

## **An Integrated Mathematical Model of Production Planning Considering Order Acceptance, Production and Customer Delivery at Marun Petrochemical Company**

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**Abstract** –The scheduling and batch delivery problem has been studied by many researchers as one of the classic operation sequence problems. However, this study attempted to address this problem by adding order acceptance determination and sequence-dependent setup. In this study, a mathematical model has been presented in Marun Petrochemical Company considering the number of orders received for production to maximize the organization's profit by selecting the order and planning the production based on the order received. Therefore, to realize this goal in the small production space, the mathematical model was coded in GAMS software and solved using the CPLEX solving method. Then, to investigate the larger scale of the model under study, the validation of 23 generated problems was carried out by the exact solution in GAMS software and genetic algorithm in MATLAB software, and in the end, the comparative evaluation was performed. The evaluation showed that the meta-heuristic solution on a small scale has a small deviation from the exact solution, and the mathematical model is solved in a proper time by the meta-heuristic algorithm. As the problem's size grows up, the exact solution loses its efficiency in terms of time, and the application of the exact solution algorithm to solve the model becomes inadequate. The genetic algorithm achieves an acceptable solution in a proper time with reasonable deviation. So, this algorithm can replace the exact solution properly.

**Keywords**–Production Planning, Scheduling, Order Acceptance, Production, Customer Delivery, Increasing Profit.

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### **I. INTRODUCTION**

Production planning is generally carried out in three long-term, mid-term and short-term time horizons. Scheduling involves production planning decisions in the short-term time horizon that allocate production capacity to production goals. Its output is the precise determination of what product, when, and what machine will be processed. In other words, the scheduling problem determines the exact start and end time of all jobs and their sequence on each machine. This classic problem has been of great interest to researchers over the past decades due to its many applications in manufacturing and service units (Mokhtari, 2015). Decision-making about selecting the order batch and delivering them to customers, which is a short-term process, is one of the critical decisions of each manufacturing unit and the issues of

supply and production. One of the delivery approaches is batch delivery. The delivery process is always done after the production process, so the production plan always influences the delivery policies.

On the other hand, observing the customer's delivery deadline requires a production schedule in line with the customer's delivery deadline. As a result, the order batch selection and the production and delivery are entirely interdependent, so it is advised to deal with this problem in an integrated manner. By considering the scheduling problem in production and the delivery approach in production planning, the integrated problem will include simultaneous scheduling and delivery considering its profit (Iranpoor et al., 2014). Therefore, this study aims to determine the order batch and the optimal sequence of jobs, allocating the jobs to the batches and delivering the batches to the customers to maximize their profit. Each order can be transported to the customer immediately after processing, meaning a batch can contain only one job. In this case, delivery costs will rise sharply. It can also remain in the system to be sent together with the next job or jobs. So, the holding cost and time will increase (Park et al., 2013). Therefore, the selection and scheduling and batch delivery problem approach balance the costs of holding the order in the system until the batch completion (maintenance, delay), and the cost of delivery depends on the number of batches formed. Sometimes the sequence of jobs can be changed so that two or more jobs are sequenced and sent in one batch. One of the essential assumptions that existed in most studies on scheduling and batch delivery is that only the jobs belonging to one customer can be in a batch (Parsaei et al., 2014). In the short-term time horizon, the production system's available capacity is constrained and usually less than the capacity required for processing all orders received. Therefore, proper decision-making for production planning, in addition to determining the scheduling and sequence of jobs, involves selecting the best orders from the orders received by the system (Rasti Barzoki & Hejazi, 2013).

The approach of selecting orders from all received orders is called order acceptance. The scheduling and order acceptance problem simultaneously determine which orders to produce and accept and how the scheduling and sequence of processing these orders should be (Moghaddam et al., 2015). The primary assumption is that there is not enough capacity to process all orders received, and the manufacturer has to choose from several demands. There are two main approaches to scheduling and order acceptance: In the first approach, the production system is penalized for not accepting each order, so the objective function of these problems will be to minimize total costs and increase organization profits. Generally, four categories of costs can be identified in this production system: production cost, sequence-dependent setup cost, holding cost, and batch delivery cost, which depend on the number of batches sent and are independent of the number of orders in the batch.

The central assumption is that there is not enough capacity to process all orders received, and the manufacturer has to choose from several demands. There are two main approaches to scheduling and order acceptance: In the first approach, the production system is penalized for not accepting each order, so the objective function of these problems is to minimize total costs. In the second approach, the production system earns revenue by accepting each order, so the objective function is to maximize profits resulting from the difference between revenue and costs. This research's main idea is to present an integrated problem for simultaneously identifying orders accepted for production, determining production sequence, and delivering orders to customers with a batch delivery approach. In the problem under investigation, many orders reach the production system by several customers. The manufacturer wishes to accept the best production orders considering the available capacity and the revenue of each order and the costs (including the cost of producing each order, the cost of holding each order, the sequence-dependent setup cost, and the delivery cost). The production sequence, the products' allocation to the batches, and sending the batches should be determined simultaneously. According to the reviewed studies, the integrated approach to production scheduling, batch delivery, and order acceptance has not been implemented yet. One of the objectives of this study is to develop approaches to production scheduling based on received orders and production, which this study will investigate these concepts in the form of a mathematical model. Considering the previous studies on scheduling and order acceptance problem (Silva et al., 2018), adding the batch delivery problem to the above study, adding the order acceptance problem to the scheduling and batch delivery problem (Yin et al., 2013), providing an integrated model for investigating the scheduling and batch delivery problem simultaneously (Ahmadizar & Farhadi, 2015); adding the order acceptance approach, exploring the benefits of an integrated approach to the problem by using exact and meta-heuristic solution methods, and problem

analysis and achieving useful managerial outcomes are of the innovations of this research.

So the central questions of this research are:

- How is production planning of received orders?
- Which batch of orders should be accepted and produced according to the profitability of the system?
- How is batch delivery to customers?
- Which algorithm performs well on the proposed mathematical model?
- What are the study's fundamental and strategic parameters, and what is the mathematical model's sensitivity?

The innovations of the paper are as follows:

- Considering that the problem of scheduling and order acceptance in the production planning concept
- Considering the failure to address the issue of order selection for production, adding the issue of order acceptance to the problem of scheduling and batch delivery
- Customizing Genetic algorithm for solving the proposed mathematical model

The paper includes the following issues: In the first and second sections, the introduction and model formulation are presented, respectively. Solving method of the model, in Section 3, and the results and research findings are presented in Section 5. Finally, the conclusion is presented in Section 8.

### ***A. Research literature***

Browne et al. (1999) were among the first to introduce sequence-dependent setup time. Coleman (1992) presented a simple model to minimize the sum of the earliness and tardiness costs of a single-machine problem with setup time constraints. Its objective was a linear function of the sum of the earliness and tardiness costs, and it was assumed that all jobs were ready at the same time (Tavakoli Moghaddam et al., 2012). He also presented computational results for problems with more than eight jobs.

Ayough and Khorshidvand (2019) presented a model for implementing a cellular manufacturing system. Their main objective was minimizing the costs regarding a limited number of cells. Considering dynamic production times and uncertainty demands in designing cells were their main contributions. The quality of the two algorithms has been compared. The Simulated Annealing (SA) and Particle Swarm Optimization (PSO) algorithms have been used to solve their problem.

Ayough et al. (2020) presented a new job rotation scheduling and line-cell conversion problems. They investigated the effect of rotation frequency on flow time of a Seru system. Invasive weed optimization (IWO) has been used to solve their problem. Presenting improved IWO equipped with shake enforcement was their main contribution. The results show nonlinear behavior of flow time versus number of rotation periods. Also ability of presented method to generate clusters of equivalent solutions was shown.

