Using Metaheuristic Algorithms Combined with Clustering Approach to Solve a Sustainable Waste Collection Problem

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Abstract— Sustainability is a monumental issue that should be considered in designing a logistics system. In order to incorporate sustainability concepts in our study, a waste collection problem with economic, environmental, and social objective functions was addressed. The first objective function minimized overall costs of the system, including establishment of depots and treatment facilities. Addressing environmental concerns, greenhouse gases emission was minimized by the second objective function and the third one maximized distances between each customer and treatment facilities. Treatment facility is noxious for human health and should be located in the maximum distance from the urban area. Initially, the locations of depots and treatment facilities were determined. Then, heterogeneous vehicles started to collect waste from the location of each customer and take it to treatment facilities. The problem included two types of open and close routes. Moreover, each vehicle had a capacity restriction, servicing time, and route length. There were different types of waste and each vehicle had a different capacity for them. Three metaheuristic algorithms combined with clustering approach were proposed to look for the best solutions in rational time. The Non-dominated Sorting Genetic Algorithm-II (NSGA-II), improved Strength Pareto Evolutionary Algorithm (SPEA-II), and Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) were compared in terms of performance metrics. According to the results, NSGA-II outweighed other algorithms in the presented model.

Keywords— Facility location problem, Vehicle routing, Waste collection, Sustainability, Metaheuristic algorithms.

I. INTRODUCTION

The aim of Vehicle Routing Problem (VRP) is finding optimal routes for collection or delivery from one or multi depots to customers by consideration of the existing constraints (Laporte, 1992). Different types of VRP models have been presented in the literature so far in order to consider real situations, e.g., capacitated VRP, periodic VRPs (Uchoa et al., 2017), VRP with time windows (Pecin et al., 2017), pickup and delivery problems (Mahmoudi and Zhou, 2017), vehicle routing with multiple depots (Azadeh and Farrokhi-Asl, 2019), vehicle routing with split deliveries (Chen et al., 2017), and green vehicle routing (Braekers et al., 2016). These problems have different applications and versions in real life. We will review some aspects of these problems in the following.

Some studies in the literature have considered only economic goals. Bula et al. (2017) proposed a VRP model with heterogeneous vehicles for hazardous commodities transportation. They concentrated on VRP with Heterogeneous Fleets (HFVRP) in the context of hazardous materials. Their main goal was to specify an assortment of routes that reduced the total expected routing risk. They used variable neighborhood search to tackle the problem. Finally, the algorithm was examined with different sets of HFVRP examples. The examples were changed in order to include risk factors. The results
were acceptable regarding computational time and quality of the solutions. Yu et al. (2017) minimized the total routing cost of capacitated VRP by using Symbiotic Organisms Search (SOS) heuristic. The results showed that the introduced approach generated rational VRP solutions in a rational running time, proving that it was a good alternative algorithm for solving the capacitated vehicle routing problem.Todosijević et al. (2017) investigated swap-body VRP and minimized total cost of using vehicles. The fleet of vehicles including trucks, semi-trailers, and swap bodies was at one depot to meet a specific number of customers. In order to meet demand of the customers, some might utilize either a truck-handling one-swap body or a train-handling two-swap body. In another study, Silvestrin and Ritt (2017) defined Multi-Compartment VRP (MCVRP) with a heuristic approach to tackling the problem. In various test problems, they examined the performance of the algorithm and compared the results. They found that the approach could generate solutions that were better than the available heuristic approaches.

In addition to the economic costs, the environmental and social effects of supply chains have been considered in the literature. In some researches, green concepts were incorporated into VRP, leading to a new version of the problem called Green VRP (GVRP). GVRP is an important and common issue for many researchers, e.g., Yu et al. (2017). They considered hybrid power source (electric and fuel) vehicles in a GVRP. Yin and Chuang (2016) studied fuel efficiency and carbon emissions in cross-docking distribution. They concentrated on vehicles that utilized a hybrid power source, called Plug-in Hybrid Electric Vehicle (PHEV), and presented a novel mathematical formulation to minimize the total cost of the whole system by driving PHEV. Moreover, Cheng et al. (2017) considered environmental effect in green inventory routing problem. They introduced a mixed-integer programming mathematical model and then, conducted numerical experiments to analyze the outcomes of applying comprehensive objective and heterogeneous fleets. Afterwards, managerial insights were given based on parameter analyses. Vincent et al. (2016) presented a pollution routing model to dwindle the total operational and environmental costs with continuous speed of vehicles. In addition, they developed a mathematical formulation and utilized a Simulated Annealing (SA) metaheuristic for solving this problem. The performance of the presented metaheuristic was validated by applying benchmark data related to the Pollution Routing Problem (PRP). The obtained results demonstrated that SA performed better than the previous methods in 7 test problems. In another paper, Kumar et al. (2016) introduced Production and Pollution Routing Problems with Time Window (PPRP-TW) and capacitated vehicles. In this research, a new version of VRP was developed. A set of alike capacitated vehicles departed plants to serve a set of customers. Moreover, in the routing phase of PPRP-TW, carbon footprint was analyzed. Finally, a hybrid Self-Learning Particle Swarm Optimization (SLPSO) heuristic algorithm in the framework of multi-objective optimization was applied to tackling this problem. Comparisons with well-known algorithms, like genetic algorithm, showed the superiority of this algorithm.

