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Discounting Strategy in Two-Echelon Supply Chain with Random Demand and Random Yield

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Abstract-- *This paper analyzes different pricing strategies in a two-echelon supply chain including one supplier and two retailers. The supplier and the retailers face random yield and random demand, respectively. Moreover, coordination or non-coordination of retailers in receiving the discount is investigated. Game theory is used to model and analyze the problems. The supplier as a leader of Stackelberg specifies quantity discount and an initial wholesale price. Then, retailers determine their optimal order quantity in which their profit is maximized. Finally, the supplier decides on the quantity of the input for production. Coordination of the retailers in receiving discount quantity enhances their profit and improves supply chain performance. However, the supplier gains more profit by escalating competition between customers/retailers. Numerical examples are shown to explain the results.*

Keywords: *Discount Strategy, Supply Chain, Coordination, Non-coordination.*

I. INTRODUCTION

Random yield and random demand are uncertainty factors in the supply chain that force members who intend to reduce their impact to employ procedures such as coordination and cooperation in costs. For example, (Gurnani et al. 2007) studied coordination in an assemblage system with multiple suppliers and a producer with a Stackelberg game. (He Xu 2010) considered manufacture and supplies management in a supply chain with stochastic demand. He formulated the problem as a Stackelberg game, with the producer being the leader. Moreover, (Güler 2015) proposed four contracts which had fewer payment schemes than the current coordination contracts of assemblage systems in the researches. Furthermore, they showed that the contracts could coordinate the chain under forced compliance. (Yin and Ma 2015) improved the service level in a random-yield supply chain with bonus contracts. (Giri and Bardhan 2015) investigated a contract mechanism to coordinate the supply chain and found threshold statuses in which the coordinated model would collapse. (Nouri et al. 2018) inspected the coordination of a manufacturer-retailer chain in which the manufacturer innovated in the producing process and the retailer applied advertisement efforts. Various pricing strategies, including wholesale price and discount, have been considered as coordination approach in uncertainty production and random demand. These factors are investigated in the following.

An urgent issue in uncertainty supply chain is that how and when prices are determined. (Li et al. 2009) assessed collection price and selling price before random yield and demand randomness were realized. (Zhu 2013) investigated a common decision problem for replenishment, production, and pricing policies when both supply and demand were uncertain. (Li et al. 2015) studied two strategies of First-Remanufacturing-Then-Pricing (FRTP) and First-Pricing-Then-Remanufacturing (FPTR) when both remanufacturing yield and demand for remanufactured products were random. (Hsieh and Lin 2016) considered relevant parameters of pricing policies such as rents, the expected rate of rising in land prices, land proprietorship values, risks, and royalties for leasing non-public-use land based on real alternatives analysis. (Liu et al. 2016) identified optimal support decisions for power generation with a particular level of economic punishments in case of violation of real coal-pricing contracts between coal suppliers and power factories.

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Wholesale price contract is one of the useful pricing strategies in order to coordinate supply chain members in case of uncertainty production and random demand. For example, (He and Zhao 2012) proposed a return policy used by the supplier and the retailer in which they considered a wholesale price contract applied for raw-material supplier and the manufacturer. (Güler and Keskïn 2013) studied five well-known contracts including wholesale price, buy-back, income apportion, quantity discount, and quantity flexibility. (Yuyin and Haishen 2017) investigated the mechanism of impact between the manufacturer's wholesale price and retailers' marketing objective with network externality by expanding an evolutionary game model. (Amrouche and Yan 2017) showed that the wholesale price incentive motivating the retailer to finance further in promoting was not preferred as expected, and all supply chain members were better off without prior information about the supplier's behavior in the base of branding contest and promoting-level related motivation.

Quantity discounts present a workable basis for inventory coordination in supply chains (Shin and Benton, 2007). For instance, Sam Walton established a retail store network working at 6500 areas globally with the employing of 1.8 million people. The requirement of the best quality of merchandise and services at the lowest affordable prices for consumers is the main policy of Walmart (Natto, 2014). Giant Walmart stores run a chain between discount department stores and warehouse stores (Rajesh Kumar, 2016). (Hökelekli et al. 2017) provided some results showing that discounters benefited from the more growth in their national brand offerings as well as from price cut in their traditional retailers' private label from the point of view of the discounters. (Obara and Park 2017) introduced a general class of time discounting, which might exhibit present bias or future bias, to repeated games with perfect monitoring. A strategy profile was described an agent subgame perfect equilibrium if there was no profitable one-shot deviation by any player at any history. They studied symmetric agent subgame perfect equilibria for repeated games with asymmetric stage game, perfectly. (Gabler et al. 2017) investigated the regularly growing discount pricing strategy with mine products scarcity against a future discount and compelled customers to make a choice between cost savings and the possible risk of losing the buying chance. (Haga et al. 2018) detected that discount rates were positively associated with income-increasing earnings administration. This means that administrators amend both accrual-based and real earnings administration when discount rates are above. (Li et al. 2018) addressed a dynamic model capturing discount pricing in word-of-mouth (WOM) marketing and the problem of finding an optimal discount strategy boiled down to an optimal control problem. Neglecting impacts of the interaction between low echelons of the supply chain to receive the discount is a salient concern of these models. However, regarding uncertain demand and production, production management is affected significantly by the interactions' down-stream in the supply chain.

