

## **Cooperative Advertising and Pricing in a Supply Chain: A Bi-level Programming Approach**

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**Abstract-** Nowadays, coordination between members in a supply chain has become very important and beneficial to channel members. Through cooperative advertising, manufacturers and retailers can jointly participate in promotional programs. This action not only reduces the cost of advertising, but also is important to create a link with local retailers in order to increase immediate sales at the retail level. In this article, the problem of cooperative advertising and pricing decisions in a multi-product manufacturer-retailer (oligopoly market) supply chain is investigated. Stackelberg game with leadership of the manufacturer is proposed to model the problem. In order to find optimal prices and advertising expenditure, the bi-level programming approach is implemented. Solutions for the first level are determined by a genetic algorithm and best responses of retailers to the generated solutions of the manufacturer are calculated by CPLEX. Finally, numerical experiments and sensitivity analysis are conducted in order to assess the efficiency of models and solution procedures. Results show that competition will lead to a lower retail price, which is preferable from the consumers' point of view. Also, profit of the manufacturer and retailers will decrease if competition effect increases.

**Keywords:** Cooperative advertising, Pricing, Stackelberg game, Genetic algorithm, Bi-level programming

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### **I. INTRODUCTION**

The literature on channel coordination demonstrates that channel members benefit from cooperation (Li & Liu, 2015); (Mirzaee et al., 2012); (Taheri et al., 2015). The cooperative advertising has become a popular strategy that is used by many manufacturers to boost sales. The fraction of the retailer's advertising expenditure, which is supplied by the manufacturer, is determined by the participation rate. Cooperative advertising is the relationship between a manufacturer and a retailer, in which the manufacturer pays a proportion of the advertising costs of the retailer in order to cooperate with the retailer to advertise locally. In the marketing and economics literature, cooperative advertising models in a supply chain between a manufacturer and a retailer generally focus on a game in which the manufacturer is the leader followed by the retailer. This means that the manufacturer has great power and complete control over the retailers' behavior.

Cooperative advertising is a very common practice in the United States. Recent reports indicate that US firms offered about \$36billion (12% of their total advertising costs) to their retailers in 2015 (Borrell Associates Report, 2015). The participation rates of the manufacturers vary heavily from 25% at General Motors to 50% at IBM and 75% at Apple (Xie & Wei, 2009). These values show that in most of the cases, the manufacturer determines their participation rate arbitrarily, rather than through detailed analysis, and undermines the necessity of theoretical examination.

Growing trend of co-op ad programs in developed countries expresses importance and application of these programs in order to increase profit of the supply chain members. As there is an economic downturn in most of the developing countries, co-op ad programs could help the manufacturers and retailers to increase their sale and profit as well as their business continuity. On the other hand, it could be expressed that in developing countries, there are some cooperative advertising programs between the manufacturer and retailers, but scientific determination of the participation rate and

other variables is neglected.

Numerous studies have examined the key impacts of co-op ad in supply chains with one manufacturer and one retailer. The results of these researches indicate that co-op ad activities are powerful motivations that increase advertising of the retailer. Thus, consumers' demand will expand and, eventually, enhance profits of both the manufacturer and the retailer (Aust, 2015); (Aust and Buscher, 2014); (Huang et al., 2002); (Jørgensen and Zaccour, 2014); (Zhang et al., 2013). Almost the entire literature on cooperative advertising confirms the effectiveness of collaborative advertising activities in increasing profits of members.

The analysis in most of the studies on cooperative advertising with static environment is limited only to advertising decisions of channel members (Ahmadi-Javid and Hoseinpour, 2015); (Amir Farshbaf-Geranmayeh, 2017a); (Amir Farshbaf-Geranmayeh, 2017b); (Wang et al., 2011); (Yang et al., 2013); (Zhang et al., 2013). However, in some studies, in addition to advertising, analysis of other variables such as price (Aust, 2015); (Karray & Surti, 2016); (SeyedEsfahani et al., 2011); (Zegordi & Mokhlesian, 2015); (Zhang and Zhang, 2016) and quality (Marchi and Cohen, 2009) in the channel is considered. Moreover, some studies have investigated cooperative advertising in a dynamic environment (Jørgensen et al., 2000); (Jørgensen et al., 2001); (Jørgensen et al., 2003); (Jørgensen et al., 2006); (Karray and Zaccour, 2005). Recently, two comprehensive reviews of cooperative advertising literature were provided by (Jørgensen and Zaccour 2014) and (Aust and Buscher 2014a). The literature indicates that, in a bilateral monopoly, cooperative advertising program effectively increases advertising efforts of the retailer and leads to higher demand and profit for both the retailer and the manufacturer. A classification of the cooperative advertising literature (static models) is illustrated in Table I.

**TABLE I.** Classification of cooperative advertising literature

Study	Decisions		Market Type	Number of products	Game structure	Advertising budget constraint
	Advertising	Pricing				
Yang et al. (2013)	✓		Monopoly	Single	SM	
Ahmadi-Javid and Hoseinpour (2015)	✓		Monopoly	Single	N, SM	
Amir Farshbaf-Geranmayeh (2017b)	✓		Monopoly	Single	N, SM	
Amir Farshbaf-Geranmayeh (2017a)	✓		Monopoly	Single	N, SM, CO	
Alaei and Setak (2016)	✓		Duopoly	Single	SM	
Wang et al. (2011)	✓		Duopoly	Single	N, SM	
Karray and Surti (2016); Martín-Herrán and Sigué (2017)	✓	✓	Monopoly	Single	SM	
SeyedEsfahani et al. (2011)	✓	✓	Monopoly	Single	N, SM, SR, CO	
Zhang and Zhang (2016); Aust and Buscher (2014b)	✓	✓	Duopoly	Single	SM	
Aust (2015);	✓	✓	Duopoly	Single	N, SM, SR, CO	
Taleizadeh and Charmchi (2015)	✓	✓	Monopoly	Two	SR, SM	
Zegordi and Mokhlesian (2015)	✓	✓	Monopoly	Multiple	SM	
This Study	✓	✓	Oligopoly	Multiple	SM	✓

\*N: Nash; SM: Stackelberg (manufacturer as a leader); SM: Stackelberg (retailer as a leader); CO: Cooperation (joint profit)