Tirkolaee et al. (2020) presented a new production planning bi-objective model in manufacturing organization. Their main purposes were simultaneously minimizing the total cost of the production system and total energy consumption. The  $\epsilon$ -constraint has been used to solve the proposed mathematical model exactly. The interactive fuzzy solution technique and a self-adaptive artificial fish swarm algorithm (SAAFSA) have been used to solve their problem. The results demonstrate the high efficiency of the proposed method in comparison with CPLEX solver in different problem instances.

Yang and Xu (2020) presented the flow shop scheduling problem with flexible assembly and batch delivery. Research objectives include minimizing assembly costs and delays and design batch allocation strategies. Three methods, including a variable neighborhood descent (VND) algorithm, and two greedy algorithms, are proposed to solve the problem. The results indicate the proper performance of the proposed model after the solution.

Li et al. (2020) presented a real-time order acceptance and scheduling problem in metal additive manufacturing. In this research, the manufacturer decides with multiple machines about accepting orders and scheduling machines. Their main goal is to maximize factory profits during the planning period. The contributions considered in this study are capacity and due date constraints. Finally, the proposed model is solved with a metaheuristic approach, and the results are compared with the best and the worst approach.

Petering et al. (2019) presented the algorithm solving for cyclic inventory control problem with multiple flexible batch supply and demand processes. Their main objective is to minimize multiple batch supply processes during a cycle time. Finally, the proposed model is solved with a genetic algorithm, simulated annealing, and a random algorithm. The results showed that the genetic algorithm performed better than other methods.

Sarvestani et al. (2019) presented a model to investigate order acceptance, order scheduling, supplier selection, and due to date assignment. The primary purpose of this study is to maximize production profits and minimize transportation and holding costs. The proposed model is solved by two genetic algorithms and Variable Neighborhood Search, and the results indicate the appropriate and positive performance of the proposed model after the solution.

Chen et al. (2019) considered the Order acceptance and scheduling (OAS) in their study. They selected orders according to their capacity constraints and then complete these orders by their due dates. The main contributions considered in this study are considering carbon emission cost and the time-of-use electricity cost. This research aims to maximize the total revenue of the accepted orders and then subtract the carbon emission cost under different periods.

Jiang et al. (2017) presented an Order acceptance and scheduling model with batch delivery. Their objective is to minimize the weighted sum of the accepted orders' maximum lead time and the total cost of rejecting and delivering orders. They presented two approximation algorithms for the model. Their study manufacturer can reject some orders placed by the customer, process the others on parallel machines, and then deliver them to the customer in batches.

However, in our research, the objective function of the problem is to maximize profit. This profit is derived from the difference between sales revenue and costs (maintenance, setup, production, and delivery). Considering the failure to address the issue of order selection for production, adding the issue of order acceptance to scheduling and batch delivery and customizing the Genetic algorithm for solving the proposed mathematical model are our research contributions. A genetic approach to multi-objective methods is suitable due to its exact answer. Therefore, the approach presented in this study does not have the problem of selecting the desired Pareto point compared to other multi-objective studies.

Thus, the observed research gap is:

Considering that scheduling and order acceptance has been studied before, not paying attention to the batch delivery in production planning concepts is the missing link in the production chain to supply. Considering the failure to address the issue of order selection for production, adding the issue of order acceptance to scheduling and batch delivery is a research innovation. Failure to present an integrated model to address simultaneous scheduling, batch delivery, and order acceptance is a research gap. According to the previous studies, the integrated approach of production scheduling, batch delivery, and order acceptance has not been implemented so far, which this research fills the identified research gap in terms of presenting a model and solution method.

The different classifications in the literature review table are as follows:

- Complete-time the last job: The completion time of the whole project or the total production time is equal to the completion time of the last production job.
- Total time in flow: This is equal to the sum of the total time of completed jobs.
- Total weighted time in flow: This is equal to the sum of the total time of weighed completed jobs.
- Total Delay: Equals to negative deviation (delay) from the expected delivery time

- Total early: Equals to positive deviation (early) from the expected delivery time
- Maximum delays: The maximum negative deviation (delay) from the expected delivery time
- The weighted sum of delays: Equals to the sum of weighed negative deviation (delay) from the expected delivery time
- Inventory holding: This fee includes the costs incurred by the store for the storage of the goods, including the following: Insurance costs, warehouse
- fuel cost, Tax costs
- Batch Delivery: Batch Delivery is a feature that sends the commodities in timed batches rather than to whole customers at once.

Table I. Summary of empirical research literature

<i>Author</i>	<i>Year</i>	<i>complete time the last job</i>	<i>Total time inflow</i>	<i>Total weighted time inflow</i>	<i>Total Delay</i>	<i>Total early</i>	<i>Maximum delays</i>	<i>The weighted sum of delays</i>	<i>Inventory holding</i>	<i>Batch Delivery</i>
Lu et al.	2008	√								√
Tavakoli Moghaddam [19]	2012								√	√
Ahmadizar et al.	2014				√	√			√	√
Sultani	2016				√	√			√	√
Asadi	2016	√								√
Badri	2017		√			√			√	√
Glory	2017						√			√
Cramatello	2017				√	√			√	√
Xiang	2017			√				√		√
Raeisi	2017								√	√
Wang	2018	√			√					√
Silva	2018	√							√	√

## II. Model formulation

In this research, by presenting a mathematical model in Marun Petrochemical Company and the number of orders received for production, order selection is performed and proportional to the order received. Production planning is done to maximize the organization's profit. In this study, designing the simultaneous scheduling, order acceptance, and

delivery model considering the capacity and sequence-dependent setup constraints and batch delivery approach is discussed, which this model is based on Mixed Integer Linear Programming (*MILP*).

### **A. Problem Assumptions**

- Orders are received from customers (each order can include several goods that are produced).
- Each order can be sent directly to the customer after the production or after all the customer orders are completed.
- Only one order can be processed at each moment, and no order can be accepted in the decision-making system.
- The delivery of orders is done by the batch delivery approach.
- The manufacturer is obliged to deliver orders to customers by the end of the period at the latest.
- The order acceptance is considered in this study. The approach of selecting orders from all received orders is called order acceptance. The scheduling and order acceptance problem simultaneously determines which orders to produce and accept and how the scheduling and sequence of processing these orders should be
- In the short-term period of  $T$ , the manufacturer wishes to select orders from the orders received to maximize profits due to time constraints, system costs, and earnings per order.
- Work interruption is not allowed.
- Before processing each order, setup is required which its duration is sequence-dependent.
- One or more orders are delivered to the customer each time.
- System costs include the cost of production (processing), the cost of holding (the holding time is the difference between the time each order is delivered to the customer and its completion time), the cost of setup (it is considered as a multiple of setup time), cost of delivery (depends on the number of batches formed).

The following indices, parameters, and variables are used in the model to describe the above model:

### **B. Indices**

- $I$             The index of order.
- $J$             The index of the customer.
- $K$             The index of the batch.

