a high time and cost. To solve this problem, lifetime tests are used. In this research, one and two-sided design of the Shewhart control chart has been studied under the censored life test assumption (Barlow & Proschan, 1975).

Censoring is an important and non- ignorable fact that may be observed in real- life situations. In the presence of censored data, the usual statistical process monitoring techniques become less effective because they are not designed for handling the situations where censoring occurs. Meanwhile, some approaches have been developed to monitor lifetime data, but they do not consider censoring in the procedures either. Censored data can be classified as type- I, type- II, hybrid censoring, progressively type- I, and progressively type- II. Details about censoring can be found in (Meeker & Escobar, 2014).

Censoring during a reliability test means that the correct failure time is not known due to the long duration of the test or cost concerns. Different types of censoring schemes are proposed to conduct reliability and life testing. Generally speaking, the censoring schemes can be classified into censoring schemes with replacement and censoring schemes without replacement. In this article, the censoring schemes without replacement are used. Without Replacement during a censoring schemes means that once observing a failure item, it is not replaced by a new one (Pham, 2006).

Often, the underlying assumption of designing Shewhart control charts is that the quality characteristics in question follow a normal distribution. However, if in a process, the desired quality characteristics do not follow the normal distribution, the use of a control chart designed to monitor the quality characteristics with the normal distribution is incorrect and causes an incorrect warning in the process or indicates the change in the process with a delay. Therefore, a late warning about the altered process will lead to an increase in non-compliant or defective cases. To solve this problem, control charts must be designed with the distribution that models the qualitative characteristic. One of the characteristics that often follow non-normal distributions is life, so the normal distribution has limited application in life data. The most important distributions used to model quality characteristics in reliability include Weibull, exponential, log-normal, Burnham Sanders, Gamma, and Pareto distributions (Birolini, 1994).

(Rasay & Alinezhad, 2022) develop a novel life test according to the sequential probability ratio test of the Bernoulli/binomial distribution, which can be simply, straightforwardly and effectively adapted for life testing of different continuous distributions. (Rasay, Naderkhani, & Golmohammadi, 2020) Designed variable sampling plans based on the lifetime performance index under failure censoring reliability.

The application of control charts is now widespread in various fields of engineering, management, services, biology, healthcare, and finance. (Gharib, Amiri, & Jalilibal, 2021) Designed a multivariate exponentially weighted moving average control chart with measurement errors. (Jafarian-Namin, Fallahnezhad, Tavakkoli-Moghaddam, & Mirzabaghi, 2019) Examined the Economic-Statistical design of acceptance control charts using a robust optimization approach. (Rasay, Fallahnezhad, & Zaremehrjerdi, 2019)Considered a two-stage dependent process and defined a qualitative characteristic for each stage. Then, at each stage, they monitored the process by control charts. Finally, they presented an integrated model for maintenance and statistical quality control.

In general, the research performed so far has been on the type of control charts and life tests in various distributions. For example, (Huang, Yang, Xie, & International, 2017) assumed that the qualitative characteristic follows an exponential distribution. They presented a control chart under the failure censoring test and used the mean time to alert (ATS) to evaluate the performance of their proposed control chart. (Rao, 2018) studied the design of an NP control chart under the truncated life test when the product lifetime follows an Exponentiated half logistic distribution.

An np quality control chart, under the truncated life test was designed by (Aslam, Khan, & Jun, 2016) assuming that the product lifetime follows the Pareto type II distribution. A quantitative control chart under the time truncated life test for Weibull distribution is also presented (Khan, Aslam, Khan, & Jun, 2018). (Shafqat, Hussain, Al-Nasser, & Aslam, 2018) presented a statistical model for the control chart and obtained how to design a attribute control chart for different distributions and introduced the distribution that performed best. (Rao, 2018) designed a np control chart under the truncated life test when the product lifetime follows an exponential half-logistic distribution. (Dickinson, Roberts, Driscoll, Woodall, & Vining, 2014) presented a cusum control chart to monitor the lifetime quality characteristic and compared his proposed control chart with an ewma control chart. (S. Balamurali & Jeyadurga, 2018) researched the economic design of a np control chart to monitor average lifetime based on multiple deferred state sampling.

