Journal of Quality Engineering and Production Optimization

Vol. 7, No. 1, Winter & Spring 2022

http://jqepo.shahed.ac.ir

Research Paper

DOI: 10.22070/JQEPO.2022.15248.1210

Applications of Lifetime Performance Index in Acceptance Sampling plans under censoring

Shima Falahi[1], Hiwa Farughi[1] * & Hasan Rasay[2]

[1] Department of Industrial Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran
[2] Department of Industrial Engineering, Kermanshah University of Technology, Kermanshah, Iran

Corresponding Author: *Hiwa Farughi (Email: hfarughi@gmail.com)*

Abstract *– In this research, lifetime performance index (LPI) data are used to present a quick switching sampling (QSS) plan based on a type- II censoring life test and the assumption that the lifetime of units follows the Weibull distribution. In this proposed QSS plan, it is also assumed that the sample size (n) and the acceptance criterion (k) are the same for both the normal and the tightened inspections of the QSS plan, but the failures (r) during the normal and tightened inspections are different in number. The equations needed to calculate the operating characteristic (OC) curve are presented for the proposed QSS along with an optimization model to minimize the average failure number (AFN). In this regard, the constraints of producer and consumers' risks are incorporated into the model. To show the performance of the proposed QSS plan, numerical analyses are performed and the studies conducted in this field are compared. The introduced QSS sampling plan can significantly reduce the cost of manufacturers at the level of industrial organizations.*

Keywords*– Life testing, Censoring, Reliability, Lifetime performance index, Acceptance sampling plan, Lifetime*

I. INTRODUCTION

There are various statistical tools to analyze qualitative problems and improve the performance of production processes. According to Montgomery (Montgomery, 2020), there are three major statistical methods of quality control including (i) acceptance sampling plans, (ii) statistical process control (SPC) and (iii) design of experiment.

The acceptance sampling plan is one of the oldest facets of quality assurance that's subject to examination and decision-making regarding the products. In the 1930s and 1940s, the acceptance sampling plan was a significant element of statistical quality control practically used to inspect input materials. This method often provides sufficient data about controlling production processes (Montgomery, 2020). There are many strategies to classify sampling plans. A basic classification depends on the kind of data and postulates variable and attribute sampling plans. In variable sampling plans, the quality characteristic is measurable and can be represented on a numerical measure. In attribute sampling plans, nevertheless, the taxonomies of units are just based on conforming or non-conforming. Attribute qualitative characteristics are the concepts that cannot be expressed numerically, hence expressed based on the method of acceptance and rejection (Montgomery, 2020). Both variable and attribute sampling plans can be implemented through different modes of sampling: single sampling, double sampling, multiple sampling, multiple state repetitive sampling, sequential sampling, dodge- Romig sampling, resubmitted sampling, repetitive group sampling, chain sampling, and quick switching sampling.

Manuscript Received:14- Feb -2022 & Revised: 27- May -2022 & Accepted: 29- June -2022
ISSN: 2423-3781

(Cha & Badía, 2021) proposed a variables acceptance reliability sampling plan for the units that are subject to the reverse Gaussian degradation process. (Fallahnezhad et al., 2020) proposed three mathematical models for the economic single-sampling plans using Bayesian inferences, while inspection errors are taken into consideration. (FallahNezhad et al., 2015) investigated a mathematical model for designing single-stage and double-stage sampling plans.

Dodge (1967) was the first one to propose the Quick switching sampling plan as a new type of sampling method. In line with his study, the present research makes use of LPI data to develop a quick switching sampling plan QSS(r_N, r_T, k) under a type- II censoring life test. The QSS plan consists of two subsets one of which is done based on normal inspection and the other one based on tightened inspection. A Quick switching sampling plan begins via normal inspection. The normal inspection of the submitted batch continues until the batch is rejected. Then, the plan switches to tightened inspection. The inspection based on tightened conditions continues long as a batch is accepted. Then, the plan switches back to normal inspection for the next batch. It should be noted that the conditions for the acceptance of a batch based on tightened inspection are tougher than those based on a normal inspection. QSS plans have been widely studied by researchers. For instance, Romboski (1969) investigated quick switching acceptance sampling systems AQSS-1(n, c_N, c_T). Soundararajan & Arumainayagam (1990) discussed a set of master tables to select a reformed quick-switching system indexed by different sets of parameters. Balamurali & Usha (2012) investigated a variable quick switching sampling system VQSS(n, c_N, c_T) for measurable quality characteristics with double specification limits beyond which a unit would be considered non-conforming. Senthilkumar et al. (2012) designed a quick switching variables sampling system QSVSS(n, k_N, k_T) indexed by crossover points. Liu & Wu (2016) developed a new QSS plan under the process yield index for batch determination and the quality characteristic followed normal distribution with two specification limits. Senthilkumar & Raffie (2015) designed a quick switching variable sampling system SSQVSS(n_N, n_T, k) based on six sigma quality levels.

Nowadays, the main goal of manufacturers who seek to make high-quality and reliable products is the adoption of policies to minimize consumers risk and producers' risks. A combination of reliability and acceptance sampling plans leads to a new class of acceptance sampling plans called reliability acceptance sampling plans (RASPs). A RASP is a reliability life test. The life test is a type of sampling plan in which the desired quality characteristic is the lifetime of the units. However, there are several challenges to reliability testing. A major challenge of a life test is its duration. There have been various methods proposed to decrease the time and cost of life tests. In this regard, one may refer to truncated life tests, type- II censoring, type- I censoring, progressive censoring, hybrid censoring, and sequential probability ratio life test. Rasay & Alinezhad (2022) developed a novel life test according to the sequential probability ratio test of the Bernoulli/binomial distribution. The test is simple, straightforward and effective enough to be adapted for the life testing of different continuous distributions. Mohammadipour et al. (2021) designed EWMA control charts based on type- II censoring reliability tests. Shrahili et al. (2021) presented acceptance sampling plans based on life tests and the percentiles of the new Weibull-Pareto distribution. Chakrabarty et al. (2020) made an optimal reliability acceptance sampling plan RASP using type-I generalized hybrid censoring plan for non-repairable products sold based on the general rebate warranty. Rasay et al. (2022) introduced resubmitted sampling plans for truncated life testing. Goodarzi & Amiri (2017) employed an accelerated failure time (AFT) model and two control charts are presented to monitor the quality characteristic in the second stage of a production process, while censored and non-censored reliability data are analyzed.