### **C. Parameters**

- $T$             Time horizon.
- $PC_i$         The cost of producing order  $i$
- $SC_{ii'}$       The cost of setting up order  $i'$ , if order  $i$  is before it
- $HC_i$         The holding cost of order  $i$
- $R_i$          The sales revenue of order  $i$
- $P_i$          The processing time of order  $i$
- $DC_j$         The cost of each delivery to customer  $j$
- $O_{ij}$         Equals to one if the order  $i$  belongs to customer  $j$ ; otherwise, it is zero
- $ST_{ii'}$       The time of setting up order  $i'$ , if order  $i$  is before it

**D. Variables**

- $s_i$             The start time of order  $i$
- $c_i$             The completion time of order  $i$
- $del_i$          The delivery time of order  $i$
- $c_k$             The completion time of batch  $k$
- $\alpha_j$          The number of batches delivered to customer  $j$
- $x_i$             Equals to one if the order  $i$  is accepted; otherwise, it is zero
- $b_{ik}$           Equals to one if the order  $i$  is allocated to batch  $k$ ; otherwise, it is zero
- $z_{jk}$           Equals to one if the batch  $k$  is allocated to customer  $j$ ; otherwise, it is zero
- $y_{ii'}$          Equals to one if the order  $i'$  is processed immediately after the order  $i$ ; otherwise, it is zero

The mathematical model of the problem is presented in the following:

$$Max Z = \sum_i (R_i - PC_i) x_i - \sum_i \sum_{i'} SC_{ii'} y_{ii'} - \sum_i (del_i - c_i) HC_i - \sum_j DC_j \alpha_j \tag{1}$$

**Subject to:**

$$c_i \leq T x_i \qquad \forall i \tag{2}$$

$$c_i \geq s_i + P_i \qquad \forall i \tag{3}$$

$$s_{i'} \geq c_i + ST_{ii'} - M(1 - y_{ii'}) \qquad \forall i, \forall i' \tag{4}$$

$$\sum_i \sum_{i'} y_{ii'} = \sum_i x_i - 1 \tag{5}$$

$$\sum_{i'} y_{i'i} \leq x_i \qquad \forall i \tag{6}$$

$$\sum_{i'} y_{ii'} \leq x_i \qquad \forall i \tag{7}$$

$$\sum_k b_{ik} = x_i \qquad \forall i \tag{8}$$

$$\sum_j z_{jk} = 1 \qquad \forall k \tag{9}$$

$$\sum_i b_{ik} O_{ij} \leq M z_{jk} \quad \forall j, \forall k \quad (10)$$

$$\sum_i b_{ik} O_{ij} \geq -M z_{jk} \quad \forall j, \forall k \quad (11)$$

$$c_k \geq c_i - M(1 - b_{ik}) \quad \forall i, \forall k \quad (12)$$

$$del_i \geq c_k - M(1 - b_{ik}) \quad \forall i, \forall k \quad (13)$$

$$\alpha_j = \sum_k z_{jk} \quad \forall j \quad (14)$$

$$\begin{aligned} c_i, \quad s_i, \quad c_k \geq 0 \\ x_i, \quad b_{ik}, \quad z_{jk}, \quad y_{i'j} = 0, 1 \\ \alpha_j = Integer \end{aligned} \quad \forall i, \forall j, \forall k, \forall i' \quad (15)$$

Equation (1) shows the objective function of the problem that is to maximize profit. This profit is derived from the difference between sales revenue and costs (holding, setup, production, and delivery).

Equation (2) shows that if order  $i$  is accepted ( $x_i = 1$ ), the order completion time must be before the end of the time horizon  $T$ . If order  $i$  is not accepted ( $x_i = 0$ ), the completion time has no meaning and is considered zero.

Equation (3) shows that if the order  $i$  is accepted, the time to complete the order  $i$  is equal to the start time plus its processing time, and if the order  $i$  is not accepted ( $x_i = 0$ ) the start time is equal to the end time and is assumed to be zero, the same as the previous constraint.

Equation (5) states that the number of times the setup is performed is equal to the number of orders received except for the first job for which the setup is not performed.

Equation (6) indicates that if order  $i$  is accepted ( $x_i = 1$ ), it certainly has a place in the production sequence, and if the order  $i$  is not accepted ( $x_i = 0$ ), it has no place in the production sequence.

Equation (7) indicates that if order  $i'$  is accepted ( $x_{i'} = 1$ ), it certainly has a place in the production sequence, and if the order  $i'$  is not accepted ( $x_{i'} = 0$ ), it has no place in the production sequence.

Equation (8) shows that if order  $i$  is accepted ( $x_i = 1$ ), it must be allocated to a batch such as  $k$ , and if order  $i$  is not accepted ( $x_i = 0$ ), it has no place in any delivery batch.

Equation (9) shows that each batch such as  $k$  can only contain one customer's orders, such as  $j$ .

Equations (10 and 11) state that only if batch  $k$  belongs to customer  $j$ , the orders such as  $i$  belonging to customer  $j$  can be allocated to batch  $k$ , and vice versa if orders such as  $i$  belonging to customer  $j$  are in batch  $k$ , then batch  $k$  will belong to customer  $j$ .

Equation (12) shows that the completion time of batch  $k$  is at least equal to the time of completion of the last job inside the batch (provided that batch  $k$  contains order  $i$ ).

Equation (13) shows that the delivery time of order  $i$  is at least equal to the completion time of batch  $k$ . (provided that the batch  $k$  contains the order  $i$ ).

Equation (14) represents the total number of batches delivered to customer  $j$ .

Equation (15) shows the types of variables.

### ***E. Verification of the mathematical model***

The concept of validation is more general than the concept of verification. Validation means that the results obtained after the system run correspond to the results that the system was designed to achieve (does the good work). Nevertheless, verification means the system's complete correspondence to the system's provided description (does the work rightly). After developing the mathematical model of the organization's production status under study, it is necessary to evaluate the mathematical model's output using the company's actual process to verify the performance of the mathematical model. Therefore, first, the normality of the studied data and simulation data is evaluated, and then, a parametric or non-parametric test is done (if the data is standard, the parametric test will be used, and if it is not normal, non-parametric tests will be used).

Marun Petrochemical Company was established in 1998 and was put into operation in 2006, and transferred to the private sector in 2008. Marun Petrochemical Company, having an ethane recovery plant, uses ethane to recycle ethane from natural gas to produce chemical and polymer products. The company is one of the largest petrochemical companies in Iran and the world, with an annual production capacity of 1,100,000 tons of ethylene, 200,000 tons of propylene, 300,000 tons of heavy polyethylene, 300,000 tons of polypropylene, and more than 443,000 tons of glycols.

The following process is implemented to verify the mathematical model:

- Gathering actual data from the company under study
- Running the Kolmogorov-Smirnov (K-S) test to show the normality or non-normality of the data
- Doing hypothesis test to compare the mean and variance of the studied data and the simulation model

A small-scale problem is solved by GAMS software to test the verification and validation of the proposed model (since the mathematical model is programmed in *LP* mode, so this software provides appropriate analyzes of model sensitivities). After the mathematical model is solved by GAMS software, which is operations research software, the proposed model is investigated with a numerical solution example (based on information presented in the paper of Silva (2018) and random data). Therefore, for solving the proposed model, GAMS software edition 22 and CPLEX solver will be used. Then, using meta-heuristic algorithms to solve the mathematical model, the accuracy of the mathematical model's performance will be evaluated, and mathematical model parameters will be modified to evaluate the numbers' sensitivity on the right side of the constraints. The algorithm proposed in this research is a genetic algorithm, in which parameter adjustment for the genetic algorithm will be fully illustrated. The reasons for using this algorithm are as follows:

1. It is easy to implement and does not require complicated procedures to solve problems.
2. Due to competition, responses, and selection of the best from the population will most likely reach the global optimal point.
3. Due to the simplicity of the search process, it is speedy and efficient.
4. In this algorithm, all the answers are accurate, and there are no approximations.

## **III. Solving method of the model**

### ***A. Genetic Algorithm***

In this study, to solve the model on a large scale, the genetic algorithm is used. The flowchart of the genetic algorithm, which displays an overview of how the algorithm is executed, is shown in Fig. 1.

- **Display of the chromosome**

The first step after determining the technique used to convert each solution to a chromosome is to create an initial chromosome population. At this stage, the initial solution is usually generated randomly. Here, for example, the chromosome of the variable  $z_{jk}$  is as figure 3. If batch  $k$  is allocated to customer  $j$ , then the numbers inside each gene will equal one.

- **Genetic operations**

Genetic operations imitate the inherited gene transfer process for the creation of new children in each generation. In general, this operation is performed by two major operators: mutation operator and crossover operator.