(Mohammadipour, Farughi, Rasay, & Arkat, 2021) design EWMA control charts under failure censoring reliability tests. They assumed the lifetime follows the Weibull distribution with a fixed shape parameter and a variable scale parameter. (S Balamurali, Jeyadurga, & Engineering, 2019) designed an attribute np control chart to monitor the mean lifetime of type-II Pareto distribution through truncated life tests and multiple deferred state sampling. (Gadde, Fulment, Josephat, & Engineering, 2019) Assumed that the quality characteristic of lifetime follows the Daugm distribution and then designed attribute control charts under truncated life tests. (Goodarzi & Amiri, 2017) used the accelerated failure time (AFT) model and two control charts are presented to monitor the quality characteristic in the second stage under the censored and non-censored reliability data.

According to the above mentioned literature, the design and application of a control chart to monitor the average lifetime of products under the lifetime tests have become an important issue in the application of lifetime tests in industrial engineering. Product lifetime is one of the characteristics that its reduction is often considered by the manufacturer. Therefore, in this research, in addition to two-sided control charts, how to design one-sided control charts is also presented. In this study, we used the failure censoring test to obtain the average lifetime of products. we design one-sided and two-sided Shewhart control charts under the failure censoring test to monitor the average lifetime of products when the lifetime of the Weibull distribution follows the fixed shape parameter and the variable scale parameter.

In this research, to design control charts, first the relationships of control limits are presented and the values of these limits are obtained for different parameters. Then, how to get ARL and its relationships are also presented. The performance of the proposed control chart will be assessed using ARL in the out-of-control state. The tables of ARLs presented for various shift constants and specified parameters. Then, based on ARL values in the out-of-control state, sensitivity analysis was performed. Finally, applications of the control charts are shown by practical examples.

## 2. PROBLEM STATEMENT

It is assumed that the product lifetime, which is considered its main qualitative characteristic, is X and follows the Weibull distribution with a constant shape parameter and an unknown scale parameter. The cumulative distribution function X is as follows:

$$F_x(x) = 1 - exp[-(x\theta)^m] \tag{1}$$

In the above relation, m is the shape parameter and θ is the scale parameter. The average lifetime of Weibull distribution is as follows:

$$\mu = \frac{\Gamma\left(\frac{1}{m}\right)}{\theta m} \tag{2}$$

According to Equation (2), monitoring the average lifetime in the case where the shape parameter is constant will be the same as monitoring the scale parameter. The scale parameter is equal to $\theta_0$ when the process is in control, and when the process is affected by an assignable cause and gets out of control, it will become $\theta_1 = c\theta_0$, which is called a constant c value. The statistics obtained from the failure censoring test are as follows:

$$V_i = \sum_{i=1}^{r} \left(\frac{x_i}{\mu_0}\right)^m + (n-r)\left(\frac{x_r}{\mu_0}\right)^m \tag{3}$$

In the above relation $x_i$ is the failure time of component i and the $\mu_0$ is equal to the average life expectancy and has a fixed and definite value. According to studies by John et al. (Jun, Lee, Lee, & Balamurali, 2010), the value of $V_i$ follows the gamma distribution with parameters r and $W_0$, the value of W0 is considered as follows:

$$W_0 = (\theta_0\mu_0)^m = \left(\frac{\Gamma(1/m)}{m}\right)^m \tag{4}$$

Also, due to the relationship between gamma distribution and chi-square distribution, the value of $2VW_0$ follows the chi-square distribution with 2r degree of freedom (Jun et al., 2010).

## 3. Designing Shewhart control chart

In order to design a control chart, it is needed to first determine the control limits of the chart and then create a chart. In the next step, sampling will be done to monitor the process and the desired statistics will be calculated. In designing a one-sided and two-sided control chart, a one-sided and two-sided design has been studied. First, how to design a one-sided control chart and then how to design a two-sided control chart are presented.

### 3.1 One-sided Shewhart control chart

Suppose only the deterioration or improvement of a qualitative characteristic is considered in a process. In this case, one-sided control charts are used. If the deterioration of quality characteristics or reduction of lifetime is considered in a process, a lower control limit is used and if improvement of quality characteristics or increase of lifetime is considered, upper control limit is used. Lifetime is one of the characteristics that its deterioration or reduction is considered. Therefore, if we want to monitor only the deterioration of the process or the reduction of life in the one-sided control chart, we use the lower control limit. In this case, since we have only one control limit, the α error is as follows:

$$P(V_i < LCL \mid \theta = \theta_0) = \alpha \tag{5}$$

In the above relation, $V_i$ is the same statistic obtained from the lifetime test which is obtained from equation (3). Now, according to what has been said about $V_i$, we can rewrite Equation (5) as follows to obtain a lower control limit:

$$\begin{gathered} \mathrm{P}(2\mathrm{W}_0\mathrm{V}_\mathrm{i} < 2\mathrm{W}_0 \times \mathrm{LCL}) = \alpha \\ \mathrm{P}(\chi^2_{2\mathrm{r}} < 2\mathrm{W}_0 \times \mathrm{LCL}) = \alpha \\ \mathrm{LCL} = \frac{\chi^2_{2\mathrm{r},1-\alpha}}{2\mathrm{W}_0} \end{gathered} \tag{6}$$