Process capability indexes (PCI) play an important role in quality engineering. So far, various such indexes have been introduced to measure quality levels in production processes, i.e., to check the performance of operations in manufacturing and service industries. Most PCIs are based on quality characteristics with normal distribution. Compared to these characteristics, lifetime is a qualitative feature distinct due to being a non-negative random variable and a larger-the-better type of quality characteristic. (Montgomery, 2020) developed the LPI to measure this quality characteristic. LPI is a type of PCI that evaluates the performance of a process with a non-negative random variable of lifetime (T) for which a specification limit (L) is set. In general, it is necessary to have a lifetime performance index greater than L per unit time to have profitable and satisfactory customers. L is a lower lifetime limit. Many researchers have done life testing by considering LPI data. For example, Rasay et al. (2020) presented two sampling plans, RGS and RS, based on the type-

II censoring life test, and assumed that lifetime followed the Weibull distribution. Aslam et al. (2019) presented a multiple dependent state repetitive (MDSR) sampling plan under LPI data. Rasay & Naderkhani (2020) presented a quick switching sampling QSS(r, k_N, k_T) plan in the in the field of type- II censoring reliability tests by considering the LPI data. Abd El- Monsef & Hassanein (2020) evaluated the MLE of the lifetime performance index(C_L) based on the failure progressive censoring sample for the normal Kumaraswamy distribution. Ahmadi & Doostparast (2019) sought to evaluate the performance of a process subject to a given lower specification limit. They performed a Pareto analysis for LPI under progressive first-failure-censored data that followed the Pareto distribution. Badr et al. (2019) considered the independent lifetimes of Chen products with specified one-shape parameters to evaluate the C_L of the performance of a process. Wu et al. (2018) adopted a LPI to make acceptance-sampling plans for an exponential population with and without censoring. S.-F. Wu & Hsieh (2019) utilized the maximum likelihood estimator of C_L to develop a hypothesis-testing procedure with respect to lower specification limits through progressive type-I interval censoring. Hu & Gui (2020) obtained the MLE of C_L with two unknown parameters in the Lomax distribution based on a progressive type-I interval censored sample. Bhattacharya & Aslam (2019) developed plans to sample variables under the LPI for the exponentially distributed hybrid censored data. Ahmadi et al. (2013) obtained the maximum likelihood estimate of LPI based on progressive first-failure-censored data.

The present paper introduces a quick switching sampling (QSS) plan in the field of type- II censoring reliability life tests by considering lifetime performance index data. Lifetime is taken into consideration as a qualitative characteristic, and it is assumed that the lifetime of units follows the Weibull distribution. The purpose is to minimize the average number of failures (AFN) and to ensure that the lifetime, as a qualitative characteristic, meets the corresponding constraints of producer's and consumer's risks. Also, equations are presented to calculate OC curves.

The paper is organized into several sections. The second section provides the equation of the LPI in the Weibull distribution as well as the minimum variance unbiased estimator of that index. Section 3 presents the equations for the single sampling (SS) plan. Then, a Quick switching sampling plan is developed in Section 4. Section 5 is dedicated to a case study, numerical analyses with two real examples, sensitivity analyses and comparisons with different sampling plans. In the end, the conclusion of the study is brought in Section 6.

II. LIFETIME PERFORMANCE INDEX (LPI) FOR THE WEIBULL DISTRIBUTION

At the beginning of this section, the life time performance index equations are formulated for the Weibull distribution. Next, a minimum variance unbiased estimator (MVUE) is provided for C_L.

The lifetime performance index C_L for the random variable T is as follows:

$$C_L = \frac{\mu_T - L}{\sigma_T} \tag{1}$$

where μ_T and σ_T denote the mean and the standard deviation of T, and L is the lower lifetime limit specified for T. It is assumed that T follows the Weibull distribution and cumulative distribution function (c.d.f) for T is as follows:

$$F(t; \lambda, \nu) = 1 - exp\{-(\lambda t)^\nu\} \tag{2}$$

where ν is the shape parameter and λ is scale parameter of the Weibull distribution. In this study, it is assumed that the shape parameter can be estimated based on historical data. As the numerical analyses in Section 5 show, several statistical tests have been proposed in this regard; goodness of fit tests are among them. The mean and the variance of the random variable T are as follows:

$$\mu = \frac{\Gamma\left(\frac{1}{\nu}\right)}{\nu\lambda} \tag{3}$$

$$\sigma^2 = \frac{1}{\lambda^2}\left[\Gamma\left(1+\frac{2}{\nu}\right) - \left\{\Gamma^2\left(1+\frac{1}{\nu}\right)\right\}\right] \tag{4}$$

where $\Gamma(.)$ represents the Gamma function. Substituting Equations (3) and (4) in Equation (1) yields to the following equations for C_L:

$$C_L = \frac{1}{\lambda\sigma}\left(\frac{\Gamma\left(\frac{1}{\nu}\right)}{\nu} - L\lambda\right) = \frac{1}{B}\left(\frac{\Gamma\left(\frac{1}{\nu}\right)}{\nu} - L\lambda\right) \tag{5}$$

And

$$B = \left(\Gamma\left(1+\frac{2}{\nu}\right) - \left\{\Gamma^2\left(1+\frac{1}{\nu}\right)\right\}\right)^{0.5} \tag{6}$$

where $-\infty < C_L < \frac{\Gamma\left(\frac{1}{\nu}\right)}{B\nu}$.

The lifetime of an unit is likely to be less than L, and the probability is calculated as follows:

$$\delta = P(T < L) = 1 - exp\left\{-\left(\frac{\Gamma\left(\frac{1}{\nu}\right)}{\nu} - C_L B\right)^{\nu}\right\} \tag{7}$$

The value of δ is generally referred to as the non-conforming rate in the QC literature. Accordingly, the value of the conforming rate is $1-\delta$. Obviously, there is a positive relationship between the lifetime performance index C_L and the conforming rate. Thus, the higher the C_L index, the higher the conforming rate.

At this stage, the MVUE of C_L is computed. A test of type- II censoring life is conducted in a certain procedure. For beginning, *n* units are randomly chosen from the batch and put to the test concurrently. The test continues long as the first *r* failures occure. In this case, $r \leq n$. During the test, the number of the failures is recorded for each unit as $t_{(1)}$, $t_{(2)}$,…, $t_{(r)}$. It is axiomatic that the values of $t_{(1)}$, $t_{(2)}$,…, $t_{(r)}$ form the order statistics. Hence , the minimum variance unbiased estimator of C_L is derived according to the characteristics of that order statistics. As (C.-W. Wu et al., 2018) proved, the MVUE of C_L is as follows:

$$\hat{C}_L = \frac{1}{B}\left[\Gamma\left(1+\frac{1}{\nu}\right) - \frac{L.\Gamma(r)}{D^{\frac{1}{\nu}}.\Gamma\left(r-\frac{1}{\nu}\right)}\right] \tag{8}$$

where

$$D = \sum_{i=1}^{r}(n-i+1)\left(t_{(i)}^{\nu} - t_{(i-1)}^{\nu}\right) \tag{9}$$

