Journal of Quality Engineering and Production Optimization

Vol. 4, No. 2, Summer & Autumn 2019

http://jqepo.shahed.ac.ir

DOI: 10.22070/JQEPO.2020.4499.1109

Fuzzy Double Variable Sampling Plan under Uncertainty

Robab Afshari [1*], Bahram Sadeghpour Gildeh [1], Adel Ahmadi Nadi [1]

[1] *Department of Statistics, Faculty of Mathematical Sciences , Ferdowsi University of Mashhad, Mashhad, Iran*

[*] ***Corresponding author:*** *Robab Afshari (Email: robab.afshari@mail.um.ac.ir)*

Abstract *– Double sampling plan is an examination with a certain parameter, so it cannot decide about manufactured products whose portion parameter* ($\boldsymbol{p}$) *is not certain. The main goal of this survey is to introduce double variable plan when* $\boldsymbol{p}$ *is indefinite to examine manufacturing products when concerned characteristics are normally distributed. Plan parameters are achieved by an optimization manner. Sum of fuzzy customer and producer's risks and contract's commitments are assumed as a goal function and restrictions, respectively, in this manner. Optimum values of parameters are provided to be employed in industry for variant compositions of demands. A simulation study is also conducted to represent that the presented approach becomes traditional one as* $\boldsymbol{p}$ *is not imprecise. In addition, conclusions display that the proposed method is more economical than the existing scheme. At the end, an industrial example is given in real situations.*

Keywords *– Double acceptance sampling plan, Producer risk, Consumer risk, Fuzzy numbers.*

I. INTRODUCTION

Acceptance sampling plans are usually employed to check if a submitted lot should be allowed. Acceptance sampling schemes are graded into attribute and variable. An essential preference of variable sampling scheme is that it gets equivalent operating characteristic (OC) curve using less sample information. So, it may be convenient to palliate assessment cost inexpensive conditions (Montgomery, 2012; Schilling & Neubauer, 2017).

Other procedure to group the sampling plans is derived from the type of taking a sample containing single, double, sequential, multiple, etc. Among these, variable single sampling plan (VSSP) and double variable sampling plan (DVSP) are favorites among users. Although VSSP is more common material than DVSP due to its easiness in practice, DVSP uses less average sample number (ASN) to assess the submitted lot. Also, DVSP provides a second chance to recheck the lot before refusal (Sommers, 1981). Fallahnezhad et al. (2015) designed an optimum double sampling plan with a model applied to recognize the optimum tolerance bounds and sample size. Some studies on DVSP can be seen in some related articles (Butt et al., 2019; Razmkhah et al., 2017).

Mostly, the process quality p is supposed a precise value to plan customary sampling schemes. But in practice, it may not be recorded certainly. So, it should be described based on fuzzy logic. Chakraborty (1992) investigated single plans based on fuzzy optimizing. A plan was introduced by Tong & Wang (2012) when one deals with quality characteristics measured uncertainly. Baloui Jamkhaneh et al. (2011) represented attribute fuzzy single sampling plan

Manuscript Received:11-July-2019 & Revised: 23-Sep-2019 & Accepted: 11-Dec-2019
ISSN: 2423-3781

(FSSP). A plan by applying a sequential sampling method was displayed by (Baloui Jamkhaneh & Sadeghpour Gildeh, 2013). Fuzzy multiple deferred state attribute approach was introduced by Afshari et al. (2017) as the inspection is not perfect. Also, Afshari & Sadeghpour Gildeh (2017a) provided tables of optimum parameters for fuzzy multiple deferred state plan by applying two-point strategy. Afterward, Afshari & Sadeghpour Gildeh (2018) extended fuzzy attribute multiple deferred state plan to the status in which quality characteristics are measurable, and they also proposed fuzzy variable single sampling plan (FVSSP) to compare their suggested methods. To obtain more information about fuzzy schemes refer (Kahraman et al., 2016; Divya 2012; Baloui Jamkhaneh & Sadeghpour Gildeh 2012; Turanoglue et al., 2012; Afshari et al., 2018; Afshari & Sadeghpour Gildeh, 2017b; and Venkateh & Elango, 2014).

Although classical DVSP uses small ASN to test submitted lot, it is not helpful to assess manufactured productions whose quality is not certain. Hence in this paper fuzzy DVSP (FDVSP) is studied to examine manufactured productions with normal-distributed quality characteristics. We survey the conditions as standard deviation is known or not. Besides, it is illustrated that introduced plan is an extension of existing DVSP by implementing an extensive simulation study. Further, comparisons between the introduced plan and existing FVSSP are made.

The rest of the paper is organized as follows. In a further section, we remind several statements of fuzzy sets theory. We fuzzify DVSP in section III and provide optimum plan parameters. Section IV deals with fuzzy ASN. Analysis and comparison of the new plan with the existing FVSSP are made in section V. An industrial example is demonstrated in section VI. Finally, section VII represents the conclusions.

II. CONCEPTS

Now, several notions of fuzzy sets are remembered (see Dubois & Prade, 1978).

Definition 1. Fuzzy subset $\tilde{A}$ of real line, with membership function $\mu_{\tilde{A}}: R \to [0,1]$ is a fuzzy number if (a) $\tilde{A}$ is normal, (b) $\tilde{A}$ is fuzzy convex, (c) $\mu_{\tilde{A}}$ is upper semi-continuous and (d) support($\tilde{A}$) is bounded.

Definition 2. A trapezoidal fuzzy number $\tilde{A}$ is a fuzzy number whose membership function is determined by four values, $a_1 \le a_2 \le a_3 \le a_4$ so that:

$$\mu_{\tilde{M}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x < a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 < x \le a_4 \\ 0 & a_4 < x \end{cases}$$

A trapezoidal fuzzy number $\tilde{A}$ with $a_2 = a_3$ is called triangular fuzzy number shown by $\tilde{A} = (a_1, a_2, a_4)$.

Definition 3. γ-cut of a fuzzy number $\widetilde{A}$ $(\gamma \in [0,1])$ is a non-fuzzy set defined as $\tilde{A}[\gamma] = \{x \in R, \mu_A(x) \ge \gamma\}$, so that $\tilde{A}[\gamma] = [A^L[\gamma], A^U[\gamma]]$, where $A^L[\gamma] = inf\{x \in R, \mu_{\tilde{A}}(x) \ge \gamma\}$ and $A^U[\gamma] = sup\{x \in R, \mu_{\tilde{A}}(x) \ge \gamma\}$.

Definition 4. Let $\widetilde{N}[\gamma] = [N^L[\gamma], N^U[\gamma]]$ and $\widetilde{M}[\gamma] = [M^L[\gamma], M^U[\gamma]]$ be γ-cut of two fuzzy numbers $\widetilde{N}$ and $\widetilde{M}$, respectively. In one way of ordering fuzzy numbers, $\widetilde{N} \lessapprox \widetilde{M}$ if and only if $\forall\, \gamma \in [0,1]$, $N^U[\gamma] \le M^L[\gamma]$.

III. OPERATING PROCEDURE OF FUZZY DVSP

Assume that interesting quality characteristic follows normal distribution with unknown mean μ and standard deviation σ, so that it has upper specification limit U. In Appendix A, the operating method of DVSP is recalled in customary case. Here, authors want to study DVSP in a fuzzy case when nonconforming proportion p is ambiguous. In order to be simple in computation, let p be a triangular fuzzy number $\tilde{p} = (a_1, a_2, a_3)$. $\forall\, \gamma \in [0,1]$, the γ-cut of $\tilde{p}$ is:

$$\tilde{p}[\gamma] = [a_1 + (a_2 - a_1)\gamma, a_3 - (a_3 - a_2)\gamma]. \tag{1}$$

A. Known standard deviation case

Operating procedure in known standard deviation fuzzy DVSP (KSD-FDVSP) is similar stages displayed in Appendix A. Thus, KSD-FDVSP is identified by three parameters $k_{a\sigma}$, $k_{r\sigma}$ *and* n_σ. By employing relation (A-2) and Buckley's manner (Buckley 2003; 2006), γ-cut of lot acceptance fuzzy probability is

$$\tilde{P}_{a\sigma}(\tilde{p})[\gamma] = \{P(A_1) + P(A_2) \mid \; p \in \tilde{p}[\gamma]\} = \left[P^L_{a\sigma}[\gamma], P^U_{a\sigma}[\gamma]\right] \tag{2}$$

in which, $P^L_{a\sigma}[\gamma] = \min \tilde{P}_{a\sigma}(\tilde{p})[\gamma]$ and $P^U_{a\sigma}[\gamma] = \max \tilde{P}_{a\sigma}(\tilde{p})[\gamma]$.