The amount of error β is obtained similarly, except that in this case, the process has changed and the scale parameter has become $\theta_1 = c\theta_0$. So in this case, we will use the value $W_1$, instead of $W_0$, which is equal to:

$$W_1 = (\theta_1 \mu_0)^m = c^m W_0 \tag{7}$$

Therefore, the β-error relation is obtained as follows:

$$\beta = P\left(V_i > LCL|\theta_1 = C\theta_0\right)$$
$$\beta = 1 - P(\chi^2_{2r} < 2c^m w_0 \times LCL) \tag{8}$$

Given the β value obtained, the value of $\mathrm{ARL}_1$ will be obtained from the following equation:

$$ARL_1 = \frac{1}{1 - \left(P\left(c^m * \chi^2_{2r,1-(\alpha/2)} < \chi^2_{2r} < c^m * \chi^2_{2r,\alpha/2}\right)\right)} \tag{9}$$

### 3.2. Two-sided Shewhart control chart

Because the chart in Section 3-1 uses only a lower control limit due to its design, it is not able to report process improvements and deteriorations (increase and decrease in life) at the same time. Now suppose that the deterioration and improvement of the qualitative characteristic are considered in a process. In this case, two-sided control charts are used. Like the one-sided control chart, we use the concept of α error to obtain the control limits. The probability of error α is as follows.

$$\alpha = P(V_i > UCL|\theta = \theta_0) + P(V_i < LCL\ |\theta = \theta_0) \tag{10}$$

Assuming that half of the α error is above the upper control limit and the other half is below the lower control limit, the above relation can be rewritten as follows and the UCL relation can be obtained as a one-sided control chart:

$$P(V_i > UCL|\theta = \theta_0) = \frac{\alpha}{2}$$
$$UCL = \frac{\chi^2_{2r,\alpha/2}}{2W_0} \tag{11}$$

Similarly, Equation (10) will be rewritten as follows and the LCL relation will be obtained:

$$LCL = \frac{\chi^2_{2r,1-(\alpha/2)}}{2W_0} \tag{12}$$

In this case, the error ratio β will be as follows:

$$\beta = P\left(LCL < V_i < UCL\ |\theta_1 = C\theta_0\right)$$
$$\beta = P(2c^m W_0 \times LCL < 2W_1 V_i < 2c^m W_0 \times UCL) \tag{13}$$

The value of $\mathrm{ARL}_1$ will also be obtained from Equation (14):

$$ARL_1 = \frac{1}{1 - \left(P\left(c^m * \chi^2_{2r,1-(\alpha/2)} < \chi^2_{2r} < c^m * \chi^2_{2r,\alpha/2}\right)\right)} \tag{14}$$

## 4- Computational results and analysis

In this section, we will first obtain the control limits of these two graphs for different parameters and then the process of changing the control limits for these parameters will be analyzed. Next, $ARL_1$ is used to evaluate the performance of the proposed control charts. The changes trend of $ARL_1$ for different parameters is then discussed and analyzed. As an example in Table (1), the lower control limit values for m = 0.5,1 and the $ARL_0$=200,370 and the different values of r are shown.

Table 1. The LCL limits of the one-sided control charts for different parameters

| m=0.5 | | | | | | | m =1 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ARL_0$= 370 | | | | | | | $ARL_0$=370 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 | r | 1 | 2 | 3 | 4 | 5 | 6 |
| LCL | 0.001 | 0.053 | 0.191 | 0.398 | 0.657 | 0.955 | LCL | 0.003 | 0.08 | 0.31 | 0.65 | 1.08 | 1.57 |
| $ARL_0$= 200 | | | | | | | $ARL_0$=200 | | | | | | |
| LCL | 0.003 | 0.073 | 0.238 | 0.475 | 0.762 | 1.086 | LCL | 0.005 | 0.10 | 0.33 | 0.67 | 1.07 | 1.53 |

By examining the table (1), the following results can be achieved:

- By increasing the r value, the LCL value also increases.
- By increasing the $ARL_0$ value, the LCL value decreases.
- By increasing the m value, the value of LCL also increases and this trend is true for different values of r and $ARL_0$.

