Journal of Quality Engineering and Production Optimization

Vol. 4, No. 2, Summer & Autumn 2019

http://jqepo.shahed.ac.ir

DOI: 10.22070/JQEPO.2020.5087.1123

A forecasting system by considering product reliability, POQ policy, and periodic demand

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Abstract *– This paper presents an economic production quantity (EPQ) model with a periodic order quantity (POQ) policy, product reliability and periodic demand. The machine reliability has decreased over time; therefore, the rates of perfect and defective products reduce and increase over time, respectively. A fixed percentage of these products are reworked while the rest is wasted. Some equipment in their early days operates with excellent efficiency, but its performance deteriorates over time; therefore, operating costs increase. It is assumed the machine is inspected and repaired at the end of each production cycle; then, the machine returns to its original state. The demand for the final products is discrete, periodic and constant for each period. We use the POQ policy to meet customers' demand. In POQ procedures, orders for replenishment occur at fixed intervals. A Mixed Integer Non-Linear Program (MINLP) model is suggested. Then, a computational experiment is presented to discuss the optimally of the profit function. By analyzing the data, it is found that under different conditions, the manufacturer can use either Single Setup Single Period (SSSP) or Single Setup Multi-Period (SSMP) policy.*

Keywords *– Periodic order quantity (POQ), Reliability, Defective product, Inventory production system.*

I. INTRODUCTION

The EPQ model has some assumptions which sometimes are not applicable in the real system. In some real systems, the quality of the final product is dependent on the technology, quality of raw material and labor. The quality of raw material directly affects perfect products. The low quality of raw material decreases the number of perfect products in each period.

The EPQ model was developed by (Harris, 1913). One of the assumptions of EPQ models is that the annual demand is continuous and constant. However, in the real system, the demand may be periodic, dynamic or uncertain. A number of researches have studied the inventory systems with multi-period demands (Wagner & Whitin, 1958) discuss the multi-period lot sizing problem in which the demand is discrete and periodic. Sajadi et al. (2009) consider the single item dynamic lot size model without allowing the backlogging. They improved Wagner Whihin's solution method. Sakaguchi (2009) considers a multi-period inventory model with varying continuous demand. Heibatolah Sadeghi

Manuscript Received:13-July-2019 & Revised: 26-Aug-2019 & Accepted: 19-Dec-2019
ISSN: 2423-3781

(2019) studies economic production quantity with a periodic demand. He used a proposed mathematical model and heuristic method to achieve the optimal solution, but he did not consider the periodic order policy (POQ) for EPQ models. He assumed that the production rate is constant and operation and maintenance costs are negligible.

One of new interesting researches in the economic production models is the reliability of products. In classical EPQ model, it is assumed that all produced items are of perfect quality, but at real systems the reliability of a product is dependent to some items such as raw materials, technology being used and manpower. Low quality of raw material reduces the reliability of products. Perfect manpower increases the number of items in control. The major literature in this area is described as follows.

Tripathy (2003) considered the economic order quantity model with defective products while taking into account the reliability of the production process. The defective products are not reworkable and thus are discarded. Also, it is assumed that the reliability is inversely related to the demand rate and shortage is not allowed. Sana (2010) used a product reliability and imperfect production process for an EPQ model with reworkable defective items. He also assumed that the shortage is not allowed, and time horizon is infinite. Sarkar (2011) considered a single-product EPQ model in which the demand depends on time, and the production rate depends on time and reliability. They also used a reworkable defective item. Sarkar and Sarkar (2013) developed an EPQ model with stochastic deterioration rate of product, where the demand is dependent on the time and time horizon is finite. The raw material cost is directly related to the reliability. Manna et al. (2017) employed an economic production model with demand dependence the advertisement and production rate dependent defective rate. They modeled the problem in an infinite time horizon and assumed that the production rate is variable because of the quality of raw material, labor experience, etc. Khara et al. (2017) utilized an EPQ model with an imperfect production process. They assumed that the demand is dependent on the reliability and selling price. They developed a mathematical model considering the reworking the out of control items. Among others, inventory models of supply planning items with dependent demand system. Cunha et al. (2018) examined the EPQ inventory model with partial backordering and discount for defective items. The defective items are not reworkable, and it is assumed that the planning horizon is finite. Tsao et al. (2019) developed economic production quantity (EPQ) models considering the imperfect items, the reworking of defective products and predictive maintenance.

Shah and Naik (2020) considered the inventory production model dealing with the product's sell price-stock as well as reliability-dependent demand; also, they assumed that the production process is imperfect and during the production time some of products are perfect quality and the imperfect products undergo the reworking process owing a cost to become a perfect product.

Pasandideh et al. (2013) developed a multi-product economic production model considering the defective and reworkable items and limited orders. They supposed that a fixed percent of the production items is defective with a possibility to be reworked and the demand is constant. A mathematical model is suggested, and a meta-heuristic algorithm is used to achieve the optimal solution. Gouiaa-Mtibaa et al. (2018) employed an integrated maintenance-quality policy for the manufacturing system assuming that the random failure rate is increased over the time and propose reworking activities for defective items.

Polotski et al. (2019) analyzed the corrective maintenance in a failure-prone manufacturing system under a variable demand. They assumed the demand is varying periodically over time. This is often the case for repeated variations in seasonal demands. The production rate in this paper is constant, and backordering is not allowed.