Another part of the research on VRP deals with its combination with the facility location problem, called Location Routing Problem (LRP). There are more logical solutions to the LRP model than only considering routing and location problems separately. Therefore, many studies have been carried out in this field (Farrokhi-Asl et al., 2017). For example, Ceselli et al. (2014) proposed an LRP for the health care system in which distribution centers should be located. The best routes for distributing vaccines or drugs were found. Zhao and Verter (2015) minimized environmental risk and total cost in LRP by locating depots and disposal facilities and finding the best routes for oil transportation.

Collection, recycling, and disposal of waste and finding a rational location for treatment facilities or disposal centers have significant effects on environmental and social aspects. Thus, attention to these issues is important for human health. Among different types of waste, management of hazardous waste, e.g., medical waste, is the most critical consideration. Medical waste is one of the most hazardous types of waste and carries a large number of pathogenic bacteria. Therefore, these materials can pose a threat to the human health and environment if not recycled and disposed properly. He et al. (2016) proposed medical waste problem and recycled it in disposal company. They analyzed a collection network consisting of the infrastructures and flow of medical waste between different facilities. In addition, solutions were proposed to tackle the available problem in hand.

Finally, some suggestions about regulations and tracking methods during the collection process have been given. Akhtar et al. (2017) minimized the length of collection routes with capacitated vehicles by considering environmental
factors. They proposed a modified Backtracking Search Algorithm (BSA) for solving a CVRP model with the smart bin concept to obtain the optimum waste collection route. Also, a new VRP model for waste collection problem with some real restrictions and multiple transfer stations was investigated by Rabbani et al. (2016). In another study, Samanlioglu (2013) minimized total risk and costs of hazardous waste location routing problem. Flows between different facilities including recycling, disposal, and generation nodes were investigated in this paper. Solid waste management is regarded as a special type of complex optimization problems. Decision makers should make short-, medium-, and long-term decisions considering the multi-stage supply chain of waste network including producing, treatment, and disposal facilities. In these conditions, neglecting the uncertainty of waste generation rates leads to catastrophes. Gambella et al. (2019) investigated the tactical problem of waste flow assignment with the goal of reducing total management cost and increasing the net of possible profits gained by recycling operation. Rabbani et al. (2016) minimized total cost of a waste collection problem with heterogeneous vehicles and mixed close-open routes. In the extended version of the previous work, Rabbani et al. (2017) considered economic and social effects of waste location routing problem.

In the current study, a sustainable location routing model is applied to waste collection framework. There are three objective functions. The first one evaluates the economic aspects of the network and tries to minimize total network cost, including transportation cost and the establishment cost of depots and treatment facilities. The second objective function considers environmental issues of sustainability, that is, it minimizes total fuel consumption based on vehicle speeds and loads. The third one addresses social concept of sustainability by maximizing distances between customers and treatment facilities, which are categorized as undesirable facilities, because they have negative effects on human health. Furthermore, Non-dominated Sorting Genetic Algorithm-II (NSGA-II), improved Strength Pareto Evolutionary Algorithm (SPEA-II), and Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) are applied in this paper, since they can achieve suitable solutions in an acceptable time for multi-objective problems (Farrokhi-Asl et al., 2017; Mahmoudsoltani et al., 2018). To attain better results by the algorithms, their parameters are tuned by Taguchi method. Finally, three performance metrics are used to compare the developed metaheuristic algorithms.

The rest of the paper is organized as follows. The mathematical formulation of the presented problem is introduced in Section II. The presented solution methodology for solving the proposed model is provided in Sections III. Tuning of the parameters of the algorithms is addressed in Section IV. The computational results and discussion are given in Section V. Finally, the concluding remarks and future research directions are discussed in Section VI.

II. PROBLEM DESCRIPTION

The model consists of three objective functions, including minimization of total cost, minimization of fuel consumption, and maximization of distance between treatment facilities and city areas. Since treatment facility is noxious for human health, it should be located in the maximum distance from the urban area. The presented model determines the optimal location of depots and treatment centers as well as routes for waste collection from generation nodes. Demands and locations of customers (i.e., generation nodes) are known and deterministic. There are several potential locations for establishing different facilities and decision makers should select appropriate locations for them. Moreover, two types of vehicles, including internal and external, are assigned to collect waste from generation nodes. Note that internal vehicles are different from the external ones in that they should come back to the depots, while external vehicles are assumed to be free after unloading the waste in the last treatment facility. Therefore, this model comprehends two types of open and close routes. Moreover, each vehicle has a capacity restriction.