III. THE SINGLE SAMPLING (SS) PLAN

The basis of designing a QSS plan is the single sampling (SS) plan. This section presents the equations of the SS plan for the type- II censoring life test. The design of this plan is based on LPI, as described in detail by (C.-W. Wu et al., 2018). A brief review of the issue is presented here. For the variable SS plan, *n* units are randomly chosen from the batch and put to the test concurrently. The test continues as long as the first *r* ($r \leq n$) failures occur. During the test, the number the failure is recorded for each unit. According to the test data and using Equation (8), the value of $\boldsymbol{\hat{C}_L}$ is calculated. If it is bigger than or equal to an acceptance criterion, which is represented by *k*, the batch is accepted. Otherwise, the batch is rejected. So, the suggested SS plan has two main parameters including *r* and *k*. For a given value of $\boldsymbol{C_L}$, the probability of acceptance of a batch can be computed as follows:

$$P_a(r,k|C_L=c)=P\left(\hat{C}_L>k\middle|C_L=c\right)=P\left\{\chi^2_{2r}>\frac{2\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bc\right]^{\upsilon}.\Gamma^{\upsilon}(r)}{\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bk\right]^{\upsilon}\Gamma^{\upsilon}\left(r-\frac{1}{\upsilon}\right)}\right\} \tag{10}$$

where χ^2_{2r} represented a chi-squared distribution with $2r$ degrees of freedom. According to Equation (7), the relationship of C_L and the conforming rate δ can be presented as follows:

$$c=B^{-1}\left[\Gamma(1+\upsilon^{-1})-[-ln(1-\delta)]^{\frac{1}{\upsilon}}\right] \tag{11}$$

The probability of acceptance for a given value of the non-conforming rate δ can be calculated with placing Equation (10) into Equation (11), leading to the following equation:

$$P_a(r,k|\delta)=P\left\{\chi^2_{2r}>\frac{-2ln(1-\delta).\Gamma^{\upsilon}(r)}{\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bk\right]^{\upsilon}\Gamma^{\upsilon}\left(r-\frac{1}{\upsilon}\right)}\right\} \tag{12}$$

IV. THE QUICK SWITCHING SAMPLING PLAN (r_N, r_T, k)

The QSS plan contains two SS plans one of which is done under normal inspection and the other one based on tightened inspection. The QSS plan begins with a normal inspection, and the normal inspection of the submitted batch continues until a batch is rejected. Now, the plan switches to tightened inspection, and it continues as long as a batch is accepted. In the QSS(r_N, r_T, k) sampling plan based on the type- II censoring life test, it is assumed that the sample size *n* and the acceptance criterion *k* are the same in both normal and tightened inspections, but the number of failures *r* during the two types of inspections differs. So, r_N is the number of failures in normal inspection, and r_T is the one in tightened inspection. In this research, it is also assumed that the number of failures based on tightened inspection in the QSS(r_N, r_T, k) plan is greater than that in normal inspection ($r_N < r_T$). The number of failures in normal and tightened inspections must be less than the sample size *n*.

As widely discussed in the QC literature, the conditions for designing the sampling plans are according to the following:

1. The acceptance probability of a batch at the acceptable quality level (AQL) must be bigger than the producer's risk represented by α. It means a batch at the quality level AQL must be accepted with a minimum of $(1-\alpha)$%.
2. The acceptance probability of a batch at the rejectable quality level (RQL) must be less than consumer's risk represented by β. It means a batch at the quality level RQL must be accepted with a minimum of β%.

Therefore, in the QSS(r_N, r_T, k), the values of $(\alpha.\delta_{AQL})$ and $(\beta.\delta_{RQL})$ must be determined in advance. There are the two special points of (AQL, $1-\alpha$) and (RQL, β) to consider on the OC curve. In this study, an optimal QSS plan is proposed under these two points in the OC curve to minimize the value of AFN as the objective function.

The steps of implementing the QSS(r_N, r_T, k) plan based on the type- II censoring reliability test with LPI considered are explained below. Moreover, these steps are shown as a flowchart in Figure (1).

1. Determine the values of producer's risk α, consumer's risk β, δ_{AQL}, and δ_{RQL}.
2. The QSS(r_N, r_T, k) plan begins with a normal inspection, a sample with size *n* is randomly taken from the batch, and, based on the type- II censoring test, the units are tested concurrently as long as the first r_N failures occur. According to the test data, the value of $\hat{C}_L$ is calculated with the following equation (Rasay & Naderkhani, 2020):

$$\hat{C}_L = \frac{1}{B}\left[\Gamma\left(1+\frac{1}{\nu}\right) - \frac{L.\Gamma(r_N)}{D^{\frac{1}{\nu}}.\Gamma\left(r_N-\frac{1}{\nu}\right)}\right] \quad (13)$$

where

$$D = \sum_{i=1}^{r_N}(n-i+1)\left(t_{(i)}^{\nu} - t_{(i-1)}^{\nu}\right) \quad (14)$$

Also, B is calculated using Equation (6).

3. If the relation $\hat{C}_L \geq k$ is established, the batch is accepted and the normal inspection continues for the next batch. Otherwise, if $\hat{C}_L < k$, the batch is rejected and the plan switches to tightened inspection for the next batch.
4. Under tightened inspection, n units are randomly chosen from the batch. According to the type- II censoring reliability test, the units are tested concurrently as long as the first r_T failures occur. According to the test data, the value of $\hat{C}_L$ is calculated with the following equation (Rasay & Naderkhani, 2020):

$$\hat{C}_L = \frac{1}{B}\left[\Gamma\left(1+\frac{1}{\nu}\right) - \frac{L.\Gamma(r_T)}{D^{\frac{1}{\nu}}.\Gamma\left(r_T-\frac{1}{\nu}\right)}\right] \quad (15)$$

where

$$D = \sum_{i=1}^{r_T}(n-i+1)\left(t_{(i)}^{\nu} - t_{(i-1)}^{\nu}\right) \quad (16)$$

Also, B is calculated using Equation (6)

5. If the relation $\hat{C}_L \geq k$ is established, the batch is accepted and the plan switches back to the normal inspection for the next batch. Otherwise, if $\hat{C}_L < k$, the batch is rejected and the tightened inspection is still applied for the following batch.

The overall acceptance probability of a batch for the QSS(r_N, r_T, k) sampling plan can be calculated by either of two methods, namely the use of the lifetime performance index C_L or the non-conformance rate δ.