This paper analyzes different pricing strategies for a supply chain involving one supplier and two rival/cohort retailers. The supplier faces random yield and rival/cohort retailers face random demands. The supplier as a Stackelberg leader proposes different wholesale prices and discounting strategies to two rival/cohort retailers. Then, retailers as followers determine their quantity and discount percentage. Finally, the supplier produces according to the retailers' orders. Combining supplier's discounting strategies and retailers' options, three strategies are conceivable;

(1) Placing various wholesale prices to each retailer and retailers that are not coordinated.

(2) Placing the identical wholesale price to both retailers, although retailers are not coordinated on receiving discount.

(3) Placing the identical wholesale price to both retailers, while retailers are coordinated on receiving discount.

Some questions arise:

(a) What are the optimal retailers' orders, the supplier's wholesale price, and the supplier's quantity for each discounting strategy?

(b) What is the effect of supplier's discounting strategies on the supply chain, including retailers' and supplier's decisions and profits, as well as the entire supply chain profit?

(c) Which discounting strategy is the best for the supplier and which one is optimal for the retailers?

These issues are investigated in this paper, moreover, the supplier's impact on supply chain decisions and performance considering various discounting strategies. Boundaries of discount percentage provided by the retailers' profit functions are also studied. The supplier's discounting strategy considering wholesale price-dependent demands is focused on in this paper. Compared with the previous researches, this study evaluates changeable pricing by the supplier considering the retailers' gaming interaction and its power in decisions and performance of the supply chain when all main members are influenced by discount percentage-dependent demands. This is an exclusive contribution because there are few analyses which have performed such a linking.

In the following, Part 2 presents notations and problem formulations. All discounting strategies are exhibited in Part 3. Part 4 reviews the similarities and dissimilarities between these discounting strategies. Part 5 uses numerical examples to illustrate the results of discounting strategies. Finally, results are provided in Part 6.

II. NOTATIONS AND PROBLEM FORMULATION

This section introduces all decision variables, input parameters, and assumptions regarding the models.

A. Decision variables

Q_m	Production quantity of the strategy m ($m = 1, 2, 3$)
q_i	Order quantity of the retailer i ($i = 1, 2$)

B. Input parameters

m	Number of strategies ($m = 1, 2, 3$)
i	Number of retailers ($i = 1, 2$)
U	Random yield of production
x_i	Random demand of the retailer i ($i = 1, 2$)
w_i	Initial wholesale price of the retailer i ($i = 1, 2$) (The wholesale price which the supplier suggests to the retailer i at first)
w_i'	Final wholesale price of the retailer i ($i = 1, 2$) (The wholesale price which the retailer i pays at final)
c	The supplier's production cost per unit
c_e	Emergency production cost ($c_e > c$)
p_i	Selling price of the retailer i
${\pi_m}^s$	Profit function of the supplier
${\pi_m}^{R_i}$	Profit function of the retailer i for the strategy m
${\pi_m}^T$	Total profits of the supplier and the retailers
$g(x_i)$	Uniform distribution function of the demand of retailer i
$f(u)$	Uniform distribution function of supplier production
E_{x_i}	Expected demand from retailer i

C. Assumptions

The proposed models in this paper are based on the following assumptions:

1) u is a random variable with a probability density function of $f(u)$ and mean of μ. uQ is the yield of production.

2) The production quantity of supplier (Q) is independent of the yield distribution ($f(u)$).

3) Retailers face random demand with distribution function $g(\mathrm{x_i})$ and $E[\mathrm{x_i}] = d$.

4) Set-up and holding costs are negligible.

5) In shortage conditions, the order quantities of retailers are met by the supplier on time even with an emergency production cost (c_e).

6) Final wholesale price covers the supplier's cost, therefore $w_i' \geq \frac{c+c_e}{2} + \varepsilon_0$ and $\varepsilon_0 \geq 0$ [Baumol and Bradford, 1970].

III. PROPOSED MODELS

Proposing different pricing strategies to the retailers, the supplier tries to maximize their profit in case of the decline in product sales, to recreate interest in a product, or to get rid of old stock, especially in single-period inventories. In the first model, the supplier offers dissimilar wholesale prices to the retailers and they are not coordinated. In the second and the third models, the wholesale prices are identical; however, in the second model, the retailers are not coordinated and in the third model, they are.

A. Dissimilar initial wholesale prices and the retailers' non-coordination of quantity discounts

When the supplier has the power of pricing, it is possible to propose different pricing strategies depending upon factors such as retailers' degree of loyalty and receipt of individual orders. For example, Dell, a leading company in the computer industry, uses an auto-responder, which checks the best customers/retailers in a list of buyers for product discounting, in such a way that only the people who have not purchased a product yet are able to see the offer (Mendelson, 2000). Therefore, in the first model, the supplier chooses dissimilar initial wholesale prices for retailers to make the best customers/retailers feel special in this discounting strategy. Retailers determine their order quantities and, consequently, the supplier determines its optimal production quantity. Therefore, the order quantity of retailers is increased regarding their good feeling and the discount strategy. The following steps describe it.

