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Systems Risk Analysis UsingHierarchical Modeling

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Abstract- *A fresh look at the system analysis helped us in finding a new way of calculating the risks associated with the system. The author found that, due to the shortcomings of RPN, more researches needed to be done in this area to use RPNs as a new source of information for system risk analysis. It is the purpose of this article to investigate the fundamental concepts of failure modes and effects analysis to propose a conceptual hierarchically based model for calculating the risk associated with a system in general. To do so, the author developed a chance constrained goal programming model for solving the problem. A sample problem is provided to show the calculation process of risk evaluation. The findings of this article can be used for calculating the level of risk associated with the entire system provided that the RPN of each unit of subsystems is known beforehand. This model helps the managers to calculate the system risk from the perspective of management, because it is a computer aided decision making (CADM) tool.*

Keywords: *Failure Modes and Effects Analysis, CADM, Goal Programming, Chance Constrained Programming, RPN, Risk, System Risk Calculations.*

I. INTRODUCTION

Systems differ in size, complexity, strategies, goals, environments, products, competitions, and other dimensions of importance to them. It does not matter how different or unique a system is; there is no doubt that every system deals with some degrees of uncertainty and, hence, there is no risk-free system. Regarding this, the leader should pay high attention to key problems and put more efforts into accomplishing the system goals when the system engages in high risks. A system is a set of interrelated components, within a boundary, all working together to accomplish the goal(s) of the system. A system has 9 characteristics, namely, (1) components, (2) interrelated components, (3) boundary, (4) objective(s), (5) environment, (6) inferences, (7) constraints, (8) inputs, and (9) outputs. There are many forms of systems, namely, physical (an ecosystem, a weapons system), conceptual (the Metric System, a betting system), and combined (a nation's justice system, a traffic system). All systems are functional, or they would not be identified as system.

Customers' high expectations of receiving goods and the changing technology have led to the real competition requiring true attention of the management. This requires the management to improve the quality of goods, reduce unit cost, improve service level, and remove any kind of faults related to the product. To be sure that the final result is consistent with the standards, the use of Failure Mode and Effect Analysis (FMEA) is suggested. This tool helps management to identify potential failures in the system, in the process, in the products, and in the services as well.

FMEA is a widely used technique helping to identify and eliminate known or potential failures. This means that FMEA by itself is able to enhance the reliability and safety of all systems, more specifically the complex system decisions (Stamatis, 1995). Taking FMEA literature into consideration, we can see that it is classified into: (1) Design FMEA, (2) Process FMEA, (3) System FMEA, (4) Service FMEA, and (5) Tools FMEA.

It is always possible to have multiple failure modes or causes and effects in a system, design, process, or service. When this is the case, each failure mode or cause must be assessed and prioritized using the risk level with which each is dealing. The rule is that the failure modes with higher risks should obtain higher priorities and the least risky failure mode gets the lowest priority. Chang, Wei, & Lee (1999) took fuzzy linguistic terms of very low, low, moderate, high, and very

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high into consideration for evaluating the value of *O* (Occurrence), *S* (Severity), and *D* (Detection). They used grey relational analysis to determine the risk priorities of potential causes as well. As the traditional approach to RPN computation has its own critics, such as Ben-Daya & Raouf (1996), Bowles (2004), and Pillay & Wang (2003), the proposed fuzzy rule based approach for dealing with this problem is highly suitable. Braglia et al. (2003a, 2003b) proposed a risk function in which fuzzy if-then-else rules could be generated in an automatic way.

In 1963, NASA started to use failure modes and effects analysis in their work. Soon after that, industries became interested in the approach and it reached its highest popularity nationwide. As Puente et al. (2002) indicated, car manufacturing became interested in FMEA for identifying and quantifying the possible potential defects at the design stage of a product. Stamatis (1995) claimed that FMEA was accepted by small and large companies throughout a wide variety of industries all around the world to find possible failures and eliminate them.

The process of failure analysis involves inferring the global effects on a system caused by the existence of one or more localized faulty components, each of which may have a diverse range of failure modes (Lee, 1999). Examples of failure analysis methodologies include FMEA (Failure Mode Effects Analysis), FTA (Fault Tree Analysis), PRA (Probabilistic Risk Analysis), HAZOP (Hazard Operability study), and SHERPA (Systematic Human Error Reduction and Prediction Approach). A good introduction to these topics can be found in Terry's book (Terry, 1991).

The traditional method of calculating risk priority number (RPN) is through investigating the Occurrence (O), Severity (S), and Detection (D). This type of calculation, which generates a crisp RPN, has its own critics all around the world. Some of them are: Ben-Daya & Raouf (1996), Bowles (2004), Braglia, Frosolini, & Montanari (2003), Chang, Liu, & Wei (2001), Gilchrist (1993), Pillay & Wang (2003), and Sankar & Prabhu (2001).