As can be seen, most of the papers in this area considered monopoly market type with a single product in order to simplify the model. Therefore, considering oligopoly market with multiple products would increase complexity of calculus, considerably, which must be solved by rendering proper computer-based algorithms or meta-heuristics (Yu & Huang, 2010). The structure of a bi-level programming problem (BLPP) facilitates the formulation of the problems that involve a hierarchical decision making process. Therefore, in the literature, bi-level programming is widely applied for modeling interactions among the decision makers (Aviso et al., 2010); (Fang et al., 2013); (Jenabi et al., 2013); (Sun et al., 2008). However, in cooperative advertising area, only a few papers, e.g., (Zegordi & Mokhlesian 2015), considered the bi-level programming approach in formulation of advertising decisions of supply chain members. In addition to (Zegordi & Mokhlesian 2015), the most significantly related research to this study is that of (Sadigh et al. (2012), which applied bi-level programming in order to determine the optimal equilibrium prices, advertising expenditures, and production policies considering multi products in a manufacturer-retailer supply chain. (Sadigh et al. 2012) and (Zegordi & Mokhlesian 2015) applied Imperialist Competitive Algorithm (ICA) and Particle Swarm Optimization (PSO) in order to solve the first-level decisions, respectively.

In this study, unlike (Sadigh et al. 2012), we consider  $m$  retailers and the demand of each retailer is dependent on not only their own prices and advertising level, but also the prices of other retailers for the products. Also, cooperative advertising (national advertising and participation rate), which is the main focus of this study, was not considered in the study of (Sadigh et al. 2012).

In this study, cooperative advertising and pricing decisions in a multi-product manufacturer-retailer supply chain with retail competition are investigated. Since leadership of the manufacturer in the Stackelberg game is the common scenario in the related literature (see Table I and (Jørgensen & Zaccour 2014)), we assume that the manufacturer is the Stackelberg leader in the channel. In order to find the optimal prices and advertising expenditure, bi-level programming approach is implemented. Therefore, solutions for the upper level are determined by a genetic algorithm and the best responses of retailers to the generated solutions of the manufacturer are calculated by CPLEX.

The rest of the paper is organized as follows: in Section II, the main assumptions, notations, and formulation of the problem are presented. Bi-level formulations of the Stackelberg game are presented in Section III and solution procedures are proposed in Section IV. Computations and sensitivity analysis are reported in Sections V and VI, respectively. Finally, the conclusion and some future research directions are provided in Section VII.

## II. MODEL FRAMEWORK

In this section, key assumptions of the problem and its mathematical formulation are presented.

### A. Model assumptions

- The supply chain includes one manufacturer and  $m$  retailers, which are competing with each other. It means that each retailer's demand depends on not only their own prices, but also prices determined by other retailers for the products.
- The manufacturer produces  $n$  different products and retailers sell them.
- The manufacturer spends national advertising expenditure to create an image of the brand and enhance their sale. On the other hand, the retailer can separately advertise each product in order to increase their sale.
- A part of retailers' marketing efforts is paid by the manufacturer. Participation rates for all retailers are equal.
- Demand for products depends on retail price of the product as well as advertising expenditure of the manufacturer and retailers.

### B. Problem description

#### Indices:

$i$ : Index of products  $(1, 2, \dots, n)$

$r$ : Index of retailers  $(1, 2, \dots, m)$

#### Parameters

$D_0$ : Base demand

$k_1$ : Effectiveness of global advertising of the manufacturer

$k_2$ : Effectiveness of local advertising of retailers

$\alpha_i$ : Parameter to measure scale market for the  $i^{\text{th}}$  product

$\beta_{ir}, \gamma_{ir}$ : Price elasticity of demand for the  $i^{\text{th}}$  product

$c_i$ : Production costs per unit of each product

$B_m$ : Advertising budget of the manufacturer

$B_r$ : Advertising budget of the  $r^{\text{th}}$  retailer

#### Variables:

$w_i$ : Wholesale prices for the  $i^{\text{th}}$  product

$p_{ir}$ : Retail price of the retailer  $r$  for the  $i^{\text{th}}$  product

$a_{ir}$ : Advertising expenditure of the retailer  $r$  for the product  $i$

$A$ : Expenditure on national advertising from the manufacturer

$D_{ir}$ : Demand for product  $i$  from retailer  $r$

$D_i$ : Total demand for product  $i$

$t$ : Participation rate

$\Pi_m$ : Profit function of the manufacturer

$\Pi_r$ : Profit function of the retailer  $r$

Demand is in general given by  $D(a, A, p)$  where  $a$  and  $A$  are advertising expenditures by retailer and manufacturer and  $p$  is the retail price. Demand function  $D$  is concave and increasing in  $a$  and  $A$ , decreasing in  $p$  (Jørgensen & Zaccour, 2014). In most of the papers in cooperative advertising literature, demand function is multiplicatively separable in advertising and price:  $D(a, A, p) = D_0 d(p) g(a, A)$  (Aust, 2015; (Chaab & Rasti-Barzoki, 2016); (SeyedEsfahani et al., 2011); (Xie & Neyret, 2009); (Xie & Wei, 2009)). Since, in this study, there are  $m$  retailers and  $n$  products, demand function of each retailer for each product is as Eq. (1):

$$D_{ir}(a_{ir}, A, p_{ir}) = D_0 d(p_{ir}) g(a_{ir}, A) \quad (1)$$

By considering Eq. (1), total demand for each retailer could be calculated as Eq. (2).

$$D_i = \sum_{r=1}^m D_{ir} \quad (2)$$

Aust & Buscher (2014b) showed that in the case of duopoly market (two competitive retailers) in which the retailers sell a single product,  $d(p_r)$  in Eq. (1) equals  $\alpha_i - \beta_i p_r + \delta p_{3-r}$ . Therefore, for the proposed problem,  $d(p_r)$  would be obtained from Eq. (3).

$$d(p_{ir}) = \alpha_i - \beta_i p_{ir} + \sum_{c \neq r}^n \delta_{ic} p_{ic} \quad (3)$$

Furthermore, it is assumed that local advertising of one retailer will also influence the demand of their competitor (Aust & Buscher, 2014b); (Karray & Zaccour, 2007). Combining these characteristics, the following sales response function is obtained, which is valid for every channel member:

$$g(a_{ir}, A) = k_1 \sqrt{A} + k_2 \sqrt{a_{ir}} \quad (4)$$

By considering Eqs. (1)-(4), demand function for the  $i^{\text{th}}$  product of retailer  $r$  could be calculated as Eq. (5).