Step 1. Based on their own initial wholesale prices, each retailer maximizes their profits such that $\left(w_1{}' = w_1 - aq_1, w_2{}' = w_2 - bq_2, a > 0 \text{ and } b > 0\right)$

$$\pi_1{}^{R_1} = p_1 E_{x_1}[\min\{q_1, x_1\}] - (w_1 - aq_1)q_1 \quad (1)$$

$$\pi_1{}^{R_2} = p_2 E_{x_2}[\min\{q_2, x_2\}] - (w_2 - bq_2)q_2 \quad (2)$$

Proposition 1. The retailers' profit functions are concave on q_1 and q_2. Regarding supplier's discounting strategy, the retailers' optimal q_1 and q_2 satisfy:

$$p_1 \int_{q_1}^{\infty} g(x_1)dx_1 + 2aq_1 = w_1 \quad (*)$$

$$p_2 \int_{q_2}^{\infty} g(x_2)dx_2 + 2bq_2 = w_2 \quad (**)$$

Proof. See Appendix (I).

Step 2. Supplier determines optimal production quantity by maximizing their profit.

$$\pi_1{}^{S} = (w_1 - aq_1)q_1 + (w_2 - bq_2)q_2 - c_e E_u[(q_1 + q_2 - uQ_1)^+] - cQ_1 \quad (3)$$

Where $z^+ = z$ if $z \geq z$ and 0 if $z < 0$. The optimal production of supplier and some interesting results are obtained by the following lemma.

Lemma 1. The supplier's profit function is concave on Q_1, and the optimal production $Q_1{}^*$ satisfies:

$$\int_0^{\frac{q_1+q_2}{Q_1}} u\, f(u)\, du = \frac{c}{c_e}; \quad (I)$$

$Q_1{}^* = k_1(q_1 + q_2)$, where k_1 is a constant factor specified by c, c_e, and f(u). (II)

Proof. See appendix (II).

B. Similar initial wholesale prices and the retailers' non-coordination of quantity discounts

In the second model, the supplier proposes similar wholesale prices to retailers. However, based on each retailer's order quantity, different discounts are determined for them. For example, (He and Zhao 2012) proposed a return policy used by the supplier and the retailer in which they considered a wholesale price contract applied for raw-material supplier and manufacturer. They proved that their system was able to perfectly coordinate a supply chain. Walmart Stores, the US multinational retail giant, operates a chain of discount department stores and warehouse stores. Walmart, or other big supermarkets, reduce product prices according to quantity discount (Natto, 2014). Retailers order their quantities based on equal initial wholesale prices; after that, the supplier determines the production quantity, which is described in the following steps:

Step 1. By the same wholesale prices, each retailer maximizes its profit so that ($w_1 = w_2 = w$,; therefore, $w_1{}' = w - aq_1$, $w_2{}' = w - bq_2$).

$$\pi_2{}^{R_1} = p_1 E_{x_1}[\min\{q_1, x_1\}] - (w - aq_1)q_1 \tag{4}$$

$$\pi_2{}^{R_2} = p_2 E_{x_2}[\min\{q_2, x_2\}] - (w - bq_2)q_2 \tag{5}$$

Proposition 2. Retailers' profit functions are concave on q_1 and q_2. Regarding supplier's discounting strategy 2, the retailers' optimal q_1 and q_2 satisfy;

$$p_1 \int_{q_1}^{\infty} g(x_1)dx_1 + 2aq_1 = w \tag{*}$$

$$p_2 \int_{q_2}^{\infty} g(x_2)dx_2 + 2bq_2 = w \tag{**}$$

Proof. Proof is same as Proposition 1.

Step 2. The supplier determines the optimal production quantity by maximizing their profit.

$$\pi_2{}^s = (w - aq_1)q_1 + (w - bq_2)q_2 - c_e E_u[(q_1 + q_2 - uQ_2)^+] - cQ_2 \tag{6}$$

Lemma 2. The supplier's profit function is concave on Q_2, and the optimal production $Q_2{}^*$ satisfies;

$$\int_0^{\frac{q_1+q_2}{Q_2}} u\, f(u)\, du = \frac{c}{c_e}\ ; \tag{*}$$

$Q_2{}^* = k_2(q_1 + q_2)$, where k_2 is a constant factor and is specified by c, c_e, and $f(u)$; (**)

Proof. It is proved similar to Lemma 1.

A. Similar initial wholesale prices and the retailers' coordination of quantity discounts

In coordination situation, the supplier receives retailers' total order quantities, then proposes the same discount and wholesale price to retailers; therefore, each retailer takes the advantage of another retailer's quantity discount in addition to their quantity discount. Retailers' order quantities and the supplier's production quantity are obtained through the following steps:

Step 1. By the same wholesale prices, each retailer maximizes their profit in such a way that ($w_1 = w_2 = w,\ w_1{}' = w - aq_1 - bq_2$, and $w_2{}' = w - aq_1 - bq_2$, therefore $w_2{}' = w_1{}'$).

$$\pi_3{}^{R_1} = p_1 E_{x_1}[\min\{q_1, x_1\}] - (w - aq_1 - bq_2)q_1 \tag{7}$$

$$\pi_3{}^{R_2} = p_2 E_{x_2}[\min\{q_2, x_2\}] - (w - aq_1 - bq_2)q_2 \tag{8}$$

Proposition 3. Retailers' profit functions are concave on q_1 and q_2. By assuming supplier's discounting strategy 3, the retailers' optimal q_1 and q_2 satisfy;

$$p_1 \int_{q_1}^{\infty} g(x_1)dx_1 + 2aq_1 + bq_2 = w \tag{*}$$

$$p_2 \int_{q_2}^{\infty} g(x_2)dx_2 + 2bq_2 + aq_1 = w \tag{**}$$

Proof. Proof is same as Proposition 1.