The acceptance probability of a batch in normal and tightened inspections, represented by P_a^N and P_a^T, for a given value of C_L are computed as follows:

$$P_a^N(r_N, k|C_L = c) = P\left(\hat{C}_L > k \middle| C_L = c\right) = P\left\{\chi_{2r_N}^2 > \frac{2\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bc\right]^{\upsilon} \!,\Gamma^{\upsilon}(r_N)}{\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bk\right]^{\upsilon}\Gamma^{\upsilon}\left(r_N-\frac{1}{\upsilon}\right)}\right\} \quad (17)$$

$$P_a^T(r_T, k|C_L = c) = P\left(\hat{C}_L > k \middle| C_L = c\right) = P\left\{\chi_{2r_T}^2 > \frac{2\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bc\right]^{\upsilon} \!,\Gamma^{\upsilon}(r_T)}{\left[\Gamma\left(1+\frac{1}{\upsilon}\right)-Bk\right]^{\upsilon}\Gamma^{\upsilon}\left(r_T-\frac{1}{\upsilon}\right)}\right\} \quad (18)$$

Finally, the overall acceptance probability of a batch for the QSS(r_N, r_T, k) according to a given value of the life performance index, $C_L = c$ is calculated as follows:

$$\pi_a(r_N, r_T, k|C_L = c) = \frac{P_a^T(r_T, k|C_L = c)}{1-P_a^N(r_N, k|C_L = c)+P_a^T(r_T, k|C_L = c)} \quad (19)$$

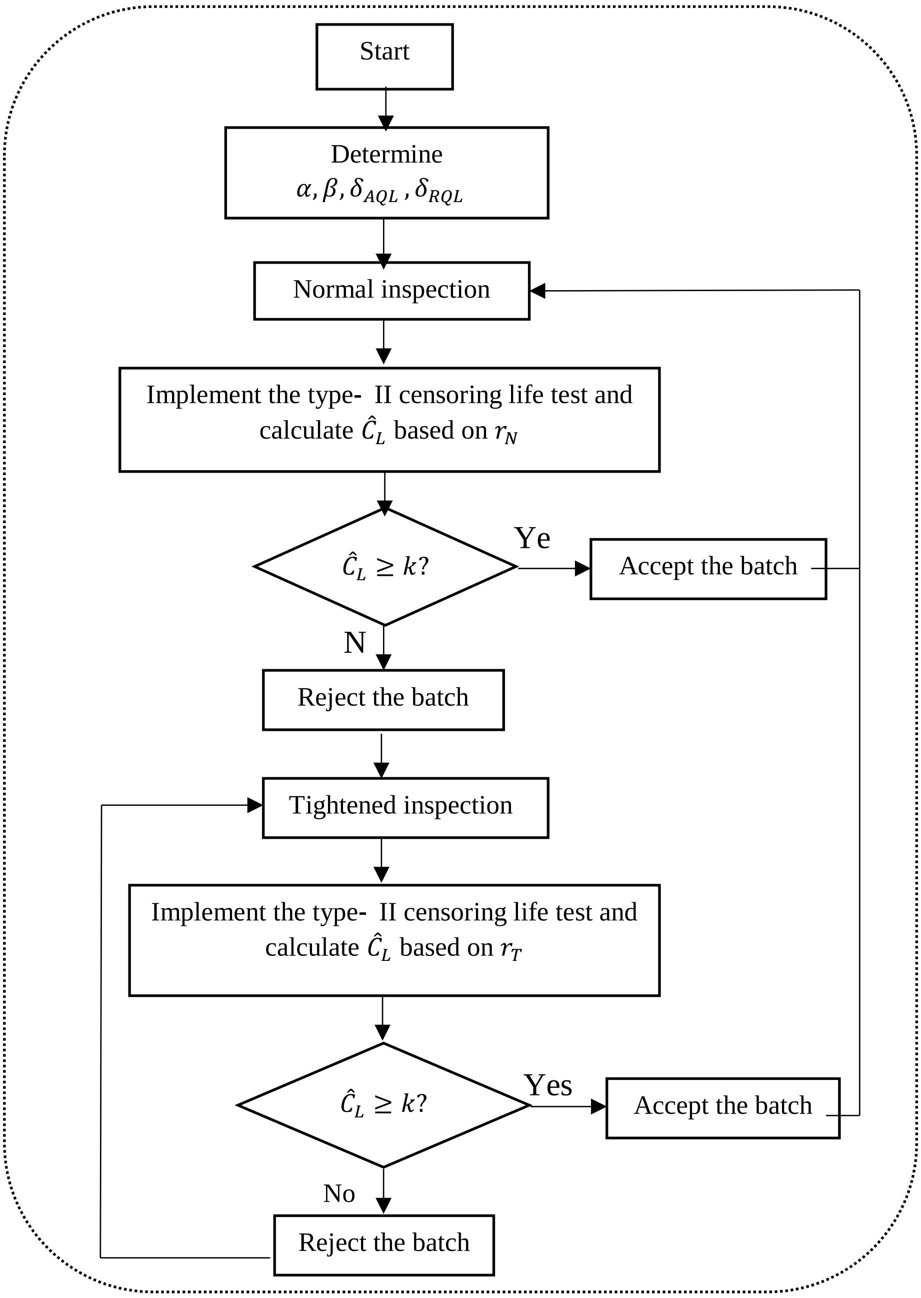

Fig 1. The flowchart of the QSS(r_N, r_T, k) plan

In QSS(r_N, r_T, k), the acceptance probability of a batch in the normal inspection for a given value of δ can be computed by the substitution of Equation (11) in Equation (17) as follows:

$$P_a^N(r_N, k|\delta) = P(\hat{C}_L > k|\delta) = P\left\{\chi^2_{2r_N} > \frac{-2ln(1-\delta),\Gamma^{v}(r_N)}{\left[\Gamma\left(1+\frac{1}{v}\right)-Bk\right]^{v}\Gamma^{v}\left(r_N-\frac{1}{v}\right)}\right\} \tag{20}$$

Similarly, the acceptance probability of a batch in the tightened inspection for a given value of $\boldsymbol{\delta}$ is computed by the substitution of Equation (11) in Equation (18) as follows:

$$P_a^T(r_T, k|\delta) = P(\hat{C}_L > k|\delta) = P\left\{\chi^2_{2r_T} > \frac{-2ln(1-\delta),\Gamma^{v}(r_T)}{\left[\Gamma\left(1+\frac{1}{v}\right)-Bk\right]^{v}\Gamma^{v}\left(r_T-\frac{1}{v}\right)}\right\} \tag{21}$$

Finally, the overall acceptance probability of a batch, according to a given value of the non-conformance rate δ, for QSS(r_N, r_T, k) is calculated as follows:

$$\pi_a(r_N, r_T, k|\delta) = \frac{P_a^T(r_T, k|\delta)}{1-P_a^N(r_N, k|\delta)+P_a^T(r_T, k|\delta)} \tag{22}$$

The operating characteristic (OC) curve is one of the important criteria for evaluating sampling plans. This curve shows the power of differentiation of sampling plans. The OC curve of the QSS(r_N, r_T, k) plan represents the overall acceptance probability of a batch with regard to its quality. This probability can be calculated by two methods, using either the lifetime performance index (C_L) or the non-conforming rate (δ). Therefore, Equation (19) or (22) can be employed to calculate the overall acceptance probability of a batch in the OC curve. The steps for drawing the OC curve of the QSS(r_N, r_T, k) plan are as follows:

1. Determine the values of producer's risk α, consumer's risk β, δ_{AQL} and δ_{RQL} as well as the shape parameters of the Weibull distribution υ.