Periodic Order Quantity (POQ) is an order policy which can be used all over the supply chain levels. This policy is so useful when the demand is periodic. By applying this policy, orders for replenishment occur at fixed intervals. In other words, the periodic order quantity is a standard number of units to be ordered over a fixed period of time. (Haibatolah Sadeghi, Makui, & Heydari, 2013) considered a multi-stage production system that lead time at each stage is a random variable. They suggested an MRP approach using a Periodic Order Quantity (POQ) policy. The analyzed results of this paper show that the fixed setup cost is low and the result of POQ policy and Lot-For-Lot is the same, but by increasing the fixed setup time, the POQ policy provides a better result. Haibatolah Sadeghi et al. (2014; 2016) considered a multi-period and multi-level supply planning model with lead-time uncertainties based on the POQ policy. They proposed a mathematical model using the MRP approach.

Louly and Dolgui (2013); Louly et al. (2012) utilized the Material Requirement Planning (MRP) under uncertainties. They used the MRP approach and Periodic Order Quantity (EOQ) policy in the proposed model. Ben-Ammar et al. (2019) proposed a single-stage and multi-period supply planning under stochastic lead times and a dynamic demand. They used the Lot-For-Lot policy to model the purposed problem. Also, a genetic algorithm (GA) is developed to find the optimal solution and minimize the expected cost.

This paper uses an inventory control system with the POQ policy to find the optimal replenishment. It is assumed that the quality of the product depends on the raw materials, the type of technology and the manpower skill. The higher the quality of the raw material used, the lower the number of imperfect products. Nonetheless, the cost of higher quality raw material will be more expensive. Moreover, the more skilled the manpower, the human error is reduced, and as a result, the imperfect products will be reduced although the skilled manpower wages will be higher. Using equipment increases their operating costs over time. This paper assumes that at the end of each production cycle, the machine will be inspected and the necessary repairs are carried out on the machine; thus, the machine is restored to its initial state. Corrective maintenance reduces the operating cost. Because of the reduced reliability of the product over time, the imperfect product increases over time. A fixed percentage of these products are reworked, and the rest is wasted.

The objective of the proposed system is to optimally determine how many items should be produced in each cycle. According to the knowledge, there is no inventory model in the related literature addressing the item reliability, discrete demand, and POQ policy, simultaneously, in an EPQ system. The shortage for final product is not allowed and the planning horizon is considered infinite. The contributions of the paper are briefly described as follows:

a) Formulating a production-inventory system through the POQ policy and the reliability of items,

b) Proposing an optimal framework which addresses the discrete and periodic demand, and

c) The operation cost increased by the time

The rest of the paper is organized as follows. Definition of the problem is provided in Section 2. In Section 3, the mathematical model of the production system with the POQ policy and the periodic demand are formulated. Section 4 examines the effectiveness of the model through numerical experimentation. And, finally, Section 5 provides conclusions and opportunities for future research.

II. PROBLEM DEFINITION

This paper considers a single-machine production system with the reliability of the product and POQ order policy. The demand for final products is periodic and constant. The rate of the perfect product is decreased over the production system because the machine performance deteriorates over time as do the efficiency of machines, technology, the skill of labor and quality of raw materials. The better quality of raw material decreases the imperfect product. A fixed percentage (α) of imperfect items is reworkable, and the rest is sold at a lower price less than the unit production cost of a product.

Fig. (1) shows the inventory level of the system. The manufacturer has used a POQ policy to fulfill his request. By applying this policy, the manufacturer meets the need for the next k periods. In addition to the unit production cost denoted by $\Pr(t,r)$, dependent on the raw material quality, skill of labor and manufacturing system and operation cost. The operating cost increases when the machine is used. To reduce the operating cost of the machine at the end of each production cycle, it is inspected and, according to the corrective repair policy, the defects are repaired and the machine returns to its original condition. The operating cost is denoted by $C(r,t)$, where r is the reliability of the final product. The manufacturer used the push system; therefore, the manufacturer forecasts the demand for each cycle based on the previous data. Since the demand variation is very low, the manufacturer decides to use the push system to meet customer needs. The other assumptions and notations are as follows.

- ***Assumptions and notations***

The mathematical model of the inventory production system here is based on the following assumptions:

a) Lead time is zero.

b) Shortages are not allowed.

c) Demand for the final product is discrete and periodic.

d) The manufacturer employs a POQ policy.

e) The reliability of the machine decreases over time.

f) The imperfect units can be rejected, reworked, refunded or sold at a lower price.

g) The operating cost increases over time.

- ***Notations***

λ	The reliability parameter of the system
$1-r$	Present of imperfect items
η	Production Rate
D	Demand rate for each period
$I(t)$	On hand inventory level at time t
α	Percentage of defective products that can be reworkable
ϕ	Rework cost per unit product
h	Unit holding cost
T	Inventory cycle length
$R.M(r)$	The raw material cost
$P.C(r)$	The production cost
υ	Unit selling price for perfect items

υ_1	Unit selling price for defective items
$\Psi(K)$	Total annual profit
c(t)	The operating cost per unit time at time t after replacement.

Independent design Variable

K	Periodic order quantity (decision variable)

Dependent Design Variable

Q	The Production quantity for each cycle
t_p	Production cycle length
t_d	The time between two sequence demand

III. MATHEMATICAL MODEL

This paper considers the production system with the reliability of the product and a POQ policy. The reliability of the production system decreases over the production system. Therefore, to prevent the degradation in reliability, we consider a development cost. The development cost is increased over time because the machine maintenance costs increase. The demand for the final product is periodic and constant. The order policy adopted is a periodic order quantity policy. In the POQ policy, the manufacturer produces the final product to meet the needs of next periods acquiring customer requirements.