Step 2. The supplier determines the optimal production quantity by maximizing profit regarding retailers' order quantities so that:

$$\pi_3{}^s = (w - aq_1 - bq_2)(q_1 + q_2) - c_e E_u[(q_1 + q_2 - uQ_3)^+] - cQ_3 \tag{9}$$

Lemma 3. The supplier's profit function is concave on Q_3, and the optimal production $Q_3{}^*$ satisfies;

$\int_0^{\frac{q_1+q_2}{Q_3}} u\, f(u)\, du = \frac{c}{c_e};$ (*)

$Q_3^* = k_3(q_1 + q_2)$, where k_3 is a constant factor and is specified by c, c_e, and f(u); (**)

Proof. It is proved similar to Lemma 1.

IV. MANAGERIAL INSIGHTS

In this section, the obtained results and managerial implications are compared in order to elaborate how the supply chain works under different discounting strategies.

Lemma 4. Constant coefficients are equal in all strategies.

$k_3 = k_2 = k_1$

Proof. It is proved by Lemmas 1, 2, and 3.

k_m is evaluated as the level of random yield risks that the supplier owns for various discounting strategies. Also, k_m is the number of units of input required to get one unit of qualified product or supplier responsibility for delivering the exact amount of order quantities. Lemma 4 shows that different discounting strategies do not influence the supplier's responsibility for production and random yield quantity risks. Therefore, the change in variance production for all of the strategies is the same. In other words, these price strategies do not have any effect on quality production. Discount percentage only influences the amount of order the production, not production process.

Lemma 5. There are comparisons between retailers' profit function and supplier profit function when each retailer orders the same quantity in all models.

$\pi_2^{R_1} < \pi_3^{R_1}$ (*)

$\pi_2^{R_2} < \pi_3^{R_2}$ (**)

$\pi_3^{s} < \pi_2^{s}$ (***)

Proof. Profit functions can be obtained by (4), (7), (5), (8), (6), and (9).

(*) Let $\pi_2^{R_1} < \pi_3^{R_1}$; since $(w - aq_1) > (w - aq_1 - bq_2)$, $\pi_2^{R_1} < \pi_3^{R_1}$.

(**) Let $\pi_2^{R_2} < \pi_3^{R_2}$; since $(w - bq_2) > (w - aq_1 - bq_2)$, $\pi_2^{R_2} < \pi_3^{R_2}$.

(***) Let $\pi_3^{s} < \pi_2^{s}$; since $k_3 = k_2$ and $(a + b)q_1 q_2 > 0$, $\pi_3^{s} < \pi_2^{s}$.

As it is proved in Lemma 5, when the retailers are coordinated on order quantity discount, their profits rise while, on the contrary, the supplier's profit decreases. In other words, retailers with discount coordination lead to a condition in which every retailer, in addition to their discount, benefits from another retailer's discount. In the third model, the final wholesale price for retailers becomes lower rather than the same retailers' order in the second or the first model.

V. NUMERICAL EXAMPLE

In this section, the obtained results are elaborated by numerical examples. The parameters are included as conform; $c = 1$, $c_e = 3$, and U is a uniformly distributed function, where $U \sim U(0.5, 1.5)$ and $k_3 = k_2 = k_1 = 1.04$.

For all strategies, optimal solutions (Q_m^*, q_1^*, q_2^*) and chain profits $(\pi_m^{S}, \pi_m^{R_1}, \pi_m^{R_2}, \pi_m^{T})$ are obtained regarding different values of (a, b) and w. In all tables, ε_0 is the lowest possible amount in order to compare the results. In comparison with recent papers (Obara and Park, 2017); (Gabler et al., 2017); (Haga et al., 2018); and (Li et al., 2018), in this paper, the interaction between low echelons is considered when they choose a variety of price strategies to receive discount. As it is seen, the initial wholesale price similar to retailers' coordination is the best strategy for retailers and the first model is the most beneficial for the supplier. It is observed that the competition is more beneficial in down-stream of the supply chain than in its upstream. We also prove that the quantity discount does not affect the quality production of a supplier (Lemma 4. $k_3 = k_2 = k_1$).

TABLE I. Dissimilar initial wholesale prices and the retailers' non-coordination of quantity discounts