$$D_{ir}(a_{ir}, A, p_{ir}) = D_0 \left( \alpha_i - \beta_i p_{ir} + \sum_{c \neq r} \delta_{ic} p_{ic} \right) (k_1 \sqrt{A} + k_2 \sqrt{a_{ir}}) \quad (5)$$

Profit function of the manufacturer and retailers is equal to the difference between sales revenue and advertising cost (considering participation rate of the manufacturer). Therefore, the profit of the channel members could be calculated as Eqs. (6)-(7).

$$\Pi_m = \sum_{i=1}^n (w_i - c_i) D_i - t \sum_{j=1}^m \sum_{i=1}^n a_{ij} - A \quad (6)$$

$$\Pi_r = \sum_{i=1}^n (p_{ir} - w_i) D_{ir} - (1-t) \sum_{i=1}^n a_{ir}, \quad r = 1, \dots, m \quad (7)$$

Advertising budget constraints of the manufacturer and each retailer are defined as Eq. (8) and Eq. (9), respectively.

$$t \sum_{r=1}^m \sum_{i=1}^n a_{ir} + A \leq B_m \quad (8)$$

$$(1-t) \sum_{i=1}^n a_{ir} \leq B_r, \quad r = 1, \dots, m \quad (9)$$

### III. METHODOLOGY

In Stackelberg manufacturer game, the manufacturer is considered as leader and the retailer as follower in the channel. In this asymmetric relationship, Stackelberg game can be applied to model the problem, in which the manufacturer decides on wholesale price of each product and participation rate by taking into account the best response decision of the retailer about their advertising and retail price.

#### A. Bi-level programming

Bi-level programming approach provides a framework to deal with situations in which a leading firm incorporates within its decision process the reaction of a following firm into its course of action (Marcotte & Savard, 2005). Bi-level problems are closely associated with Stackelberg games and Mathematical Programs with Equilibrium Constraints (MPEC) that are both characterized by two levels of optimization problems in which the constraint region of the upper level problem is implicitly determined by the lower level optimization problem (Colson et al., 2007).

Bi-level programming is a hierarchy of optimization problems (the problem of high-level and low-level or leader and follower). Every decision maker optimizes the objective function regardless of the other decision maker's objective function; however, decisions of each side affect the optimality of the other side as well as the decision space. In recent years, meta-heuristic approaches have widely been used to solve bi-level and multilevel programming.

In this problem, profit function of the manufacturer is assumed as the first and second levels, simultaneously. Bi-level programming can be considered to formulate this model, in which  $(w_1, w_2, \dots, w_n)$ ,  $A$ , and  $t$  at upper level are determined by the manufacturer subject to constraints of advertising budget. At lower level, reaction of the retailer would be choosing optimal prices and advertising policies, which are based on their own optimization model. Accordingly, the model can be

represented as follows:

**Upper-level problem:**

$$\text{Max}_{w_i, A} \Pi_m = \sum_{i=1}^n (w_i - c_i) D_i - t \sum_{r=1}^m \sum_{i=1}^n a_{ir} - A \quad (10)$$

$$\text{Subject to: } t \sum_{r=1}^m \sum_{i=1}^n a_{ir} + A \leq B_m, \quad (11)$$

**Lower-level problem:**

$$\text{Max}_{p_{ir}, a_{ir}} \Pi_r = \sum_{i=1}^n (p_{ir} - w_i) D_{ir} - (1-t) \sum_{i=1}^n a_{ir} \quad (12)$$

$$\text{Subject to: } (1-t) \sum_{i=1}^n a_{ir} \leq B_r \quad (13)$$

The stackelberg equilibrium is the optimal solution derived from this structure.

#### IV. SOLUTION PROCEDURES

**Consider a bi-level model:**

$$\text{Max}_{x,y} f(x, y)$$

$$\text{Subject to: } (x, y) \in X \quad (14)$$

$$y \in S(x), \quad (15)$$

where:

$$S(x) = \underset{y}{\text{argmax}} g(x, y) \quad (16)$$

$$\text{Subject to: } (x, y) \in Y \quad (17)$$

In the proposed solution procedure, the first step is to generate upper-level decision value  $x$  and, then, to move this initial solution through an exploratory process in order to get a new solution. After every iteration, solving the lower level problem results in the optimal reaction  $y^*$  to be obtained and returned to the upper level model. This procedure goes on until it reaches an optimal or non-linear solution to the upper level problem.

Due to the NP-hard nature of the bi-level models, a solution procedure based on Genetic Algorithm (GA) is proposed to search the equilibrium solutions. Fig (1) demonstrates the flowchart of the solution procedure for solving bi-level programs. The chromosome applied in GA for solving the upper level problem within the range of each variable is shown in Fig (2). As can be seen in Fig (1), solutions to the first level are determined by a genetic algorithm and best responses of retailers to the generated solutions of the manufacturer are calculated by CPLEX. Recently, some researchers applied the combination of GA and CPLEX for solving optimization problems. For instance, (Rabbani et al. 2016) used GA to obtain the values of the integer variables and calculated the values of continuous variables by solving the corresponding linear programming by CPLEX.