This paper assumes that the production machine is inspected and repaired at the end of each production cycle and then returns to its original state. When new, some equipment operates with excellent efficiency, but as it ages its performance deteriorates, increasing the machine operating costs over time. Fig. (1) shows the inventory level of the manufacturer versus time.

At this system, when the manufacture starts to produce products, a certain fraction ($1-r$) present of items is defective because of the efficiency of machines, technology, skill of labor and quality of raw materials. A fixed percentage (α) of the defective item can be reworkable, and the rest is sold at a lower price less than the unit production cost of a product. The reliability decreases over time. We used $e^{-\lambda\times t}$ to show the reliability rate. The production rate is η, the in-control production rate is $\eta\times e^{-\lambda\times t}$ and the rate of out-of-control production is $\eta\times(1-e^{-\lambda\times t})$, where λ is defined as follows:

$$\lambda=\frac{\text{number of defective items}}{\text{Total number of produced items within a time interval}}$$

The reliability of the product is dependent on raw material, manufacturing system and skill of labor. So raw material with better quality is increased the reliability rate of production system; therefore raw material is dependent on the reliability. This paper considers the raw material cost for increasing function of the reliability.

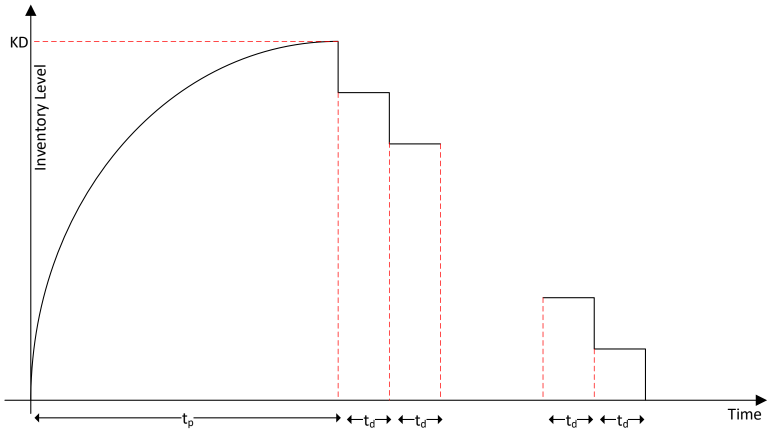

Fig. 1: Inventory level of the manufacturer vs. time

$$MC(r) = R_1 + R_2 \times e^{\lambda \times R_3} \tag{1}$$

where R_1, R_2 and R_3 are fixed parameters independent of the time.

The operating cost increases over time and depends on the reliability of the machine.

$$C(r,t) = C_1 - C_2 \times e^{-\lambda \times t} \tag{2}$$

The production cost includes raw material, operation and manpower cost. Therefore the production cost for per unit is as follows:

$$\Pr(r,t) = P_0 + \frac{MC(r)}{Q} + \frac{C(r,t)}{Q} = P_0 + \frac{R_1 + R_2 \times e^{\lambda \times R_3}}{Q} + \frac{C_1 - C_2 \times e^{-\lambda \times t}}{Q} \tag{3}$$

where P_0 is a time-independent fixed parameter.

The differential equation for this model at production time is $\frac{\partial I(t)}{\partial t} = \eta \times r$ and for non-production time demand rate is discrete and constant. By solving the differential equations for this model at production time, the inventory rate with boundary conditions $I(0)$ =0 is as follows:

$$\frac{\partial I(t)}{\partial t} = \eta \times r = \eta \times e^{-\lambda \times t} \xrightarrow{I(0)=0} I(t) = \frac{\eta}{\lambda}(1 - e^{-\lambda \times t}) \tag{4}$$

The manufacturer should meet the need of K periods, so concerning the boundary conditions $I(t_p) = K \times D$, the production time is found as follows,

$$I(t_P) = \frac{\eta}{\lambda}(1 - e^{-\lambda \times t_P}) = K \times D \Rightarrow \frac{\eta}{\lambda}(1 - e^{-\lambda \times t_1}) = K \times D$$

$$t_P = -\frac{1}{\lambda} Ln\left(1 - \frac{K \times D \times \lambda}{\eta}\right) \tag{5}$$

In order to derive the profit function, the system costs and revenue earned by sales should be accounted.

a) Holding cost

The holding cost for production time is given as follows:

$$TC_1 = h \times \int_{t=0}^{t_p} \frac{\eta}{\lambda}(1 - e^{-\lambda \times t})dt = h \times \left(\frac{\eta}{\lambda} \times \left(t_p + \frac{1}{\lambda} \times e^{-\lambda \times t_p} - \frac{1}{\lambda}\right)\right) \tag{6}$$

The holding cost of the non- production time is given by

$$TC_2 = h \times \left((K-1) \times D \times t_d + (K-2) \times D \times t_d + ... + D \times t_d\right) = h \times \frac{D \times t_d \times K \times (K-1)}{2} \tag{7}$$