Example 1. Consider KSD-FDVSP with $n_\sigma = 40, k_{r\sigma} = 2.04$, $k_{a\sigma} = 2.10$ and $\tilde{p} = (0.011, 0.012, 0.013)$. According to (2), we have $\tilde{P}_{a\sigma}[0] = [0.8755, 0.9393]$, $\tilde{P}_{a\sigma}[1] = 0.9108$. That means for every 10000 submitted lots, nearly 9108 ones are accepted. Fig. 1 indicates plots of member functions of $\tilde{p}$ and $\tilde{P}_{a\sigma}$.

Example 2. Assume all hypotheses in the previous example. Authors want to draw OC curve of proposed plan that plots lot acceptance probability against proportion of defective items (Montgomery, 2012). To draw OC curve of KSD-FDVSP, $\tilde{p} = (a_1, a_2, a_3)$ is rewritten by $\tilde{p}_t$ as

$$\tilde{p}_t = (t, b_2 + t, b_3 + t), \tag{3}$$

where $b_j = a_j - a_1$ $(j = 2, 3)$ and $t \in [0, 1 - b_3]$. Then

$$\tilde{p}_t[\gamma] = [t + b_2\gamma, b_3 + t - (b_3 - b_2)\gamma]. \tag{4}$$

Let $\tilde{p} = (0.011, 0.012, 0.013)$, then by using (3) it results $\tilde{p}_t = (t, 0.001 + t, 0.002 + t)$, $t \in [0, 0.998]$. Therefore, for $\gamma \in [0,1]$, $\tilde{p}_t[\gamma] = [t + 0.001\gamma, 0.002 + t - 0.001\gamma]$. By using $\tilde{p}_t$ for $\tilde{p}$ in (2), the γ –cuts of lot acceptance fuzzy probability is computed for some values of t when $\gamma = 0$ and 1. The obtained values are given in Table I. From Table I, it results lot acceptance fuzzy probability decreases as the proportion of defective items (t or equivalently $\tilde{p}_t$) increases, such that lot acceptance probability approximately equals 1 when the process quality becomes large (or t closes to zero). Fig. 2 presents OC curves plotted by applying (2) for different values of γ (=0, 0.2. 0.6, 1). Since proportion of nonconforming goods or process quality ($\tilde{p}_t$) is vague, lot acceptance probability is imprecise. Thus, as observed in Fig. 2, the plotted OC curves appear like a band with lower and upper bounds. Then, one should name it fuzzy operating characteristic (FOC) band. From Fig. 2, we conclude that quantity of uncertainty of the proportion parameter (γ) impresses on the width of FOC band. By comparing Figs. 2(a)-(d), it is clear that the bandwidth of FOC band becomes less as γ increases. Fig. 2(d) presents that lower and upper bounds coincide together for $\gamma = 1$ (or when process quality is crisp). As a result, proposed plan results existing KSD-DVSP by reducing amount of uncertainty of process quality.

Each fuzzy sampling plan prepares at least lot acceptance probability $1-\tilde{\alpha}$ as p̃ is at $\widetilde{AQL}$ level, and it also prepares at most probability of lot acceptance $\tilde{\beta}$ when p̃ is at $\widetilde{LQL}$ ($\widetilde{AQL}$ and $\widetilde{LQL}$ are fuzzy acceptance and limiting quality levels, in order. $\tilde{\alpha}$ and $\tilde{\beta}$ are fuzzy manufacturer and customer's risks, in order (Afshari & Sadeghpour Gildeh, 2018)). Hence, for defined $\widetilde{AQL}$ and $\widetilde{LQL}$, parameters of KSD-FDVSP are detected under these conditions:

$$1-\tilde{\alpha} \le \tilde{P}_{a\sigma}\left(\widetilde{AQL}\right), \quad \tilde{P}_{a\sigma}\left(\widetilde{LQL}\right) \le \tilde{\beta}. \tag{5}$$

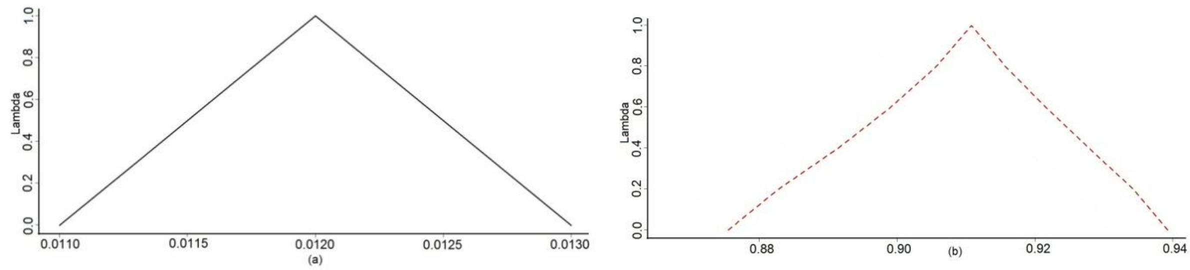

Fig. 1. Membership functions (a) $\tilde{p} = (0.011, 0.012, 0.013)$, (b) $\tilde{P}_{a\sigma}$ at $\tilde{p}$.

TABLE I. γ-CUTS OF LOT ACCEPTANCE FUZZY PROBABILITY AS γ=0, 1 FOR KSD-FDVSP

t	$\tilde{p}_t$	$\tilde{P}_{a\sigma}[0]$	$\tilde{P}_{a\sigma}[1]$
0.000	(0.000, 0.001, 0.002)	[1.0000, 1.0000]	1.0000
0.001	(0.001, 0.002, 0.003)	[1.0000, 1.0000]	1.0000
0.002	(0.002, 0.003, 0.004)	[1.0000, 1.0000]	1.0000
0.003	(0.003, 0.004, 0.005)	[0.9997, 1.0000]	0.9999
0.004	(0.004, 0.005, 0.006)	[0.9987, 0.9999]	0.9996
0.005	(0.005, 0.006, 0.007)	[0.9961, 0.9996]	0.9986
0.006	(0.006, 0.007, 0.008)	[0.9906, 0.9985]	0.9958
0.007	(0.007, 0.008, 0.009)	[0.9808, 0.9955]	0.9901
0.008	(0.008, 0.009, 0.010)	[0.9653, 0.9896]	0.9800
0.009	(0.009, 0.010, 0.011)	[0.9430, 0.9791]	0.9640
0.010	(0.010, 0.011, 0.012)	[0.9131, 0.9627]	0.9412
0.011	(0.011, 0.012, 0.013)	[0.8755, 0.9393]	0.9108
0.012	(0.012, 0.013, 0.014)	[0.8306, 0.9084]	0.8726
0.013	(0.013, 0.014, 0.015)	[0.7792, 0.8697]	0.8272
0.014	(0.014, 0.015, 0.016)	[0.7229, 0.8239]	0.7755
0.015	(0.015, 0.016, 0.017)	[0.6631, 0.7717]	0.7188
0.016	(0.016, 0.017, 0.018)	[0.6016, 0.7148]	0.6589
0.017	(0.017, 0.018, 0.019)	[0.5401, 0.6547]	0.5973
0.018	(0.018, 0.019, 0.020)	[0.4800, 0.5931]	0.5359
0.019	(0.019, 0.020, 0.021)	[0.4226, 0.5317]	0.4759

We may get a class of KSD-FDVSP contenting mentioned circumstances. Across these schemes, one chooses an individual scheme involving minimum sum of risks for given $\widetilde{AQL}$ and $L\widetilde{Q}L$. Then, optimization subject to recognize parameters is formulated as follows:

minimize $\tilde{\alpha}'_{\sigma} + \tilde{\beta}'_{\sigma}$,

subject to:

$1 - \tilde{\alpha} \leq \tilde{P}_{a\sigma}(\widetilde{AQL}), \quad \tilde{P}_{a\sigma}(L\widetilde{Q}L) \leq \tilde{\beta},$

$n_{\sigma} \geq 2, \qquad k_{a\sigma} > k_{r\sigma} > 0,$

here, $\tilde{\alpha}'_{\sigma} = 1 - \tilde{P}_{a\sigma}(\widetilde{AQL})$ and $\tilde{\beta}'_{\sigma} = \tilde{P}_{a\sigma}(L\widetilde{Q}L)$.

