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Monitoring Lognormal Reliability Data in a Two-Stage Process Using Accelerated Failure Time Model

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Abstract- *The reliability data is getting used to monitor and improve the quality of products or services. Nowadays, most of products or services are the results of processes with dependent stages referred to as multi-stage process. In these processes, the quality characteristics are affected by the quality characteristics in the previous stages, called as cascade property. In some cases, it is not possible to collect all the lifetime data due to resource limitations. Thus, the control charts have been compared under two different scenarios; censored and non-censored data. In this paper, the accelerated failure time (AFT) model is used and two control charts are presented to monitor the quality characteristic in the second stage under the censored and non-censored reliability data. The exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts are used based on the proposed residuals. The performance of the proposed control schemes is evaluated in terms of zero-state and steady-state average run length criteria through extensive simulation studies. The results generally show that CUSUM control chart performs better than the EWMA control chart for monitoring lognormal reliability data in a two-stage process. However, the EWMA control chart outperforms the CUSUM control chart under small shifts when there is no censoring in reliability data or in the censoring rate of 20%.*

Keywords: *Accelerated failure time (AFT) model, Cascade property, Censored data, CUSUM control chart, EWMA control chart.*

I. INTRODUCTION

In the manufacturing systems and service operations, usually the products are the result of the multi-stage processes. In multi-stage processes, the quality characteristics of outgoing products are influenced by quality characteristics of former stages. This feature is called cascade property in the literature (Zhang, 1984). Cause-selecting chart (CSC) is the most commonly used control charts to monitor the multistage processes first introduced by Zhang (1984). The basic concept of the CSC chart for normal quality characteristics has been reviewed by Wade and Woodall (1993). In some cases, the quality characteristics do not follow the normal distribution. Hence, the regression-adjusted process monitoring schemes based on generalized linear models (GLMs) have been proposed with including the exponential family distribution to relax the normality assumption by Jearkpaporn et al. (2003, 2005, 2007), skinner et al. (2003, 2004).

By the aim of improving the product reliability in some industrial or service systems, the quality characteristics are evaluated and monitored. There are some examples of product reliability such as the skein strength of spun cotton or the breaking of a weld which represented by Asadzadeh et al. (2014) and also there are lots of instances in the healthcare such as surgical outcomes in which each patient has different mortality is represented by Sego et al. (2009).

Reliability data cause difficulties in the monitoring or surveillance of quality characteristics. The first feature is that the reliability data follow the parametric distribution which is named location-scale or log-location-scale. These distributions have been proposed by Meeker and Escober (1998). Lawless (2003) represented that extreme value, weibull and lognormal distributions are common distributions used to model the reliability data. Censoring is the second feature which should be considered because it is impossible to have complete reliability data due to lack of resources

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such as time or expenses. Xie et al. (2002) have proposed two new techniques to monitor the time required to observe a fixed number of failures. The control charts are extended to monitor the failure process of components or systems which make the reliability monitoring effective. Nichols and Padgett (2006) designed the new bootstrap control chart to detect a shift of a percentile under Weibull distribution. Batson et al. (2006) suggested the approach to monitor mean time between failures (MTBF) for exponential, lognormal and weibull data which are converted to normal distribution data.

Khoo and Xie (2009) presented the monitoring schemes for failure process of regularly maintained systems. In their paper, the statistical process monitoring techniques are caused to take maintenance decision easily by studying the time-between-events. Also, the time-between-events follow Weibull distribution. A three parameters Weibull distribution is considered to model inter-failure times and monitor the cumulative time elapsed between failures, by Surucu and Sazak (2009). Also in this paper, the distribution of sum of independent Weibull random variables is assumed to be unknown. At the end, Zhang et al. (2011) developed an economic model to monitor time-between-events data when the data follows the exponential distribution. It is significant that censoring issue is not included in above-mentioned research although the location-scale and log-location-scale distributions have been addressed. Censored observations at fixed level are considered in monitoring process by Steiner and Makay (2001a). Moreover, Steiner and Makay (2001b, 2000) developed monitoring schemes to control the quality characteristics when the censoring occurs at variable levels due to the variable competing risk. Zhang and Chen (2004) monitored the mean of censored weibull lifetimes with the exponentially weighted moving average (EWMA) control charts. Pascaul and Li (2011) proposed control charts to monitor the weibull shape parameter under Type II (failure) censoring. Li et al. (2011) developed a control chart to monitor the time-to-failure data in the presence of right censoring with using rank tests. They derived the generic formula for the operating characteristic functions of the control chart to show the relationship between Type I error probability, Type II error probability, sample size and hazard rate change.