Then by summarizing Eqs. (7)and (8), the total holding cost of each cycle is obtained:

$$TC_3 = h \times \left(\frac{\eta}{\lambda} \times \left(t_p + \frac{1}{\lambda} \times e^{-\lambda \times t_p} - \frac{1}{\lambda}\right)\right) + \frac{K(K-1)}{2} \times D \times t_d \times h \tag{8}$$

b) Production cost

The production cost per unit of the product is calculated using Eq.(3). The production time at each cycle is t_p, therefore, the total production cost is given by Eq. (9).

$$TC_4 = \eta \times \int_0^{t_p} \left(P_0 + \frac{R_1 + R_2 \times e^{\lambda \times R_3}}{k \times D} + \frac{C_1 - C_2 \times e^{-\lambda \times t}}{k \times D}\right)dt = \eta\left(P_0 + \frac{R_1 + R_1 + R_2 \times e^{\lambda \times R_3}}{k \times D}\right)t_p + \frac{C_2 \times \left(e^{-\lambda \times t_p} - 1\right)}{\lambda \times k \times D} \tag{9}$$

c) Reworking cost

To rework the unit imperfect product, the manufacture pays ϕ unit of cost. The imperfect product is produced by rate $(1 - e^{-\lambda \times t})$ at the time and α is the percent of its reworkable product; therefore the total reworkable cost per cycle time is as follows:

$$TC_5 = \phi \times \eta \times \int_0^{t_p} \alpha \times (1 - e^{-\lambda \times t})dt = \phi \times \eta \times \alpha \times \left(t_p + \frac{1}{\lambda}(e^{-\lambda \times t_p} - 1)\right) \tag{10}$$

d) Revenue sale of perfect products

The selling price of unit perfect product is υ, the perfect product is produced at each cycle by a rate of $\eta \times e^{-\lambda \times t}$ and α is the percent of the reworkable imperfect product; therefore the revenue of the perfect product at each cycle is as follows:

$$\upsilon\times\left(\int_0^{t_p}\eta\times e^{-\lambda\times t})dt+\int_0^{t_p}\alpha\times\eta\times(1-e^{-\lambda\times t})dt\right)=\upsilon\times\frac{\eta}{\lambda}\left(1-e^{-\lambda\times t_p}\right)+\upsilon\times\alpha\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right) \tag{11}$$

e) Revenue sale of imperfect products

$\left(1-\alpha\right)$ percent of imperfect product is not reworkable; therefore it will be sold by a salvage value price, the unit price of which is υ_1.

$$\upsilon_1\times\left(\int_0^{t_p}\left(1-\alpha\right)\times\eta\times(1-e^{-\lambda\times t})dt\right)=\upsilon_1\times\left(1-\alpha\right)\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right) \tag{12}$$

A. TOTAL PROFIT

The total profit for each cycle time is the difference between revenue sale and the system costs and given as follows:

$$\begin{aligned}Total\ \Pr ofit &=\upsilon\times\frac{\eta}{\lambda}\left(1-e^{-\lambda\times t_p}\right)+\upsilon\times\alpha\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right)\\ &+\upsilon_1\times\left(1-\alpha\right)\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right)-A-h\times\left(\frac{\eta}{\lambda}\times\left(t_p+\frac{1}{\lambda}\times e^{-\lambda\times t_p}-\frac{1}{\lambda}\right)\right)\\ &-\frac{K(K-1)}{2}\times D\times t_d\times h_0-\left(\eta\left(P_0+\frac{R_1+R_1+R_2\times e^{\lambda\times R_3}}{k\times D}\right)t_p+\frac{C_2\times\left(e^{-\lambda\times t_p}-1\right)}{\lambda\times k\times D}\right)\\ &-\phi\times\eta\times\alpha\times\left(t_p+\frac{1}{\lambda}(e^{-\lambda\times t_p}-1)\right)\end{aligned} \tag{13}$$

The annual demand is $\frac{D}{t_d}$. Therefore, the total profit is as follows:

$$\Psi(K)=\frac{1}{T}\times Total\ \Pr ofit=$$

$$\frac{1}{t_d\times K}\left(\begin{aligned}&\upsilon\times\frac{\eta}{\lambda}\left(1-e^{-\lambda\times t_p}\right)+\upsilon\times\alpha\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right)+\upsilon_1\times\left(1-\alpha\right)\times\eta\left(t_p+\frac{1}{\lambda}\left(e^{-\lambda\times t_p}-1\right)\right)-A\\ &-h\times\left(\frac{\eta}{\lambda}\times\left(t_p+\frac{1}{\lambda}\times e^{-\lambda\times t_p}-\frac{1}{\lambda}\right)\right)-\frac{K(K-1)}{2}\times D\times t_d\times h-\eta\left(P_0+\frac{R_1+C_1+R_2\times e^{\lambda\times R_3}}{k\times D}\right)t_p\\ &-\frac{1}{\lambda}\times\frac{C_2\times\left(e^{-\lambda\times t_p}-1\right)}{k\times D}-\phi\times\eta\times\alpha\times\left(t_p+\frac{1}{\lambda}(e^{-\lambda\times t_p}-1)\right)\end{aligned}\right) \tag{14}$$