In this research, time of servicing to customers and the lengths of routes between different nodes are determined. Overall traveling time of each route should not trespass on the maximum acceptable servicing time. As it is shown in Fig. (1), vehicles begin their route from depots and proceed to the locations of customers in order to collect waste in these places. Then, they move to the compatible treatment facilities (i.e., there is a special treatment center for each type of waste). In addition, each type of waste occupies a separate segment in each vehicle. Finally, it is assumed that emission of CO2 gas is directly proportional to the total fuel consumption.
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**Sets:**

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
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<tbody>
<tr>
<td>$D$</td>
<td>Set of potential depots</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of customers</td>
</tr>
<tr>
<td>$F$</td>
<td>Set of potential treatment centers</td>
</tr>
<tr>
<td>$W$</td>
<td>Set of waste types and the corresponding treatment technology</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of indices for vehicles</td>
</tr>
<tr>
<td>$S$</td>
<td>Fleet type (internal or external)</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of aggregated depots and customer nodes</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of aggregated customers and potential treatment centers</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of all nodes</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of indices for discretized speed</td>
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</table>

**Parameters:**

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<td>$t_{ijsk}$</td>
<td>Loading Time spent per unit of waste $w$ in the location of customer $c$ by vehicle $k$ belonging to fleet $s$</td>
</tr>
<tr>
<td>$f_1$</td>
<td>Fixed cost of using external vehicle</td>
</tr>
<tr>
<td>$VC_k$</td>
<td>Variable cost of using vehicle $k$ per unit of time</td>
</tr>
<tr>
<td>$q_{iw}$</td>
<td>Demand of customer $i$ for treatment of waste type $w$</td>
</tr>
<tr>
<td>$cap_{kw}$</td>
<td>Maximum capacity of vehicle type $k$ for waste type $w$</td>
</tr>
<tr>
<td>$L$</td>
<td>Allowable route length</td>
</tr>
<tr>
<td>$T$</td>
<td>Allowable route duration</td>
</tr>
<tr>
<td>$\Omega_d$</td>
<td>Capacity of depot $d$</td>
</tr>
<tr>
<td>$\pi_{iw}$</td>
<td>Establishment cost of treatment center with technology $w$ in location $f$</td>
</tr>
</tbody>
</table>

Figure 1. Schematics of collecting and recycling the waste

Sets:

- $D$: Set of potential depots
- $C$: Set of customers
- $F$: Set of potential treatment centers
- $W$: Set of waste types and the corresponding treatment technology
- $K$: Set of indices for vehicles
- $S$: Fleet type (internal or external)
- $N$: Set of aggregated depots and customer nodes
- $P$: Set of aggregated customers and potential treatment centers
- $A$: Set of all nodes
- $R$: Set of indices for discretized speed

Parameters:

- $t_{ijsk}$: Loading Time spent per unit of waste $w$ in the location of customer $c$ by vehicle $k$ belonging to fleet $s$
- $f_1$: Fixed cost of using external vehicle
- $VC_k$: Variable cost of using vehicle $k$ per unit of time
- $q_{iw}$: Demand of customer $i$ for treatment of waste type $w$
- $cap_{kw}$: Maximum capacity of vehicle type $k$ for waste type $w$
- $L$: Allowable route length
- $T$: Allowable route duration
- $\Omega_d$: Capacity of depot $d$
- $\pi_{iw}$: Establishment cost of treatment center with technology $w$ in location $f$
\( \pi'_d \) Establishment cost of depot in location \( d \)

\( VR_{ij} \) Speed level between nodes \( i \) and \( j \)

\( M \) Big value

\( \omega \) Curb weight (kilogram)

\( \varepsilon \) Fuel to air mass ratio

\( \delta \) Friction level of vehicles (kilojoule/rev/liter)

\( B \) Speed of the engine of the vehicle (rev/second)

\( \mu \) Displacement of the engine of the vehicle (liters)

\( g \) Gravity constant (meter/second\(^2\))

\( Cd \) Aerodynamic drag coefficient

\( \rho \) Density of air (kilogram/meter\(^3\))

\( A_r \) Frontal area (meter\(^2\))

\( Cr \) Rolling resistance rate

\( n_{tr} \) Efficiency rate of drive train

\( \eta \) Efficiency rate of diesel engine

\( k \) Heating of a diesel fuel (kilojoule/gram)

\( \psi \) Conversion parameter (gram/second to liter/second)

\( v^l \) Lower speed limit

\( v^u \) Upper speed limit

\( WC_c \) Penalty cost per unit time of waiting at node \( c \)

**Decision variables:**

\( x_{ijk} \) 1 if vehicle type \( k \) fleet \( s \) moves directly from node \( i \) to node \( j \); 0 otherwise 0

\( y_{iw} \) 1 if treatment center with technology \( w \) is established in location \( i \); 0 otherwise

\( z_{isk} \) 1 if vehicle \( k \) fleet \( s \) is assigned to customer \( i \); 0 otherwise

\( O_i \) 1 if depot is established in location \( i \); 0 otherwise

\( U_{iskw} \) Continuous variable representing the load of compartment \( w \) vehicle \( k \) fleet \( s \) after leaving node \( i \)

\( T_{ij} \) Continuous variable that represents the total time between nodes \( i \) and \( j \)

\( d_{ij} \) Distance between nodes \( i \) and \( j \)

\( v^r \) Non-decreasing speed levels over \( R \)

\( z_{r|ijk} \) 1 if vehicle \( k \) fleet \( s \) moves from node \( i \) to node \( j \) at speed level \( r \), 0 otherwise