The merit of FMEA is that it can be used to address diverse areas of applications, such as medication delivery, traffic congestion, emergency room patient flow, product assembly line functioning, etc. Effective use of FMEA methodology helps managers to identify system shortfalls right before adverse events occur. FMEA is a preventative approach directing the leadership of an organization in identifying and resolving potential problems right before they impact the system, products, and customers. FMEA is used for analyzing possible failures so that safety factor and costumer satisfaction can be guaranteed. FMEA differs from other quality management methods in the sense that it is an active method rather than a passive one. Offering reaction is costlier to the system than using a proactive approach. FMEA can oversee the potential failure problems and the levels of risk associated with them. Then, it helps in managing to decide upon the actions leading to reduction or elimination of the risk. It should be noted that determining the preventive actions in the early phases of development demands lower costs and time than only managing reactions does. Several industrial FMEA standards, such as those set up by the Society of Automotive Engineers, the US Military of Defense, and the Automotive Industry Action Group, employ Risk Priority Number (RPN) to measure the risk and severity of failures (Rhee & Ishii, 2003).

As Su & Chou (2008) indicated in their studies, FMEA is comprised of two phases:

Phase 1: identifying the potential failure modes and deciding on the right values of Severity, Occurrence, and Detection.

Phase 2: making recommendations by the manager to correct actions, and re-calculating the RPN after correct actions.

A number of FMEA extensions or modifications have been developed to make FMEA more representative and workable, for instance, incorporating the fuzzy set theory into FMEA (Garcia et al., 2005 and Wang et al., 2009), embedding fuzzy data envelopment analysis in FMEA type decision making (Garcia et al., 2005), developing a multi-attribute decision-making approach, called fuzzy TOPSIS approach, for FMECA (Braglia et al., 2003b), the joint use of QFD and FMEA as a tool, utilizing economic based FMEA, and applying multi-failure mode FMEA. Faezy Razia et al. (2013) proposed a hybrid method for detecting the most important failure items as well as the most effective alternative strategy to cope with possible events. They employed grey technique to rank various alternatives (Zare Mehrjerdi, 2014) and FMEA technique to find important faults. Table I lists some of the researches conducted and appearing in the literature on quality management with regard to failure modes and effects analysis considering traditional FMEA with a new approach to make risk analysis more soundly and worthy.

TABLE I. List of FMEA types and its extentions

Row	Researcher(s)	FMEA and its extensions	Remarks
1	Alejandro, et al. (2006)		Reliability evaluation of a power supply
2	Almannai, et al. (2008)	FMEA, QFD and decision support system	
3	Atkinson et al. (1992)	Fault analysis	Hydraulic systems
4	Braglia (2003a, b)	Fuzzy and FMEA	
5	Braglia (2007)	FMEA and QFD concept integration	House of reliability -
6	Bevilacqua et al. (2000)	Monte Carlo Simulation approach and FMECA	Power plant
7	Bowles and Pelaez (1995)	FMEA and fuzzy logic	
8	Bowles (2004)	An assessment of RPN	
9	Ben-Daya et al. (1996)	Revised FMEA	
10	Brown, et al. (2007)	FMEA	Global positioning system
11	Cassanellia et al. (2006)	FMEA	Microelectronics
12	Chang et al. (2001)	FMEA and Grey theory	
13	Chang et al. (1999)	FMEA, fuzzy method and gray theory	
14	Chin K-S, et al. (2006)	FMEA and Product design	
15	Chin K-S, et al. (2008)	Fuzzy FMEA	
16	Chin K-S, et al. (2008)	FMEA and group based evidential reasoning approach	
17	Chen L-H, et al. (2009)	Fuzzy LP, with QFD and FMEA	
18	Colvin, et al. (2008)	Time behavior trees for FMEA	In time critical system
19	Duwe et al. (2001)	FMEA	Intensive care unit (ICU)
20	Guh et al. (2001)	Fuzzy and linear programming	
21	Hu, et al. (2008)	FMEA, FAHP and expert systems	
22	Hawkins, P.G. (1998)	FMEA of complex engineering systems using functional models	
23	Huang, et al. (1999)	Web-based FMEA	
24	Hughes et al. (1999)	Automating mechanical FMEA	
25	Hunt, J.E., (1993)	Automating FMEA process	
26	Iravani A (2006)	FMEA and QFD approach	Robot design optimization using FMEA and QFD
27	Pillay et al. (2003)	Modified FMEA and approximate reasoning	
28	Shahin, A (2007)	FMEA and Kano model	
29	Sharma, et al. (2005)	FMEA and fuzzy linguistic	
30	Sharma (2007)	FMEA, root cause analysis (RCA) and non-homogenous Poisson -point-process (NHPPP)	
31	Su et al. (2008)	FMEA, AHP and Six-Sigma	Semiconductor foundry
32	Ashley, L., Gerry Armitage, Maria Neary, Gillian Hollingsworth. (2010)	FMEA in Health care management	
33	T.H. Varzakas*, G. Zakynthinos* and I.S. Arvanitoyannis (2010)	FMEA	
34	Sachin Kumar a, Eli Dolev a, Michael Pecht (2009)	FMEA	
35	Ahsen, A.V. (2002)	Cost-oriented failure mode and effects analysis,	
36	Azienda USL Valle d'Aosta, Policlinico di Monza (2010)	Clinical risk analysis with failure mode and effect analysis (FMEA) model in a dialysis unit	
37	Day S, Dalto J, Fox J, Turpin M.	. Failure mode and effects analysis as a performance improvement tool in trauma.	
38	Garcia, P.A.A., R. Schirru & P. F. Frutuoso E Melo, (2005).	A Fuzzy data envelopment analysis approach for FMEA	FMEA and DEA
39	Garcia, P.A.A., Ilton Curty Leal Junior, Murilo Alvarenga Oliveira (2013)	A weight restricted DEA model for FMEA risk prioritization	DEA and FMEA
40	Mirghafoori,S.H., Asadian Ardakani,F., Azizi, F. (2014)	Developing a Method for Risk Analysis in Tile and Ceramic Industry Using Failure Mode and Effects Analysis by Data Envelopment Analysis	Fuzzy FMEA and DEA