In all of the mentioned studies, a single-stage process has been considered. However, the products come from the processes which can possess more than one stage. Asadzadeh and Aghaie (2012) proposed the regression-adjusted control charts to monitor a quality characteristic in the presence of censoring including fixed and variable competing risks with two-stage processes under the assumption of Weibull distribution.

There is a critical issue that the quality characteristics of the last stage are affected by the quality characteristics in the previous stages and in the out-of-control condition, it is not obvious whether the problem is related to the current stage or the previous stages. To handle this issue, the residuals are usually used to effectively monitor the reliability-related quality characteristics. Asadzadeh et al. (2013) proposed two regression-adjusted control schemes based on Cox-Snell residuals with the right censored observations at fixed level under a two-stage process when the data follow Weibull distribution. As mentioned at the beginning of this section, the lognormal distribution is widely used to model the reliability data. For example, the survival times of the lung cancer patients follow the lognormal distribution. In this example, the survival times of these patients must be adjusted for effects of some influential factors, such as the condition or type of tumor. To the best of authors' knowledge and based on the literature review, there is no research on monitoring lognormal reliability data in multi-stage processes. Hence, the lognormal distribution is considered in this paper as reliability data in the second stage of a two-stage process. Then, the statistics based on the residuals are developed under both censoring and no censoring situations. Finally, two monitoring procedures are used, namely EWMA and CUSUM control charts, to monitor the lognormal reliability data in the second stage in phase II.

The paper is organized as follows: The model and assumptions are presented in the next section. The monitoring procedures under the absence and presence of censoring are proposed in section 3. The performance of the proposed control charts are studied and compared in terms of the average run length (ARL) criterion in section 4. At the end, in section 5, concluding remarks are presented and some future researches are suggested.

II. MODEL AND ASSUMPTIONS

Consider a two-stage manufacturing or a service process in which the outgoing quality characteristic is affected by the quality characteristic in the previous stage. The quality characteristic of the first stage is denoted by X and the output quality characteristic of the second stage indicates the reliability of products (services), shown by Y as provided in Figure 1. Moreover, the reliability data may be censored due to the cost or time limitations. The presence of a

censoring mechanism prevents to have all lifetime data associated to the output quality characteristic.

The relationship between reliability data and explanatory variables has been represented by the survival analysis regression models. There are two major approaches in this case. The first one is the accelerated failure time (AFT) model which relates the distribution parameters of reliability data to the covariates. The other one is the proportional hazard model. In this model, the associated hazard function is influenced by the covariates in the previous stages (Lawless, 2003). AFT is the most widely used models among the parametric models. Therefore, in this paper, the AFT model is considered to relate the parameter of output reliability data to the quality characteristic in the first stage.

The probability density and the survival function of the lognormal distribution are given in Equations (1) and (2), respectively as follows:

$$f\left(y\right)=\frac{1}{\sqrt{2\pi}\sigma y}e^{-\left(\frac{1}{2}\left(\frac{logy-\mu}{\sigma}\right)^{2}\right)} \qquad y>0\,, \tag{1}$$

$$s\left(y\right)=1-\phi\left(\frac{logy-\mu}{\sigma}\right), \tag{2}$$

where μ and σ are the location and shape parameters of the lognormal distribution, respectively. These parameters respectively are mean and standard deviation of the corresponding normal distribution as well. Also $\phi\left(.\right)$ is the cumulative distribution function of normal standard distribution. We assumed that only the location parameter μ depends on the covariate in the two-stage process and its dependency is shown as

$$\mu_{y|x}=\beta_0+\beta_1 x\,, \tag{3}$$

and the AFT-based survival function is written as

$$s\left(y|x\right)=1-\phi\left(\frac{logy-\beta_0-\beta_1 x}{\sigma}\right), \tag{4}$$

where β_0 and β_1 are the parameters of regression model. The upstream quality characteristics in the survival regression models most widely follow the normal distribution i.e. $x\sim N(\mu_x\,,\sigma_x^2)$. In the next section, the monitoring procedures are elaborated based on the AFT model.