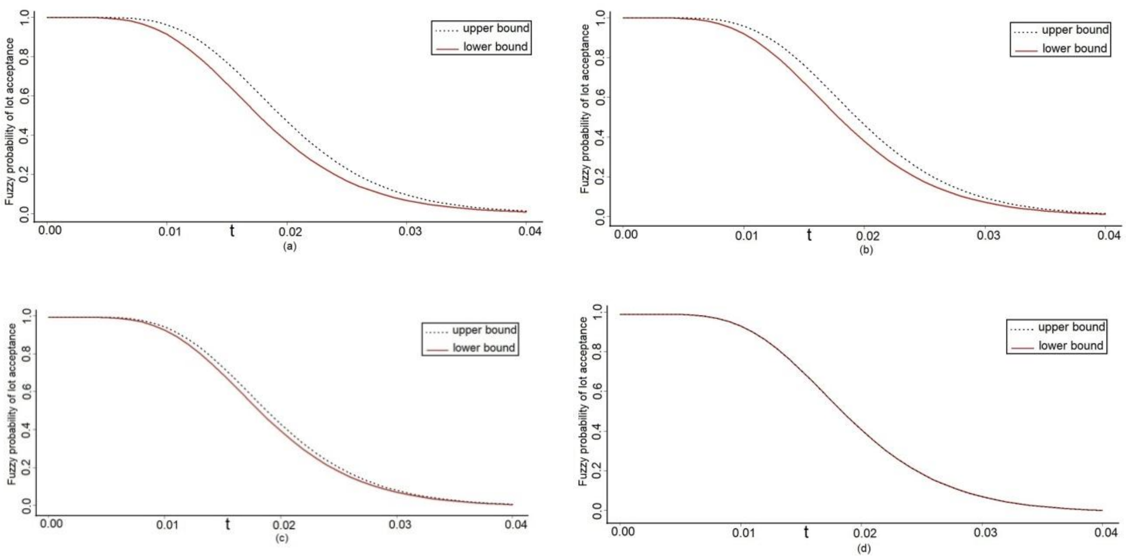

Fig. 2. FOC in KSD-FDVSP for (a) $\gamma = 0$, (b) $\gamma = 0.2$, (c) $\gamma = 0.6$, (d) $\gamma = 1$.

To solve nonlinear optimization issue function ‘fmincon’ is used through the article. The obtained optimum parameters $k_{a\sigma}$, $k_{r\sigma}$ and n_{σ} are reported in Table II for various couples ($\widetilde{AQL}$, $L\widetilde{Q}L$) when $\tilde{\alpha}$ and $\tilde{\beta}$ almost equal 0.05 and 0.1, in order.

B. Unknown standard deviation case

Operating algorithm in unknown standard deviation fuzzy DVSP (USD-FDVSP) is like USD-DVSP explained in Appendix A. Then, by applying relation (A-6) and Buckley’s manner it results:

$$\tilde{P}_{as}(\tilde{p})[\gamma] = \{P(A'_1) + P(A'_2) \mid p \in \tilde{p}[\gamma]\} = [P^L_{as}[\gamma], P^U_{as}[\gamma]] , \tag{6}$$

so that, $P^L_{as}[\gamma] = \min \tilde{P}_{as}(\tilde{p})[\gamma]$ and $P^U_{as}[\gamma] = \max \tilde{P}_{as}(\tilde{p})[\gamma]$.

Applying the same procedure explained in subsection A, optimum parameters of USD-FDVSP, k_{rs}, k_{as} and n_s are obtained that are also recorded in Table II. For example, if the design points are set in $\widetilde{AQL} = (0.0149, 0.015, 0.0151)$ and $\widetilde{LQL} = (0.0399, 0.04, 0.0401)$ when the manufacturer and customer's risks are approximately 0.05 and 0.1, then from Table II the optimal parameters are obtained as $(n_s, k_{rs}, k_{as}) = (101, 1.89, 2.02)$ and $(n_\sigma, k_{r\sigma}, k_{a\sigma}) = (37, 1.89, 2.02)$ under the operation of USD-FDVSP and KSD-FDVSP, respectively. It means that by applying USD-FDVSP to decide about the manufacturing lot, lot is passed if, under-sample information with size 101, $\hat{V}_s \geq 2.02$, but this lot is not admitted when $\hat{V}_s \leq 1.89$. Otherwise, if $1.89 < \hat{V}_s < 2.02$, second sample is taken to achieve ultimate intention. The lot is admitted if, under the second sample of size 101, $\hat{U}_s \geq 1.89$, otherwise the lot is rejected.

From Table II, two appealing trends are seen in the behavior of optimum values. (i) Required sample size decreases by increasing the distance between $\widetilde{AQL}$ and $\widetilde{LQL}$ in both known and unknown standard deviation cases. It means that one needs less sample information to decide about the production lot when the gap between the rejectable and acceptable quality levels becomes large. (ii) The required sample size to make an accurate decision about productions for USD-FDVSP is more than KSD-FDVSP.

TABLE II. OPTIMUM PARAMETERS AS $\tilde{\alpha}$ AND $\tilde{\beta}$ ARE ALMOST 0.05 AND 0.1, IN ORDER.

		KSD-FDVSP			USD-FDVSP		
$\widetilde{AQL}$	$\widetilde{LQL}$	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	n_s	k_{rs}	k_{as}
(0.0009, 0.001, 0.0011)	(0.0149, 0.015, 0.0151)	10	2.49	2.61	30	2.46	2.83
	(0.0199, 0.02, 0.0201)	7	2.43	2.58	25	2.41	2.64
	(0.0249, 0.025, 0.0251)	6	2.36	2.53	20	2.35	2.61
	(0.0299, 0.03, 0.0301)	5	2.28	2.56	16	2.28	2.64
	(0.0349, 0.035, 0.0351)	5	2.28	2.42	15	2.26	2.52
	(0.0399, 0.04, 0.0401)	5	2.28	2.34	15	2.26	2.41
	(0.0499, 0.05, 0.0501)	4	2.15	2.33	11	2.15	2.45
(0.0024, 0.0025, 0.0026)	(0.0199, 0.02, 0.0201)	12	2.31	2.49	43	2.31	2.50
	(0.0249, 0.025, 0.0251)	10	2.27	2.41	34	2.26	2.43
	(0.0299, 0.03, 0.0301)	8	2.2	2.41	25	2.18	2.5
	(0.0349, 0.035, 0.0351)	7	2.15	2.38	22	2.15	2.41
(0.0049, 0.005, 0.0051)	(0.0249, 0.025, 0.0251)	19	2.18	2.29	56	2.16	2.37
	(0.0299, 0.03, 0.0301)	14	2.12	2.28	43	2.11	2.33
	(0.0349, 0.035, 0.0351)	12	2.09	2.22	37	2.08	2.24
	(0.0399, 0.04, 0.0401)	10	2.04	2.22	28	2.02	2.31
	(0.0499, 0.05, 0.0501)	8	1.97	2.16	21	1.95	2.26
(0.0099, 0.01, 0.0101)	(0.0249, 0.025, 0.0251)	47	2.08	2.21	-	-	-
	(0.0299, 0.03, 0.0301)	40	2.04	2.1	98	2.02	2.2
	(0.0349, 0.035, 0.0351)	28	2.01	2.07	70	1.98	2.16
	(0.0399, 0.04, 0.0401)	19	1.94	2.13	55	1.94	2.13
	(0.0449, 0.045, 0.0451)	16	1.9	2.12	44	1.9	2.13
(0.0149, 0.015, 0.0151)	(0.0399, 0.04, 0.0401)	37	1.89	2.02	101	1.89	2.02
	(0.0449, 0.045, 0.0451)	28	1.85	2.02	75	1.85	2.02

Continue TABLE II. OPTIMUM PARAMETERS AS $\tilde{\alpha}$ AND $\tilde{\beta}$ ARE ALMOST 0.05 AND 0.1, IN ORDER.