The main purpose of this article is to design and develop an interactive multiple-objective goal programming model that allows the management to determine the reliability of the entire system taking risk priority numbers (RPN) and costs into consideration. What makes this article different from the previous work is two-fold. First, it takes the reliability study of a system into consideration using the concepts of system decomposition and hierarchical structuring. Second, it integrates risk priority numbers and costs into a single decision making model for calculating weights to compute overall reliability of the system.

The paper is organized as follows: Section II gives an overview of the FMEA methodology and briefs the FMEA type model development and decision making. The model of the problem is discussed in Section III. The overall model along with its three phases is discussed in Section IV. A sample hypothetical problem along with the discussion on sensitivity analysis is given in Section V. Section VI discusses application areas for the proposed model. Finally, the conclusion is given in Section VII.

I. MODEL DEVELOPMENT

A. Chance constrained programming

Chance constrained programming (CCP) is a well-defined methodology for treating probabilistic-type constraints. Taking the following double-inequality-type constraint into consideration, we can call $(1-\alpha)$ the constraint reliability while α is the risk level associated with it. This kind of probabilistic-type constraint was initially introduced into the literature of stochastic programming

by Charnes & Cooper [10-12] and since then, it has been developed and applied by Kataoka [71], Sengupta [37], Seppala [83], and Seppala & Orpana [84], to mention a few. A sample chance constraint can be shown as:

$P(AX <= b) = (1-\alpha)$

where α is the level of risk associated with the problem constraint.

B. Goal programming

Linear goal programming was first introduced by Charnes & Cooper (1962a, 1962b). One can solve a GP model either regularly or interactively. To solve a GP model interactively, one needs to develop a computer program that can allow the decision maker to be a part of the solution process. Goal programming and interactive goal programming (Park,1984; Zare Mehrjerdi, 1986) are chosen to solve multi-criteria programming problems of various types for the following primary reasons:

1. They are computationally efficient and make modeling easier;
2. Concepts can easily be linked to the decision;
3. It is flexible enough to address problems in an MCDM form.

There are many extensions to the GP that can be named, e.g., weighted GP, lexicographical GP, integer GP (Lee, 1979), nonlinear GP, stochastic GP, fractional GP, interactive GP, fuzzy GP, chance constrained GP, Min-max GP, and GP with intervals (Lee 1972, 1979; Ignizio, 1976; Chen et al., 2006; Zare Mehrjerdi, 2016; Zeleny, 1984). A goal programming model can be solved using one of the many techniques available in the literature, which are listed below: (1) Archimedean Goal programming, (2) Preemptive (Lexicographical) Goal Programming, (3) Partitioning Goal Programming, (4) Multiple-Criteria Function Goal Programming, (5) Iterative Goal Programming, (6) Dual Simplex method for Goal programming, and (7) Revised Simplex method of Goal Programming.

C. Proposed model

The model proposed here is capable of breaking down the entire system into subsystems and, then, further breaking the subsystems into other subsystems as long as it is necessary for making the study of the reliability of the whole system possible. As it is well understood, by breaking up a system into subsystems, a hierarchy of subsystems would be constructed. It is through the study of these hierarchies that we can determine the risk level associated with each branch and, then, the entire system. To implement the proposed model, the analyst should take the following steps one at a time:

1. Break the system into as many subsystems as possible to generate a hierarchy, as shown in Fig (1);
2. Identify costs associated with each failure for each subsystem;
3. Apply Monte Carlo simulation to determine the cost level of C^* for all possible failures in the system;

4. Identify levels of *O* (Occurrence), *S* (Severity), and *D* (Detection) for each failure in each subsystem;
5. Calculate the risk priority number (RPN) for each failure in each subsystem;
6. Check with management whether the RPN of the entire system is most likely acceptable for the their team. This RPN can be a random variable with known distribution function.

D. Notations

The notations used in model building are:

x_j = The percentage of failure to be considered in the modeling of the problem

c_j = The cost associated with the *jth* failure

C^* = The threshold cost level for all risks combined

O = Probability of failure mode occurrences

S = Severity of failure effect

D = Probability of the failure being detected

PRN = Priority risk factor

E. Goals and priorities

The managerial goal priorities are defined as:

P1: The level of cost of C^* identified by the management should be satisfied not to be overachieved.

P2: The level of RPN determined for each sub-system should be kept intact.

P3: Total percentage of all failures is only one.

F. Goal constraints

Two types of goal constraint, namely, cost and risk priority number, are discussed in this section. Notes on the system type constraints are given after the model is developed.

1. Cost goal constraint

Three types of failure cost have been recognized by Rhee & Ishi (2003) that are: labor cost, material cost, and opportunity cost. Labor cost and opportunity cost can be measured in terms of time and can be further broken up into four different stages: detection time, fixing time, delay time, and recovery time (Rhee & Ishi, 2003), which are briefly described here.