By replacing the production cycle length (t_p) in Eq. (14), the total profit is modified as follows:

$$\Psi(K)=\left(\frac{Dh}{2}+\frac{D\upsilon}{t_d}+\frac{Dh}{t_d\times\lambda}-\frac{D\alpha\upsilon}{t_d}+\frac{D\alpha\phi}{t_d}-\frac{D\upsilon_1}{t_d}+\frac{D\alpha\upsilon_1}{t_d}\right)-\frac{DhK}{2}-\frac{A}{Kt_d}+\frac{C_2}{Kt_d\eta}$$
$$+\frac{1}{K}\times\left(\frac{h\eta}{t_d\lambda^2}+\frac{P_0\eta}{t_d\lambda}-\frac{\alpha\eta\upsilon}{t_d\lambda}+\frac{\alpha\eta\phi}{t_d\lambda}-\frac{\eta\times\upsilon_1}{t_d\lambda}+\frac{\alpha\eta\times\upsilon_1}{t_d\lambda}\right)\times Ln[1-\frac{DK\lambda}{\eta}] \tag{15}$$
$$+\frac{1}{K^2}\times\left(\frac{\eta\times C_1+}{DK^2t_d\lambda}+\frac{\eta\times R_1}{DK^2t_d\lambda}+\frac{\eta\times R_2\times e^{\lambda\times R_3}}{DK^2t_d\lambda}\right)\times Ln[1-\frac{DK\lambda}{\eta}]$$

Therefore, the total profit will be summarized as follows:

$$Max\ \Psi(K)=B_1-\frac{DhK}{2}-\frac{A}{Kt_d}+\frac{C_2}{Kt_d\eta}+\frac{B_2}{K}\times Ln[1-\frac{DK\lambda}{\eta}]+\frac{B_3}{K^2}\times Ln[1-\frac{DK\lambda}{\eta}]$$

$St.$

$$B_1=\left(\frac{Dh}{2}+\frac{D\upsilon}{t_d}+\frac{Dh}{t_d\times\lambda}-\frac{D\alpha\upsilon}{t_d}+\frac{D\alpha\phi}{t_d}-\frac{D\upsilon_1}{t_d}+\frac{D\alpha\upsilon_1}{t_d}\right)$$
$$B_2=\left(\frac{h\eta}{t_d\lambda^2}+\frac{P_0\eta}{t_d\lambda}-\frac{\alpha\eta\upsilon}{t_d\lambda}+\frac{\alpha\eta\phi}{t_d\lambda}-\frac{\eta\times\upsilon_1}{t_d\lambda}+\frac{\alpha\eta\times\upsilon_1}{t_d\lambda}\right) \tag{16}$$
$$B_3=\left(\frac{\eta\times C_1+}{DK^2t_d\lambda}+\frac{\eta\times R_1}{DK^2t_d\lambda}+\frac{\eta\times R_2\times e^{\lambda\times R_3}}{DK^2t_d\lambda}\right)$$

Although the objective function can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find the optimal solution in explicit form. Thus, the model will be solved approximately by using a truncated Taylor series expansion for the exponential term, i.e

$$Ln[1-\frac{DK\lambda}{\eta}]\approx-\frac{D\lambda K}{\eta}-\frac{(D^2\lambda^2)K^2}{2\eta^2} \tag{17}$$

With the above approximation, the annual net profit function can be rewritten as

$$\Psi(K)=B_1-\frac{DhK}{2}-\frac{A}{K\times t_d}+\frac{C_2}{K\times t_d\eta}+\frac{B_2}{K}\times\left(-\frac{D\lambda K}{\eta}-\frac{(D^2\lambda^2)K^2}{2\eta^2}\right)+$$
$$\frac{B_3}{K^2}\times Ln[1-\frac{DK\lambda}{\eta}]-\frac{B_4}{K^2}\times\left(\frac{e^{\lambda R_3}\eta R_2}{2Dt_d\lambda^2}\right)\times\left(\frac{D^2\lambda^2K^2}{\eta^2}\right) \tag{18}$$

B. SOLUTION ALGORITHM:

To solve the problem of primitive consideration of the constraints, the problem is solved, and its optimal solution is determined. Given the concatenation of the profit function, it can be expected that integer solutions are around the optimal point obtained in the continuous state. Therefore, based on the proposed algorithm, we try to search for those values of decision variables around the continuous optimal solution. In the following, first, the proof of the concavity of the total profit function is expressed. Then a heuristic algorithm is used to determine the discrete optimal solution.

Theorem 1: the profit function is concave.

Proof

The first and the second derivatives of $\Psi(K)$ with respect to K are:

$$\frac{\partial\Psi(K)}{\partial K}=\frac{2D\eta\lambda B_2\times t_d-D^2K^2\lambda^2B_3\times t_d+\eta(2A\eta-DhK^2\eta\times t_d-2C_2)}{2K^2\eta^2\times t_d} \tag{19}$$

and

$$\frac{\partial^2\Psi(K)}{\partial K^2}=-\frac{2(A\eta+Dt_d\lambda B_2-C_2)}{K^3\times t_d\times\eta} \tag{20}$$

If $(A\eta+Dt_d\lambda B_2-C_2)\geq 0$ then $\frac{\partial^2\Psi(K)}{\partial K^2}$ for all positive K is not positive. Therefore, $\Psi(K)$ is concave if $(A\eta+Dt_d\lambda B_2-C_2)\geq 0$.

■

the $\Psi(K)$ is a concave function of K, and there exists a unique value K, which maximizes $\Psi(K)$ as follows:

$$\frac{\partial\Psi(K)}{\partial K}=0\Rightarrow K^*=\frac{\sqrt{2}\sqrt{A\eta^2+Dt_d\eta\lambda B_2-\eta C_2}}{\sqrt{Dht_d\eta^2+D^2t_d\lambda^2B_3}}=\frac{\sqrt{2\eta}\times\sqrt{A\eta+Dt_d\lambda B_2-C_2}}{\sqrt{Dht_d\eta^2+D^2t_d\lambda^2B_3}} \tag{21}$$

K^* is a positive value ($K^*\in R^+$), therefore $\left(A\eta+Dt_d\lambda B_2-C_2\right)$ is a positive.