2. The overall acceptance probability of a batch in the OC curve of the QSS(r_N, r_T, k) plan can be calculated using Equation (19) or (22).

Considering that the QSS(r_N, r_T, k) plan includes the two quality levels of AQL and RQL, the optimization model of the QSS(r_N, r_T, k) plan should be presented preferably by the use of the average quality level δ_M to calculate the AFN. The average quality level is obtained through the following equation:

$$\delta_M = \frac{\delta_{AQL}+\delta_{RQL}}{2} \tag{23}$$

Finally, the optimization model of the QSS(r_N, r_T, k) plan can be expressed as follows:

$$\min AFN(r_N, r_T, k) = \frac{P_a^T(r_T, k|\delta_M).r_N+\left(1-P_a^N(r_N, k|\delta_M)\right).r_T}{1-P_a^N(r_N, k|\delta_M)+P_a^T(r_T, k|\delta_M)}$$

St:

$$\pi_a\left(r_N, r_T, k\middle|\delta_{AQL}\right) \geq 1-\alpha$$

$$\pi_a\left(r_N, r_T, k\middle|\delta_{RQL}\right) \leq \beta$$

$$n > r_T \geq r_N$$

$$c_{RQL} < k < \frac{\Gamma\left(\frac{1}{\upsilon}\right)}{B,\upsilon} \tag{24}$$

In this model, minimizing the average failure number represented by AFN is considered the objective function. There are four constraints in this model. The first constraint ensures that, for a batch with the non-conforming rate δ_{AQL}, the acceptance probability is more than $1-\alpha$. The second constraint ensures that the acceptance probability at the non-conforming rate δ_{RQL} is less than the consumer's risk β. The third constraint regards the relationship between the number of failures in normal and tightened inspections. It also takes into account the condition that the number of defects in both normal and tightened inspections is smaller than the sample size *n*. The fourth constraint determines the lower and upper bounds for *k*. The decision variables of the model that describe the QSS(r_N, r_T, k) plan are *k*, r_N, and r_T.

V. CASE STUDY AND NUMERICAL ANALYSIS

To show the actual application and advantage of the suggested QSS(r_N, r_T, k) plan, two real datasets are derived from the Weibull distribution with the shape parameters v = 1 and v = 2.102. The data specify the period within which each specimen fails (or breaks down) in a life test.

There are 19 pieces of data recorded for electrical insulating fluids. The data are adapted from (C.-W. Wu et al., 2018). They include 0.19, 0.78, 0.96, 1.31, 2.78, 3.16, 4.15, 4.67, 4.85, 6.50, 7.35, 8.01, 8.27, 12.06, 31.75, 32.52, 33.91, 36.71 and 72.89. Based on the goodness-of-fit test, the Weibull distribution with v = 1 is an appropriate distribution to fit the data.

Table I. Input parameters for v = 1

parameters	α	β	δ_{AQL}	δ_{RQL}	v
value	0.05	0.05	0.05	0.2	1

Based on the input parameters presented in Table 1, the optimization problem is solved for the QSS(r_N, r_T, k) plan to achieve the following results:

Table II. Output results for v = 1

output	r_N	r_T	k	***AFN***
value	5	6	0.8949	5.5228

The QSS(r_N, r_T, k) plan begins with a normal inspection, a sample size of 19 is randomly taken from the batch, and the units are tested concurrently as long as the first $r_N = 5$ failures occur. This is done by means of a type- II censoring reliability test. Based on the test data and using Equation 22, the lifetime performance index ($\hat{C}_L$) during the normal inspection is found to be 0.9074. According to the flowchart in Figure (1), since $\hat{C}_L > k$ and $0.9074 > 0.8949$, the batch is accepted and the normal inspection continues for the following batch.

Besides, 23 failures of deep-grove ball bearings (in millions of revolution) are recorded based on the case study presented by (C.-W. Wu et al., 2018). The failure times are 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.40, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 27.92, 128.04, 173.40. Based on the goodness-of-fit test, the Weibull distribution with v = 2.102 is an appropriate distribution to fit the data.

Table III. Input parameters for v = 2.102

parameters	α	β	δ_{AQL}	δ_{RQL}	v
value	0.01	0.01	0.005	0.025	2.102

Based on the input parameters presented in Table 3, the optimization problem is solved for the QSS(r_N, r_T, k) plan to achieve the following results:

Table IV. Output results for v = 2.102

output	r_N	r_T	k	***AFN***
value	9	10	1.7216	9.6514

The QSS(r_N, r_T, k) plan begins with a normal inspection, a sample size of 23 is randomly taken from the batch, and the units are tested concurrently as long as the first $r_N = 9$ failures occur. This occurs through a type- II censoring reliability test. Based on the test data and using Equation 22, the value calculated for the lifetime performance index ($\hat{C}_L$) during the normal inspection is found to be 1.4125. According to the flowchart in Figure (1), since $\hat{C}_L < k$ and $1.4125 < 1.7216$, the batch is rejected and the plan switches to the tightened inspection for the following batch.

The values of the important parameters in the QSS(r_N, r_T, k) plan have been changed, and the effects of the change on the variables and the average number of failures (AFN) have been investigated based on the various values of the shape parameters of the Weibull distribution (v = 1, 2, 3). The results are shown in Tables (5) to (7). It should be noted that to solve the designed optimization model, a grid search algorithm is used and coded in MATLAB software. The following results are inferred from the sensitivity analyses reported in the tables:

- For various values of δ_{AQL} and δ_{RQL}, an increase in the (α) and (β) risks causes to a decrease in the values of AFN, r_N and r_T.
- An increase in the (α) risks causes an increase in the value of k.
- An increase in the (β) risks causes a decrease in the value of k.
- An increase in the value of δ_{AQL} causes an increase in the values of AFN, r_N and r_T but a decrease in the value of k.
- An increase in the value of δ_{RQL} decreases the values of AFN, r_N, r_T and k.
- An increase in the values of the shape parameters in the Weibull distribution causes an increase in k.

In this research, first, the QSS(r_N, r_T, k) plan is compared to QSS(r, k_N, k_T) proposed by (Rasay & Naderkhani, 2020), and then a comparison is performed of the QSS(r_N, r_T, k) plan and the SS plan suggested by Wu et al (2018).