Detection time: Time to realize and identify the occurrence of a certain type of failure and diagnose the exact location.

Fixing time: Time to fix the problem; it is the actual fixing time for each individual component.

Delay time: Time incurred for non-value activities such as waiting for response, set-up time, and mailing/shipping time.

Recovery time: Time to have the system up and running in its original state.

Labor cost can be derived from the time information obtained from the cost-based FMEA Table using the following equation (Rhee & Ishi, 2003), where labor rate is the labor's wage per hour for doing the task:

Labor cost= Occurrence * {[detection time
* labor rate * no. of operators]
+ [fixing time * labor rate
* no. of operators] + [delay time
* labor rate * no. of operators]}.

Component replacement due to failure is considered as material cost. Material cost is obtained using the following equation (Rhee & Ishi, 2003):

Material cost = Occurrence *x* cost of part.

Opportunity cost is the cost incurred when a failure inhibits the main function of the system and prevents any creation of value. The following equation can be used to calculate the opportunity cost (Rhee & Ishi, 2003), in which labor rate is the labor's wage per hour of doing the task:

Opportunity cost = downtime x hourly opportunity cost.
where,
Downtime = {detection time + fixing time + delay time}.

The total cost of occurrence of all failures is $\sum_{j=1}^{n} c_j x_j$, which should be leveled with the total cost that management can accept due to the new risk taking phenomenon. A goal constraint of the following type can be added to the model of the problem:

$$\sum_{j=1}^{n} c_j x_j + d_1^- - d_1^+ = C^* . \tag{1}$$

The objective function to be minimized is

Minimize $P_1(d_1^+)$. (2)

2. RPN goal constraint

Risk priority Number (RPN) is calculated as follows (Zare Mehrjerdi & Dehghan, 2010, 2013):

RPN = $S * O * D$. (3)

Since RPN can get a value between 1 and 1000, we have the following inequality intact:

1 $\leq$ RPN = $S * O * D \leq$ 1000. (4)

With regard to the type of failure in association with which the management needs to identify the risk, let us assume that management is capable of determining the value we call RPN*. Using the following double-inequality allows us to relate the waited average of RPN to the goal level of RPN the management has taken into consideration:

$$\mathrm{P}\left(\sum_{j=1}^{n} RPN_j * x_j \Big/ \sum_{j=1}^{n} RPN_j\right) \leq RPN^* \} \geq \alpha . \tag{5}$$

The equivalent deterministic form of the above double-inequality is as follow:

$$\sum_{j=1}^{n} RPN_j * x_j \Big/ \sum_{j=1}^{n} RPN_j \leq \mu_{RPN^*} + F^{-1}(1-\alpha) * \sigma_{RPN^*}^{1/2} . \tag{6}$$

which can as well be rewritten as follows:

$$\sum_{j=1}^{n} RPN_j * x_j \leq \mu_{RPN^*} * \sum_{j=1}^{N} RPN_j + F^{-1}(1-\alpha) * \sigma_{RPN^*}^{1/2} * \sum_{j=1}^{n} RPN_j . \tag{7}$$

A goal constraint of the above problem is as shown below:

$$\sum_{j=1}^{n} RPN_j * x_j + d_2^- - d_2^+ = \mu_{RPN^*} * \sum_{j=1}^{N} RPN_j + F^{-1}(1-\alpha) * \sigma_{RPN^*}^{1/2} * \sum_{j=1}^{n} RPN_j . \tag{8}$$

The objective function to be minimized is

Minimize $P_1(d_2^- + d_2^+)$. (9)

II. OVERALL ALGORITHM

The algorithm provided here looks at the risk associated with the entire system. In this model, the management should provide two numbers that would be used for calculating weights of the subsystems. The model proposed here is comprised of three phases as briefly discussed below:

Phase 1

Break the system under study into subsystems Fig (1). Now, do the following:

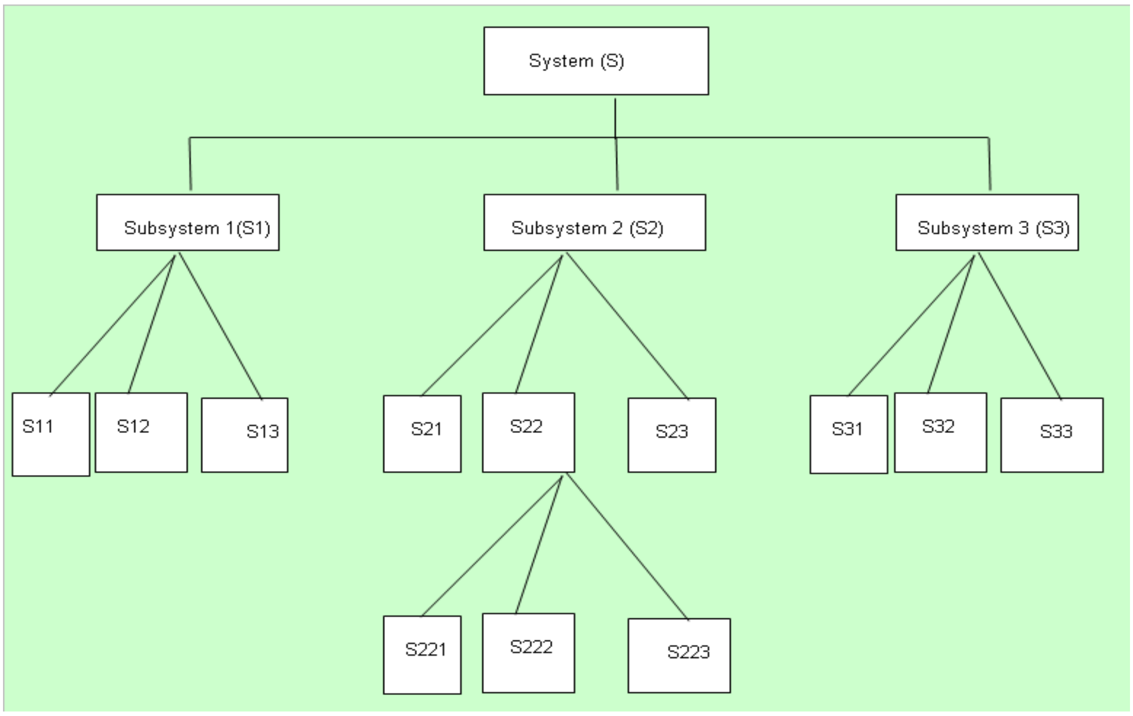

Fig 1. Hierarchical structure of system S

1. Break the system into K subsystems of $S_1, S_2, S_3, \ldots, S_k$, where each subsystem may deal with only one failure;
2. Identify the values of O_j, S_j, and D_j for each failure to calculate RPN_j for all j = 1, 2,.., n;
3. Find an estimate of C* by the consulting management;
4. Ask the management to provide the mean and variance of RPN* of the entire system along with the distribution function that are acceptable to the company's management.

Phase 2

Now, solve the following goal programming problem to determine the value of x_j (Zare Mehrjerdi, 2011, 2012, 2015).

Minimize $P_1(d_1^+)$

Minimize $P_2\{d_2^- + d_2^+)$

Minimize $P_3(d_3^- + d_3^+)$ (10)

S.t.

$$\sum_{j=1}^{n} c_j x_j + d_1^- - d_1^+ = C^* \quad (11)$$

$$\sum_{j=1}^{n} RPN_j * x_j + d_2^- - d_2^+ = \mu_{RPN^*} * \sum_{j=1}^{N} RPN_j + F^{-1}(1-\alpha) * \sigma_{RPN^*}^{1/2} * \sum_{j=1}^{n} RPN_j \quad (12)$$

$$\sum_{j=1}^{n} x_j + d_3^- - d_3^+ = 1 \quad (13)$$

$$l_j \le x_j \le u_j \text{ , for all } j=1,\ldots, 10 \quad (14)$$

$$x_j \ge 0 \quad (15)$$

$$d_j \ge 0 forallj = 1, 2, ..., 10 \quad (16)$$

The value of x_j determined by the goal programming model of Phase 2 can be used as a guide for determining the percentage of each of these failures that must be covered up in the designing stage of the product. The processes of Phases 1 and 2 can be repeated as long as the management is not satisfied with the final results obtained for the product design. Once the result is satisfactory, the process may come to end.

Using the value of x_j, the management can determine the appropriateness of the weights considered in the first phase of the algorithm. Using x_j and RPN_j, we can determine the overall RPN of the system according to the following formula, in which T represents the number of the subsystems of the system:

$$RPN_{si} = \sum_{g=1}^{T} x_j * RPN_j \ . \quad (17)$$

Also, the RPN of the overall system can be calculated using the weighted average formula as

$$RPN_s = \sum_{i=1}^{K} W_i * RPN_{si} / \sum_{i=1}^{K} W_i \qquad . \quad (18)$$

These three phases are shown in Fig (3).