\( m_{ijk} \) Material flow from node \( i \) to node \( j \) by vehicle \( k \) of fleet \( s \)

The problem is formulated as follows
\[
\text{min} \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{A}} f_i x_{ij1} + \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}} V C_k \left( \left( d_{ij} / v^r \right) + t_{ijsk} q_{iw} \right) z_{isk}^r + \sum_{i \in \mathcal{I}} t_{ij0} x_{ij0} + \sum_{i \in \mathcal{I}} \pi_i y_{iw} + \sum_{i \in \mathcal{I}} \pi'_i O_i \\
\text{min} \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{A}} \delta B \mu d_{ij} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} z_{isk}^r / v^r \\
+ \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \omega y \lambda d_{ij} x_{isk} + \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \gamma \lambda d_{ij} x_{isk} \\
+ \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} \beta y \lambda d_{ij} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} z_{isk}^r (v^r)^2
\]
\[
\text{Max Min}_{i \in \mathcal{I}, j \in \mathcal{J}, w \in \mathcal{W}} \left( d_{ij} y_{jw} \right)
\]
subject to

\[
\sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{isk} = 1 \quad \forall j \in \mathcal{C}
\]
\[
\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{isk} = 0 \quad \forall s \in \mathcal{S}, k \in \mathcal{K}
\]
\[
\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left( d_{ij} / VR_{ij} \right) x_{isk} + \sum_{w \in \mathcal{W}} t_{ijw} z_{isk} \leq T \quad \forall s \in \mathcal{S}, k \in \mathcal{K}
\]
\[
\sum_{i \in \mathcal{I}} x_{isk} = z_{isk} \quad \forall j \in \mathcal{C}, s \in \mathcal{S}, k \in \mathcal{K}
\]
\[
\sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} x_{isk} \leq \Omega_i o_i \quad \forall i \in \mathcal{D}
\]
\[
U_{iskw} = 0 \quad \forall i \in \mathcal{D}, s \in \mathcal{S}, k \in \mathcal{K}
\]
\[
U_{iskw} + q_{jw} - M(1 - x_{isk}) \leq U_{iskw} \quad \forall s \in \mathcal{S}, k \in \mathcal{K}, w \in \mathcal{W}, i, j \in \mathcal{C}
\]
\[
q_{iw} \leq \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} U_{iskw} \leq \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}} x_{isk} c_{apkw} \quad \forall w \in \mathcal{W}, i \in \mathcal{C}
\]
\[
\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} d_{ij} x_{isk} \leq L \quad \forall s \in \mathcal{S}, k \in \mathcal{K}
\]
\[ \sum_{i \in C} q_{iw} z_{isk} \leq cap_{kw} \quad \forall s \in S, k \in K, w \in W \quad (13) \]

\[ \sum_{s \in S} \sum_{k \in K} \sum_{j \in F} \sum_{i \in D} x_{ijsk} = 0 \quad (14) \]

\[ \sum_{s \in S} \sum_{k \in K} \sum_{j \in E} \sum_{i \in C} x_{ijsk} = 0 \quad (15) \]

\[ \sum_{s \in S} \sum_{k \in K} \sum_{j \in E} \sum_{i \in F} x_{ijsk} = 0 \quad (16) \]

\[ \sum_{s \in S} \sum_{k \in K} \sum_{i \in F} \sum_{j \in E} \sum_{d} x_{ijsk} = \sum_{s \in S} \sum_{k \in K} \sum_{p \in P} x_{pfsk} \quad \forall f \in \mathcal{F} \quad (17) \]

\[ \sum_{i \in F} y_{iw} = 1 \quad \forall w \in W \quad (18) \]

\[ \sum_{w \in W} y_{iw} \leq 1 \quad \forall i \in \mathcal{F} \quad (19) \]

\[ \sum_{r \in R} z_{ijsk}^{r} = x_{ijsk} \quad \forall i, j \in \mathcal{A}, s \in S, k \in K \quad (20) \]

\[ x_{ijsk} = \{0,1\} \quad \forall i, j \in \mathcal{A}, s \in S, k \in K \quad (21) \]

\[ y_{iw} = \{0,1\} \quad \forall i \in \mathcal{F}, w \in W \quad (22) \]

\[ o_{i} = \{0,1\} \quad \forall i \in \mathcal{D} \quad (23) \]

\[ U_{iskw} \geq 0 \quad \forall i \in \mathcal{N}, s \in S, k \in K, w \in W \quad (24) \]

\[ m_{ijsk} \geq 0 \quad \forall i, j \in \mathcal{A}, s \in S, k \in K \quad (25) \]

Objective function (1) minimizes total cost. Objective function (2) minimizes total fuel consumption. Fuel and CO2 emissions according to the study of Barth et al. (2005) are calculated as follows:

\[ FR = \varepsilon \left( \frac{\delta \mu}{\eta} + \frac{P}{\eta} \right) \quad (26) \]

where \( P \) is the engine power output per second (in kW), which is evaluated as follows:
\[
p = \frac{\text{p}_{\text{tract}}}{{n}_{t} + p_a}
\]