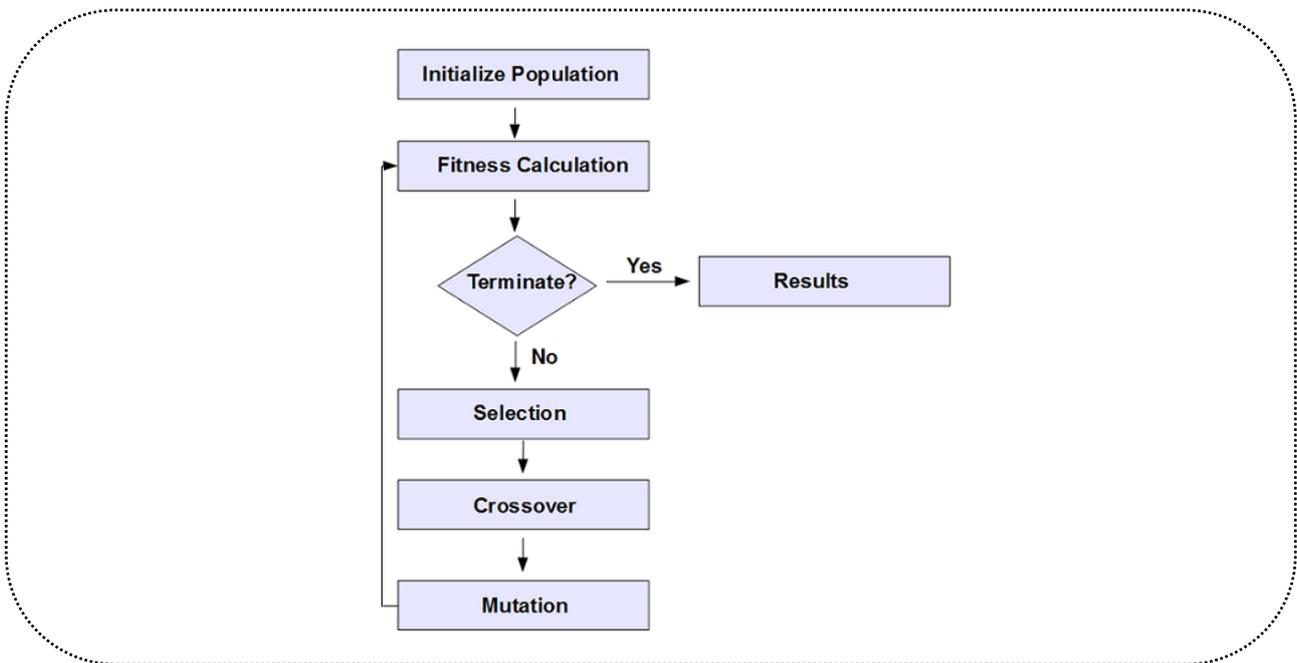


Fig. 1. The structure of the algorithm

Also, the Pseudocode is as figure 2:

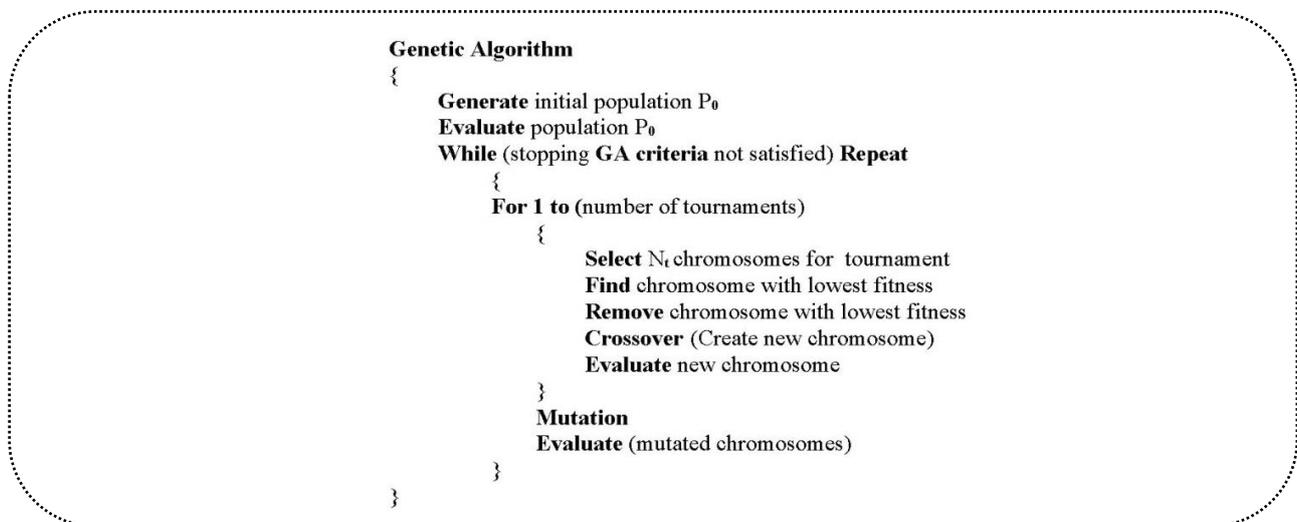


Fig. 2. Pseudocode

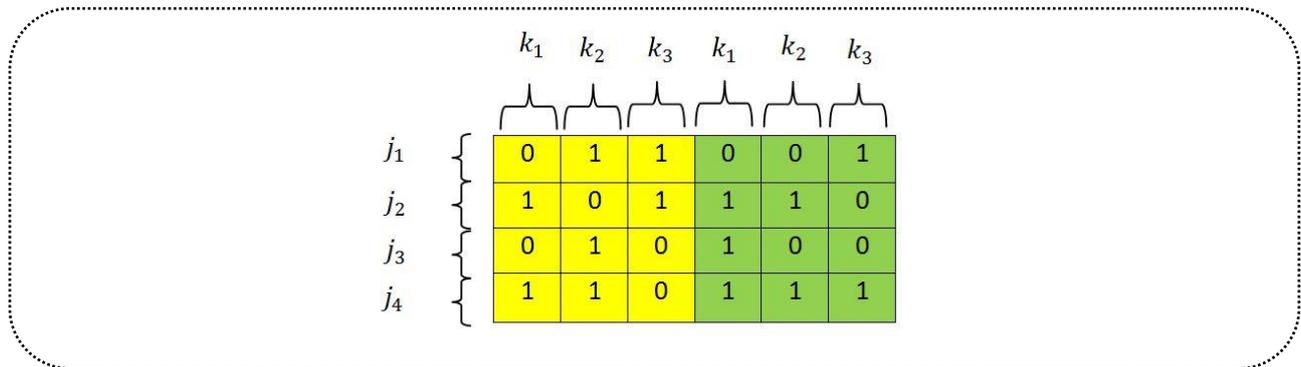


Fig. 3. Chromosome representation

• **Crossover operators**

They are the operators that select one or more points from two or more solutions and change their values. These operators consider a solution, exchange some of the solution locations with other solutions and create new solutions. These operators are called crossover operators. Here in Figure 4, a two-point crossover is used.