III. EXAMPLE PROBLEM

The model under study is a system comprised of two main departments of

1. Surgery
2. Disease control

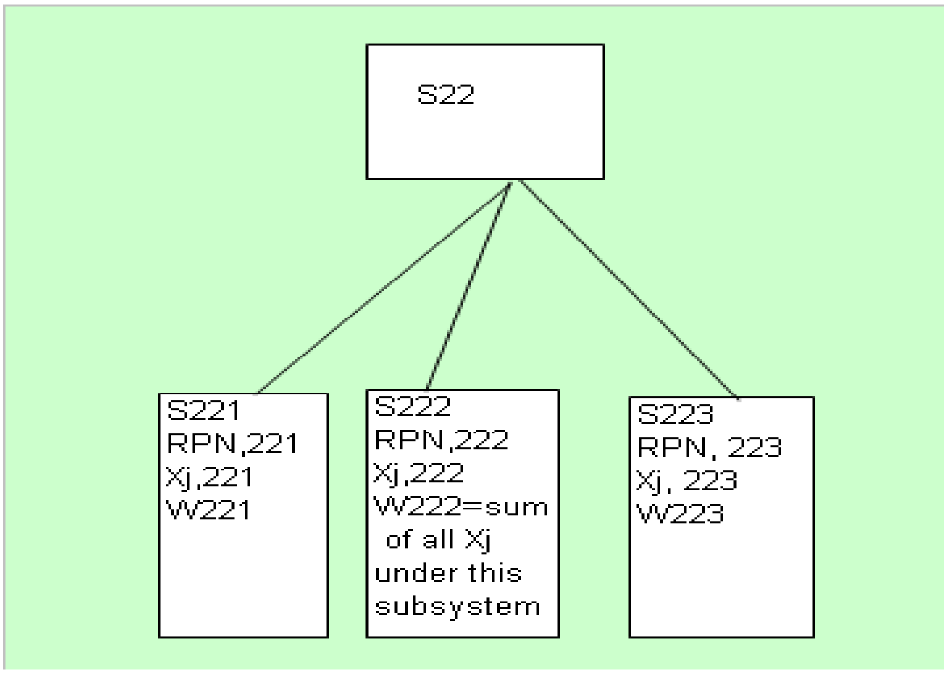

Fig 2. Demonstration of a system and its subsystems

where the surgery department can be further subdivided into brain surgery, heart surgery, and lung surgery departments. The heart surgery can be broken into open heart surgery, closed heart surgery, and general heart problems (heart repairing) of any kind. The disease control portion of this system is comprised of three departments of: (1) Aids, (2) Cancer, and (3) Non-manageable diseases. The non-manageable diseases department is further broken up into type A, B, and C diseases. The final hierarchy of the system under study is shown below:

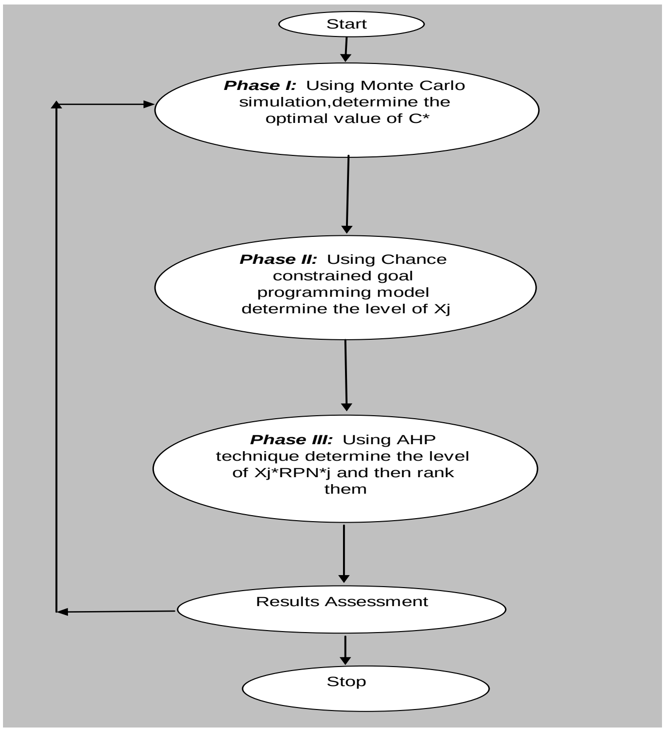

Fig 3. The three-phase method of system risk analysis

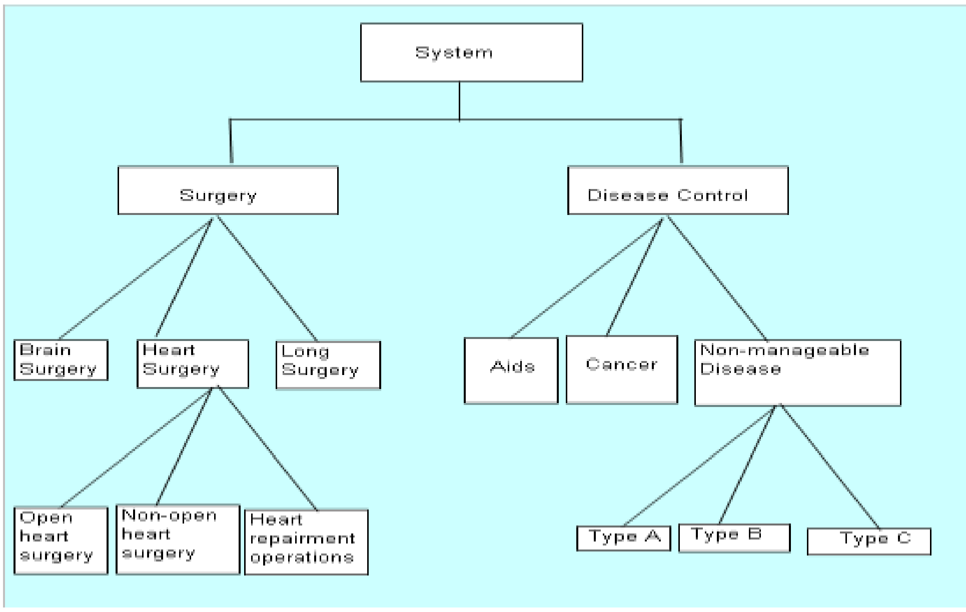

Fig 4. Demonstration of the sample system with its subsystems

TABLE II. Costs and RPN of each section

	Brain Surgery	Long Surgery	Open H-S	Non-open H-S	Heart R-O	Aids	Cancer	Type A	Type B	Type C
C_j	1.50	2.00	2.20	1.20	1.20	0.56	0.75	0.50	1.00	1.00
RPN_j	20	25	25	30	40	45	50	55	60	65

Knowing that C_j (j=1,...,10) and RPN values are given in Table I, we can formulate the CC-GP model of the problem to calculate the value of x_j .