Note that the periodic order quantity (*K*) is an integer number, therefore if the optimal value of *K* achieved from $\frac{\partial\Psi(K)}{\partial K}=0$ is not integer, the optimal value of *K* is equal to $K_1^*=\left[K^*\right]$ or $K_2^*=\left[K^*\right]+1$. If $\psi(K_1^*)\geq\psi(K_2^*)$, then K_1^* is the optimal solution; otherwise, K_2^* is the optimal solution.

Based on the above properties, the following solution algorithm is developed to determine the optimal solution of the approximate model.

Solution algorithm

Step 1: Determine K^* from (21). If K^* is an integer value, go to **Step 3**, else go to **Step 2.**

Step 2: $K_1^*=\left[K^*\right]$ and $K_2^*=\left[K^*\right]+1$, and if $\Psi(K_1^*)\geq\Psi(K_2^*)$, then K_1^* is the optimal solution; and replace K^* by K_1^*; otherwise, K_2^* is the final optimal solution and replace K^* by K_2^*.

Step 3: Replace K^* in Eq. (15) and find the optimal total profit.

IV. NUMERICAL EXAMPLE

In the following, a numerical experiment has been carried out to illustrate the performance of the synchronized cycles algorithm. Assume that a producer has an annual production rate of 3500 units and the customer's weekly demand rate for the manufacturer products is 40 units.

It is assumed that the price function of raw material is dependent on the reliability which set of $100+50\times e^{(10\times\lambda)}$ \$ and that the operation cost is an increasing function over time in the form of $200-40\times t\times e^{(-t\times\lambda)}$. Therefore, the unit production cost equals $50+\frac{100+50\times e^{(10\times\lambda)}}{Q}+\frac{200-40\times t\times e^{(-t\times\lambda)}}{Q}$ \$ per unit product. Other parameters are set as follows:

- Selling price per unit of a perfect and nonreworkable products equals \$350 and \$30, respectively.
- 80% of the defective product is reworked to convert into perfect.
- The cost per unit reworkable product is \$40.
- The setup cost is \$1000.

If the reliability parameter of the system is 0.05, the unit holding cost is \$50. The optimal production quantity should be found to achieve the maximum profit.

Solution

Step 1: Determine K^* from (21).

$$K^*=\frac{\sqrt{2}\sqrt{A\eta^2+Dt_d\eta\lambda B_2-\eta C_2}}{\sqrt{Dht_d\eta^2+D^2t_d\lambda^2B_3}}=6.9922$$

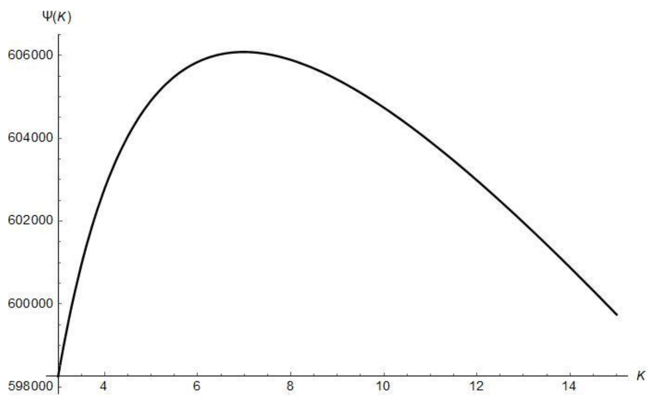

Fig. 2: Relationship between the total profit and the periodic order quantity

Step 2:

$K_1^* = \lfloor K^* \rfloor = 6$, $\Psi(K_1^*) = 605842.0184$ and $K_2^* = \lfloor K^* \rfloor + 1 = 7$, $\Psi(K_2^*) = 605891.7114$

$\Psi(K_2^*) \geq \Psi(K_1^*)$ then K_2^* is the optimal solution;

Fig. (2) shows the relationship between the total profit and the periodic order quantity. It is clear that the profit function is concave for the proposed example.

A. ANALYSIS OF RESULTS

This section studies the effects of changes in the system parameters including reliability parameter (λ), setup cost(A) and unit holding cost(h) on optimal values of K^* and the optimal total profit $\psi(K^*)$. The sensitivity analysis is performed by changing each of the parameters by +75%, +50%, +25%, +10%, −10%, −25%, −50%, and else, taking one parameter at a time and keeping the remaining parameters unchanged. The results based on the proposed example are shown in Tables I-III.

As shown in Table I, by increasing the reliability parameter (λ), the optimal periodic order quantity (K) and the optimal order quantity are increased. When the value of the reliability parameter is very small, it means that it needs better quality of the raw material and more skilled manpower; therefore lower reliability parameter imposes a higher cost on the system. On the other hand, the lower reliability reduces the number of defective products and increases perfect products.

It is shown that the system revenues and costs vary for different reliability parameter values. The question is what kind of raw material and technology should be used for manufacturing products?

Fig. (3) shows the relationship between % Change Reliability Parameter and the total profit. As it is illustrated in this figure, as the reliability parameter value increases, the earnings trend increases initially and then decreases from one value to the next. This is due to the balance between the reliability costs imposed by raw materials and technology as well as the number of defective products.