		KSD-FDVSP			USD-FDVSP		
$\widetilde{AQL}$	$\widetilde{LQL}$	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	n_s	k_{rs}	k_{as}
	(0.0499, 0.05, 0.0501)	26	1.84	1.92	62	1.82	1.98
	(0.0599, 0.06, 0.0601)	18	1.77	1.90	45	1.77	1.91
	(0.0699, 0.07, 0.0701)	14	1.72	1.87	33	1.71	1.91
(0.0199, 0.02, 0.0201)	(0.0449, 0.045, 0.0451)	48	1.81	1.96	-	-	-
	(0.0499, 0.05, 0.0501)	38	1.78	1.91	98	1.78	1.91
	(0.0599, 0.06, 0.0601)	27	1.73	1.84	63	1.72	1.88
	(0.0699, 0.07, 0.0701)	20	1.67	1.82	46	1.67	1.84
	(0.0799, 0.08, 0.0801)	15	1.62	1.83	35	1.62	1.82

Note: Main assumption to make introduced method is that distribution of interesting quality characteristic must be normal, which might not be valid in practical world. To solve this problem, Box-Cox conversions are helpful in modifying non-normal data to normal.

IV. FUZZY AVERAGE SAMPLE NUMBER

Fuzzy average sample number ($\widetilde{\text{ASN}}$) specifies inexact average number of samples taken from lot to assess it. In existing FVSSP (see Afshari & Sadeghpour Gildeh, 2018), sample size is fixed, whilst in introduced FDVSP, sample size to make final decision depends on entering lot quality. Firstly, assume known standard deviation case. Hence, average sample size for KSD-DVSP equals product of the first sample size (n_σ) in probability of existing one sample (P_I), plus size of the combined samples ($n_\sigma + n_\sigma$) times probability of existing a second sample (1-P_I). Consequently, according to Buckley's way, γ-cut of $\widetilde{\text{ASN}}$ of KSD-FDVSP ($\widetilde{\text{ASN}}_\sigma[\gamma]$) is equal to:

$$\widetilde{\text{ASN}}_\sigma[\gamma] = \{n_\sigma P_I + (n_\sigma + n_\sigma)(1 - P_I) \;|\; p \in \tilde{p}[\gamma]\} = [ASN_\sigma^L[\gamma], ASN_\sigma^U[\gamma]], \tag{7}$$

in which, $ASN_\sigma^L[\gamma] = \min \widetilde{\text{ASN}}_\sigma[\gamma]$, $ASN_\sigma^U[\gamma] = \max \widetilde{\text{ASN}}_\sigma[\gamma]$,

and $P_I = 1 - \Phi\left((z_p - k_{r\sigma})\sqrt{n_\sigma}\right) + \Phi((z_p - k_{a\sigma})\sqrt{n_\sigma})$. Identically, γ-cut of $\widetilde{\text{ASN}}$ of USD-FDVSP ($\widetilde{\text{ASN}}_s[\gamma]$) is

$$\widetilde{\text{ASN}}_s[\gamma] = \{n_s P_I + (n_s + n_s)(1 - P_I) \;|\; p \in \tilde{p}[\gamma]\} = [ASN_s^L[\gamma], ASN_s^U[\gamma]], \tag{8}$$

in which,

$$P_I = 1 - \Phi\left((z_p - k_{rs})\sqrt{\frac{n_s}{1+\frac{k_{rs}^2}{2}}}\right) + \Phi\left((z_p - k_{as})\sqrt{\frac{n_s}{1+\frac{k_{rs}^2}{2}}}\right).$$

Example 3. Consider the KSD-FDVSP and USD-FDVSP with their parameters $(\text{n}_\sigma, \text{k}_{\text{r}\sigma}, \text{k}_{\text{a}\sigma}) = (12, 2.31, 2.49)$ and $(\text{n}_\text{s}, \text{k}_{\text{rs}}, \text{k}_{\text{as}}) = (43, 2.31, 2.50)$, respectively. We want to sentence the submitted lot under the operation of above-mentioned plans when the incoming quality is equal to $\tilde{\text{p}} = (0.0199, 0.02, 0.0201)$. By using Eqs. (7) and (8), we have:

$\widetilde{ASN}_\sigma[0] = [13.35, 13.58]$, $\widetilde{ASN}_\sigma[1] = 13.46$,

$\widetilde{ASN}_s[0] = [48.05, 48.89]$, $\widetilde{ASN}_s[1] = 48.46$.

It means that one needs a sample of size approximately 13 in average to decide about the considered lot under the operation of KSD-FDVSP, while it increases to approximately 48 as the standard deviation is unknown.

V. ANALYSIS AND ANALOGY OF INTRODUCED SCHEME

This part, we indicate that presented fuzzy plan includes an existing customary double variable sampling plan (DVSP). Moreover, an analogical study is made between the introduced and available plans in terms of ASN.

A. Description of FOC band

In section III, the authors derived that introduced scheme converts to the existing conventional double sampling plan when the process quality (p) is not vague (see Example 2). But a simulation study is needed to investigate whether the obtained result is valid or not in general. To this, we do a simulation study for the reported plans in Table II. In the simulation method, we consider that $\mu = 0$ and $U = 3$. Under these conditions, the standard deviation is obtained by equation (A-1). The following steps are conducted for each sampling plans in unknown standard deviation case with fixed values of γ, $\tilde{\alpha}$, $\tilde{\beta}$, $\widetilde{AQL}$, $\widetilde{LQL}$, n_s, k_{rs} and k_{as}.

TABLE III. RESULTS ON $\tilde{\alpha}'_\sigma$ AND $\tilde{\beta}'_\sigma$ FOR THE PROPOSED KSD-FDVSP WITH $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$ AND $\tilde{\beta} = (0.0999, 0.1, 0.1001)$

		KSD-FDVSP	
$\widetilde{AQL}$	$\widetilde{LQL}$	$\tilde{\alpha}'_\sigma$	$\tilde{\beta}'_\sigma$
(0.0009, 0.001, 0.0011)	(0.0149, 0.015, 0.0151)	(0.0259, 0.0295, 0.0332)	(0.0962, 0.0971, 0.0980)
	(0.0199, 0.02, 0.0201)	(0.0375, 0.0416, 0.0457)	(0.0976, 0.0982, 0.0988)
	(0.0249, 0.025, 0.0251)	(0.0344,0.0379, 0.0415)	(0.0987, 0.0991, 0.0996)
	(0.0299, 0.03, 0.0301)	(0.0333, 0.0363, 0.0395)	(0.0970, 0.0992, 0.0978)
	(0.0349, 0.035, 0.0351)	(0.0328, 0.0358, 0.0389)	(0.0979, 0.0982, 0.0985)
	(0.0399, 0.04, 0.0401)	(0.0324, 0.0354, 0.0384)	(0.0968, 0.0970, 0.0973)
	(0.0499, 0.05, 0.0501)	(0.0283, 0.0307, 0.0331)	(0.0993, 0.0995, 0.0997)
(0.0024, 0.0025, 0.0026)	(0.0199, 0.02, 0.0201)	(0.0420, 0.0445, 0.0470)	(0.0978, 0.0986, 0.0995)
	(0.0249, 0.025, 0.0251)	(0.0440, 0.0463, 0.0487)	(0.0957, 0.0963, 0.0969)
	(0.0299, 0.03, 0.0301)	(0.0428, 0.0449, 0.0469)	(0.0974, 0.0978, 0.0983)
	(0.0349, 0.035, 0.0351)	(0.0410, 0.0428, 0.0447)	(0.0980, 0.0984, 0.0988)
(0.0049, 0.005, 0.0051)	(0.0249, 0.025, 0.0251)	(0.0421, 0.0438, 0.0454)	(0.0959, 0.0968, 0.0976)
	(0.0299, 0.03, 0.0301)	(0.0445, 0.0460, 0.0475)	(0.0984, 0.0991, 0.0997)
	(0.0349, 0.035, 0.0351)	(0.0465, 0.0479, 0.0493)	(0.0983, 0.0988, 0.0993)
	(0.0399, 0.04, 0.0401)	(0.0458, 0.0471, 0.0484)	(0.0970, 0.0974, 0.0979)
	(0.0499, 0.05, 0.0501)	(0.0440, 0.0451, 0.0462)	(0.0989, 0.0992, 0.0996)
(0.0099, 0.01, 0.0101)	(0.0249, 0.025, 0.0251)	(0.0467, 0.0483, 0.0498)	(0.0950, 0.0964, 0.0979)