The results of the QSS(r, k_N, k_T) plan for the Weibull distribution with v = 2 are presented in Table 8. The comparisons suggest the following:

- For the same values of α, β, δ_{AQL}, $\delta_{RQL}$and v, the AFN has decreased in the QSS(r, k_N, k_T) plan, as compared to the QSS(r_N, r_T, k) plan. For example, for v = 2, $\alpha = 0.01$, $\beta = 0.05$, $\delta_{AQL} = 0.005$ and $\delta_{RQL} = 0.025$, the AFNs in QSS(r, k_N, k_T) and QSS(r_N, r_T, k) are 4 and 7.4897, respectively.
- For the same values of α, β, δ_{AQL}, $\delta_{RQL}$and v, the acceptance criterion k in the QSS(r_N, r_T, k) plan is equal to something between the values of the normal and tightened acceptance criteria of the QSS(r, k_N, k_T) plan. For example, for v = 2, $\alpha = 0.01$, $\beta = 0.05$, $\delta_{AQL} = 0.005$ and $\delta_{RQL} = 0.025$, the acceptance criterion in the QSS(r_N, r_T, k) plan is equal to 1.6586. For QSS(r, k_N, k_T), the acceptance criterion in the normal and tightened inspections is 1.5936 and 1.7022, respectively.
- The number of failures in QSS(r, k_N, k_T) is lower than that in the QSS(r_N, r_T, k) plan.

To provide more comparison, the results of the SS plan in the Weibull distribution with v = 2 are reported in Table 9. The comparison suggests the following:

- For the same values of α, β, δ_{AQL}, $\delta_{RQL}$and v, the AFNs in the QSS(r, k_N, k_T) plan have slightly decreased compared to the SS plan. For example, with$\alpha = 0.01$, $\beta = 0.01$, $\delta_{AQL} = 0.01$ and $\delta_{RQL} = 0.03$, the AFNs in the QSS(r_N, r_T, k) and SS plans are 18.6060 and 19, respectively.
- For the same values of α, β, δ_{AQL}, δ_{RQL} and v, the acceptance criterion k in the QSS(r_N, r_T, k) plan is approximately equal to that in SS plan. For example, with v = 2, $\alpha = 0.05$, $\beta = 0.05$, $\delta_{AQL} = 0.02$ and $\delta_{RQL} = 0.06$, the acceptance criterion in the QSS(r_N, r_T, k) and SS plans is 1.502 and 1.501, respectively.
- For the same values of α, β, δ_{AQL}, $\delta_{RQL}$and v, the number of failures (r) in the SS plan is higher than that in the

normal inspection (r_N) and lower in the tightened inspection (r_T). For example, there are $r_N = 4$ and $r_T = 6$ calculated for $v = 2$, $\alpha = 0.05$, $\beta = 0.05$, $\delta_{AQL} = 0.005$ and $\delta_{RQL} = 0.025$, and five failures (r) are recorded for the SS plan.

Table V. Different QSS(r_N, r_T, k) plans (v = 1)

δ_{AQL}	δ_{RQL}	*v=1*											
		$\alpha = 0.01$ *and* $\beta = 0.01$				$\alpha = 0.01$ *and* $\beta = 0.05$				$\alpha = 0.05$ *and* $\beta = 0.05$			
		r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*
0.005	0.02	11	13	0.9894	12.2120	8	13	0.9878	10.0955	5	8	0.9898	6.6238
	0.025	9	10	0.9882	9.6547	7	8	0.9867	7.4727	3	7	0.9877	4.8874
	0.03	7	8	0.9867	7.6484	6	8	0.9845	6.9431	3	5	0.9875	4.0893
0.01	0.03	18	19	0.9821	18.5962	14	17	0.9800	14.6948	7	13	0.9815	9.8866
	0.05	8	10	0.9757	9.2490	7	9	0.9727	7.9369	3	7	0.9757	4.9742
	0.06	7	8	0.9731	7.6433	5	8	0.9681	6.3232	2	6	0.9723	3.7179
0.02	0.06	17	19	0.9636	18.1703	13	16	0.9601	14.3020	7	12	0.9631	9.4377
	0.08	11	12	0.9576	11.6215	9	10	0.9526	9.4565	5	7	0.9586	6.0503
	0.1	8	10	0.9506	9.2484	6	9	0.9426	7.2851	4	5	0.9506	4.5371

Table VI. Different QSS(r_N, r_T, k) plans (v = 2)

δ_{AQL}	δ_{RQL}	*v=2*											
		$\alpha = 0.01$ *and* $\beta = 0.01$				$\alpha = 0.01$ *and* $\beta = 0.05$				$\alpha = 0.05$ *and* $\beta = 0.05$			
		r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*
0.005	0.02	12	13	1.6902	12.6417	9	10	1.6782	9.4865	6	7	1.6916	6.5641
	0.025	9	10	1.6736	9.6505	7	8	1.6586	7.4897	4	6	1.6716	5.0818
	0.03	7	8	1.6603	7.6650	6	7	1.6413	6.4980	3	6	1.6563	4.6702
0.01	0.03	18	19	1.6223	18.6060	14	15	1.6093	14.4662	9	10	1.6243	9.5515
	0.05	9	10	1.5762	9.6724	7	8	1.5517	7.4891	4	6	1.5742	5.1233
	0.06	7	8	1.5521	7.6533	6	7	1.5261	6.4937	3	5	1.5511	4.0908
0.02	0.06	18	19	1.4991	18.6008	13	16	1.4773	14.3371	9	10	1.5021	9.5487
	0.08	11	12	1.4627	11.6277	9	10	1.4352	9.4712	6	7	1.4627	6.5589
	0.1	8	10	1.4284	9.2929	7	8	1.4014	7.5015	4	5	1.4374	4.5642

Table VII. Different QSS(r_N, r_T, k) plans (v = 3)

δ_{AQL}	δ_{RQL}	*v=3*											
		$\alpha = 0.01$ *and* $\beta = 0.01$				$\alpha = 0.01$ *and* $\beta = 0.05$				$\alpha = 0.05$ *and* $\beta = 0.05$			
		r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*	r_N	r_T	k	*AFN*
0.005	0.02	12	13	2.0733	12.6531	9	11	2.0433	9.6981	6	7	2.0723	6.5666
	0.025	9	10	2.0357	9.6433	7	8	2.0107	7.5078	4	7	2.0277	5.5882
	0.03	7	8	2.0132	7.6780	6	7	1.9812	6.5306	4	5	2.0052	4.5689
0.01	0.03	18	19	1.9412	18.6123	14	15	1.9172	14.4751	9	10	1.9432	9.5570
	0.05	9	10	1.8536	9.6680	7	8	1.8166	7.5103	4	6	1.8516	5.1520
	0.06	7	8	1.8178	7.6753	5	8	1.7528	6.4004	4	5	1.8205	4.6022
0.02	0.06	18	19	1.7278	18.6145	14	15	1.6948	14.4685	9	10	1.7278	9.5510
	0.08	11	12	1.6653	11.6327	9	10	1.6203	9.4772	6	7	1.6603	6.5509
	0.1	9	10	1.6112	9.6580	7	8	1.5662	57.5072	4	5	1.6192	4.5717