The results obtained from the proposed model with respect to the amount of reliability parameter are summarized in Table I.

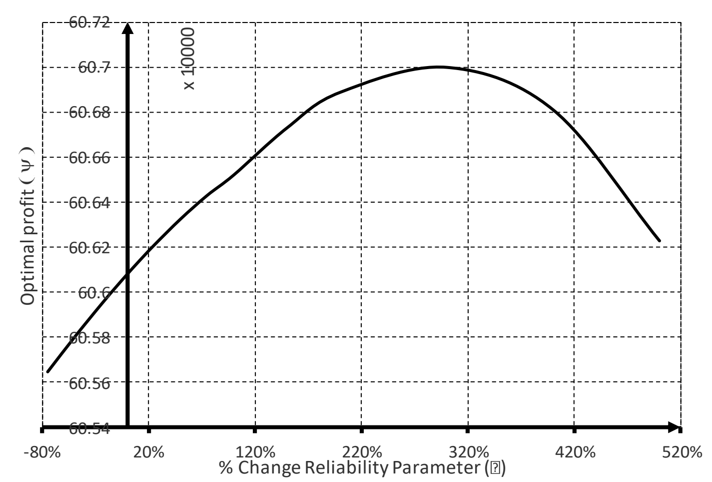

Fig. 3: Relationship between % Change Reliability Parameter and the total profit

TABLE I: SENSITIVITY ANALYSIS OF THE RELIABILITY PARAMETER OF THE SYSTEM.

Parameter	*% Change*	*Value*	*Periodic order quantity (K)*	*Optimal order quantity (K×D)*	*Production cycle length (t_p)*	*Inventory cycle length (T)*	*Optimal profit (Ψ)*
reliability parameter (λ)	−75%	0.0125	7	280	0.0800	0.2143	605646.0013
	−50%	0.025	7	280	0.0800	0.2143	605798.8019
	−25%,	0.0375	7	280	0.0801	0.2144	605944.3317
	−10%,	0.045	7	280	0.0801	0.2144	606027.7453
	00%,	0.05	7	280	0.0801	0.2144	606081.5817
	+10%,	0.055	7	280	0.0802	0.2144	606133.9121
	+25%,	0.0625	7	280	0.0802	0.2144	606209.4079
	+50%,	0.075	7	280	0.0802	0.2145	606326.5133
	+75%	0.0875	7	280	0.0802	0.2145	606431.4271
	+100%	0.1	7	280	0.0803	0.2146	606522.4821
	+150%	0.125	8	320	0.09196	0.2454	606731.9069
	+200%	0.15	8	320	0.0921	0.2455	606890.1374
	+300%	0.2	9	360	0.1039	0.2765	607001.0888
	+400%	0.25	10	400	0.11595	0.3077	606812.0886
	+500%	0.3	12	480	0.1400	0.3702	606228.984

The sensitivity of results to the reliability parameter has been tested to provide more confidence in the model. The result of this model is dependent on the setup cost and unit holding cost. The results obtained from the proposed model with respect to the amount of Unit Holding Cost are summarized in Table II.

Table II: SENSITIVITY ANALYSIS OF UNIT HOLDING COST

Parameter	*% Change*	*Value*	*Periodic order quantity (K)*	*Optimal order quantity (K×D)*	*Production cycle length (t_p)*	*Inventory cycle length (T)*	*Optimal profit (Ψ)*
Unit Holding Cost (h)	−75%	12.5	16	640	0.1837	0.4905	617012.0424
	−50%	25	10	400	0.1146	0.3064	612229.5424
	−25%,	37.5	8	320	0.0916	0.2451	608837.1929
	−10%,	45	8	280	0.0802	0.2144	607659.0003
	00%,	50	7	280	0.0801	0.2144	606081.5817
	+10%,	55	7	280	0.0802	0.2144	605063.3231
	+25%,	62.5	6	240	0.0687	0.1838	603696.0925
	+50%,	75	6	240	0.0687	0.1838	601550.1665
	+75%	87.5	5	200	0.0572	0.1531	599676.6213
	+100%	100	5	200	0.0572	0.1531	597930.301398
	+150%	125	4	160	0.0458	0.1225	594706.4187
	+200%	150	4	160	0.0458	0.1225	592012.76273
	+300%	200	3	120	0.0343	0.0918	586886.6408

When the unit holding cost is low, the manufacturer will favor producing for more periods to avoid paying the high holding cost. However, as the cost increases, the tendency to produce for more periods also decreases. Fig. (4) illustrates the relationship between % Change Unit Holding Cost and the total profit. As the figure shows, by increasing the unit holding cost, total profit will decrease.