Continue TABLE III. RESULTS ON $\tilde{\alpha}'_\sigma$ AND $\tilde{\beta}'_\sigma$ FOR THE PROPOSED KSD-FDVSP WITH $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$ AND $\tilde{\beta} = (0.0999, 0.1, 0.1001)$

		KSD-FDVSP	
$\widetilde{AQL}$	$L\widetilde{QL}$	$\tilde{\alpha}'_\sigma$	$\tilde{\beta}'_\sigma$
	(0.0299, 0.03, 0.0301)	(0.0349, 0.0360, 0.0371)	(0.0970, 0.0980, 0.0990)
	(0.0349, 0.035, 0.0351)	(0.0472, 0.0484, 0.0496)	(0.0966, 0.0974, 0.0981)
	(0.0399, 0.04, 0.0401)	(0.0477, 0.0487, 0.0497)	(0.0983, 0.0989, 0.0995)
	(0.0449, 0.045, 0.0451)	(0.0457, 0.0465, 0.0474)	(0.0974, 0.0979, 0.0984)
(0.0149, 0.015, 0.0151)	(0.0399, 0.04, 0.0401)	(0.0457, 0.0466, 0.0475)	(0.0953, 0.0961, 0.0970)
	(0.0449, 0.045, 0.0451)	(0.0469, 0.0478, 0.0486)	(0.0964, 0.0971, 0.0978)
	(0.0499, 0.05, 0.0501)	(0.0470, 0.0478, 0.0486)	(0.0966, 0.0971, 0.0977)
	(0.0599, 0.06, 0.0601)	(0.0461, 0.0467, 0.0474)	(0.0989, 0.0993, 0.0997)
	(0.0699, 0.07, 0.0701)	(0.0475, 0.0481, 0.0487)	(0.0980, 0.0984, 0.0987)
(0.0199, 0.02, 0.0201)	(0.0449, 0.045, 0.0451)	(0.0476, 0.0485, 0.0493)	(0.0964, 0.0974, 0.0983)
	(0.0499, 0.05, 0.0501)	(0.0475, 0.0483, 0.0491)	(0.0979, 0.0986, 0.0993)
	(0.0599, 0.06, 0.0601)	(0.0477, 0.0483, 0.0490)	(0.0976, 0.0981, 0.0986)
	(0.0699, 0.07, 0.0701)	(0.0446, 0.0451, 0.0456)	(0.0987, 0.0991, 0.0995)
	(0.0799, 0.08, 0.0801)	(0.0486, 0.0491, 0.0496)	(0.0978, 0.0981, 0.0984)

1. Generate two sets of random samples with size n_s from normal distribution with zero mean and standard deviation equivalent to AQL and LQL by (A-1).
2. Using the sample information, compute the estimation of statistics $\hat{V}_s$ and $\hat{U}_s$. Then, according to the introduced operating algorithm, check how the submitted lot is accepted or rejected by the corresponding k_{rs} and k_{as}.
3. Repeat the above steps for 100000 times. Based on the $\hat{P}(A'_1), \hat{P}(A'_2)$ and also relation (6), compute the empirical producer and consumer's risks $\tilde{\alpha}'_s$ and $\tilde{\beta}'_s$.

Note: In case of known standard deviation, three above simulation steps are done based on the sample of size n_σ and critical values $k_{r\sigma}$ and $k_{a\sigma}$ to obtain the empirical producer and consumer's risks $\tilde{\alpha}'_\sigma$ and $\tilde{\beta}'_\sigma$ by applying relation (2).

To save the capacity of the paper, the obtained results are reported in Tables III and IV for some combinations of requirements when pre-specified risks are $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$ and $\tilde{\beta} = (0.0999, 0.1, 0.1001)$ as the standard deviation is known and unknown, in order. From Tables III and IV, it concludes that all the sampling plans meet the pre-specified risks well. Tables III and IV display both empirical risks $(\tilde{\alpha}'_\sigma, \tilde{\alpha}'_s)$ and $(\tilde{\beta}'_\sigma, \tilde{\beta}'_s)$ are smaller than the pre-specified risks $\tilde{\alpha}$ and $\tilde{\beta}$. For example, when $\widetilde{AQL} = (0.0099, 0.01, 0.0101)$ and $L\widetilde{QL} = (0.0299, 0.03, 0.0301)$, from Table IV we notice that the empirical risks are $\tilde{\alpha}'_s = (0.0422, 0.0433, 0.0445)$ and $\tilde{\beta}'_s = (0.0976, 0.0987, 0.0998)$ which are smaller than the pre-specified risks $\tilde{\alpha}$ and $\tilde{\beta}$.

TABLE IV. RESULTS ON $\tilde{\alpha}'_s$ AND $\tilde{\beta}'_s$ FOR THE PROPOSED USD-FDVSP WITH $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$ AND $\tilde{\beta} = (0.0999, 0.1, 0.1001)$