Table VIII. Different QSS(r, k_N, k_T) plans (v = 2)

δ_{AQL}	δ_{RQL}	v=2 QSS(r, k_N, k_T)											
		$\alpha = 0.01$ and $\beta = 0.01$				$\alpha = 0.01$ and $\beta = 0.05$				$\alpha = 0.05$ and $\beta = 0.05$			
		k_N	k_T	r	AFN	k_N	k_T	r	AFN	k_N	k_T	r	AFN
0.005	0.02	1.6242	1.7342	5	5	1.6222	1.7142	5	5	0.9798	1.7402	3	3
	0.025	1.5856	1.7262	4	4	1.5936	1.7022	4	4	0.9747	1.7502	2	2
	0.03	1.5413	1.7292	3	3	1.5533	1.7032	3	3	1.5993	1.7302	2	2
0.01	0.03	1.5363	1.6767	7	7	1.5423	1.6257	7	7	0.9715	1.6827	4	4
	0.05	1.4702	1.6447	4	4	1.4722	1.6107	4	4	0.6927	1.6827	2	2
	0.06	1.3981	1.6497	3	3	1.4091	1.6127	3	3	1.4801	1.6507	2	2
0.02	0.06	1.3881	1.5742	7	7	1.3941	1.5402	7	7	0.9481	1.5842	4	4
	0.08	1.3377	1.5482	5	5	1.3377	1.5062	5	5	0.9166	1.5582	3	3
	0.1	1.2794	1.5282	4	4	1.2954	1.4772	4	4	0.9798	1.7402	3	3

Table IX . Different single sampling (SS) plans (v = 2)

δ_{AQL}	δ_{RQL}	v=2					
		$\alpha = 0.01$ and $\beta = 0.01$		$\alpha = 0.01$ and $\beta = 0.05$		$\alpha = 0.05$ and $\beta = 0.05$	
		k	AFN	k	AFN	k	AFN
0.005	0.01	1.73	46	1.725	35	1.731	23
	0.02	1.692	12	1.678	10	1.691	7
	0.025	1.677	9	1.658	8	1.679	5
0.01	0.02	1.654	46	1.647	35	1.655	23
	0.03	1.623	19	1.61	15	1.624	10
	0.05	1.577	9	1.558	7	1.58	5
0.02	0.04	1.545	45	1.535	34	1.546	23
	0.06	1.502	18	1.484	14	1.501	10
	0.08	1.463	12	1.442	9	1.473	6

Here is a point to make about parameter n in the proposed QSS. According to Equations (17) to (24), which calculate the overall acceptance probability of a batch and the average failure number (AFN), it is clear that sample size (n) does not impact these two basic characteristics of QSS plans. The proposed QSS model (Equation 24) determines the values of r_T, r_N and k as the output of the model, so that AFN can be minimized. After that, the value of n can be arbitrarily chosen to satisfy $n \geq r_T$ and $n \geq r_N$. Hence, the value of n does not affect the other parameters of the QSS.

For examine the performance of the suggested QSS(r_N, r_T, k) plan, the OC curves of the QSS(r_N, r_T, k) and QSS(r, k_N, k_T) plans are compared. The graphs in Figure 2 show the overall acceptance probability of a batch against various values of the non-conforming rate δ. As it can be seen,the overall acceptance probability of a batch in the QSS(r_N, r_T, k) plan is lower than that in the QSS(r, k_N, k_T) plan, but the difference is not significant. For the same value of the non-conforming rate δ, the overall acceptance probability is almost equal in the two plans. The values of the input and output parameters corresponding to Figure 2 are presented in Tables 10 and 11, respectively.

Table X. Input parameters

parameters	α	β	δ_{AQL}	δ_{RQL}	v
value	0.01	0.05	0.05	0.2	1

Table XI. Output results

output / plan	r_N	r_T	k	*AFN*	k_N	k_T	r
QSS(r_N, r_T, k) plan	8	10	0.8729	8.8449	-	-	-
QSS(r, k_N, k_T) plan	-	-	-	4	0.8019	0.9247	4

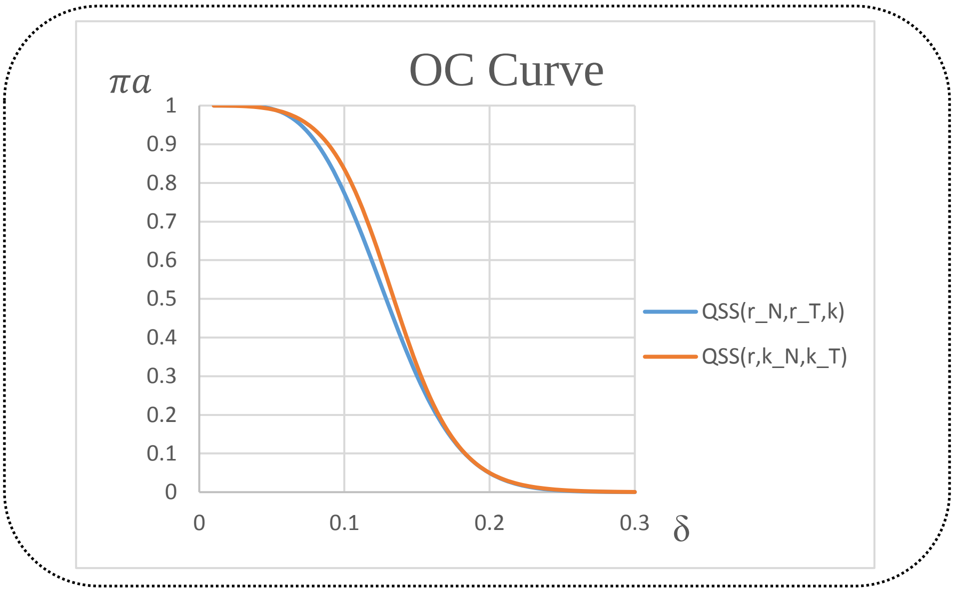

Fig 2. OC curves of the QSS(r_N, r_T, k) and QSS(r, k_N, k_T) plans

VI. CONCLUSION

This study has been conducted to evaluate the quality of a batch of units with a focus on lifetime as a qualitative characteristic. A hypothesis test should be performed on the average lifetime of the units in the batch to determine whether the average lifetime of the units is greater than or less than a specified limit. For this purpose, a QSS(r_N, r_T, k) plan is used based on type- II censoring life test. It is assumed that the lifetime of units follows the Weibull distribution. With the aim of minimizing the AFN, a mathematical model is also presented along with the equations to calculate the OC curve for the proposed QSS (r_N, r_T, k). This plan is then compared with a QSS(r, k_N, k_T) plan. The results show that, for the same value of the non-conforming rate δ, the QSS(r_N, r_T, k) plan has a lower overall acceptance probability, but the difference is not significant; the OC curves of these two sampling plans are not significantly different. Moreover, the AFN in the QSS(r, k_N, k_T) plan is found lower than that in the QSS(r_N, r_T, k) plan. Finally, using two real datasets from the Weibull distribution with the shape parameters of v = 1 and v = 2.102, the performance of the QSS(r_N, r_T, k) plan is examined.