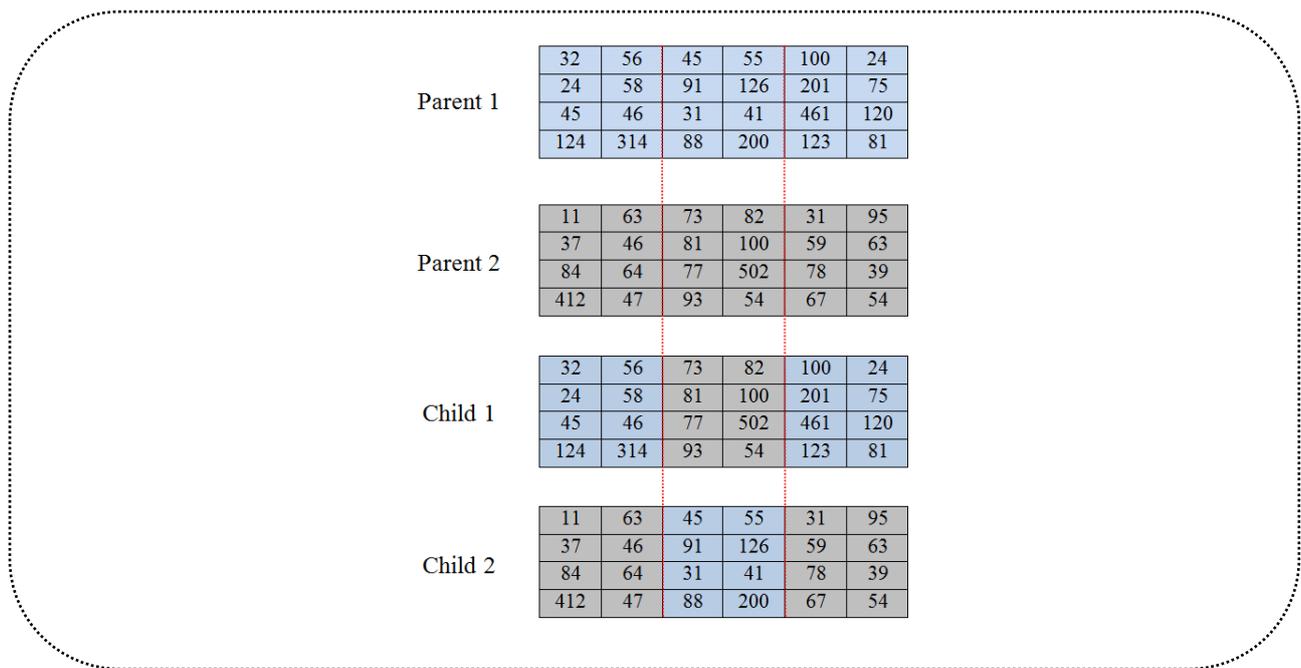


Fig. 4. Crossover operator

• **Mutation operators**

When the population converges towards a particular solution, the probability of a mutation must be increased to prevent this, and vice versa. If the population has non-identical solutions, the probability of mutation must be reduced. Here, for a mutation operator, a row is randomly selected and reversed.

In this part, the effective range is determined for each of the genetic algorithm parameters. Then, by designing multi-factor experiments using the Taguchi method, the interactive effects of the parameters are analyzed, and finally, the optimal combination is determined.

Parent	81	54	46	62	71	48
	6	61	28	95	401	100
	47	44	57	401	461	75
	100	321	32	63	123	91
Child	81	54	46	62	71	47
	100	401	95	28	61	6
	47	44	57	401	461	75
	100	321	32	63	123	91

Fig. 5. Mutation operator

One of the most critical parameters of a genetic algorithm is the size of the initial population. Small sizes do not guarantee to obtain the desired solution due to their small search range. Moreover, large sizes reduce the probability of obtaining the proper solution at a proper time. So, several tests were performed for different levels of this parameter. As shown in Figure (6), the solutions' quality increases up to size 90, and then, they lose their qualitative effect, and the solutions converge to a constant value. According to Figure (7), increasing the size of the population increases the solving time.

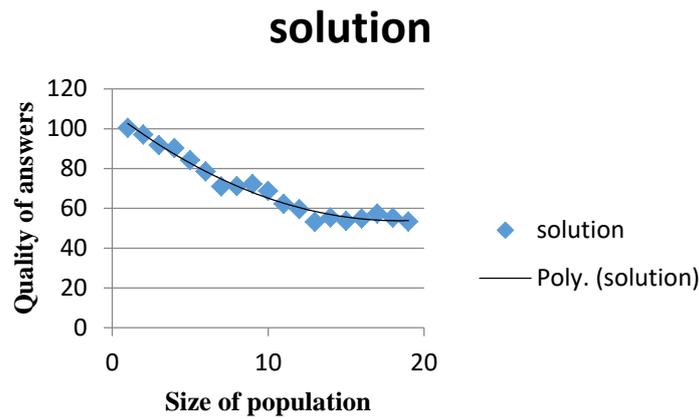


Fig. 6. The quality of answers and the size of the population.

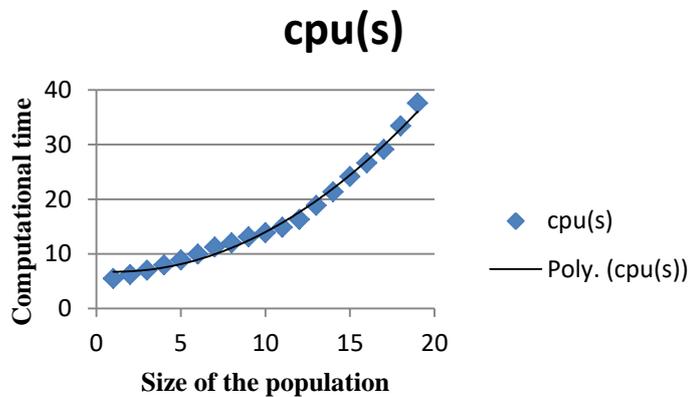


Fig. 7. Computational time and size of the population.

Tests performed to determine the crossover rate range from  $0.45$  to  $0.90$ . The results show that the solutions' quality increases with the crossover rate increase up to  $0.7$  and then converges to a constant value. As the crossover rate increases, the solving time also slightly increases.

Tests performed to determine the mutation rate are  $0.05$ ,  $0.08$ ,  $0.10$ ,  $0.15$ ,  $0.18$  and  $0.20$ , respectively. The best solutions are seen in the range of  $0.20$  to  $0.40$ .

As the number of generations increases, the solving time increases. An appropriate range of this parameter is from  $200$  to  $400$ .

The ranges obtained for the parameters are listed in Table (III).

**Table II. Control parameters of the genetic algorithm and their effective range.**

<i>Effective range</i>	<i>Parameters</i>
60-40	Pop size
0.8-0.6	Pc
0.40-0.20	Pm
400-200	iter_max

## IV. Results and research findings

### A. Evaluating the validity of the mathematical model

Twenty-three sample problems were generated to test the mathematical model's performance using an exact solution with GAMS software and CPLEX solver, which are distinguished by two parameters: number of orders ( $i$ ) and number of customers ( $j$ ).

Table (II) shows the results obtained from solving the sample problems with the exact method.

- Small problems: Problems that are solved optimally by the exact method in less than  $3600$  seconds
- Medium problems: Problems that can be solved optimally by the exact method between  $3600$  and  $4100$  seconds
- Large problems: Problems that can be solved optimally between  $4100$  and  $5000$  seconds

According to the solution of medium and large problems, it can be seen that the exact solution has lost its efficiency, and meta-heuristic algorithms should be developed to estimate the medium and large boundaries in order to increase the optimization speed while reducing the solving time. Therefore, due to the mathematical model being single-objective and introducing the heuristic genetic algorithm in evaluating the integrated mathematical model of order selection and customer delivery, this algorithm has been developed to solve the model.

As shown in Table II, the amount of gap has increased with the increasing size of problems. As can be seen, the average gap for small size problems is  $0$ . Also, the average gaps for medium and large size problems are  $0.025$  and  $0.055$ , respectively. Therefore, with increasing the problem's size, the solution time increases, while the quality of the answers decreases. It should be noted that as the gap increases, the quality of the answers decreases. Therefore, due to the low quality of GAMS answers in medium and large size and the high time of GAMS solution, we will solve the problem in large size with a genetic metaheuristic approach.