(p_1, p_2)	x_1	x_2	a	b	Q_1	$q_1{}^*$	$q_2{}^*$	$w_1{}'$	$w_2{}'$	$\pi_1{}^{R_1}$	$\pi_1{}^{R_2}$	$\pi_1{}^S$	$\pi_1{}^T$
$(5,5)$	$(0,100)$	$(0,100)$	0.020	0.020	208.000	100.000	100.000	2.000	2.000	50.000	50.000	125.540	225.540
			0.020	0.015	193.440	100.000	86.000	2.000	1.990	50.000	73.960	115.890	239.851
			0.020	0.010	181.984	100.000	74.000	2.000	2.000	50.000	84.386	109.840	244.223
	$(0,100)$	$(0,125)$	0.020	0.016	234.000	100.000	125.000	2.000	2.000	50.000	62.500	141.230	253.730
			0.020	0.015	228.800	100.000	120.000	2.000	2.000	50.000	70.000	138.090	258.090
			0.020	0.012	215.800	100.000	107.500	2.000	1.990	50.000	92.450	129.170	271.620
			0.020	0.010	208.000	100.000	100.000	2.000	2.000	50.000	100.000	125.540	275.540
			0.015	0.016	219.440	86.000	125.000	1.990	2.000	73.100	62.572	131.580	267.252
			0.015	0.015	214.240	86.000	120.000	1.990	2.000	73.100	72.000	128.450	273.550
			0.012	0.015	206.807	78.853	120.000	2.000	2.000	81.117	72.000	125.120	278.237
			0.011	0.013	198.817	76.797	114.280	2.010	1.914	82.944	91.926	110.520	285.390
			0.010	0.010	181.984	74.985	100.000	2.000	2.000	84.382	100.000	109.840	294.222
			0.009	0.010	180.050	73.125	100.000	2.000	2.000	85.690	100.000	108.800	294.490
$(5,6)$	$(0,100)$	$(0,100)$	0.020	0.020	208.000	100.000	100.000	2.000	2.000	50.000	100.000	125.539	275.539
			0.020	0.015	196.190	100.000	88.644	2.000	2.010	50.000	118.000	119.333	287.333
			0.020	0.010	187.200	100.000	80.000	2.000	2.000	50.000	128.000	112.985	290.985
	$(0,100)$	$(0,125)$	0.020	0.016	234.000	100.000	125.000	2.000	2.000	50.000	125.000	141.231	316.231
			0.020	0.015	229.945	100.000	121.100	2.000	2.000	50.000	132.400	139.202	321.602
			0.020	0.012	219.255	100.000	110.820	2.000	2.010	50.000	147.500	133.490	330.990
			0.020	0.010	213.572	100.000	105.360	2.000	2.000	50.000	155.000	128.464	333.464
			0.015	0.016	219.440	86.000	125.000	1.990	2.000	73.960	125.000	131.371	330.331
			0.015	0.015	215.385	86.000	121.100	1.990	2.000	73.960	132.400	129.564	335.924
			0.012	0.015	207.946	78.847	121.100	2.000	2.000	81.120	132.400	126.232	339.752
			0.011	0.013	199.012	76.797	114.560	2.010	1.990	82.180	143.300	119.391	344.871
			0.010	0.010	187.551	74.985	105.350	2.000	2.000	84.380	155.000	112.761	352.141
			0.009	0.010	185.629	73.125	105.360	2.000	2.000	85.690	155.000	111.734	352.424

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			0.020	0.012	219.255	100.000	110.820	2.000	2.010	50.000	147.500	133.490	330.990
			0.020	0.010	213.572	100.000	105.360	2.000	2.000	50.000	155.000	128.464	333.464
			0.015	0.016	219.440	86.000	125.000	1.990	2.000	73.960	125.000	131.371	330.331
			0.015	0.015	215.385	86.000	121.100	1.990	2.000	73.960	132.400	129.564	335.924
			0.012	0.015	207.946	78.847	121.100	2.000	2.000	81.120	132.400	126.232	339.752
			0.011	0.013	199.012	76.797	114.560	2.010	1.990	82.180	143.300	119.391	344.871
			0.010	0.010	187.551	74.985	105.350	2.000	2.000	84.380	155.000	112.761	352.141
			0.009	0.010	185.629	73.125	105.360	2.000	2.000	85.690	155.000	111.734	352.424

Table I shows that the rise in the order quantities of retailers or their prices increases the supplier profit, because rising of the order quantities and the prices leads to more production and more selling for the supplier. More discount percentage is also reasonable for a retailer whose expected demand and order quantity are more than the other ones. Also, the supplier's profit rises by increasing quantity discount percentage in the first model. Although the supplier gains more profit with more discount percentage, i.e., (0.020, 0.020) and (0.020, 0.016), total profit of supply chain is maximum in minimum discount percentage (0.009, 0.010) with equal or unequal prices of the retailers. In this model, it is beneficial for the retailers to pay less discount percentage.

TABLE II. Similar initial wholesale prices and the retailers' non-coordination of quantity discounts