REFERENCES

Abd El- Monsef, M. M. E., & Hassanein, W. A. A. E. L. (2020). Assessing the lifetime performance index for Kumaraswamy distribution under first- failure progressive censoring scheme for ball bearing revolutions. Quality and Reliability Engineering International, 36(3), 1086-1097.

Ahmadi, M. V., & Doostparast, M. (2019). Pareto analysis for the lifetime performance index of products on the basis of progressively first-failure-censored batches under balanced symmetric and asymmetric loss functions. Journal of Applied Statistics, 46(7), 1196-1227.

Ahmadi, M. V., Doostparast, M., & Ahmadi, J. (2013). Estimating the lifetime performance index with Weibull distribution based on progressive first-failure censoring scheme. Journal of Computational and Applied Mathematics, 239, 93-102.

Aslam, M., Yen, C.-H., Chang, C.-H., Al-Marshadi, A. H., & Jun, C.-H. (2019). A Multiple Dependent State Repetitive Sampling Plan Based on Performance Index for Lifetime Data with Type II Censoring. IEEE Access, 7, 49377-49391.

Badr, M. M., Shawky, A. I., & Abd-Elmougod, G. A. (2019). Hybrid censoring samples in assessment the lifetime performance index of Chen distributed products. Open Physics, 17(1), 607-616.

Balamurali, S., & Usha, M. (2012). Variables quick switching system with double specification limits. International Journal of Reliability, Quality and Safety Engineering, 19(02), 1250008.

Bhattacharya, R., & Aslam, M. (2019). Design of variables sampling plans based on lifetime-performance index in presence of hybrid censoring scheme. Journal of Applied Statistics, 46(16), 2975-2986.

Cha, J. H., & Badía, F. (2021). Variables acceptance reliability sampling plan for items subject to inverse Gaussian degradation process. Journal of Applied Statistics, 48(3), 393-409.

Chakrabarty, J. B., Chowdhury, S., & Roy, S. (2020). Optimum reliability acceptance sampling plan using Type-I generalized hybrid censoring scheme for products under warranty. International Journal of Quality & Reliability Management.

Dodge, H. (1967). A dual system of acceptance sampling. Technical Report No. 16.

FallahNezhad, M. S., Ahmadi Yazdi, A., Abdollahi, P., & Aslam, M. (2015). Design of economic optimal double sampling design with zero acceptance numbers. Journal of Quality Engineering and Production Optimization, 1(2), 45-56.

Fallahnezhad, M. S., Rasay, H., Darbeh, J., & Nakhaeinejad, M. (2020). Economic Single-Sampling Plans Based on Different Probability Distributions Considering Inspection Errors. Journal of Quality Engineering and Production Optimization, 5(1), 55-64.

Goodarzi, A., & Amiri, A. (2017). Monitoring Lognormal Reliability Data in a Two-Stage Process Using Accelerated Failure Time Model. Journal of Quality Engineering and Production Optimization, 2(1), 17-26.

Hu, X., & Gui, W. (2020). Assessing the lifetime performance index with Lomax distribution based on progressive type I interval censored sample. Journal of Applied Statistics, 47(10), 1757-1775.

Liu, S.-W., & Wu, C.-W. (2016). A quick switching sampling system by variables for controlling lot fraction nonconforming. International Journal of Production Research, 54(6), 1839-1849.

Mohammadipour, P., Farughi, H., Rasay, H., & Arkat, J. (2021). Designing Exponentially Weighted Moving Average Control Charts under Failure Censoring Reliability Tests. International Journal of Engineering, 34(11), 2398-2407. doi:10.5829/ije.2021.34.11b.03

Montgomery, D. C. (2020). Introduction to statistical quality control: John Wiley & Sons.

Rasay, H., & Alinezhad, E. (2022). Developing an adaptable sequential probability ratio test applicable for lifetime analysis of different continuous distributions. Quality Technology & Quantitative Management, 1-20.

Rasay, H., Farughi, H., & Advay, F. (2022). Truncated life testing under resubmitted sampling plans for Weibull distribution. Journal of Industrial and Systems Engineering.

Rasay, H., & Naderkhani, F. (2020). Designing a Reliability Quick Switching Sampling Plan based on the Lifetime Performance Index. Paper presented at the 2020 IEEE International Conference on Prognostics and Health Management (ICPHM).

Rasay, H., Naderkhani, F., & Golmohammadi, A. M. (2020). Designing variable sampling plans based on lifetime performance index under failure censoring reliability tests. Quality Engineering, 32(3), 354-370.

Romboski, L. (1969). An investigation of quick switching acceptance sampling systems [PhD dissertation]. New Brunswick (NJ): Rutgers–The State University.

Senthilkumar, D., & Raffie, B. E. (2015). Construction and Selection of Six Sigma Quick Switching Sampling System: Sample Size Tightening. International Journal of Innovative Research in Computer and Communication Engineering, 3(3), 1410-1418.

Senthilkumar, D., RAJ, S. S. M., & Raffie, B. E. (2012). Construction of Quick Switching Variables Sampling System Indexed by Crossover Point. International Journal of Engineering Science and Technology (IJEST), 4 (2) 2012, 419, 430.

Shrahili, M., Al-Omari, A. I., & Alotaibi, N. (2021). Acceptance Sampling Plans from Life Tests Based on Percentiles of New Weibull–Pareto Distribution with Application to Breaking Stress of Carbon Fibers Data. Processes, 9(11), 2041.

Soundararajan, V., & Arumainayagam, S. D. (1990). Construction and selection of modified quick switching systems. Journal of Applied Statistics, 17(1), 83-114.

Wu, C.-W., Shu, M.-H., & Chang, Y.-N. (2018). Variable-sampling plans based on lifetime-performance index under exponential distribution with censoring and its extensions. Applied Mathematical Modelling, 55, 81-93.

Wu, S.-F., & Hsieh, Y.-T. (2019). The assessment on the lifetime performance index of products with Gompertz distribution based on the progressive type I interval censored sample. Journal of Computational and Applied Mathematics, 351, 66-76.