		USD-FDVSP	
$\widetilde{AQL}$	$L\widetilde{Q}L$	$\tilde{\alpha}'_s$	$\tilde{\beta}'_s$
(0.0009, 0.001, 0.0011)	(0.0149, 0.015, 0.0151)	(0.0406, 0.0452, 0.0499)	(0.0979, 0.0988, 0.0998)
	(0.0199, 0.02, 0.0201)	(0.0403, 0.0444, 0.0486)	(0.0985, 0.0991, 0.0997)
	(0.0249, 0.025, 0.0251)	(0.0420, 0.0459, 0.0498)	(0.0976, 0.0981, 0.0986)
	(0.0299, 0.03, 0.0301)	(0.0425, 0.0461, 0.0497)	(0.0988, 0.0991, 0.0995)
	(0.0349, 0.035, 0.0351)	(0.0424, 0.0458, 0.0493)	(0.0990, 0.0993, 0.0996)
	(0.0399, 0.04, 0.0401)	(0.0418, 0.0452, 0.0486)	(0.0986, 0.0988, 0.0991)
	(0.0499, 0.05, 0.0501)	(0.0420, 0.0451, 0.0481)	(0.0980, 0.0982, 0.0984)
(0.0024, 0.0025, 0.0026)	(0.0199, 0.02, 0.0201)	(0.0440, 0.0465, 0.0491)	(0.0979, 0.0988, 0.0996)
	(0.0249, 0.025, 0.0251)	(0.0449, 0.0472, 0.0496)	(0.0980, 0.0986, 0.0992)
	(0.0299, 0.03, 0.0301)	(0.0444, 0.0464, 0.0485)	(0.0988, 0.0992, 0.0997)
	(0.0349, 0.035, 0.0351)	(0.0455, 0.0474, 0.0494)	(0.0982, 0.0986, 0.0990)
(0.0049, 0.005, 0.0051)	(0.0249, 0.025, 0.0251)	(0.0450, 0.0466, 0.0483)	(0.0976, 0.0985, 0.0993)
	(0.0299, 0.03, 0.0301)	(0.0454, 0.0469, 0.0484)	(0.0971, 0.0977, 0.0984)
	(0.0349, 0.035, 0.0351)	(0.0455, 0.0469, 0.0483)	(0.0984, 0.0989, 0.0994)
	(0.0399, 0.04, 0.0401)	(0.0472, 0.0485, 0.0498)	(0.0977, 0.0981, 0.0986)
	(0.0499, 0.05, 0.0501)	(0.0476, 0.0487, 0.0499)	(0.0992, 0.0995, 0.0999)
(0.0099, 0.01, 0.0101)	(0.0299, 0.03, 0.0301)	(0.0422, 0.0433, 0.0445)	(0.0976, 0.0987, 0.0998)
	(0.0349, 0.035, 0.0351)	(0.0477, 0.0488, 0.0499)	(0.0976, 0.0983, 0.0991)
	(0.0399, 0.04, 0.0401)	(0.0473, 0.0483, 0.0493)	(0.0978, 0.0984, 0.0990)
	(0.0449, 0.045, 0.0451)	(0.0474, 0.0483, 0.0492)	(0.0978, 0.0983, 0.0989)
(0.0149, 0.015, 0.0151)	(0.0399, 0.04, 0.0401)	(0.0474, 0.0484, 0.0493)	(0.0975, 0.0984, 0.0993)
	(0.0449, 0.045, 0.0451)	(0.0480, 0.0489, 0.0497)	(0.0977, 0.0984, 0.0991)
	(0.0499, 0.05, 0.0501)	(0.0471, 0.0478, 0.0486)	(0.0968, 0.0974, 0.0979)
	(0.0599, 0.06, 0.0601)	(0.0484, 0.0491, 0.0498)	(0.0984, 0.0988, 0.0992)
	(0.0699, 0.07, 0.0701)	(0.0479, 0.0485, 0.0491)	(0.0971, 0.0974, 0.0977)
(0.0199, 0.02, 0.0201)	(0.0499, 0.05, 0.0501)	(0.0477, 0.0485, 0.0492)	(0.0981, 0.0988, 0.0996)
	(0.0599, 0.06, 0.0601)	(0.0482, 0.0488, 0.0495)	(0.0978, 0.0984, 0.0989)
	(0.0699, 0.07, 0.0701)	(0.0482, 0.0488, 0.0493)	(0.0975, 0.0979, 0.0983)
	(0.0799, 0.08, 0.0801)	(0.0478, 0.0482, 0.0487)	(0.0989, 0.0992, 0.0995)

Fig. 3(a) and 3(b) illustrate the γ-cuts of FOC band for USD-FDVSP and KSD-FDVSP, respectively, when $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$, $\tilde{\beta} = (0.0999, 0.1, 0.1001)$, $\widetilde{AQL} = (0.0099, 0.01, 0.0101)$ and $L\widetilde{Q}L = (0.0299, 0.03, 0.0301)$ for $\gamma = 0, 0.2, 0.6$ and 1. In addition, Fig. 3(a) and 3(b) include the OC curve for classical USD-DVSP and KSD-DVSP, respectively, under the above-mentioned circumstances. According to Fig. 3, one can see that the lower and upper bounds of FOC band approach together when γ increases (or when the amount of ambiguity of nonconforming proportion gets smaller) such that these bounds come closer as $\gamma \to 1$ and coincide with the OC curve of the traditional DVSP. This means that the bandwidth of FOC band of the proposed plan becomes narrower when uncertainty value of process quality decreases, and FOC band is in a crisp state as ambiguity level of process quality is equal to zero (or $\gamma = 1$).

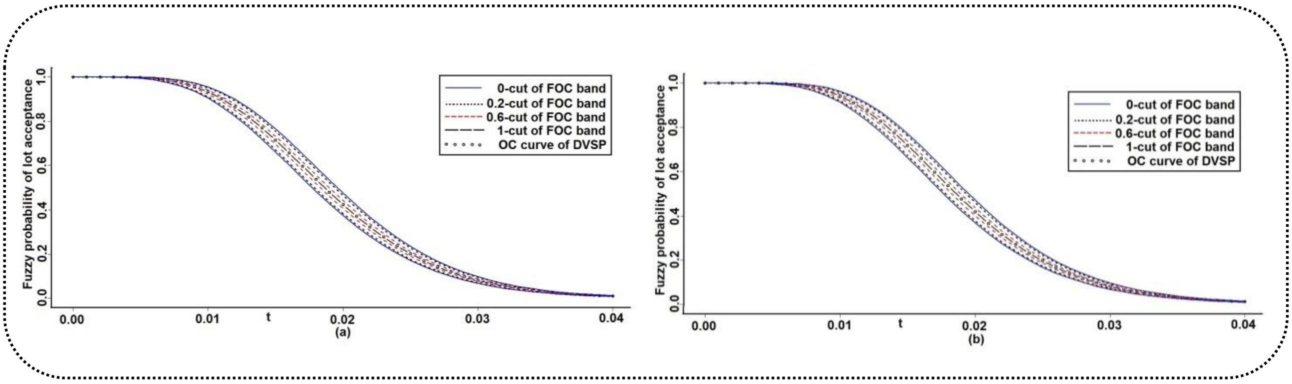

Fig. 3. OC curve of DVSP and γ-cuts of FOC band of the proposed FDVSP for $\gamma = 0,\ 0.2, 0.6, 1$ (a) Unknown standard deviation, (b) Known standard deviation.

B. Comparative study

ASN function that indicates the relation between the number of items required to be examined and the incoming quality, is widely applied as a measure to judge between sampling schemes. The plan with less ASN is preferred to another. As it was mentioned in section IV, although the needed sample size is fixed under the operation of the existing FVSSP, it depends on if one requires the information of the second sample or not for the proposed FDVSP.

TABLE V. 1-CUT OF ASN OF INTRODUCED PLAN AND FVSSP (Afashari and Sadeghpour Gildeh, 2018)

		Known Standard Deviation			UnKnown Standard Deviation		
$\widetilde{AQL}$	$\widetilde{LQL}$	FVSSP	FDVSP	Reduction%	FVSSP	FDVSP	Reduction%
(0.0024, 0.0025, 0.0026)	(0.0199, 0.02, 0.0201)	15	13	13.33	58	48	17.24
	(0.0249, 0.025, 0.0251)	12	11	8.33	44	37	15.90
	(0.0349, 0.035, 0.0351)	9	7	22.22	31	24	22.58
(0.0049, 0.005, 0.0051)	(0.0249, 0.025, 0.0251)	23	20	13.04	-	64	-
	(0.0299, 0.03, 0.0301)	18	15	16.66	60	49	18.33
	(0.0349, 0.035, 0.0351)	15	13	13.33	48	41	14.58
(0.0099, 0.01, 0.0101)	(0.0249, 0.025, 0.0251)	64	54	15.62	-	-	-
	(0.0299, 0.03, 0.0301)	43	40	7.50	136	115	15.44
	(0.0349, 0.035, 0.0351)	32	29	9.37	100	81	19.00
	(0.0399, 0.04, 0.0401)	26	21	19.23	78	63	19.23

Table V demonstrates some 1-cuts of ASN values of introduced FDVSP and available FVSSP (introduced in Afshari & Sadeghpour Gildeh, 2018) when standard deviation is known or unknown for some combinations of $(\widetilde{AQL}, \widetilde{LQL})$ as $\tilde{\alpha}$ and $\tilde{\beta}$ are approximately set 0.05 and 0.1, respectively. From Table V, the presented scheme has preferable implementation compared to FVSSP in terms of ASN. For instance, when $\widetilde{AQL} = (0.0049,\ 0.05,\ 0.0051)$ and $\widetilde{LQL} = (0.0299, 0.03, 0.0301)$, ASN of FVSSP is approximately equal to 18, while it approximately equals 15 under the operation of FDVSP (16.66% reduction) in known standard deviation case. Similarly, in unknown standard deviation case, ASN of FVSSP approximately equals 60, while the required sample size in average is approximately equal to 49 (18% reduction) under the operation of FDVSP. According to Table V, it concludes that the proposed plan in both cases is more desirable than FVSSP in terms of ASN needed to inspect the submitted lot.