Having identified the value of C^* and the mean and variance of RPN* – at the level acceptable to the management – the analyst can set up the following multiple-objective chance constrained goal programming model to determine the value of x_j . Assuming C^* is 10 and RPN* is normally distributed with the mean of 50, with the variance of 16, the management is able to work with the system if the overall cost is 10 and the mean RPN of the system is 50. We also assume that constraint reliability is predetermined and set to 0.90.

Considering the factors given above, we can formulate the model of the problem as shown below:

Minimize (19)

Minimize $P_2\{d_2^- + d_2^+)$

Minimize $P_3(d_3^- + d_3^+)$

S.t.

$1.5x_1+2x_2+2.2x_3+1.2x_4+1.2x_5+0.56x_6+0.75x_7+0.5x_8+x_9+x_{10}+d_1^- - d_1^+ = 10$

$20x_1+25x_2+25x_3+30x_4+40x_5+45x_6+50x_7+55x_8+60x_9+65x_{10} + d_2^- - d_2^+ = 415*(50+4*1.965)$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10} + d_3^- - d_3^+ = 1$

$l_j \le x_j \le u_j$, *for all j=1,..., 10*

$x_j \ge 0$

$d_i \ge 0 for all i = 1, 2, ..., 10$

A. Solution methodology

To use LINDO Software for solving a goal programming problem of type (20) with K goal priorities,

$$Minimize \sum_{r=1}^{K} P_r\{d_{ir}^- + d_{ir}^+\} \quad (20)$$

S.t.

$$G_i(x_j) + d_{ir}^- - d_{ir}^+ = c_i$$

$$x_j \ge 0, d_{ir}^+, d_{ir}^+ \ge 0$$

one needs to obtain a linear combination of all positive and negative deviational variables appearing in the goal priorities and then solve the following linear programming instead.

Min Z (21)

S.t.

$$G_i(x_j) + d_{ir}^- - d_{ir}^+ = c_i$$

$$x_j \ge 0, d_{ir}^+, d_{ir}^+ \ge 0$$

$$Z - \sum_{r=1}^{K} P_r\{d_{ir}^- + d_{ir}^+\} = 0$$

The solution to the linear programming problem (21) is a non-dominated solution point for the linear goal programming (20).

Using LINDO software, we can calculate the value of x_j as given in Table II for problem (19). Weights W_j are simply assumed to be equal to the sum of x_j of the associated branch.

TABLE III. Values of *xj* and weights

	Brain Surgery j=1	Long Surgery2	Open H-S3	Non-open H-S 4	Heart R-O5	Aids 6	Cancer 7	Type A 8	Type B 9	Type C 10
RPNj	20	25	25	30	40	45	50	55	60	65
xj	0.12	0.09	0.08	0.09	0.085	0.095	0.125	0.095	0.095	0.095

Following the discussion given on the model, we can determine the value of RNP for each subsystem as given below:

$$RPN_{si} = \sum_{j=1}^{T} x_j * RPN_j \tag{22}$$

The systems' RPN is calculated according to the following formula:

$$RPN_s = \sum_{i=1}^{K} W_i * RPN_{si} / \sum_{i=1}^{K} W_i \tag{23}$$

Using the above formula, we get:

RPN1=0.12*20=2.40

RPN_2=0.09*25= 2.25

RPN_3=0.08*25+0.09*30+0.085*40=8.225

RPN_6=0.095*45= 4.275

RPN_7=0.125*50=6.25

RPN_8=0.095*55=5.225

RPN_9=0.095*60=5.70

RPN_{10}=0.095*65=6.175

$RPN_{Surgery}$=2.599125/0.470=5.530053

$RPN_{Disease\ Control}$=2.811875/0.505=5.568069

RPN_{System}= {5.599125*0.47 + 2.811875*0.505}/ 0.975 = 5.549744

IV. SENSITIVITY ANALYSIS

Two types of sensitivity analysis can be performed on the proposed goal programming model for the problem, namely, cost sensitivity analysis and risk level sensitivity analysis. More details about sensitivity analysis issues and difficulties are provided below.

A. Model sensitivity analysis

Due to the fact that the original model is comprised of bounded constraints, values between the predetermined upper and lower bounds $l_j \le x_j \le u_j$ are enforced on variables. Therefore, the sensitivity analysis has shown no sign of changes in the values of the variables as well as the priority achievement levels. This is due to the fact that the bounded-type constraints manage the construction of feasible region. In this experiment, the author employed risk levels of 2.5%, 5%, and 10% in association with cost values of 5, 20, 50, 100, 200, 450, and 1000 and solved each situation problem. Please note that the right-hand values associated with the given risk levels are 22,993.38, 23,707.02, and 24,011.90, respectively. The solution to this problem remained same as the one obtained first and presented in Table III.

B. Alternative modeling

To carry out sensitivity analysis on the right-hand parameters including the cost and risk levels, an alternative modeling of the problem is proposed:

Minimize $P_1(d_1^+)$

Minimize $P_2\{d_2^- + d_2^+)$ (24)

$Minimize\ P_3(d_3^- + d_3^+)$

$$Minimize \sum_j P_4\{d_{j4}^+\}$$

$$Minimize \sum_j P_5\{d_{j5}^-\}$$

S.t.