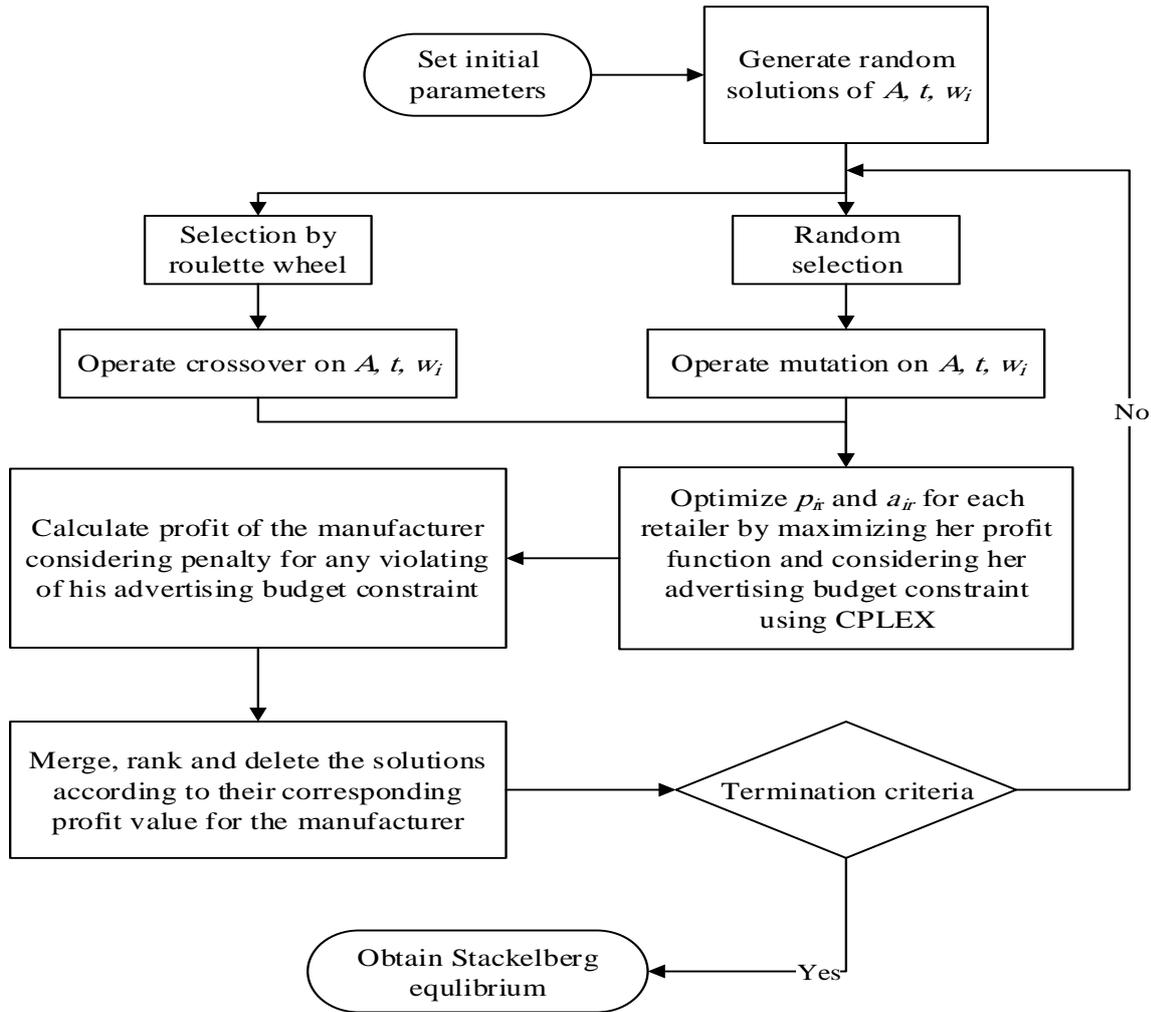


Fig 1. The flowchart of the solution procedure

**V. COMPUTATIONAL EXPERIMENTS**

Numerical experiments are conducted in this section in order to assess the ability of the solution process to find the optimum equilibrium in the manufacturer’s Stackelberg game. The experiments are implemented in two categories; firstly for test problems with single retailer and secondly for test problems with multiple retailers. Table II includes the parameters of the model, which are randomly generated to perform the test problems.

Since performance of the evolutionary algorithms strongly depends on their parameters (Bashiri & Geranmayeh, 2011),  $L27$  orthogonal array of experimental plan is chosen for parameter tuning and five replications for each experiment are considered. Fig (3) shows the effect of these factors on the mean profit of the manufacturer from which the optimum values of the significant parameters in the proposed GA for the 20<sup>th</sup> test problem are obtained (Table III). Performance of the tuned GA for optimizing the test problem is shown in Fig (4).

$A \in [0, B_m]$	$t \in [0, 1]$	$w_1 \in \left[0, \frac{\alpha}{\beta_1}\right]$	$w_2 \in \left[0, \frac{\alpha}{\beta_2}\right]$	...	$w_n \in \left[0, \frac{\alpha}{\beta_n}\right]$
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Fig 2. Chromosome of the proposed GA within the range of each variable

TABLE II. The values of parameters

$k_2$	0.5	$\gamma_{ir}$	$\sim$ Uniform(0.1,0.3)
$k_1$	0.7	$c_i$	$\sim$ Uniform(1.5,3)
$\alpha_i$	$\sim$ Uniform(10,15)	$B_m$	100
$\beta_{ir}$	$\sim$ Uniform(2.5,4.5)	$B_r$	$\sim$ Uniform(20,25)
$D_0$	100		

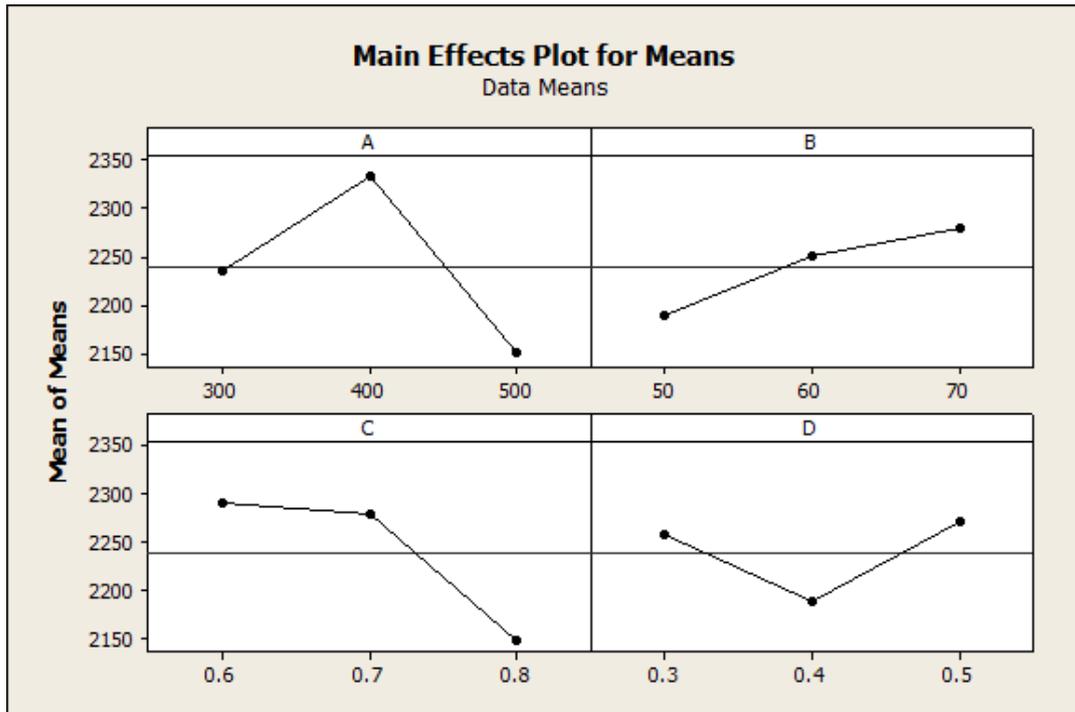


Fig 3. Results of the Taguchi method

In order to validate the proposed solution procedure, first, experiments are carried out on 5 small-size instances. Here, small-size instances are for problems with a single retailer, which are optimally solvable in GAMS software using JAMS solver. JAMS is a solver to handle Extended Mathematical Programming (EMP) problems, including bi-level programming problems. The solutions obtained from the proposed solution procedure (mixture of CPLEX solver and GA) are compared with optimal solutions to these instances to verify the efficiency of the proposed algorithm (Table IV).