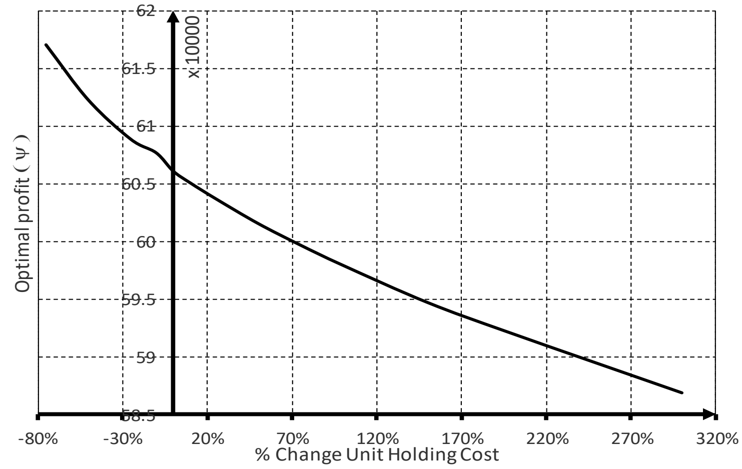

Fig. 4: Relationship between % Change Unit Holding Cost and the total profit

The impact of a fixed startup cost on the total profit is similar to the effect of unit holding cost on the total profit. In this case, by increasing the amount of fixed setup cost, the total profit decreases. This is a logical result because the increase in the order quantity and fixed setup cost results in a lower unit setup cost. Fig. (5) illustrates the relationship between % Change Fixed Setup Cost and the total profit. As the figure shows, by increasing the setup cost, the total profit will decrease.

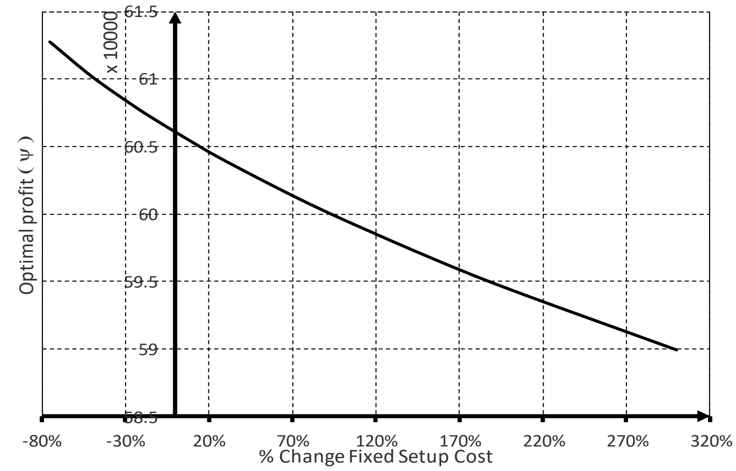

Fig. 5: Relationship between % Change Fixed Setup Cost and the total profit

It is clear from the results that if the fixed setup cost is much higher than the unit holding cost, the system tends to produce for several periods, which we called it a single-setup multi-period (SSMP). If this ratio is low, the system tends to make for one single period, called single-setup single-period (SSSP). In other words, as this ratio increases, we should produce for more periods in each cycle. In Table II and Table III, the proposed example is examined for different scenarios of startup to unit holding cost.

When the A/h ratio is low, the manufacturer decides to separately produce for individual periods due to the low fixed setup cost and high unit holding cost. However, if the ratio is high, the manufacturer decides to produce the need for several periods at one setup time.

TABLE III: SENSITIVITY ANALYSIS OF UNIT HOLDING COST

Parameter	*% Change*	*Value*	*Periodic order quantity (K)*	*Optimal order quantity (K×D)*	*Production cycle length (t_p)*	*Inventory cycle length (T)*	*Optimal profit (Ψ)*
Fixed Setup Cost (A)	−75%	250	5	200	0.0572	0.1531	612737.0094
	−50%	500	6	240	0.0687	0.1838	610187.2566
	−25%,	750	6	240	0.0687	0.1838	608014.6375
	−10%,	900	7	280	0.0802	0.2144	606826.4797
	00%,	1000	7	280	0.0801	0.2144	606081.5817
	+10%,	1100	7	280	0.0801	0.2144	605336.6838
	+25%,	1250	8	320	0.0916	0.2451	604262.2471
	+50%,	1500	8	320	0.0916	0.2451	602632.7828
	+75%	1750	9	360	0.1031	0.2757	601070.1919
	+100%	2000	9	360	0.1031	0.2757	599621.7792
	+150%	2500	10	400	0.1146	0.3064	596917.1223
	+200%	3000	11	440	0.1261	0.3371	594435.16274
	+300%	4000	12	480	0.1376	0.3678	589947.5152

V. CONCLUSION

This paper considers an inventory production system with a (POQ) order policy, reliability, operation cost, and periodic demand. The production cost and the rate of imperfect items are dependent on the reliability of the products. The reliability of the product depends on raw material, manufacturing system, skill of labor and operation cost. Thus, raw material with better quality increases the reliability rate of the production system. The production rate of the product with perfect quality reduces over time, but the operation cost increases. The demand for the final product is discrete with constant units and constant times between two consecutive demands. We used the periodic order quantity to meet the customers' demand. Also, a concave concept and partial derivatives are employed to find the optimal solution. Finally, a numerical example is presented and the sensitivity of the model parameters to the validation of the model is analyzed. The results show that if the reliability parameter (λ) is low, the manufacturer uses an SSMP policy and produces the product for several periods. Yet, for an increased reliability parameter, the number of periods that should be produced by a single setup is decreased. When the reliability parameter is large the manufacturer uses an SSSP policy. We analyzed the effect of setup cost and unit holding cost on the result of the model. The result shows that if the A/h ratio is very small, the manufacturer utilizes the SSSP policy. If the ratio is large, the manufacturer applies the SSMP policy.

The proposed model can be extended in several ways. In the actual production system, the selling price is affected by the raw material price and final demand, so taking this into account can significantly improve the proposed model. We could extend the deterministic demand function to stochastic. Finally, we could generalize the proposed model to allow for shortages, random break down the machine, quantity discounts, and others.

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