III. MONITORING PROCEDURES

In this section, the monitoring schemes are developed for two different scenarios, whether data of the response variable are censored or not. The control charts are designed based on these two scenarios to detect decreasing shifts in the reliability of product.

$$\boxed{x\sim N(\mu_x,\sigma_x)} \xrightarrow{\mu_{y|x}=\beta_0+\beta_1 x} \boxed{y\sim lognormal(\mu,\sigma)}$$

Fig. 1. A two-stage process with lognormal reliability data in the second stage

Although the proposed control charts are one-sided, it is easy to establish two-sided control charts for the sake of detecting increasing and decreasing shifts simultaneously. Notably that changing the location parameter, μ, of lognormal distribution leads to changing in the mean of lognormal distribution as well. The mean and variance of lognormal distribution is calculated as follows respectively (Elsayed, 2012):

$$\mathrm{E}(y) = e^{\mu + \frac{\sigma^2}{2}}, \tag{5}$$

$$\mathrm{Var}(y) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1), \tag{6}$$

where

$$\mu = \beta_0 + \beta_1 x. \tag{7}$$

Hence, the mean and variance of the lognormal distribution are equal to $e^{\beta_0+\beta_1 x+\frac{\sigma^2}{2}}$ and $(e^{2\beta_0+\beta_1 x+\sigma^2})(e^{\sigma^2} - 1)$, respectively. There is a critical issue that the response variable depends on the covariate of the first stage. Therefore, changing the covariates leads to changing in the location parameter of the quality characteristic in the second stage. The residuals are developed to deal with this problem and make the cause-selecting control charts. The residual is defined as follows:

$$z_i = \frac{y_i - \mathrm{E}(y_i)}{\sqrt{\mathrm{Var}(y_i)}}, \tag{8}$$

where the z_is are the standardized residuals. In this paper, two control charts are proposed under two scenarios including no censoring and censoring with the aim of immediate finding of the out-of-control situation.

A. No Censoring Scenario

In this sub-section, we concentrate on developing monitoring procedures to identify the mean shift in the outgoing quality characteristic when there are no censored reliability data. Hence, two control charts including CUSUM and EWMA are proposed. By applying the residuals in Equation (8), the one-sided CUSUM statistic is given by Montgomery (2005)

$$c_i^- = \max\left\{0, k - z_i + \mu_0 + c_{i-1}^-\right\}, \tag{9}$$

$$c_0^- = 0, \tag{10}$$

where *k* is the reference value which is calculated via $k = \frac{\vartheta}{2}$. In this formula, ϑ is the minimum shift we want to detect. Also, μ_0 is defined as the in-control mean of the residuals whose value is zero. Hence, the CUSUM statistic is computed as follows:

$$c_i^- = \max\left\{0, k - z_i + c_{i-1}^-\right\}, \tag{11}$$

$$c_0^- = 0. \tag{12}$$

The CUSUM control chart signals an out-of-control status when its statistic falls below the lower control limit shown by LCL_1. The LCL_1 is set by simulation to achieve a desirable in-control ARL criterion. It should be noted that it is necessary to carry out some corrective actions to prevent further deterioration in the product reliability when an alarm is received.

The next monitoring surveillance method is based on the EWMA control chart. The traditional one-sided EWMA statistic for detecting decreasing shift is defined as follows (Montgomery, 2005):

$$Q_i = \min\left\{\mu_0\ ,\ \lambda z_i + (1-\lambda)Q_{i-1}\right\}, \tag{13}$$

where the smoothing constant λ satisfies $0 < \lambda \le 1$. Q_0 and μ_0 are the starting point of the EWMA statistic and in-control mean of the residuals, respectively. Since the mean of the residuals is equal to zero then μ_0 is equal to zero. Also, Q_0 is equal to zero because the starting point of the EWMA statistic is equal to the mean of the z_i statistic. The general one-sided lower control limit is given by Montgomery (2005) as

$$LCL_2 = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \tag{14}$$

Since the z_i is the standardized residual $(\mu_0 = 0, \sigma = 1)$, then LCL_2 reduces to:

$$LCL_2 = -L\sqrt{\frac{\lambda}{2-\lambda}}. \tag{15}$$

The one-sided EWMA control chart generates a signal when $Q_i < LCL_2$. Also, LCL_2 is set such that a desirable in-control ARL is obtained. In the next sub-section, the proposed control charts are developed when there is censoring in the reliability data.