(p_1, p_2)	x_1	x_2	a	b	Q_2	q_1^*	q_2^*	w_1'	w_2'	$\pi_2^{R_1}$	$\pi_2^{R_2}$	π_2^S	π_2^T
$(5, 5)$	$(0, 100)$	$(0, 100)$	0.020	0.020	208.000	100.000	100.000	2.000	2.000	50.000	50.000	125.539	225.539
			0.020	0.015	156.000	100.000	50.000	2.000	3.25	50.000	25.000	156.654	231.654
			0.020	0.010	138.000	100.000	33.300	2.000	3.67	50.000	16.560	139.183	205.749
	$(0, 100)$	$(0, 125)$	0.020	0.016	234.000	100.000	125.000	2.000	2.000	50.000	62.500	141.231	253.731
			0.020	0.015	208.000	100.000	100.000	2.000	2.000	50.000	50.000	175.539	275.539
			0.020	0.012	169.000	100.000	62.500	2.000	3.250	50.000	31.250	180.125	261.375
			0.020	0.010	156.000	100.000	50.000	2.000	3.500	50.000	25.000	169.154	244.154
			0.015	0.016	182.000	50.000	125.000	3.250	2.000	25.000	62.500	172.346	259.846
			0.015	0.015	187.200	60.000	120.000	2.900	2.000	36.000	72.000	166.985	274.985
			0.012	0.015	172.800	46.160	120.000	3.246	2.000	27.500	72.000	161.803	261.313
			0.011	0.013	178.300	57.140	114.290	2.771	1.910	45.800	91.912	141.890	279.592
			0.010	0.010	173.330	74.985	100.000	2.333	2.000	68.900	100.000	126.831	295.711
			0.009	0.010	169.000	66.660	100.000	2.438	2.000	62.300	100.000	129.344	291.684
$(5, 6)$	$(0, 100)$	$(0, 100)$	0.020	0.020	208.000	62.500	100.000	2.000	2.000	50.000	100.000	125.540	275.539
			0.020	0.015	173.350	100.000	66.650	2.000	3.000	50.000	66.650	171.290	287.947
			0.020	0.010	156.000	100.000	50.000	2.000	3.500	50.000	50.000	169.150	269.154
	$(0, 100)$	$(0, 125)$	0.020	0.016	234.000	100.000	125.000	2.000	2.000	50.000	125.000	141.230	316.231
			0.020	0.015	219.560	100.000	111.120	2.000	2.330	50.000	111.500	169.540	331.006
			0.020	0.012	190.670	100.000	83.332	2.000	3.000	50.000	83.330	198.410	331.748
			0.020	0.010	178.220	100.000	71.362	2.000	3.290	50.000	71.360	199.370	320.727
			0.015	0.016	234.000	58.000	125.000	2.500	2.000	28.000	125.000	191.230	344.230
			0.015	0.015	187.530	59.100	121.220	2.930	2.000	35.000	132.200	168.200	335.430
			0.012	0.015	173.350	45.460	121.220	3.270	2.000	27.000	132.200	162.440	321.640
			0.011	0.013	175.080	54.070	114.270	2.890	2.000	41.000	143.700	153.910	338.610
			0.010	0.010	176.960	64.900	105.250	2.400	2.000	63.440	155.100	133.080	351.655
			0.009	0.010	172.740	60.840	105.250	2.510	2.000	59.000	155.100	135.070	349.156

As it is seen in Table II, in equal conditions for the retailers, a retailer who pays more percentage discount gains more profit. When the prices of retailers are equal but their demands are different, it is profitable for two retailers to pay less percentage discount. While retailers' demands are the same but their prices are different, the retailer with higher price should have more percentage discount. Therefore, it is clear that a retailer with more demand and more price gains more profit with more percentage discount. Maximum profit for the supplier does not happen in the maximum total discount percentage. It occurs about near maximum total discount percentage, for instance, (0.020, 0.015), which is lower than (0.020, 0.020). Total profit of the supply chain is the same as the supplier profit. It can be concluded that offering the same wholesale price to all retailers would not always be a good decision.

TABLE III. Similar initial wholesale price and the retailers' coordination of quantity discounts

(p_1, p_2)	x_1	x_2	a	b	Q_3	q_1^*	q_2^*	$w_1' = w_2'$	$\pi_3^{R_1}$	$\pi_3^{R_2}$	π_3^S	π_3^T
$(5, 5)$	$(0, 100)$	$(0, 100)$	0.020	0.020	208.000	100.000	100.000	2.000	50.000	50.000	125.500	225.540
			0.020	0.015	189.280	98.000	84.000	2.060	48.000	70.500	125.200	243.680
			0.020	0.010	182.000	100.000	75.000	2.000	50.000	84.370	109.800	244.220
	$(0, 100)$	$(0, 125)$	0.020	0.016	234.000	100.000	125.000	2.000	50.000	62.500	141.200	253.730
			0.020	0.015	228.800	100.000	120.000	2.000	50.000	72.000	138.100	260.090
			0.020	0.012	211.120	98.000	105.000	2.060	48.000	88.200	139.600	275.800
			0.020	0.010	208.000	100.000	100.000	2.000	50.000	100.000	125.500	275.540
			0.015	0.016	201.030	84.000	109.300	2.270	52.900	82.410	173.800	309.090
			0.015	0.015	199.680	80.000	112.000	2.200	64.000	62.720	158.900	285.610
			0.012	0.015	212.940	81.300	123.500	1.910	86.000	76.570	110.600	273.190
			0.011	0.013	194.680	77.000	110.190	2.010	82.000	86.630	119.500	288.090
			0.010	0.010	182.000	75.000	100.000	2.000	84.400	100.000	109.800	294.220
			0.009	0.010	179.930	73.100	99.904	2.000	85.700	100.100	109.100	294.930
$(5, 6)$	$(0, 100)$	$(0, 100)$	0.020	0.020	208.000	100.000	100.000	2.000	50.000	50.000	125.540	275.540
			0.020	0.015	196.440	100.000	88.890	2.010	50.000	50.000	119.830	287.460
			0.020	0.010	187.200	100.000	80.000	2.000	50.000	50.000	112.990	290.990
	$(0, 100)$	$(0, 125)$	0.020	0.016	234.000	100.000	125.000	2.000	50.000	50.000	141.231	316.230
			0.020	0.015	231.050	100.500	100.500	1.981	50.000	50.000	136.834	319.460
			0.020	0.012	219.570	100.000	111.120	2.010	50.000	50.000	133.900	330.920
			0.020	0.010	215.800	101.000	106.250	1.960	51.410	51.410	122.400	332.120
			0.015	0.016	224.120	88.000	125.000	1.920	77.440	77.440	118.028	325.520
			0.015	0.015	215.650	85.900	88.000	1.990	74.070	74.070	128.620	335.710
			0.012	0.015	209.580	79.600	121.918	1.980	82.000	82.000	121.660	337.030
			0.011	0.013	199.200	77.100	114.452	1.990	83.500	83.500	119.130	347.200
			0.010	0.010	187.780	75.100	105.420	1.990	85.020	85.020	112.320	353.360
			0.009	0.010	189.690	73.700	105.821	1.980	86.870	86.870	108.840	352.260