The designed test problems (including small- and large-size instances) and the summary of their results, which are obtained by using the proposed solution procedure, are presented in Table V. It is notable that GA is run 10 times for each designed test problem and the best results of replications are reported in the table. Standard deviation of the manufacturer profit (objective function in genetic algorithm) is shown in the last column of Table V.

TABLE III. Result of parameter tuning

Iteration number	Population size	Crossover rate	Mutation rate
400	70	0.6	0.5

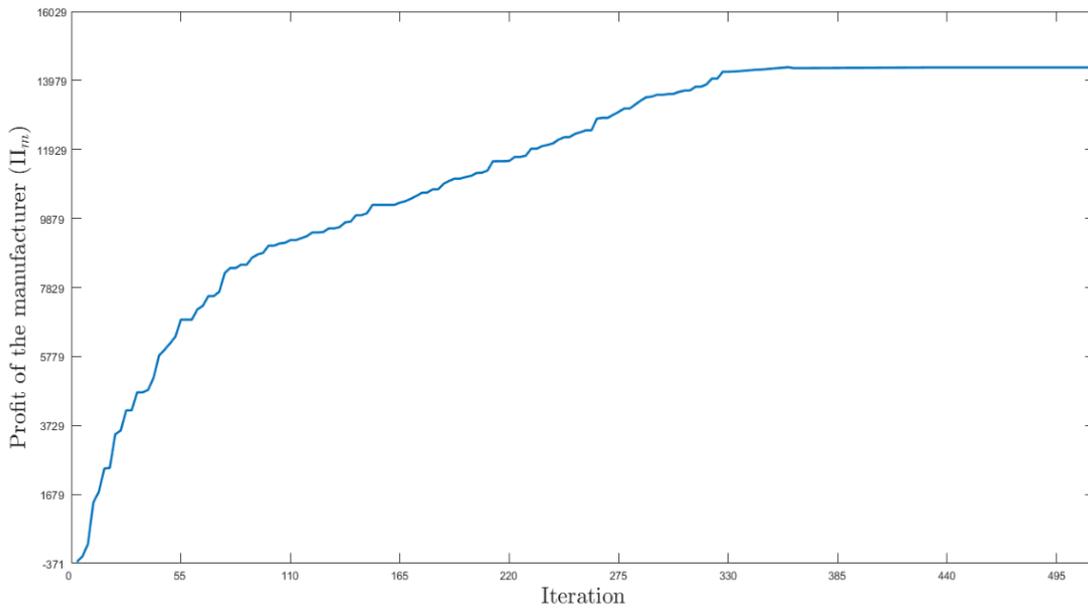


Fig 4. Performance of the proposed GA in the 20<sup>th</sup> test problem

TABLE IV. Comparison results for test problems with a single

Test Problem	Number of retailers ( $m$ )	Number of products ( $n$ )	Profit of the manufacturer ( $\Pi_m$ )		
			JAMS solver	Proposed solution procedure	Gap (%)
1	1	1	2588.17	2515.91	2.87%
2	1	3	3009.41	2899.30	3.80%
3	1	5	3560.37	3500.82	1.70%
4	1	7	5193.02	5051.06	2.81%
5	1	9	5728.14	5556.53	3.09%

VI. SENSITIVITY ANALYSIS

Since parameters associated with effect of both advertising ( $k_1$  and  $k_2$ ) and competition  $\gamma_{ir}$  are the most important parameters in the proposed model, sensitivity analysis is conducted for these parameters. For analysis of the effect of advertising, the second test problem, i.e., one retailer with three products, is considered. Parameters used for the sensitivity analysis are shown in Table 6 and results of each case appear in Table A1 in Appendix section.

Fig (5) and (6) illustrate profit sensitivity analysis of the manufacturer and the retailer to changes of local and national advertising effectiveness. As expected, due to positive coefficient of these parameters in profit of the manufacturer, it increases as  $k_1$  and  $k_2$  increase. Fig (6) shows that increase in  $k_1$  leads to increase in profit of the retailer, too. However, this is not always true for  $k_2$  from the retailer’s point of view, since the retailer is the follower in the game.

TABLE V. Summary of the results

Test Problem	Number of retailers ( $m$ )	Number of products ( $n$ )	Advertising expenditure of manufacturer ( $A$ )	Participation rate ( $t$ )	Profit of the manufacturer ( $\Pi_m$ )	Average of retailers' profit ( $E(\Pi_r)$ )	Standard deviation ( $\sigma_{\Pi_m}$ )
1	1	1	64.79	0.42	2515.91	612.34	81.8
2	1	3	85.98	0.39	2899.30	1030.22	96.8
3	1	5	62.9	0.28	3500.82	2090.15	85.0
4	1	7	67.62	0.26	5051.06	2844.44	168.4
5	1	9	62.93	0.26	5556.53	3304.99	113.8
6	2	1	73.42	0.22	4651.68	856.75	159.2
7	2	3	97.36	0.22	5819.95	1950.46	177.8
8	2	5	100	0.22	6147.94	2410.68	201.4
9	2	7	72.37	0.2	7186.89	2564.25	263.3
10	2	9	83.57	0.25	7050.18	2951.36	247.8
11	3	1	81.45	0.25	8558	502.9	294.9
12	3	3	72.38	0.23	8650.34	612.27	238.0
13	3	5	96.85	0.18	9709.01	1561.38	229.9
14	3	7	81.78	0.23	9837.53	1871.58	268.4
15	3	9	82.38	0.22	11567.95	2464.33	322.5
16	4	1	99.59	0.13	10891.2	117.62	309.8
17	4	3	93.71	0.12	13240.14	424.47	371.7
18	4	5	100	0.1	13036.97	1303.17	411.0
19	4	7	93.43	0.1	12568.32	1361.98	386.6
20	4	9	90.09	0.13	14674.34	1969.617	297.3

TABLE VI. Parameter values used for the sensitivity analysis

$\alpha_i$	[13.03, 10.84, 14.08]	$c_i$	[1.70, 1.94, 2.53]
$\beta_{ir}$	[4.30, 3.74, 2.97]	$B_m$	100
$D_0$	100	$B_r$	20

Fig (7) and (8) show changes of wholesale and retail prices of three considered products, respectively. As can be seen, both wholesale and retail prices are higher for product 3 than for others. This is due to values of parameters  $\alpha_i$  and  $\beta_{ir}$  for this product, which are greater and less than those of other products, respectively (Table VI). It means that maximum demand of this product is the greatest and its demand sensitivity to price is the least among the considered products.