Parameter \( p_a \) is the requisite engine power, \( cw \) is empty vehicle weight, \( f \) is the weight of vehicle load, and \( p_{\text{tract}} \) is wheel tractive power (in kW), which are calculated as follows:

\[
\text{p}_{\text{tract}} = \frac{(M \tau + Mg \sin \theta + 0.5 C_d \rho Av^2 + MgC_r \cos \theta)v}{1000}
\]

\[
cw = \omega + f
\]

Fuel consumption of a vehicle between nodes \( i \) and \( j \) is calculated by the following formula, where \( y, \alpha, \) and \( \beta \) are constant.

\[
F(v,cw) = \frac{\lambda ( \delta B \mu + \omega \gamma \alpha v + \gamma \alpha f v + \beta \gamma v^3) d}{v}
\]

\[
\lambda = \frac{\epsilon}{k \psi}
\]

\[
y = \frac{1}{1000 \ n_{t} \ \eta}
\]

\[
\alpha = \tau + g \sin \theta + g C_r \cos \theta
\]

\[
\beta = 0.5 \ C_d
\]

Objective (3) maximizes minimum distance between customers and treatment facilities. Eq. (4) determines that each customer should have only one route. Eq. (5) prevents direct movement between depots. Eq. (6) guarantees that serving the customers in each route is done in less than time limitation. Eq. (7) shows the relationship of two decision variables. Eq. (8) ensures that the number of vehicles leaving each depot does not exceed the capacity of the corresponding depot. Eqs. (9)-(11) show Miller–Tucker–Zemlin (MTZ) sub-tour elimination constraints (Kara et al., 2004). Eq. (12) prohibits the violation of maximum route length. Eq. (13) considers the capacity constraint. Eq. (14) prohibits travelling from depots to treatment centers. Traveling between customers and depots before visiting treatment facilities is prohibited by Eq. (15). Eq. (16) bans moving from treatment facilities to customers. Eq. (17) determines that all vehicles collecting waste should pass all treatment facilities. Eq. (18) ensures that each type of waste has one open treatment facility. Eq. (19) prohibits the participation of treatment facilities with each other. Eq. (20) ensures that traveling from node \( i \) to node \( j \) is done at only one speed level. Eqs. (21)-(25) show the ranges of the variables.

III. METHODOLOGY

In this model, the total cost and CO2 emission are minimized, and distances between the urban area and treatment centers are maximized, simultaneously. Since there exists conflict between objective functions and due to the NP-hard nature of the problem (Davis and Ray, 1969), three well-known multi-objective algorithms are applied, which have been used by many researches in the literature in this field. These algorithms are described in the following subsections.

A. Clustering approach

The metaheuristic algorithms that are used for solving this model are population-based algorithms, that is, they are initialized by a number of solutions. The solutions of the initial population should be generated by the construction algorithm. Initially, clustering of customers is performed based on the proximity coefficient, demand of each customer for collecting each type of waste, and capacity of each vehicle. Clustering through different methods can be used for
segmentation of the available nodes. In the research carried out by Barreto et al. (2007), centroid proximity measure was applied to performing clustering between customers (interested readers can refer to Barreto et al., 2007). The centroid of group R is defined as:

\[ m_R = \left( \frac{\sum_{i \in R} x_i}{|R|}, \frac{\sum_{i \in R} y_i}{|R|} \right) \]  

(35)

In this method, one customer is selected as the starting point of the group and another customer, which has the highest proximity coefficient to the starting point, is added to the group. Then, the Euclidean distance between the centroid of two clusters is calculated. The formula of Euclidean distance is the following:

\[ d(p, q) = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2} \]  

(36)

where \( p(x_p, y_p) \) and \( q(x_q, y_q) \) are two points and \( d \) is Euclidean distance between them. In addition to proximity coefficient, the capacity of vehicle is considered for adding a customer to a group. After clustering of customers, for each cluster of customers, a depot that has the biggest similarity coefficient with the group is assigned to the corresponding group.

**B. Non-dominated Sorting Genetic Algorithm (NSGA-II)**

NSGA-II was introduced by Deb et al. (2002). Then, it was developed by Rabbani et al. (2016) for solving optimization problems. The schematics of this algorithm and the steps of its process are shown in Figs. (2) and (3).
Step 1. Generate the initial random population and evaluate it.

For \( i=1 \) to maximum iterations, do

Step 2. Select parents of the population by roulette wheel operator.

Step 3. Use crossover operator to create two crossovers from parents and evaluate them. Repeat steps 2 and 3 to create \( nc \) members of children.

Step 4. Select one parent by roulette wheel operator and apply mutation operator. Then, evaluate it. Repeat step 4 to create \( nm \) members of the mutation population.

Step 5. Merge the initial population, crossover, and mutation populations.

For \( j=1 \) to last members of the merge population, do

Evaluate crowding distance

Rank each member by fast non-dominated sorting. Insert them into the Pareto front.

end

Select \( npop \) members of the merged population according to their ranks and diversity.

End

Show iteration information

Figure 3. Pseudocode of NSGA-II algorithm

C. SPEA-II

SPEA-II was introduced by Zitzler et al. (2001). The flowchart and steps of this algorithm are shown in Figs. (4) and (5).
Step 1. Generate initial random population (npop).