Fig. 4(a) and 4(b) show λ-cuts of ASN of introduced plan and FVSSP when $\gamma = 0$ and 1, respectively, as $\widetilde{AQL} = (0.0099,\ 0.01,\ 0.0101)$ and $\widetilde{LQL} = (0.0299, 0.03, 0.0301)$ in unknown standard deviation case. According to Fig. 4, it results introduced plan is always better than FVSSP besides when arriving quality is neither good nor bad. Since when entering quality is well, lot acceptance probability gets bigger at the first stage, hence one does not require the second sample. But then, when incoming quality is poor, probability of lot rejection grows at the first stage. That is, there is no demand for extra knowledge obtained from second sample to judge about lot. Consequently, proposed manner is better than existing plan due to saving lot inspection time and expense.

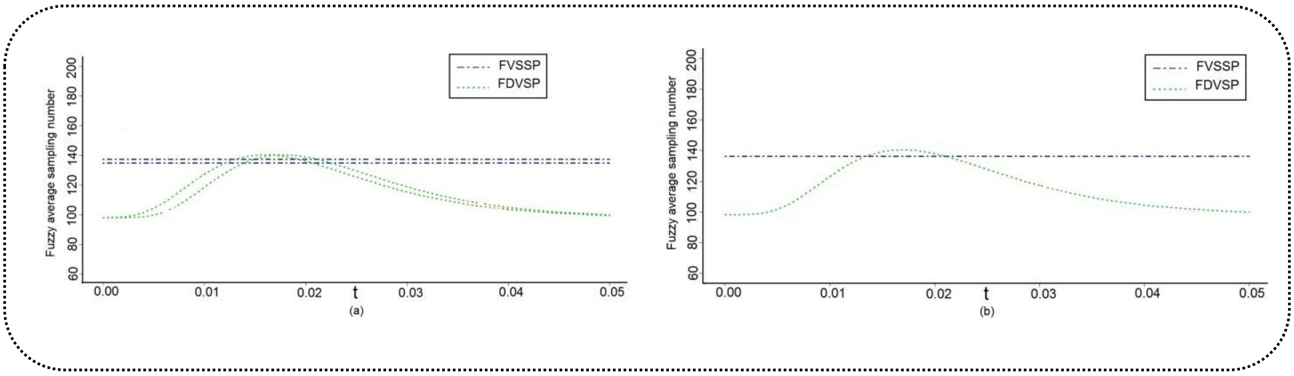

Fig. 4. $\boldsymbol{\gamma}$-cut of ASN in introduced FDVSP and available FVSSP for (a) $\boldsymbol{\gamma = 0}$, (b) $\boldsymbol{\gamma = 1}$.

VI. INDUSTRIAL APPLICATION

TABLE VI. 98 OBSERVATIONS FOR COLOUR STN DISPLAYS

11692.3	11722.6	11674.7	11681.3	11800.8	11705.9	11705.8	11705.8	11722.6
11797.2	11664.2	11664.2	11775.5	11775.5	11769.1	11667.9	11589.8	11729.2
11636.4	11722.9	11666.2	11700.2	11721.7	11647.2	11744.1	11625.6	11705.8
11726.4	11726.4	11729.2	11633.6	11721.7	11681.1	11633.6	11655.3	11757.5
11625.6	11769.1	11710.1	11726.4	11636.4	11692.3	11647.2	11633.6	11769.1
11674.7	11780.7	11655.3	11655.3	11710.1	11728.4	11636.4	11743.3	11769.1
11589.8	11712.7	11728.4	11680.5	11743.3	11636.4	11664.2	11705.8	11666.2
11698.0	11816.7	11712.7	11698.0	11797.2	11677.0	11775.5	11754.2	11773.1
11698.0	11775.5	11692.3	11728.4	11760.6	11589.8	11712.7	11655.3	11797.2
11695.9	11722.6	11671.8	11731.2	11773.1	11745.4	11705.9	11633.6	11722.9
11674.7	11775.2	11700.2	11692.3	11786.1	11727.7	11727.7	11731.2	

Here, we discuss an industry data of colour STN (super-twisted nematic) displays applied by Aslam et al. (2013). The quality characteristic has upper specification limit $U = 12500$. Assume that one would like to make a decision about the lot of mentioned products under the purchasing agreement $\widetilde{AQL} = (0.0099, 0.01, 0.0101)$, $\widetilde{LQL} = (0.0299, 0.03, 0.0301)$, $\tilde{\alpha} = (0.0499, 0.05, 0.0501)$ and $\tilde{\beta}$ =(0.0999, 0.1, 0.1001). Because of the unknown population standard deviation, USD-FDVSP is employed to assess products. From Table II, it derives $n_s = 98$, $k_{rs} = 2.02$ and $k_{as} = 2.2$. To examine the productions, a with size 98 is chosen from the submitted lot. These data are reported in Table VI. Discovered conclusion from Shapiro-Wilk test displayed $p - value = 0.2053 > 0.05$, it means the data are not from a non-normal society, then USD-FDVSP can be performed here to evaluate whether the submitted lot is accepted or rejected. Hence, to execute USD-FDVSP, we selected a sample with size 98 and calculated $\bar{x}_1 = 11707.14$, $s_1 = 51.54$ and $v_s = \frac{12500-11707.14}{51.54}$, because $v_s = 15.38 \geq 2.2$, submitted lot is passed at the first stage.

Eventually, it is worth to the point that by using FVSSP (introduced in Afshari & Sadeghpour Gildeh, 2018) to make decision about colour STN displays lot, one requires a sample of size 136, 15.44% more than ASN (=115) of the proposed FDVSP. That is introduced scheme is preferable than available FVSSP because of saving examination cost and time.

VII. CONCLUSION

Although traditional DVSP requires a small ASN to inspect productions in comparison with the available single sampling scheme, it is not an appropriate selection to decide as some plan parameters are imprecise. Here, we developed usual DVSP to a situation where the proportion of defective items is. For industrial application, optimum plan parameters were derived by considering the optimization issue. Moreover, An extensive simulation study was implemented to show that the introduced plan in a fuzzy environment includes the existing traditional one. Also, a numerical instance was given to explain how to use the proposed manner in the industrial world. We concluded that the proposed plan is more inexpensive and timesaving than the existant FVSSP. For further directions, the proposed FDVSP can be discussed when the process has multiple quality characteristics.

REFERENCES

Afshari, R., Sadeghpour Gildeh, B. & Sarmad, M. (2017). Fuzzy multiple deferred state attribute sampling plan in the presence of inspection errors. *Journal of Intelligent & Fuzzy Systems*, 33(1), 503-514.

Afshari, R. & Sadeghpour Gildeh, B. (2017a). Designing a multiple deferred state attribute sampling plan in a fuzzy environment. *American Journal of Mathematical & Management Sciences*, 36(4), 328-345.

Afshari, R. & Sadeghpour Gildeh, B., Modified sequential sampling plan using fuzzy SPRT, 5 th Iranian Joint Congress on Fuzzy and Intelligent Systems, 16th Conference on Fuzzy Systems and 14th Conference on intelligent systems, Qazvin Islamic Azad University (Iran), 7-9 March 2017b.

Afshari, R., Sadeghpour Gildeh, B. & Sarmad, M. (2018). Multiple deferred state sampling plan with fuzzy parameter. *International Journal of Fuzzy Systems*, 20(2), 549-557.

Afshari, R. & Sadeghpour Gildeh, B. (2018). Fuzzy multiple deferred state variable sampling plan. *Journal of Intelligent & Fuzzy Systems*, 34(4), 2737-2752.

Aslam, M., Azam, M. & Jun, C. H. (2013). A new lot inspection procedure based on exponentially weighted moving average. *International Journal of Systems Science*, 46(8), 1392-1400.