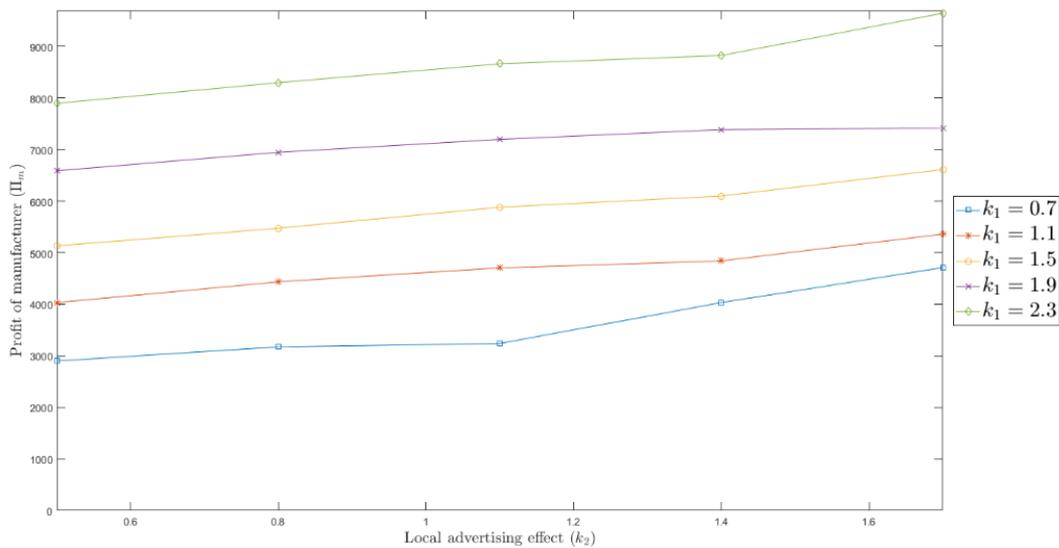


Fig 5. Sensitivity analysis of the manufacturer’s profit

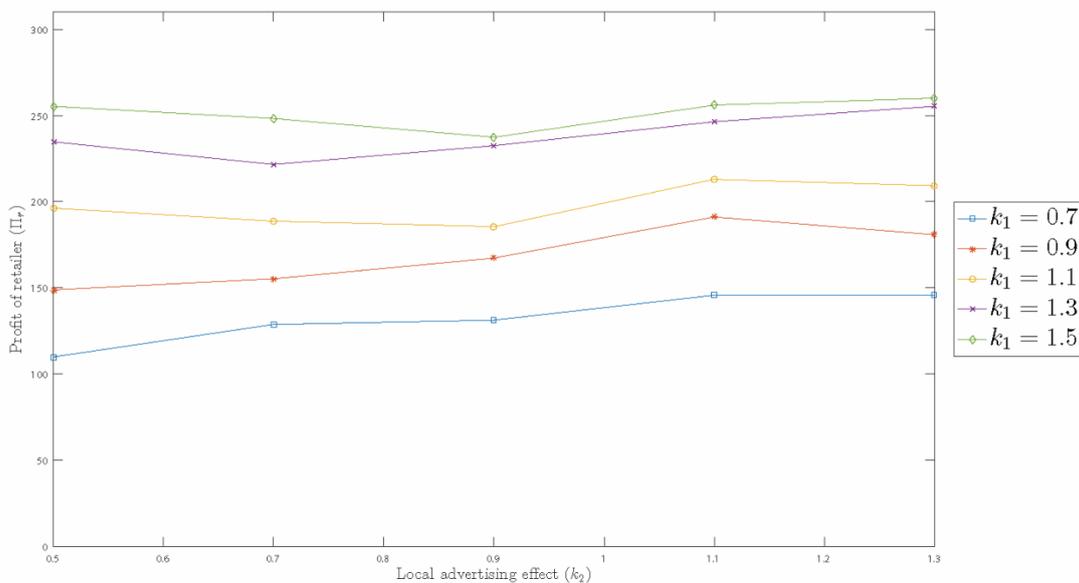


Fig 6. Sensitivity analysis of the retailer’s profit

The effects of competition ( $E(\gamma_{ir})$ ) on profits of the manufacturer and retailers in the 20<sup>th</sup> test problem are shown in Fig (9) and (10), respectively. In Fig . (10), as can be seen, profit of each retailer is illustrated in a separate plot. It is demonstrated that profits of both the manufacturer and the retailers decrease as competition effect increases. In regard to the decrease in profit of the channel members, it could be explained that intensive competition will lead to a lower retail price, which is preferable from the consumers’ point of view. Therefore, the profit of the retailers will decrease with growing competition, which would in turn lead to decrease in profit of the manufacturer.