Step 2. Evaluate objective functions.
Step 3. Calculate fitness and strength functions.
Step 4. Select non-dominated individuals and store them in the archive.

Step 5. If (Size(Archive) < Archive size)
    Store best dominated individuals in the archive.
Else, if (Size(Archive) > Archive size)
    Omit extra individuals from the archive
end

Step 6. Select parents by tournament selection operator from the archive.
    Apply crossover and mutation operators.
    Create a new population for the next iteration.
i = i + 1
End
Step 7. Return the archive.

Figure 5. Pseudocode of SPEA-II algorithm

D. MOEA/D

Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) was presented by Zhang and Li (2007). MOEA/D transforms a multi-objective optimization problem into a single-objective problem and then, optimizes it. The steps of this algorithm are shown in Fig. (6).

Step 1. Initialization
    Set EP = ∅.
    Calculate the Euclidean distances between any two weight vectors and then, work out the closest weight vectors to each weight vector.

Step 2. Generate and evaluate a random initial population.

Step 3. Update.
Reproduction:
    Generate a new solution by using genetic operators.
Improvement:
    Improve the estimation of Pareto front (EP) by the domination function.
Update the ideal point for decomposition.
Update neighboring solutions.
Update EP.

Step 4 Stopping Criteria:
If stopping criteria are satisfied, stop and output. Otherwise, go to Step 3.

Figure 6. Pseudocode of MOEA/D algorithm
**E. Crossover & Mutation**

Crossover and mutation operators are vital for the developed algorithms. They are used for exploration and exploitation solutions. In this study, Roulette wheel operators are used to select the parents. Crossover operator formula is as follows:

\[
\begin{align*}
    y_1 &= \beta x_1 + (1 - \beta)x_2 \\
    y_2 &= \beta x_2 + (1 - \beta)x_1
\end{align*}
\]

where \( y_1 \) and \( y_2 \) are offspring, \( x_1 \) and \( x_2 \) are selected parents, and \( \beta \) is a real number between 0 and 1 generated randomly.

Mutation operators basically employ three approaches of insertion, reverse, and swap mutation. Reverse mutation selects two random genes and reverses their positions. Insertion selects two random genes and the latter gene is located after the former. Swap mutation chooses two random genes and swaps them.

**IV. PARAMETER TUNING**

Adjusting the parameters of developed algorithms has significant effect on their efficiency and reliability. The Taguchi method is applied to tuning parameters. Three levels of parameters are used in Taguchi method, as shows in Table I. Taguchi design is used for the medium-scale problem in Minitab software, as shown in Figs. (7) and (8). The results of Taguchi design for all algorithm parameters are presented in Tables II-IV.

<table>
<thead>
<tr>
<th>Level</th>
<th>n\text{pop}</th>
<th>Max it</th>
<th>( p_m )</th>
<th>( p_c )</th>
<th>Archive size</th>
<th>Number of neighbors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>50</td>
<td>0.3</td>
<td>0.3</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>150</td>
<td>0.8</td>
<td>0.8</td>
<td>150</td>
<td>25</td>
</tr>
</tbody>
</table>

![Figure 7. Analysis of Taguchi design for NSGA-II parameters](image-url)
Table II. The parameters of the NSGA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Number of iterations</th>
<th>Crossover ratio</th>
<th>Mutation ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>150</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Figure 8. Analysis of Taguchi design for SPEA-II parameters

Table III. The parameters of the SPEA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Archive size</th>
<th>Maximum iterations</th>
<th>Crossover ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Figure 9. Analysis of Taguchi design for MOEA/D parameters
M. Rabbani and H. Farrokhi-Asl. Using Metaheuristic Algorithms Combined with Clustering…

Table IV. The parameters of the MOEA/D

<table>
<thead>
<tr>
<th>Population size</th>
<th>Archive size</th>
<th>Maximum iterations</th>
<th>Number of neighbor</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>150</td>
<td>50</td>
<td>20</td>
</tr>
</tbody>
</table>

V. COMPUTATIONAL RESULTS

In the following, the experimental results for solving the test problems by means of three metaheuristic algorithms, namely non-dominated sorting genetic algorithm-II, improved strength Pareto evolutionary algorithm, multi-objective evolutionary algorithm based on decomposition, are presented. The characteristics of the initial parameters for the experiments are shown in Table V. Additionally, the numbers of customer nodes and potential locations for treatment and disposal centers in small-, medium-, and large-size problems are shown in Tables VI and VII. In order to make fair comparison between algorithms regarding run-time, Table VIII demonstrates the number of function evaluations performed in the process of solving a problem by means of each algorithm. Table IX also shows the computational times of the proposed algorithms.