$1.5x_1+2x_2+2.2x_3+1.2x_4+1.2x_5+0.56x_6+0.75x_7+0.5x_8+x_9+x_{10}+d_1^- - d_1^+ = 450$

$20x_1+25x_2+25x_3+30x_4+40x_5+45x_6+50x_7+55x_8+60x_9+65x_{10} + d_2^- - d_2^+ = 24011.90$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10} + d_3^- - d_3^+ = 1$

$x_j + d_{j4}^- - d_{j4}^+ = u_j$, *for all j=1,..., 10*

$x_j + d_{j5}^- - d_{j5}^+ = l_j$, *for all j=1,..., 10*

$x_j \geq 0$

$d_i \geq 0$

$d_{j4}, d_{j5} \geq, j = 1, 2, ..., 10$

In comparison with the original goal programming model of the problem, this second model contains 40 more variables and two more goal priority levels, making the solution process more tedious and time consuming. As expected, the non-dominated solution point(s) for model (24) is different from the one(s) for model (19). The solution to problem (24) is shown in Table IV. Using the data from Table IV, the RPN values for sub-systems (Surgery and Disease Control) and the whole system are computed and presented below.

RPN1=0.05*20=1.0
RPN_2=0.05*25= 1.25
RPN_3=0.05*25+0.05*30+0.085*40=6.15
RPN_6=0.095*45= 4.275
RPN_7=0.125*50=6.25
RPN_8=0.095*55=5.225
RPN_9=0.095*60=5.70
RPN_{10}=0.095*65=6.175
$RPN_{Surgery}$=1.31175/0.295=4.4466
$RPN_{Disease\ Control}$=2.811875/0.505=5.5681
RPN_{System}= {4.4466*0.295 + 2.811875*0.505}/ 0.80 = 5.1545

The above model is set for the risk level of 10% and cost value of 450. The solutions to the above problem for cost values of 5, 10, 20, and 1000 with the risk level of 10% are also determined. The acceptable solution is the one presented in Table IV. The same problem was solved for the risk levels of 2.5%, 5%, and 10% and the cost level of 450, and the same solution was obtained.

V. OTHER SAMPLE SITUATIONS

There are many areas of application for this type of risk modeling. When management deals with upgrading and downgrading of employees in their organization, e.g., Air force, Army, Navy, and Police departments, they should consider such analysis with the organizational risk assessment policy under study to better control the situation.

TABLE IV. Values of *xj* and weights

	Brain Surgery *j*=1	**Long Surgery 2**	**Open H-S 3**	**Non-open H-S 4**	**Heart R-O 5**	**Aids 6**	**Cancer 7**	**Type A 8**	**Type B 9**	**Type C 10**
RPNj	20	25	25	30	40	45	50	55	60	65
xj	0.05	0.05	0.05	0.05	0.085	0.095	0.125	0.095	0.095	0.095

When grading of employees is under study by the management, the level of risk associated with each portion of the organization is in high demand of consideration, in such a way that the partial risk of sub-ordinates as well as the overall risk of the organization must be under the control of the management. However, more elaboration on such sample situations can be further made by real case studies. A judicial system in which sub-systems deal with sensitive cases of general benefits of people being at risk is an example. Each of these situations can be modeled individually and, then, managed to use the hierarchical risk analysis in the way it has been discussed here. Another sample situation can be an environmental case where pollutant plants contribute to the level of pollution of the city. There are many other situations that can be defined having such types of modeling in order. The practitioners must take both soft and hard constraints of the situation into consideration; risk modeling can play the soft role.

VI. CONCLUSION AND DIRECTIONS FOR FUTURE WORK

To make the design of product and its development a successful task and to bring competitive advantages to the core business, as far as reliability of a system is concerned, the management should be committed to the needs of customers and the level of their requirements. A fresh look at system analysis helped to find a new way for calculating the risk level of a system. In this article, the author investigated the fundamental concepts of the failure modes and effects analysis to propose a conceptual hierarchically based model for calculating the risk associated with a system in general. For this purpose, a chance constrained goal programming model for the problem was developed and a sample problem was solved to show the calculation process for the entire system risk evaluation. This new model is a valuable tool for decision making, because it looks into the RPNs of the subsystems for calculating the level of risk associated with each subsystem and, then, with the entire system. Since it is the first model of the type that uses chance constrained goal programming and RPN together in risk calculations and decision making, it makes a significant contribution to the FMEA, risk, and reliability analyses. The findings of this article can be used for calculating the level of risk associated with a system provided that the RPN of each unit of subsystems is known beforehand. This model helps the management to calculate risk of the system from the management perspective, because it is a computer aided decision making tool. However, there are some areas that require more research: (1) A Monte-Carlo simulation model is needed to consider the concept of “Risk optimization modeling” through RPN calculation considering key factors of: detection time, fixing time, occurrence, delay time, downtime, etc.; (2) a fuzzy-rule-based-expert-system simulation model can be designed to help determining the value of RPN more precisely as needed; (3) the concepts of FMEA, Fuzzy FMEA, and Grey FMEA can be developed to give management appropriate information for quick and on-time decision making; and (4) the concept of FMEA can help analysts to develop models for risk management.

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