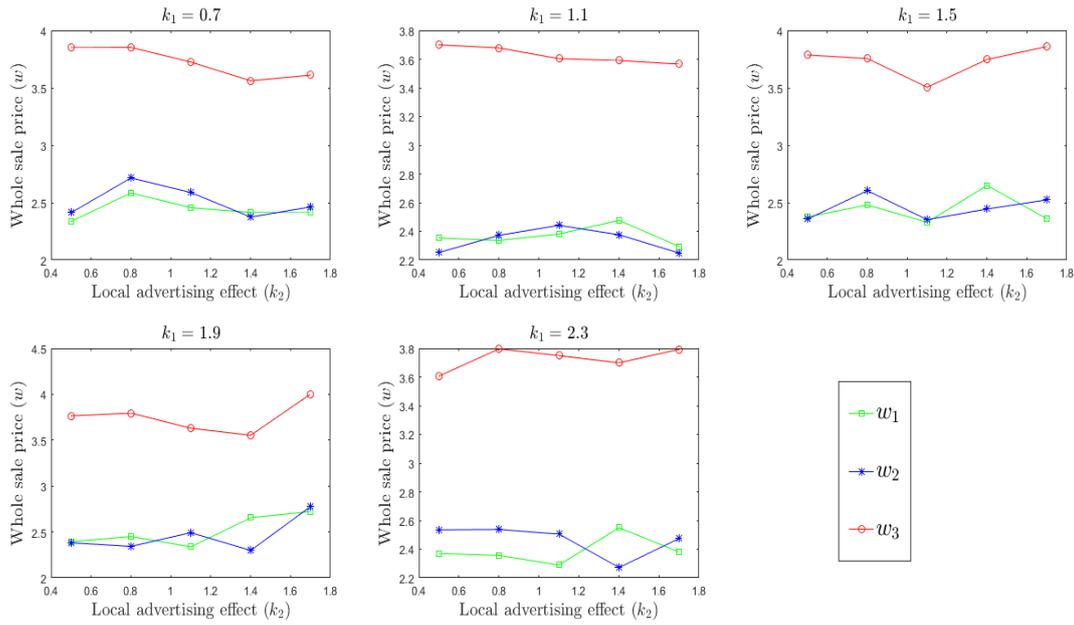


Fig 7. Sensitivity analysis of wholesale price

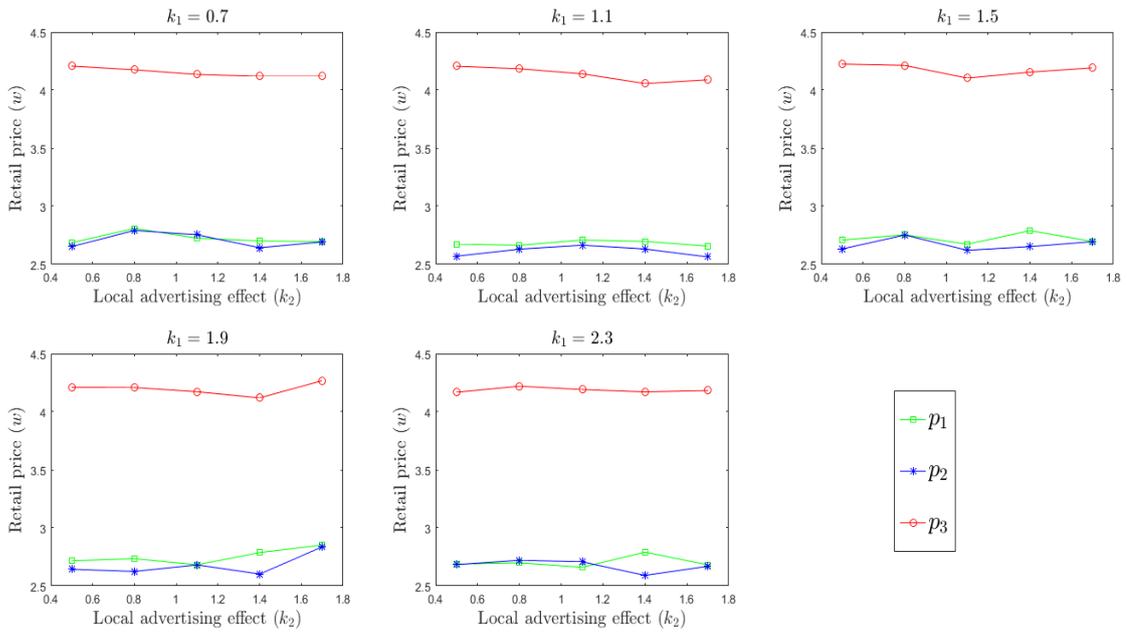


Fig 8. Sensitivity analysis of retail price

**VII. CONCLUSION**

In this paper, a multi-product manufacturer-retailer supply chain, in which the demand depended on price as well as advertising expenditure, was studied. It was assumed that there were some retailers in the market and their demands depended on prices of other retailers (oligopoly market). A Stackelberg game framework was developed, in which the manufacturer led the supply chain. The models were formulated using bi-level optimization approach in order to find the optimal equilibrium for wholesale and retail prices as well as advertising expenditures of the channel members and their

participation rates. A genetic algorithm was applied as solution procedure to solve Stackelberg game. Numerical experiments were carried out for validation and evaluation purposes. Results of the sensitivity analysis showed that profit of the manufacturer and retailers would decrease if competition effect increased.