Baloui Jamkhaneh, E., Sadeghpour Gildeh, B. & Yari, G. (2011). Acceptance single Sampling plan with fuzzy parameter. *Iranian Journal of Fuzzy Systems*, 8(2), 47-55.

Baloui Jamkhaneh, E. & Sadeghpour Gildeh, B. (2013). Sequential sampling plan using fuzzy SPRT. *Journal of Intelligent Fuzzy Systems*, 25(3), 785-791.

Baloui Jamkhaneh, E. & Sadeghpour Gildeh, B. (2012). Acceptance double sampling plan using fuzzy poisson distribution. *World Applied Sciences Journal*, 16(11), 1578-1588.

Buckley, J. J. (2003). *Fuzzy probability. New approach and application*. Physica-Verlage, Heidelberg.

Buckley, J. J. (2006). *Fuzzy probability and statistics*. Springer Verlag, Berlin Heidelberg.

Butt, K.A., Aslam, M. & Jun, C. H. (2019). Analysis of process yield in a cost-effective double acceptance sampling plan. *Communications in Statistics-Theory and Methods*, 1-13.

Chakraborty, T. K. (1992). A class of single sampling plans based on fuzzy optimization. *Quality Control & Applied Statistics*, 37(7), 359-362.

Divya, P. R. (2012). Quality interval acceptance single sampling plan with fuzzy parameter using poisson distribution. *International Journal of Advancements in Research & Technology*, 1(3), 115-125.

Dubois, D. & Prade, H. (1978). Operations of fuzzy numbers. *International Journal of Systems Science*, 9(6), 613-626.

Fallahnezhad, M. S., Ahmadi Yazdi, A., Abdollahi, P. & Aslam, M. (2015). Design of economic optimal double sampling design with zero acceptance numbers. *Journal of Quality Engineering & Production Optimization*, 1(2), 45-56.

Kahraman, C., Turanoglue Bekar , E. & Senvar, O. (2016). A fuzzy design of single and double acceptance sampling plans. *ntelligent Decision Making in Quality Management*, 179-211.

Montgomery, D. C. (2012). *Introduction to statistical quality control*. Wiley, New York.

Razmkhah, M., Sadeghpour Gildeh, B. & Ahmadi, J. (2017). An economic design of rectifying double acceptance sampling plans Via maxima nomination sampling. *Stochastics & Quality Control*, 32(2), 99-104.

Schilling, E. G. & Neubauer D. V. (2017). *Acceptance sampling in quality control*. CRC Press, Florida.

Sommers D. J. (1981).Two point double variables sampling plans. *Journal of Quality Technology*, 13(1), 25-30.

Tong, X. & Wang, Z. (2012). Fuzzy acceptance sampling plans for inspection of geospatial data with ambiguity in quality characteristics. *Computers & Geosciences*, 48(11), 256-266.

Turanoglue, E., Kaya, I. & Kahraman, C. (2012). Fuzzy acceptance sampling and characteristic curves. *International Journal of Computational Intelligence Systems*, 5(1), 13-29.

Venkateh, A. & Elango, S. (2014). Acceptance sampling for the influence of TRH using crisp and fuzzy gamma distribution. *Aryabhatta Journal of Mathematics & Informatics*, 6(1), 119-124.

APPENDIX

A. The operating algorithm of customary DVSP

Here, DVSP is briefly explained in a classical case. Assume that quality characteristic follows a normal distribution with unknown mean μ and known standard deviation σ, and it also has upper specification limit U. The operating procedure in known standard deviation DVSP (KSD-DVSP) is as follows:

Step 1. Select a sample with size $n_\sigma$from given lot $(X_1, X_2, \dots, X_{n\sigma})$ and calculate $V\sigma = (U - \bar{X}_1)/\sigma$, where $\bar{X}_1 = \sum_{i=1}^{n\sigma} X_i / n_\sigma$.

Step 2. If $v_\sigma \geq k_{a\sigma}$, then pass the lot, and if $v_\sigma \leq k_{r\sigma}$, then refuse the lot, otherwise, choose another sample with size n_σ $(X_1', X_2', \dots, X_{n\sigma}')$ and calculate $U\sigma = (U - \bar{X}_c)/\sigma$, in which $\bar{X}_c = (\bar{X}_1 + \bar{X}_2)/2$ and $\bar{X}_2 = \sum_{i=1}^{n\sigma} X_i'/n_\sigma$, then go to Step 3 ($U\sigma$ is an index to sentence the submitted lot by applying the information of both samples which are taken in Steps 1 and 2).

Step 3. If $u_\sigma \geq k_{r\sigma}$, then accept the lot. Otherwise reject the lot (v_σ and $u_\sigma$are estimated values of $V\sigma$ and $U\sigma$, respectively. Moreover, $k_{a\sigma}$ and $k_{r\sigma}$ are acceptance and rejection numbers under KSD-DVSP and $0 < k_{r\sigma} < k_{a\sigma}$).

Thus, KSD-DVSP is defined by three parameters $k_{a\sigma}$, $k_{r\sigma}$ and n_σ. Consider that p is given as

$$p = P(X > U|\mu) = 1 - \Phi((U - \mu)/\sigma), \tag{A-1}$$

in which, $\Phi(.)$ shows cumulative distribution function of standard normal variable. From Sommers (1981), lot acceptance probability is solved as follows

$$P_{a\sigma} = P(A_1) + P(A_2), \tag{A-2}$$

where,

$$P(A_1) = \Phi\big((z_p - k_{a\sigma})\sqrt{n_\sigma}\big), \tag{A-3}$$

$$P(A_2) = \frac{\sqrt{2}}{2\pi} \int_{-\infty}^{\sqrt{2n_\sigma}(z_p - k_{r\sigma})} \int_{\sqrt{n_\sigma}(z_p - k_{a\sigma})}^{\sqrt{n_\sigma}(z_p - k_{r\sigma})} f(t_1, t_2) dt_1\, dt_2, \tag{A-4}$$

and $f(t_1, t_2) = e^{-(t_1^2 - \frac{2}{\sqrt{2}} t_1 t_2 + t_2^2)}$ and z_p is the $(1 - p)$th quantile of normal standard distribution that is obtained by

$$z_p = \Phi^{-1}(1 - p). \tag{A-5}$$

If we meet unknown standard deviation DVSP (USD-DVSP), then the symbols k_{rs}, k_{as} and n_s are used for $k_{r\sigma}$, $k_{a\sigma}$ and n_σ, in order. When standard deviation is passive, sample standard deviation is applied instead of σ. In this case, the plan implements similar to known standard deviation position, and one uses $Vs = (U - \bar{X}_1)/S_1$ and $U_S = (U - \bar{X}_c)/S_c$ instead of $V\sigma$ and U_σ in Steps 1 and 2, respectively, where $S_1^2 = \frac{1}{n_s - 1} \sum_{i=1}^{n_s} (X_i - \bar{X}_1)^2$ and $S_c^2 = \frac{\sum_{i=1}^{n_s} X_i^2 + \sum_{i=1}^{n_s} X_i'^2 - 2n_s \bar{X}_c^2}{2n_s - 1}$. According to Sommers (1981), the lot acceptance probability of USD-DVSP is obtained by

$$P_{as} = P(A_1') + P(A_2'), \tag{A-6}$$

where,

$$P(A_1') = \Phi\left(\left(z_p - k_{as}\right) \sqrt{\frac{n_s}{1+\frac{k_{rs}^2}{2}}} \right), \tag{A-7}$$

$$P(A_2') = \frac{\sqrt{2}}{2\pi} \int_{-\infty}^{w_1} \int_{w_2}^{w_1} f(t_1, t_2) dt_1 \, dt_2, \tag{A-8}$$

in which $w_1 = \sqrt{\frac{2n_s}{1+\frac{k_{rs}^2}{2}}} (z_p - k_{rs})$ and $w_2 = \sqrt{\frac{2n_s}{1+\frac{k_{rs}^2}{2}}} \left(z_p - k_{as}\right)$.