Table V. The test problem data for parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>U(0,10)</td>
</tr>
<tr>
<td>Coordination</td>
<td>U(0,100)</td>
</tr>
<tr>
<td>Establishment cost of depots</td>
<td>U(10,1000)</td>
</tr>
<tr>
<td>Establishment cost of facilities</td>
<td>U(10,1000)</td>
</tr>
<tr>
<td>Fixed cost of external vehicles</td>
<td>U(10,100)</td>
</tr>
<tr>
<td>Collection time</td>
<td>U(0,10)</td>
</tr>
<tr>
<td>Capacity of vehicle</td>
<td>U(100,500)</td>
</tr>
<tr>
<td>Maximum allowable distance</td>
<td>10000</td>
</tr>
<tr>
<td>Maximum allowable travelling time</td>
<td>10000</td>
</tr>
</tbody>
</table>

Table VI. Sets for small- and medium-size problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Customers</th>
<th>Number of potential locations for depot</th>
<th>Number of potential locations for treatment centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>
Table VII. Sets for the large-size problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>Customers</th>
<th>Number of potential locations for depot</th>
<th>Number of potential locations for treatment centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>40</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>60</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>70</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>80</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>90</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>17</td>
<td>100</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>18</td>
<td>110</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Table VIII. Number of evolutionary functions

<table>
<thead>
<tr>
<th>NFE</th>
<th>NSGA-II</th>
<th>SPEA-II</th>
<th>MOEA/D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4025</td>
<td>1001</td>
<td>2551</td>
</tr>
</tbody>
</table>

The metaheuristic algorithms are compared by the following three performance metrics (Rabbani et al., 2017; Farrokhi-Asl et al., 2017).

Number of Pareto Solutions (NPS): Higher amounts of this metric imply better quality of the algorithm.

Diversity Metric (DM): How much the Pareto-optimal solution is spread in the solution space is determined by this metric through the following formula:

\[
D = \sqrt{\sum_{j=1}^{m} \max(\|p_j - q_j\|, \bar{p}, \bar{q} \in S)}
\]  

(38)

where \(S\) denotes the collection of the obtained Pareto solutions and \(m\) is equal to the number of objective functions (the dimension of the solution space).

Spacing Metric (SM): How much the Pareto-optimal solution is spread uniformly in the solution space is determined by this metric through the following formula:

\[
S = \frac{\sum_{j=1}^{M-1} |d_j - \bar{d}|}{(M - 1) \times \bar{d}}
\]  

(39)

where \(d_j\) shows the Euclidean distance between two successive solutions in the Pareto set and \(\bar{d}\) denotes the average value for all Euclidean distances. Also, \(M\) stands for the number of obtained Pareto solutions.

The qualities of the three algorithms are compared by the performance metrics in Tables X and XI. Regarding computational time, NSGA-II spends more time to obtain Pareto solutions in all test problems, no matter whether small-size or large-size. On the other hand, the computational time of MOEA/D is lower than those of other algorithms. By
investigating spacing and diversification metrics, we can find out that NSGA-II has the worst performance for all test problems. Performance of SPEA-II and MOEA/D is dependent on problems; but, in average, the best algorithm in terms of the spacing metric is MOEA/D and the best algorithm in terms of the diversification metric is SPEA-II. In contrast, NSGA-II can find the highest number of Pareto solutions. However, according to aforementioned criteria, the quality of these solutions is less than those obtained by other algorithms. It should be noted that the spacing metric for all algorithms shows that the Pareto optimal sets are uniformly distributed in the solution area. Diversity metric shows that exploration in the solution area of NSGA-II is the best. The performances of the algorithms for different problems are shown graphically in Figs. (10-12).

Table IX. Computational times (in seconds) for small- and medium-size problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>NSGA-II</th>
<th>SPEA-II</th>
<th>MOEA/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>196.29</td>
<td>67.451</td>
<td>32.286</td>
</tr>
<tr>
<td>2</td>
<td>197.34</td>
<td>68.538</td>
<td>37.896</td>
</tr>
<tr>
<td>3</td>
<td>203.104</td>
<td>75.117</td>
<td>38.041</td>
</tr>
<tr>
<td>4</td>
<td>226.350</td>
<td>86.843</td>
<td>39.117</td>
</tr>
<tr>
<td>5</td>
<td>228.839</td>
<td>90.113</td>
<td>47.854</td>
</tr>
<tr>
<td>6</td>
<td>238.763</td>
<td>97.452</td>
<td>49.850</td>
</tr>
<tr>
<td>7</td>
<td>249.934</td>
<td>101.893</td>
<td>50.204</td>
</tr>
<tr>
<td>8</td>
<td>273.504</td>
<td>112.734</td>
<td>57.902</td>
</tr>
<tr>
<td>9</td>
<td>277.269</td>
<td>115.370</td>
<td>63.081</td>
</tr>
<tr>
<td>10</td>
<td>284.741</td>
<td>115.903</td>
<td>65.497</td>
</tr>
</tbody>
</table>