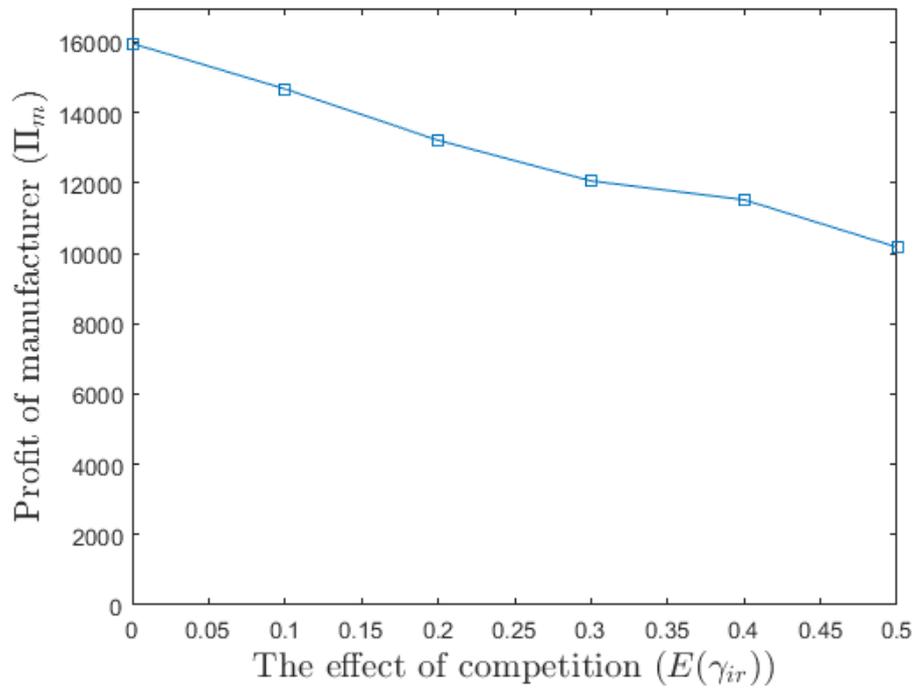


Fig 9. Impact of competition on the manufacturer’s profit

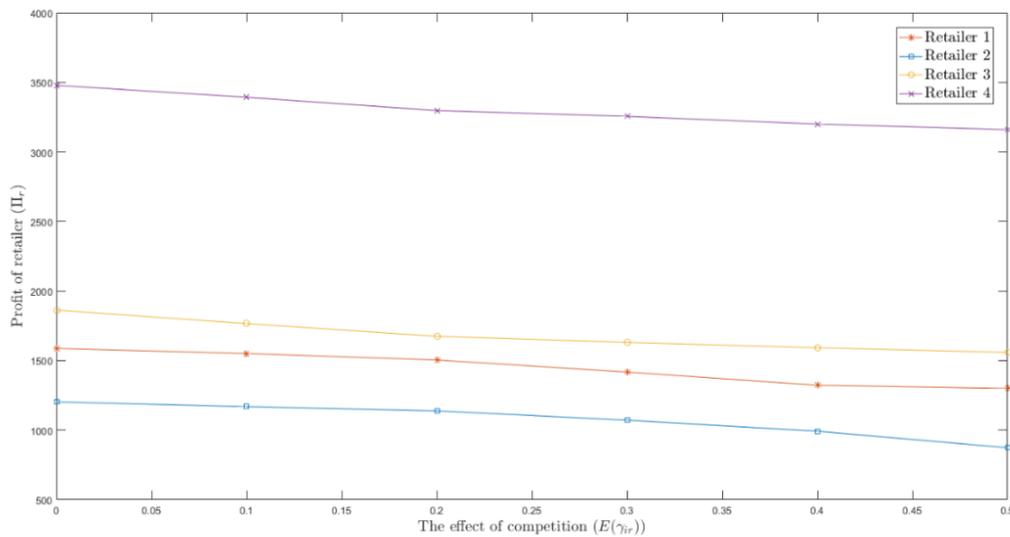


Fig 10. Impact of competition on the retailers’ profit

As a future research direction, considering more than one manufacturer and taking their competitiveness into account

would be interesting. Also, considering other types of demand functions and solving the problem with other meta-heuristic algorithms would be another research direction.

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**APPENDIX:**

**TABLE A1.** Results of the sensitivity analysis for one retailer with three products

Row	$k_1$	$k_2$	$w_1$	$w_2$	$w_3$	$t$	$A$	$\Pi_m$	$p_1$	$p_2$	$p_3$	$a_1$	$a_2$	$a_3$	$\Pi_r$
1	0.7	0.5	2.34	2.41	3.86	0.39	85.98	2899.30	2.69	2.65	4.21	4.52	0.93	27.18	1030.22
2	0.7	0.8	2.59	2.72	3.85	0.00	98.94	3174.40	2.81	2.79	4.18	0.35	0.23	19.41	725.33
3	0.7	1.1	2.46	2.59	3.73	0.26	61.44	3241.70	2.73	2.75	4.14	1.97	0.06	24.97	1084.66
4	0.7	1.4	2.41	2.37	3.56	0.76	32.73	4033.62	2.70	2.64	4.12	17.63	0.60	64.07	2065.43
5	0.7	1.7	2.41	2.46	3.61	0.67	54.01	4712.61	2.70	2.69	4.12	15.05	3.37	42.11	2111.16
6	1.1	0.5	2.35	2.25	3.70	0.00	100.00	4032.48	2.67	2.57	4.21	3.74	1.68	14.57	2054.57
7	1.1	0.8	2.34	2.37	3.68	0.00	100.00	4436.42	2.66	2.63	4.18	5.38	1.38	13.24	2110.28
8	1.1	1.1	2.38	2.44	3.60	0.24	93.17	4706.95	2.71	2.66	4.14	4.65	1.05	20.54	2282.48
9	1.1	1.4	2.48	2.38	3.59	0.74	23.84	4842.95	2.70	2.63	4.06	12.47	7.59	53.39	2010.55
10	1.1	1.7	2.29	2.25	3.57	0.21	88.53	5362.94	2.66	2.56	4.09	3.54	0.13	21.79	3044.56
11	1.5	0.5	2.38	2.36	3.79	0.39	85.88	5134.71	2.71	2.63	4.22	7.30	3.19	21.78	2174.90
12	1.5	0.8	2.48	2.60	3.76	0.34	89.37	5475.75	2.75	2.75	4.21	5.46	0.08	24.66	1911.92
13	1.5	1.1	2.33	2.35	3.51	0.25	93.28	5884.15	2.67	2.62	4.10	6.44	1.76	18.44	3502.04
14	1.5	1.4	2.65	2.44	3.75	0.33	58.16	6100.17	2.79	2.65	4.16	0.04	0.14	29.23	1715.10
15	1.5	1.7	2.36	2.52	3.86	0.69	49.26	6614.57	2.70	2.70	4.19	28.31	8.50	25.10	2155.04
16	1.9	0.5	2.39	2.38	3.77	0.17	93.66	6590.59	2.72	2.64	4.21	3.16	0.65	20.12	2728.44
17	1.9	0.8	2.45	2.34	3.80	0.29	90.38	6948.69	2.74	2.62	4.21	1.67	1.30	23.01	2645.80
18	1.9	1.1	2.34	2.49	3.63	0.00	99.09	7199.84	2.68	2.68	4.17	4.86	0.57	14.52	3486.69
19	1.9	1.4	2.66	2.30	3.55	0.00	100.00	7389.19	2.79	2.60	4.12	0.02	5.02	14.91	3522.44
20	1.9	1.7	2.72	2.78	4.00	0.29	78.21	7418.08	2.85	2.84	4.27	1.56	0.11	25.12	1129.58
21	2.3	0.5	2.37	2.54	3.61	0.00	96.26	7899.07	2.69	2.68	4.17	1.92	1.33	16.72	3680.80
22	2.3	0.8	2.36	2.54	3.80	0.00	100.00	8300.07	2.70	2.72	4.22	7.87	0.01	12.12	3178.70
23	2.3	1.1	2.29	2.51	3.75	0.01	98.24	8666.36	2.66	2.71	4.19	3.82	0.92	15.31	3746.93
24	2.3	1.4	2.55	2.27	3.70	0.63	67.09	8826.83	2.79	2.59	4.17	8.03	7.79	32.95	3516.55
25	2.3	1.7	2.38	2.47	3.79	0.54	75.95	9647.42	2.68	2.67	4.18	12.16	4.62	24.55	3381.92