Table (2) also shows the values of the control limits of the two-sided control chart as well as the one-sided control chart for different parameters m = 0.5,1 and the values of $ARL_0$ = 200,370 and different values of r.

The following results can be achieved by examining the control limits of the two-sided Shewhart control chart, which is reported as an example in the table above:

Table 2. The control limits of the twe_sided control charts for different parameters

| m=0.5 | | | | | | | m=1 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ARL_0$=370 | | | | | | | $ARL_0$=370 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 | r | 1 | 2 | 3 | 4 | 5 | 6 |
| UCL | 4.672 | 6.2934 | 7.685 | 8.966 | 10.17 | 11.33 | UCL | 7.703 | 10.37 | 12.67 | 14.78 | 16.77 | 18.69 |
| LCL | 0.001 | 0.0374 | 0.149 | 0.329 | 0.559 | 0.830 | LCL | 0.001 | 0.061 | 0.246 | 0.542 | 0.923 | 1.369 |
| $ARL_0$=200 | | | | | | | $ARL_0$=200 | | | | | | |
| UCL | 4.236 | 5.806 | 7.159 | 8.405 | 9.585 | 10.71 | UCL | 6.985 | 9.574 | 11.80 | 13.85 | 15.80 | 17.67 |
| LCL | 0.001 | 0.0512 | 0.186 | 0.390 | 0.646 | 0.940 | LCL | 0.002 | 0.084 | 0.307 | 0.643 | 1.065 | 1.551 |

- By increasing the r value, the values of the control limits also increase.
- By increasing the ARL0 value, UCL values also increase and LCL values decrease.
- Similar to one-sided control chart, as the value of m increases, the control limits also increases.

After obtaining the control limits, $ARL_1$ is used to evaluate the performance of the proposed control charts. To obtain $ARL_1$, first the β error values are obtained for different parameters and then the values of $ARL_1$ are obtained. The parameters affecting the value of $ARL_1$ in the Shewhart control chart are the shape parameter (m), in-control average run length ($ARL_0$), the number of failures in the failure censoring test (r) and the change constant (c). The trend of $ARL_1$ changes in one-sided and two-sided control charts for different parameters is exactly the same. These trends are shown as an example for a two-sided control chart, and the analyses are presented based on charts derived from $ARL_1$ values.

For example, $ARL_1$ values for different parameter values for one-sided and two-sided control charts are shown in the tables 3,4.

Table 3. The ARL $_1$ of the one-sided control charts for different parameters

| m=1 | | | | | | |
|---|---|---|---|---|---|---|
| $ARL_0$=370 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 |
| c | $ARL_1$ | | | | | |
| 1 | 370.37 | 370.37 | 370.37 | 370.37 | 370.37 | 370.37 |
| 1.1 | 336.75 | 307.62 | 283.89 | 264.40 | 248.02 | 233.97 |
| 1.2 | 308.73 | 259.78 | 223.08 | 195.10 | 173.10 | 155.30 |
| 1.3 | 285.02 | 222.46 | 178.99 | 148.02 | 125.06 | 107.44 |
| 1.4 | 264.69 | 192.77 | 146.19 | 114.99 | 93.06 | 77.00 |
| 1.5 | 247.08 | 168.76 | 121.25 | 91.17 | 71.02 | 56.88 |
| 1.6 | 231.67 | 149.07 | 101.91 | 73.57 | 55.41 | 43.14 |
| 1.7 | 218.07 | 132.70 | 86.66 | 60.30 | 44.08 | 33.48 |
| 1.8 | 205.98 | 118.96 | 74.47 | 50.11 | 35.67 | 26.52 |
| 1.9 | 195.17 | 107.30 | 64.58 | 42.16 | 29.31 | 21.39 |
| 2 | 185.44 | 97.32 | 56.47 | 35.86 | 24.41 | 17.54 |
| 2.5 | 148.45 | 63.84 | 31.90 | 18.21 | 11.52 | 7.88 |
| 3 | 123.79 | 45.44 | 20.35 | 10.86 | 6.62 | 4.47 |