Table III. Results of solving the sample problems

<i>N</i>	<i>Problem measure</i>	<i>Order number (i)</i>	<i>Customer number (j)</i>	<i>objective function observed value</i>	<i>Time (s)</i>	<i>Gap</i>
1	Small	4	2	95	10	0
2	Small	5	2	120	10	0
3	Small	7	3	170.5	20	0
4	Small	8	3	217.5	90	0
5	Small	9	4	247	110	0
6	Small	10	4	274	115	0
7	Small	12	4	409	660	0
8	Medium	17	5	523.5	3679	0.023
9	Medium	23	6	631.75	3779	0.025
10	Medium	26	7	724.5	3863	0.024
11	Medium	31	8	846	3939	0.026
12	Medium	37	9	952	4020	0.027
13	Medium	44	10	1225	4118	0.027
14	Medium	48	11	1196.75	4171	0.025
15	Large	55	12	1373	4222	0.055
16	Large	61	13	1527	4284	0.054
17	Large	69	14	1050.5	4356	0.055
18	Large	77	15	1411.5	4433	0.056
19	Large	82	16	1477	4497	0.057
20	Large	87	17	625.5	4572	0.056
21	Large	93	18	1192	4646	0.057
22	Large	98	19	2304	4733	0.053
23	Large	100	20	781	4810	0.058

### ***B. Multi-factor parameter setting***

To achieve the best combination of parameters, each of the four factors mentioned in the previous section is tested at three levels by the Taguchi method and the mean of the solutions and mean of the computational times are considered solution levels. The factors and levels of the genetic algorithm are shown in Table (IV). The data generated by the MINITAB 16 software is analyzed, and the results are presented in the following tables.

Table IV. Factors and their levels.

<i>Levels</i>	<i>Parameters</i>
40,50,60	Pop size
0.6,0.7,0.8	Pc
0.20,0.30,0.40	Pm
200,300,400	iter_max

Table V shows the analysis of variance for signal-to-noise ratios obtained by the MINITAB 16 software. As the table shows, the degree of freedom is considered 26. Also, the Seq SS is obtained 8.8500. As it is known, at a certain level of the confidence interval, the p-value is less than 0.05, and as a result, the results of the analysis of variance can be trusted.

Table V. Analysis of variance for signal-to-noise ratios.

<i>Source</i>	<i>DF</i>	<i>Seq SS</i>	<i>Adj SS</i>	<i>Adj MS</i>	<i>F</i>	<i>P</i>
Iter_max	2	3.0368	3.0368	1.51840	15.81	0.000
Pm	2	0.2773	0.2773	0.13866	1.44	0.262
Pc	2	1.6059	1.6059	0.80293	8.36	0.003
Pop size	2	2.2017	2.2017	1.10084	11.47	0.001
Residual Error	18	1.7283	1.7283	0.09602		
Total	26	8.8500				

Table VI. Estimated correlation coefficients of the model for the mean of the answers.

<i>Term</i>	<i>Coef</i>	<i>SE Coef</i>	<i>T</i>	<i>P</i>
Cnst.	27.1176	0.1841	147.314	0.000
Iter_max 70	0.3027	0.2603	1.163	0.260
Iter_max 85	1.0905	0.2603	4.189	0.001
Pm 0.1	0.2726-	0.2603	1.047-	0.309
Pm 0.15	0.4650	0.2603	1.786	0.091
Pc 0.5	0.9021-	0.2603	3.465-	0.003
Pc 0.6	0.9427	0.2603	3.621	0.002
Pop size 70	0.3600-	0.2603	1.383-	0.184
Pop size 80	1.2428	0.2603	4.774	0.000

Table (VI) shows the correlation coefficient of the mean of the answers obtained by MATLAB 2016 software. In these tables, the coefficients with more considerable absolute value are more critical. As shown in the table, at 95% confidence level, the factors of iter\_max 85, Pc 0.6, and Pop size 80 have a significant effect on solutions. Also, for the

factors, analysis of variance was performed for SN coefficients, the solutions means, and the results are listed in Tables (VI) and (VII), indicating the significant effect of three factors of iter\_max, Pc, and Pop size on the solutions at 95% level.

Table (VII) indicates the significant effect of three factors of iter\_max, Pc, the Pop size on the answers at 95% level obtained by MATLAB 2016 software. As shown in the table, at 95% confidence level, the factors of iter\_max with the degree of freedom 2, Pc with the degree of freedom 2, Pop size with the degree of freedom 2 have a significant effect on answers.

**Table VII. Analysis of variance for the mean of the answers.**

<i>Source</i>	<i>DF</i>	<i>Seq SS</i>	<i>Adj SS</i>	<i>Adj MS</i>	<i>F</i>	<i>P</i>
Iter_max	2	28.997	28.997	14.4983	15.85	0.000
Pm	2	2.948	2.948	1.4740	1.61	0.227
Pc	2	15.338	15.338	7.6688	8.38	0.003
Pop size	2	22.080	22.080	11.0399	12.07	0.000
Residual Error	18	16.468	16.468	0.9149		
Total	26	85.831				

Tables (VIII) and (IX) show a ranking of the factors according to the solution's analysis relative to SN coefficients and the mean. As shown in the table, at 95% confidence level, the factors of iter\_max with degree of rank 1, Pc with rank 3, Pop size with rank 2 and Pm with rank 4 have a significant effect on answers.

**Table VIII. SN answer.**

<i>Level</i>	<i>Iter_max</i>	<i>Pm</i>	<i>pc</i>	<i>Pop size</i>
1	28.75	28.75	28.35	28.53
2	28.99	28.79	28.95	29.04
3	28.19	28.58	28.63	28.37
Delta	0.80	0.22	0.60	0.67
Rank	1	4	3	2

**Table IX. The mean of the answers.**

<i>Level</i>	<i>Iter_max</i>	<i>Pm</i>	<i>pc</i>	<i>Pop size</i>
1	27.42	26.84	26.22	26.76
2	28.21	27.58	28.06	28.36
3	25.72	26.93	27.08	26.23
Delta	2.48	0.74	1.84	2.13
Rank	1	4	3	2

Table (IX) shows the factors' ranking according to the analysis of the answers mean. As shown in the table, at 95% confidence level, the factors of iter\_max with the degree of rank 1, Pc with rank 3, Pop size with rank two, and Pm with rank 4 have a significant effect on answers.

According to the tests, Table X shows the parameters of the genetic algorithm.

**Table X. The adjusted values of genetic parameters.**

<i>Levels</i>	<i>Parameters</i>
50	Pop size
0.80	Pc
0.3	Pm
300	iter_max

### **C. Comparing the results of GAMS and genetic algorithm**

MATLAB software has been used to design the genetic meta-heuristic method. Each problem was run at random ten times. We present the computational results of the selected problems on a larger scale. Since GAMS cannot solve the model on a larger scale, we used the proposed algorithm to solve the model. The purpose of this test is to determine the performance of the proposed algorithm under different conditions.

Table XI shows the symbols used in the model. As it shows, GAMS objective function value symbol is  $f_{opt}$ , the genetic algorithm objective function best symbol is  $f_{best}$  and finally, the genetic algorithm objective function average symbol is  $f_{avr}$ .

**Table XI. Symbols used in the model.**

<i>Time required</i>	<i>t(s)</i>
GAMS objective function value	$f_{opt}$
Genetic algorithm objective function best	$f_{best}$
Genetic algorithm objective function average	$f_{avr}$

The model was solved for the larger scale using the proposed algorithm. Given the values in Table (XII), the algorithm has reached a near-optimal solution in a reasonable time. The proposed algorithm for solving small-scale problems requires less computational time than GAMS optimization software. However, by increasing the problem's scale, the proposed algorithms' computational time is significantly less than GAMS.

Therefore, it was shown in these examples that the algorithms could achieve an acceptable solution for large-scale problems in much less time than GAMS.