Table III illustrates that the performance of the supply chain and the profit of retailers enhance by retailers' coordination; however, the supplier's profit decreases. Comparing the results of numerical examples shown in Tables III and II, one may conclude the validity of Lemma 5. Lemma 5 proves that quantity discount is beneficial for supply chain's low level when they are coordinated. Also, the profit associated with the supply chain's high level is higher when there is no coordination in the supply chain's low level. The maximum total profit occurs in the discount percentage (0.015, 0.016) when the retailers' prices are the same and the discount percentage of (0.009, 0.010) is for retailers with different prices. These points are not maximum percentage discount or maximum profit for the retailers or the supplier, but they would be Nash Equilibrium's points for the members of the supply chain.

Other four diagrams in the following depict retailers' profit in comparison with the first and the third strategies; also, the supplier's profit is investigated under the first and the second strategies, and, finally, total profit is compared for all strategies. In all these diagrams, the price, random demand, and percentage of discount for retailers are equal in a peer to peer form.

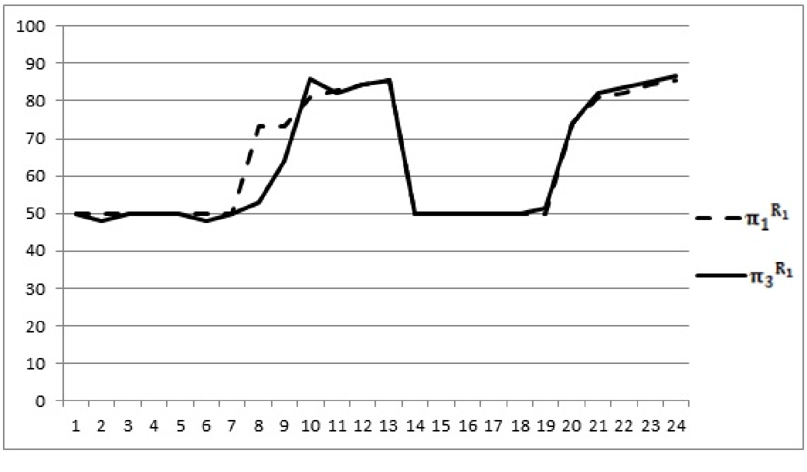

Fig 1. *Comparing the first retailer's profits in the first and third strategies in the same situation*

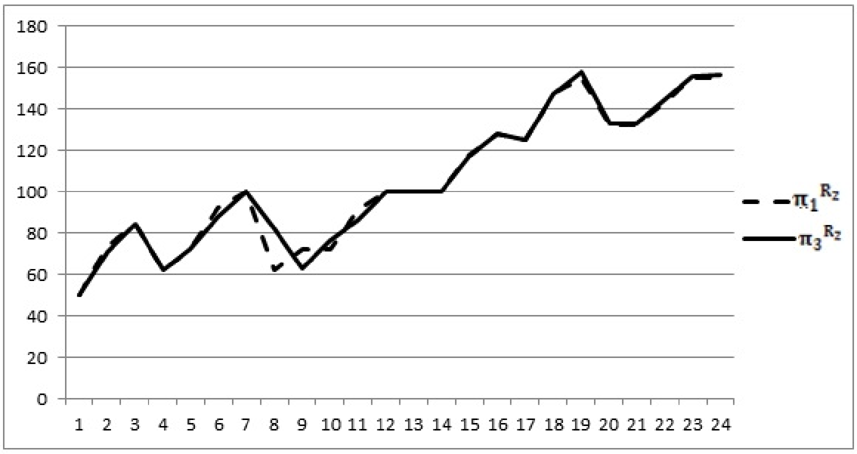

Fig 2. *Comparing the second retailer's profits in the first and third strategies in the same situation*

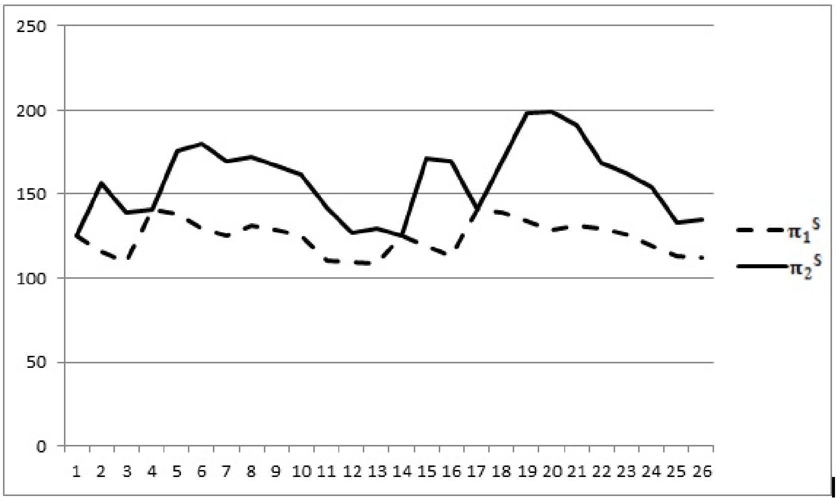

Fig 3. *Comparing supplier's profits in the first and second strategies in the same situation*