| m=1.5 | | | | | | |
|---|---|---|---|---|---|---|
| $ARL_0$=200 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 |
| c | $ARL_1$ | | | | | |
| 1 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| 1.1 | 173.42 | 151.85 | 135.32 | 122.36 | 111.87 | 103.15 |
| 1.2 | 152.27 | 118.25 | 95.20 | 78.95 | 66.96 | 57.78 |
| 1.3 | 135.10 | 94.08 | 69.22 | 53.29 | 42.45 | 34.71 |
| 1.4 | 120.93 | 76.22 | 51.76 | 37.38 | 28.27 | 22.14 |
| 1.5 | 109.09 | 62.74 | 39.67 | 27.12 | 19.65 | 14.89 |
| 1.6 | 99.07 | 52.35 | 31.06 | 20.26 | 14.18 | 10.48 |
| 1.7 | 90.51 | 44.22 | 24.78 | 15.53 | 10.58 | 7.67 |
| 1.8 | 83.11 | 37.76 | 20.10 | 12.19 | 8.13 | 5.83 |
| 1.9 | 76.68 | 32.55 | 16.56 | 9.76 | 6.41 | 4.57 |
| 2 | 71.03 | 28.30 | 13.83 | 7.97 | 5.18 | 3.69 |
| 2.5 | 50.97 | 15.62 | 6.62 | 3.62 | 2.37 | 1.77 |
| 3 | 38.90 | 9.82 | 3.88 | 2.17 | 1.52 | 1.24 |

Table 4. The ARL $_1$ of the two-sided control charts for different parameters

| m=2.5 | | | | | | |
|---|---|---|---|---|---|---|
| $ARL_0$=370 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 |
| c | $ARL_1$ | | | | | |
| 0.2 | 1.13 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.4 | 1.95 | 1.29 | 1.11 | 1.04 | 1.02 | 1.01 |
| 0.6 | 6.30 | 3.43 | 2.40 | 1.89 | 1.60 | 1.41 |
| 0.8 | 42.48 | 26.45 | 18.86 | 14.41 | 11.51 | 9.49 |
| 1 | 370.37 | 370.37 | 370.37 | 370.37 | 370.37 | 370.37 |
| 1.2 | 463.30 | 302.67 | 206.32 | 147.58 | 109.88 | 84.48 |
| 1.4 | 319.67 | 144.24 | 72.95 | 41.28 | 25.58 | 17.02 |
| 1.6 | 229.10 | 76.39 | 30.87 | 15.04 | 8.51 | 5.40 |
| 1.8 | 170.79 | 44.05 | 15.10 | 6.80 | 3.79 | 2.48 |
| 2 | 131.36 | 27.21 | 8.34 | 3.69 | 2.16 | 1.53 |
| 2.5 | 75.41 | 10.30 | 2.87 | 1.48 | 1.12 | 1.03 |
| 3 | 47.99 | 5.00 | 1.56 | 1.07 | 1.01 | 1.00 |

| m=2 | | | | | | |
|---|---|---|---|---|---|---|
| $ARL_0$=200 | | | | | | |
| r | 1 | 2 | 3 | 4 | 5 | 6 |
| c | $ARL_1$ | | | | | |
| 0.2 | 1.27 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 |
| 0.4 | 2.61 | 1.61 | 1.29 | 1.14 | 1.07 | 1.04 |
| 0.6 | 8.58 | 4.85 | 3.39 | 2.63 | 2.17 | 1.86 |
| 0.8 | 43.08 | 29.69 | 22.53 | 17.99 | 14.86 | 12.58 |
| 1 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 |
| 1.2 | 264.75 | 193.44 | 144.71 | 112.00 | 89.23 | 72.79 |
| 1.4 | 204.00 | 109.00 | 63.97 | 40.91 | 27.97 | 20.15 |
| 1.6 | 156.55 | 65.76 | 32.20 | 18.10 | 11.31 | 7.68 |
| 1.8 | 123.80 | 42.40 | 18.07 | 9.35 | 5.61 | 3.77 |
| 2 | 100.38 | 28.83 | 11.07 | 5.48 | 3.29 | 2.27 |
| 2.5 | 64.42 | 13.12 | 4.38 | 2.21 | 1.48 | 1.20 |
| 3 | 44.89 | 7.18 | 2.37 | 1.37 | 1.10 | 1.02 |

- According to Figure (1), as r increases, the value of $ARL_1$ decreases. The reason for this trend is that the higher the number of failures in the failure censoring test, the longer the test time and the more information is collected.