### **D. Sensitivity analysis of model key parameters**

Due to the importance of setting the model's basic parameters, the input parameters' sensitivity to the model needs to be examined, and appropriate policies to improve the scheduling should be carried out. The revenue and cost parameters directly affect the total profit of the system such that the reduction of 1 unit of costs in the objective function reduces the total cost to the same amount. Therefore, in this section, we perform sensitivity analysis on the parameters that do

not have affect the objective function directly and have always been an issue for the organization:

**A) Sensitivity analysis of the scheduling horizon:**

According to Figure 8, the planning horizon has no significant effect on the objective function of the organization's total profit, and if the time of the planning horizon is reduced by 25%, 1% of the organization's profit is reduced.

**B) Effect on revenue:**

As shown in Figure 9, changes in the planning period do not affect the organization's revenue. The planning horizon's effect on costs of holding, setup, production, and delivery are shown in Figure 10.

**Table XII. Comparing the results of GAMS and genetic algorithm.**

Row	GAMS		GA Algorithm		
	Fopt	t(s)	fbest	Favr	t(s)
1	95	10	97.85	106.704	2.5
2	120	10	129.6	126.072	2.6
3	170.5	20	187.55	184.3446	3.2
4	217.5	90	243.6	241.947	3.8
5	247	110	261.82	282.1728	4.3
6	274	115	306.88	307.428	5.6
7	409	660	429.45	438.039	5.4
8	523.5	3679	565.39	576.6876	6.9
9	631.75	3779	650.7025	716.1518	9.3
10	724.5	3863	746.235	805.4991	14.3
11	846	3939	922.14	957.8412	16.4
12	952	4020	1047.2	1019.7824	24.1
13	1225	4118	1372	1336.965	46
14	1196.75	4171	1268.555	1331.2647	57.4
15	1373	4222	1427.92	1583.8928	68
16	1527	4284	1710.24	1651.6032	71.7
17	1050.5	4356	1113.53	1125.0855	194
18	1411.5	4433	1496.19	1541.0757	186
19	1477	4497	1595.16	1612.5886	417
20	625.5	4572	656.775	676.47825	647
21	1192	4646	1227.76	1376.0448	732
22	2304	4733	2465.28	2659.7376	745
23	781	4810	812.24	884.873	946

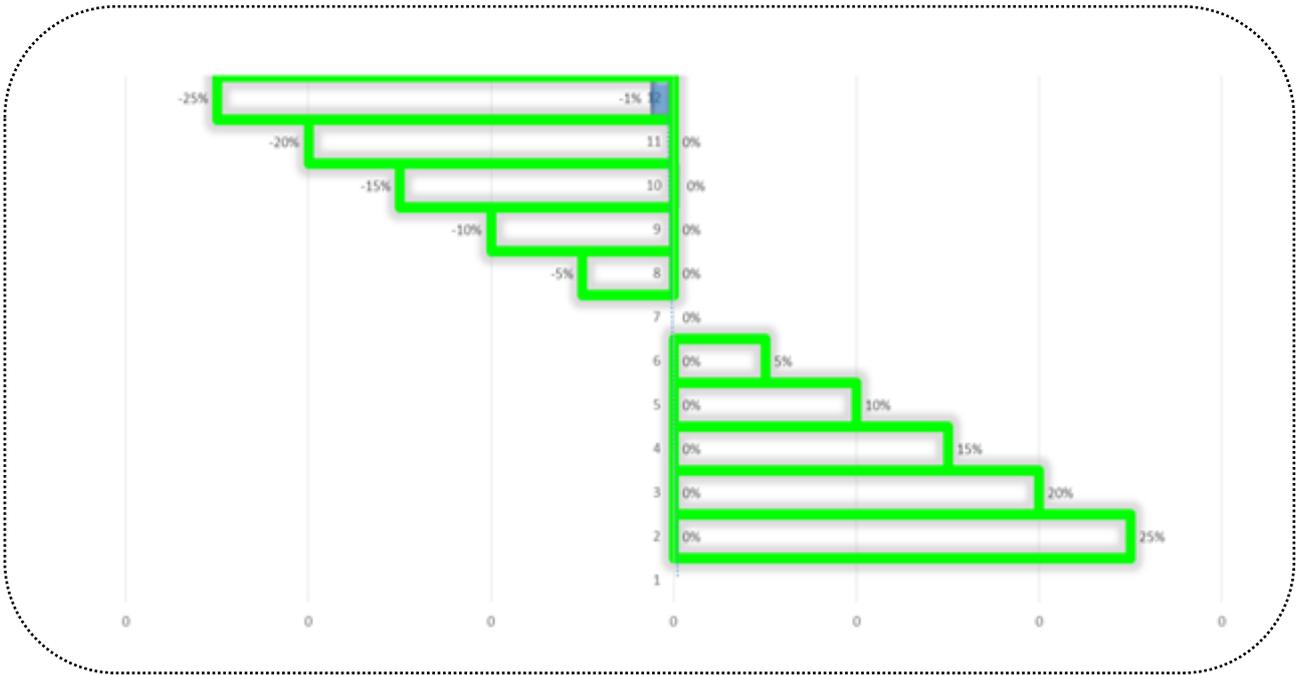


Fig. 8. Sensitivity analysis of the planning horizon.

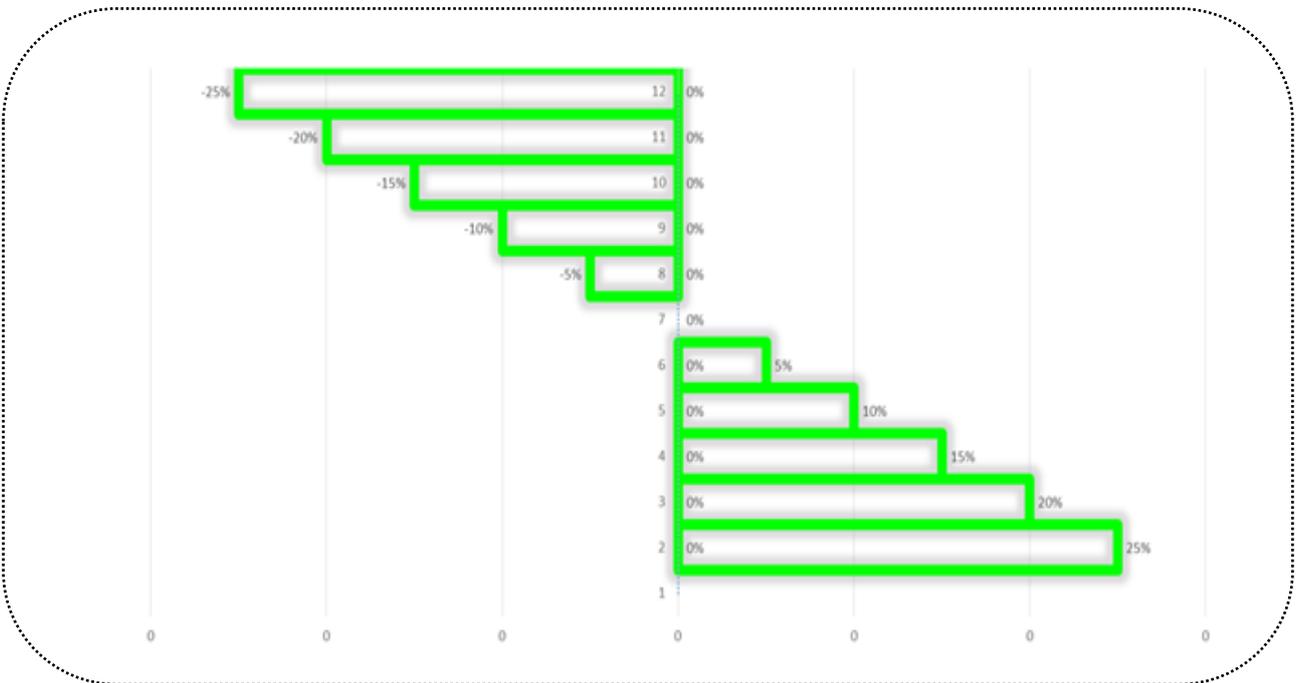


Fig. 9. Sensitivity analysis of the planning horizon on the organization's revenue.