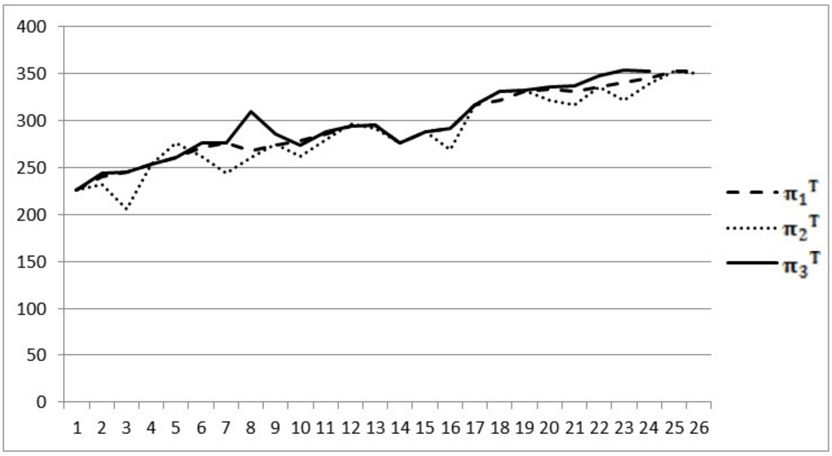

Fig 4. Comparing total profits in the third strategy in identical situations

VI. CONCLUSION

In this paper, we studied different pricing strategies in a two-echelon supply chain including one supplier and two retailers. The supplier and the retailers faced random yield and random demand, respectively. Coordination of retailers or non-coordination was considered on receiving discount quantity while the supplier offered different quantity discounts with different or equal wholesale prices to two retailers. The retailers' optimal order quantities, as well as the supplier's optimal production and profits associated with different pricing strategies, were obtained. The obtained results are summarized in the following:

Coordination of retailers on receiving discount quantity leads to the most possible profit, because the one who increases their discount quantity, grants the other an advantage, too. Supplier benefits more by escalating competition between customers/retailers, especially in initial wholesale price. In other words, escalating competition in low levels of the supply chain is profitable for high levels of supply chain. Regularly, the result supports our insight and the supply chain performance is grown as the retailers are coordinated on order quantities.

Like other studied models in the literature, a set of assumptions can be deliberated on according to the present study; for example, an important extension of this research is to study three discounting strategies by considering supplier's random yield and retailers' stochastic demands in the two-echelon supply chain. Incorporating competition between suppliers is another interesting extension. The model would be more real by considering unstable order quantity for the retailers. Another potential of this research is to consider the condition for strategies with incomplete or asymmetric information to make the model compatible with real situations. In addition, including bargain in the wholesale price model would be another motivating research area.

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Appendix

(I) Proof of proposition 1

(*)

$$\frac{\partial {\pi_1}^{R_1}}{\partial q_1} = p_1 \int_{q_1}^{\infty} g(x_1) dx_1 + 2aq_1 - w_1$$

$$\frac{\partial^2 {\pi_1}^{R_1}}{\partial {q_1}^2} = 2a - p_1 g(q_1)$$

If $\frac{\partial^2 {\pi_1}^{R_1}}{\partial {q_1}^2} < 0$, then ${\pi_1}^{R_1}$ is concave on q_1; therefore, $a < \frac{p_1 g(q_1)}{2}$. If $a < \frac{p_1 g(q_1)}{2}$, then ${q_1}^*$ is established, $\frac{\partial {\pi_1}^{R_1}}{\partial q_1} = 0$.

(**) It is proved the same way (*) is proved.

(II) Proof of Lemma 1

(I)

$$\frac{\partial {\pi_1}^{s}}{\partial Q_1} = c_e \int_0^{\frac{q_1+q_2}{Q_1}} u \ f(u) \, du - c$$

$$\frac{\partial^2 {\pi_1}^{s}}{\partial {Q_1}^2} = - c_e \frac{(q_1+q_2)^2}{{Q_1}^3} \ f\left(\frac{q_1+q_2}{Q_1}\right) < 0$$

Therefor, ${\pi_1}^{s}$ is concave on Q_1 and setting $\frac{\partial {\pi_1}^{s}}{\partial Q_1} = 0$, $\int_0^{\frac{q_1+q_2}{Q_1}} u \ f(u) \, du = c/c_e$ is derived.

(II) All we need is to show that function $T(z) = \int_0^z f(u) \, du$ is a monotone function of z, which is obvious from $\partial T / \partial z = z f(z) > 0$. Therefore, **(I)** has a unique solution and $(q_1 + q_2)/Q_1$ is a constant coefficient, denoted as k_1 here.