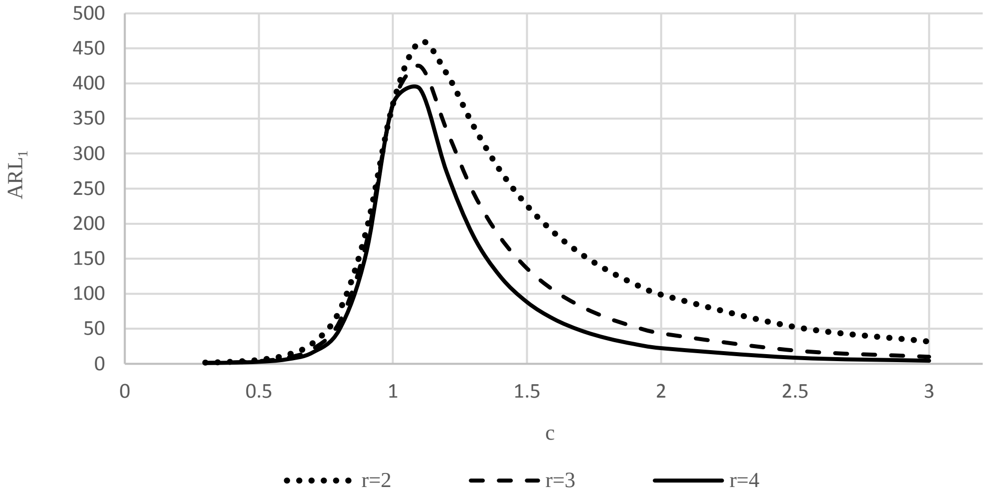


Figure 1. ARL $_1$ of the two_sided control charts for various values of *r*

- As can be seen in Figure (2), the values of $ARL_1$ decrease by decreasing the value of $ARL_0$. The reason for this trend, as explained in the one-sided control chart, is that in a two-sided control chart, the lower the $ARL_0$ value, the closer the control limits will be. This allows the control chart to detect the change sooner if there is a change in the process.

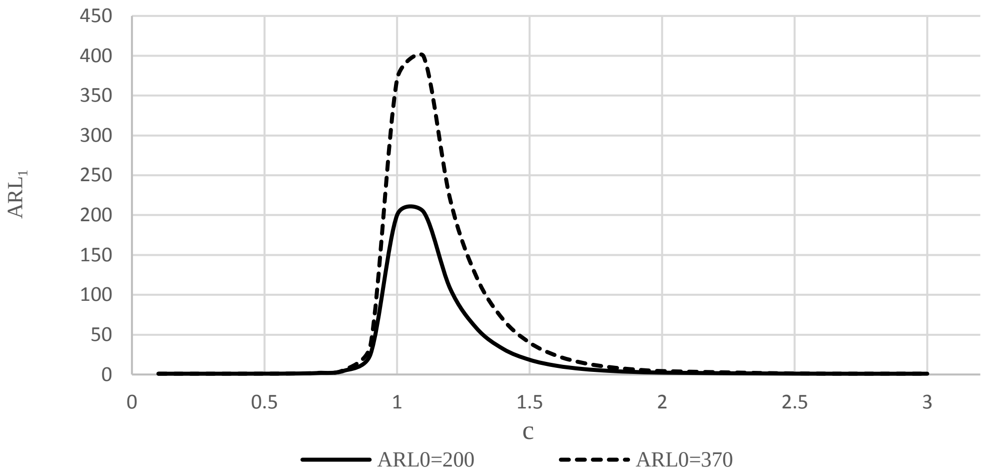


Figure 2. ARL $_1$ of the two_sided control charts for various values of ARL $_0$

- As the trend of Figure (1) shows, the relationship between $ARL_1$ and the change constant (c) is such that with increasing constant change value up to c = 1, the $ARL_1$ trend is upward and from c = 1 onwards, the trend will be downward. The reason for this trend is that the greater the amount of change in a process, the greater the probability of detecting that change and the lower the probability of type II error, and consequently the lower the value of $ARL_1$.
- By increasing the value of the parameter in Figure (2), the changes in $ARL_1$ are as follows. The reason for this trend is that as the shape parameter increases, the amount of type II error decreases and $ARL_1$ will have a decreasing trend. This trend is shown in Figure (3).