According to evaluating the three costs of holding, setup, production, and ordering, the planning horizon changes only affected the production and ordering.

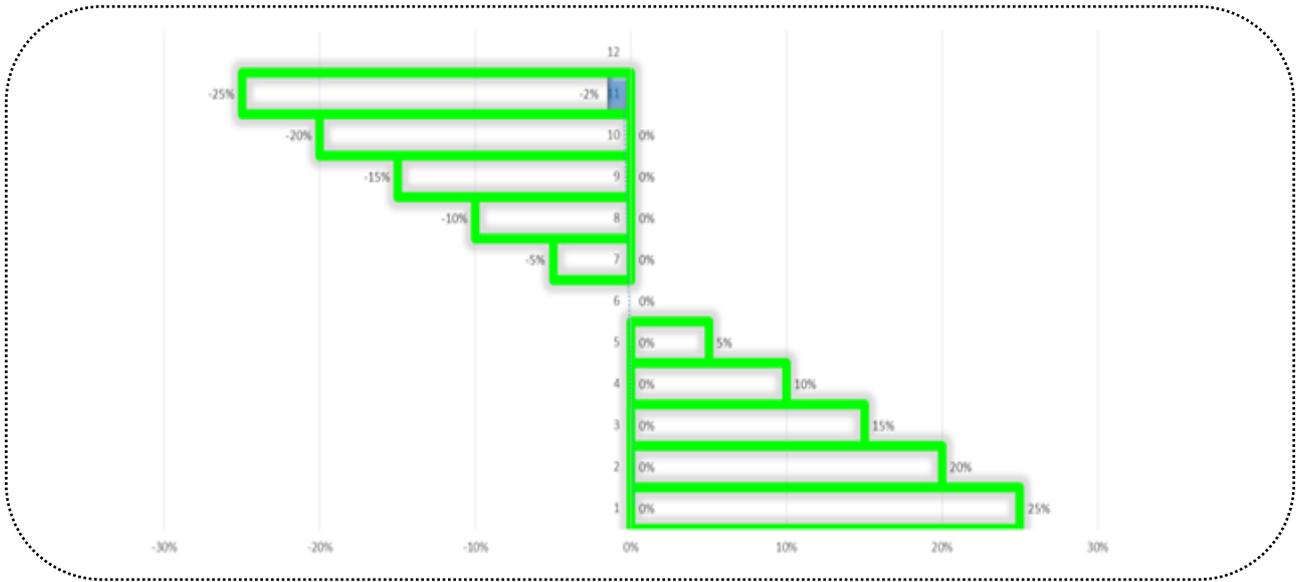


Fig. 10. Sensitivity analysis of the planning horizon on production and delivery.

**C) Sensitivity analysis of order processing time:**

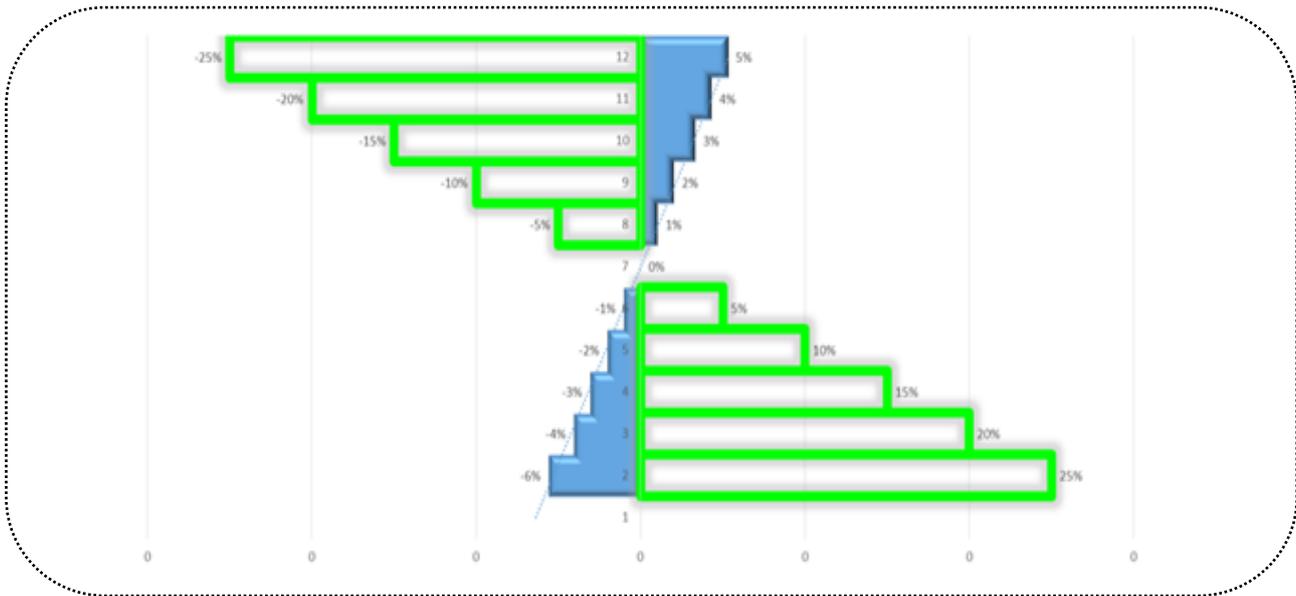


Fig. 11. Sensitivity analysis of order processing time.

Figure 11 reveals that the higher the order processing time, the lower the system profit will be, and the lower the processing time, the higher the system profit will be.

**V. Conclusions and Suggestions**

After introducing the problem of simultaneously determining order acceptance, scheduling, batch delivery, presenting the mathematical model, and developing exact and meta-heuristic solutions for 23 problems, the observed results are as follows: Since the exact solution's efficiency has been noticeably reduced in the larger scale. Therefore, simultaneous determination of order acceptance, scheduling and batch delivery is an NP-Hard problem, and a meta-

heuristic algorithm has been developed to investigate the more comprehensive solutions. The simultaneous determination of order acceptance, scheduling, and batch delivery can only be solved optimally for a small scale due to having integers and zero and one variables. The simultaneous approach reduces the cost and increases the profitability of the system compared to solving problems separately.

Therefore, to evaluate the problem under study, after modeling the problem, the mathematical model performance was evaluated using small data and then using the simulation approach of orders received to generate and solve the model under study, the validation of the proposed model was evaluated, and it was observed that the results of the simulation and the mathematical model were consistent and validated. To investigate the larger scale of the model under study, a comparative evaluation was carried out to validate 23 generated problems by the exact solution in GAMS software and genetic algorithm in MATLAB software. As observed in this evaluation, it was found that the meta-heuristic solution has a small deviation from the exact solution in small-scale problems, and the mathematical model is solved in a proper time by the meta-heuristic algorithm. As the problem's size increases, the exact solution loses its efficiency in terms of time, and the application of the exact solution algorithm for solving the model is inadequate. The genetic solution has reached the optimal solution properly with reasonable deviation, and this algorithm can replace the exact solution properly. In this study, the problem of simultaneous determination of order acceptance, scheduling, and batch delivery was presented considering the capacity and sequence-dependent setup constraints. Given the newness of this problem, it provides researchers with a new horizon for future studies. So, the result of a combination of three problems of order acceptance is scheduling and batch delivery. Due to this new problem's complexity, some of the researchers' assumptions and conditions in a separate investigation of these three problems were ignored in this study, which is the basis for defining new studies in this field. Some of the research contexts in this field are introduced in the following: In this study, it was assumed that if customers demand a product, at least their minimum demand should be met. The problem can be investigated if the production system can supply or not supply the demand, and shortages in the system are allowed. The problem can be considered under uncertainty for parameters such as demand values, processing times, availability of machinery, cost of delivery of each batch, setup times.

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