Table X. Computational times (in seconds) of the large-size problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>NSGA-II</th>
<th>SPEA-II</th>
<th>MOEA/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>301.930</td>
<td>124.904</td>
<td>71.120</td>
</tr>
<tr>
<td>12</td>
<td>306.117</td>
<td>129.638</td>
<td>74.602</td>
</tr>
<tr>
<td>13</td>
<td>313.002</td>
<td>137.004</td>
<td>76.301</td>
</tr>
<tr>
<td>14</td>
<td>323.603</td>
<td>138.027</td>
<td>79.205</td>
</tr>
<tr>
<td>15</td>
<td>331.734</td>
<td>143.081</td>
<td>85.930</td>
</tr>
<tr>
<td>16</td>
<td>330.211</td>
<td>146.802</td>
<td>87.021</td>
</tr>
<tr>
<td>17</td>
<td>338.501</td>
<td>155.112</td>
<td>96.201</td>
</tr>
<tr>
<td>18</td>
<td>340.346</td>
<td>156.720</td>
<td>97.303</td>
</tr>
<tr>
<td>19</td>
<td>469.597</td>
<td>164.392</td>
<td>100.301</td>
</tr>
<tr>
<td>20</td>
<td>474.329</td>
<td>169.067</td>
<td>105.872</td>
</tr>
</tbody>
</table>
Table XI. Experimental results for the performance metrics in small- and medium-size problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>NSGA-II</th>
<th>SPEA-II</th>
<th>MOEA/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPS</td>
<td>DM</td>
<td>SM</td>
<td>NPS</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>305.872</td>
<td>0.485</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>473.720</td>
<td>0.383</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>539.018</td>
<td>0.642</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>570.309</td>
<td>0.591</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>578.943</td>
<td>0.573</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>599.221</td>
<td>0.673</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>607.131</td>
<td>0.731</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>690.364</td>
<td>0.752</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>759.082</td>
<td>0.533</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
<td>840.937</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table XII. Experimental results for the performance metrics in the large-size problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>NSGA-II</th>
<th>SPEA-II</th>
<th>MOEA/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPS</td>
<td>DM</td>
<td>SM</td>
<td>NPS</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>1095.118</td>
<td>0.841</td>
</tr>
<tr>
<td>12</td>
<td>19</td>
<td>1238.234</td>
<td>0.742</td>
</tr>
<tr>
<td>13</td>
<td>17</td>
<td>1694.882</td>
<td>0.730</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
<td>1733.830</td>
<td>0.526</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
<td>2956.239</td>
<td>0.601</td>
</tr>
<tr>
<td>16</td>
<td>18</td>
<td>2860.754</td>
<td>0.647</td>
</tr>
<tr>
<td>17</td>
<td>21</td>
<td>3116.302</td>
<td>0.734</td>
</tr>
<tr>
<td>18</td>
<td>24</td>
<td>3740.371</td>
<td>0.590</td>
</tr>
<tr>
<td>19</td>
<td>27</td>
<td>3940.230</td>
<td>0.634</td>
</tr>
<tr>
<td>20</td>
<td>31</td>
<td>4955.027</td>
<td>0.791</td>
</tr>
</tbody>
</table>

For more details, the comparison of algorithms for test problem 10 is presented in Figs. (14-16). It should be noted that the minimum amount for the first and second objective functions belongs to NSGA-II algorithm.

VI. CONCLUSION

This paper proposed a new location routing model and applied it to the waste collection network. We considered sustainability issues in order to make our model more reliable. Three distinct goals including economic costs, fuel consumption and $\text{CO}_2$ emission, and social effects of undesirable facilities were optimized, simultaneously. Economic costs consisted of transportation, customer servicing, and opening costs of depots and treatment facilities. Fuel consumption rate was calculated based on distance and weight of material flow between two nodes as well as the speed level of vehicles. The third objective function minimized maximum distance between urban areas (customer nodes) and
treatment facility centers. The reason is that treatment facilities usually have negative effects on human health. In order to solve the problem, three widely known metaheuristic algorithms, namely SPEA-II, NSGA-II, and MOEA/D, were used. Additionally, clustering method was adopted to cluster customers and depots for these algorithms. Through this procedure, the algorithms could be initiated with higher quality solutions than the solution obtained by the simple random initialization approach. Then, the parameters of these algorithms were tuned by Taguchi design method to achieve more efficient solutions. Finally, some test examples were utilized so as to compare performance of the algorithms. From the computational results, we concluded that, despite the high running time of NSGA-II, the quality of the obtained solutions by this method was not acceptable. By investigating spacing and diversification metrics, it was concluded that NSGA-II had the worst performance for all test problems. Performance of SPEA-II and MOEA/D was dependent on the problems; but, in average, the best algorithm in terms of the spacing metric was MOEA/D and the best algorithm in terms of the diversification metric was SPEA-II. In contrast, NSGA-II could find the highest number of Pareto solutions. However, regarding the aforementioned criteria, the quality of these solutions were less than those of other algorithms. As a future research direction, we suggest using SPEA-II and MOEA/D for solving similar optimization problems, especially the location routing ones.

Figure 10. Comparison of the computational times for larg-size problems

Figure 11. Comparison of the numbers of Pareto solutions for the larg-size problem
Figure 12. Comparison of the diversity metrics for the large-size problem

Figure 13. Comparison of the spacing metrics for the large-size problem
Using Metaheuristic Algorithms Combined with Clustering…

Figure 15. Comparison of metaheuristic algorithms for problem10 in terms of cost function

Figure 16. Comparison of metaheuristic algorithms for problem10 in terms of total fuel consumption function

Figure 17. Comparison of metaheuristic algorithms for problem10 in terms of social effect function
REFERENCES