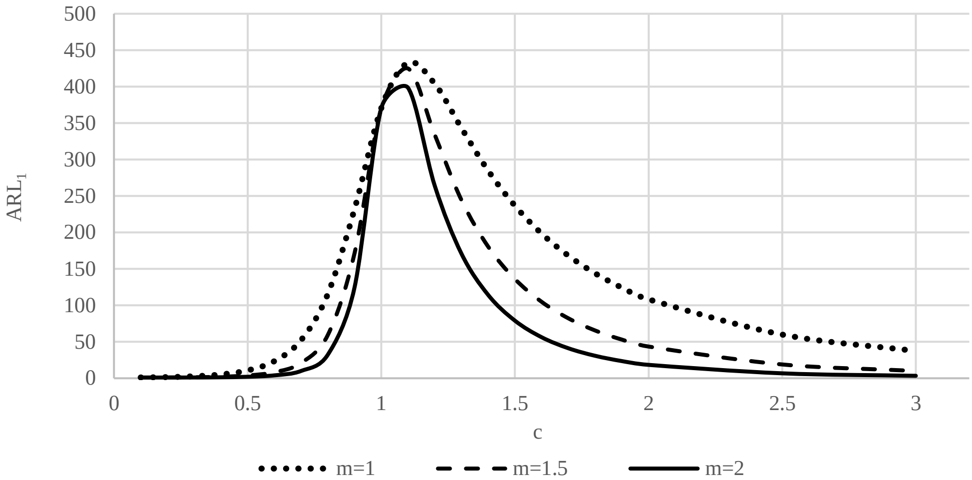


Figure 3. ARL $_1$ of the two_sided control chart for various values of *m*

Now, the $ARL_1$ in one-sided and two-sided control charts are compared. A one-sided control chart monitors the process from one direction because it has only one control limit. However, due to having two control limits, the two-sided control chart can to monitor the process from both sides. For this reason, the $ARL_1$ curve of the two-sided chart has skewness and one-sided control charts have no skewness towards $ARL_1$ and have a uniform decreasing or increasing trend, which is also shown in Figure (4).

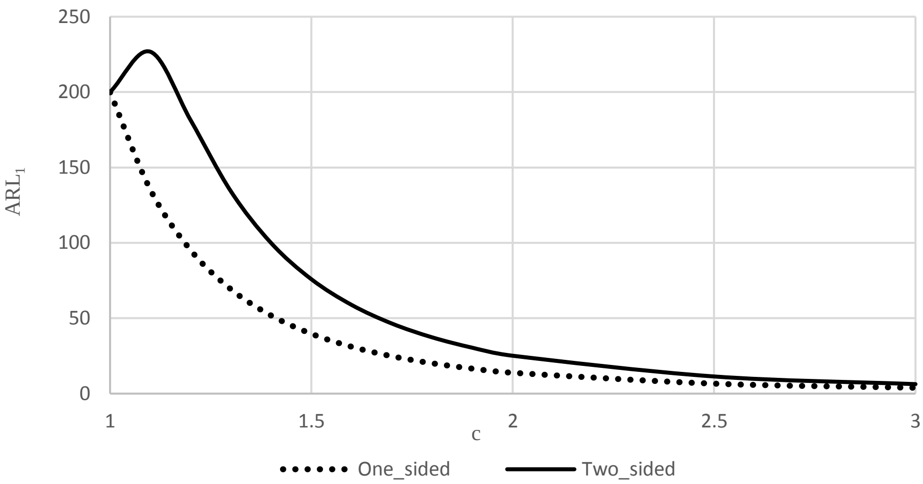


Figure 4. ARL $_1$ for one-sided and two-sided control chart for various values of c

On the other hand, in a one-sided control chart, the $ARL_1$ values are always lower than the $ARL_1$ values in the two-sided control chart. Because there is only one control limit in a one-sided control chart, it is assumed that the α error exists on only one side, and therefore, the LCL one-sided control chart is larger than the LCL two-sided control chart. This causes the one-sided control chart to detect the deviation sooner if there is a deviation in the process.

5. Designing a control chart in a practical example

In this section, the actual data of a Korean car manufacturer is used to design control charts. The data are about the operating time of a part of the machine until the breakdown in a period of one month, which follows the Weibull distribution with the shape parameter m = 2.5 and the scale parameter $\theta_0 = 1$. It is assumed that $\text{ARL}_0 = 370$ and $\text{failure number(r)} = 3$. The values of $V_i$ are reported in Table (5) (Aslam & Jun, 2015).

Table 5. The $V_{(i)}$ statistical values for real data

| sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $V_{(i)}$ | 8.2600 | 9.6870 | 3.2270 | 3.7560 | 5.1410 | 1.3090 | 2.9770 | 5.9220 | 3.4210 | 4.7180 | 4.5950 |
| sample | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| $V_{(i)}$ | 7.9940 | 9.7640 | 5.3940 | 2.4050 | 3.8080 | 3.8600 | 1.6550 | 1.1080 | 3.1160 | 4.2380 | 3.1210 |
| sample | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| $V_{(i)}$ | 7.1010 | 5.5620 | 6.4440 | 4.1890 | 3.4480 | 7.2690 | 4.6650 | 0.6960 | 1.9340 | 2.9680 | 5.0930 |
| sample | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| $V_{(i)}$ | 5.3030 | 10.2320 | 4.6860 | 3.2180 | 2.6830 | 4.6460 | 10.5180 | 2.5220 | 1.1100 | 4.0510 | 8.8780 |
| sample | 45 | 46 | 47 | 48 | 49 | 50 | | | | | |
| $V_{(i)}$ | 3.2530 | 2.0900 | 5.2360 | 1.3630 | 4.5920 | 3.1770 | | | | | |

The process is monitored with a two-sided Shewhart control chart. Figure (5), which is related to the Shewhart control chart, shows that the process is in control and no factors have caused the process to deviate.

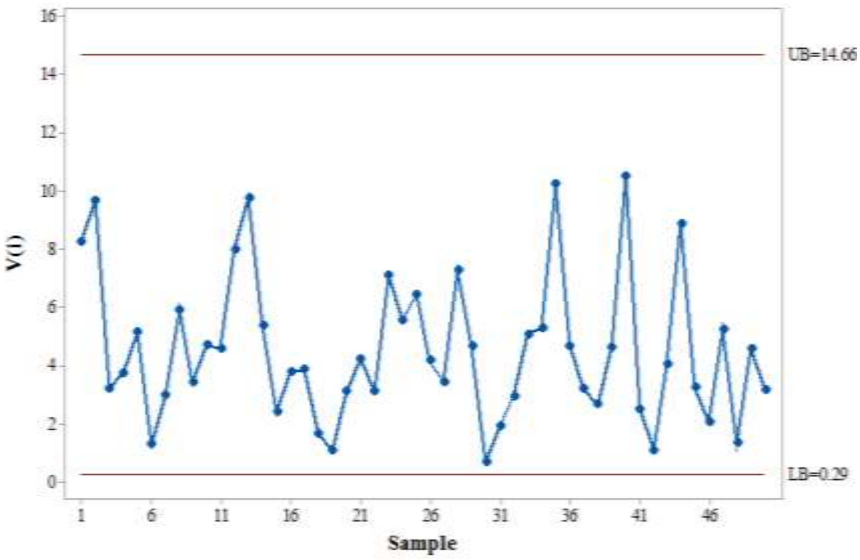


Figure 5. The Shewhart control chart proposed for real data

## 5. Conclusion

One of the most important and widely used control charts is the Shewhart control charts. The main quality characteristic in reliability is the product lifetime. Reducing lifetime is an important issue in reliability. Therefore, in this study, in addition to two-sided control charts, one-sided control charts that monitor lifetime reduction are presented. In this research, the failure censored test without replacement has been used to obtain the statistical. next, the relationships control limits, α and β errors and $ARL_1$ are presented. Then the trend of changes in ARL and control limits for different values of effective parameters are presented and analyzed. The effective parameters in the Shewhart control charts are the number of failures, the shape parameter, the change constant, and the average run length of the in-control state. According to the results, the one-sided Shewhart control chart performs better than the two-sided control chart in detecting reduced lifetime. It is recommended to use one-sided control charts in industries that only reduce lifetime is considered. In one-sided and two-sided control charts, by increasing each of the values r, m, and decreasing $ARL_0$ , the performance of the control chart increases. One of the most important parameters to increase the performance of the control chart is the number of failures of the lifetime test, and the more it is considered, the faster the chart reports the change in the process. The control charts presented in this research can be used to monitor the production process of products in industries whose product lifetime is long, such as mechanical components, electrical and electronics components. Designing CUSUM-Shewhart control charts for hybrid censoring life tests or other censoring life tests is a promising directions for future studies.

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