B. Censoring Scenario

In this sub-section, the monitoring procedures are proposed under the presence of censoring at fixed level. When the censoring occurs, the values of the process output are recorded completely unless they reach the pre-determined limit denoted by *c*. In this situation, all censored data can be replaced with their conditional expected values (*CEV*s) which are calculated with the following equation:

$$w_c = CEV(y) = \mathrm{E}(y|y > c) = \frac{\int_c^{+\infty} y.f(y)\,d_y}{\int_c^{+\infty} f(y)\,d_y}. \tag{16}$$

Solving Equation (16) by Maple, the CEV-weights can be calculated by the following equation:

$$w_c = \frac{\frac{1}{2}e^{\mu+\frac{\sigma^2}{2}} - \frac{1}{2}e^{\mu+\frac{\sigma^2}{2}}\mathrm{erf}\left(\frac{\log(c)-\mu-\sigma^2}{\sqrt{2}\sigma}\right)}{S(c)}, \tag{17}$$

where *S(.)* is the survival function of the reliability-related quality characteristic calculated by Equation (2) and erf(.) is the error function defined as follows (Gautschi, 1970):

$$\mathrm{erf}(t) = \int_0^t \frac{2}{\sqrt{\pi}} e^{-t^2} d_t, \tag{18}$$

Subsequently, the *CEV* weights can be obtained as

$$w = \begin{cases} y, & if \quad y \le c \\ w_c & if \quad y > c \end{cases}. \tag{19}$$

The *CEV*-weights can be established by the following equation:

$$w = \begin{cases} y & if \quad y \le c \\ \dfrac{\frac{1}{2}e^{\beta_0+\beta_1 x+\frac{\sigma^2}{2}} - \frac{1}{2}e^{\beta_0+\beta_1 x+\frac{\sigma^2}{2}}\mathrm{erf}\left(\frac{\log(c)-\mu-\sigma^2}{\sqrt{2}\sigma}\right)}{S(c)} & if \quad y > c \end{cases}. \tag{20}$$

After presenting the *CEV*-weights, the proposed residuals are modified as below when the censoring occurs

$$u_i = \frac{w_i - \mathrm{E}(w_i)}{\sqrt{\mathrm{Var}(w_i)}}, \tag{21}$$

where $\mathrm{E}(w)$ and Var(*w*) are the mean and variance of the *CEV*-weights under in-control situation. The mean of the *CEV*-weights is defined as below:

$$\mathrm{E}\left(w\right)=\int_{0}^{c} y.f\left(y\right)d_{y}+\int_{c}^{+\infty} w_{c}\,f\left(y\right)d_{y}\,. \tag{22}$$

By computing Equation (22), the mean of the *CEV*-weights can be derived as

$$\mathrm{E}\left(w\right)=e^{\mu+\frac{\sigma^{2}}{2}}\,. \tag{23}$$

It is clear that the mean of *CEV*-weights is similar to the mean of lognormal distribution in the absence of censoring. Although the mean of *CEV*-weights is the same in both scenarios, the variance of *CEV*-weights is different and it is calculated by the following equation:

$$\mathrm{Var}\left(w\right)=\mathrm{E}\left(w^{2}\right)-\mathrm{E}^{2}\left(w\right), \tag{24}$$

where $\mathrm{E}^{2}\left(w\right)$ is defined as

$$\mathrm{E}^{2}\left(w\right)=\int_{0}^{c} y^{2} f\left(y\right)d_{y}+\int_{c}^{+\infty} w_{c}^{2}\,f\left(y\right)d_{y}=\int_{0}^{c} y^{2} f\left(y\right)d_{y}+w_{c}^{2}.s(c)\,. \tag{25}$$

Now by calculating Equation (25), $\mathrm{E}^{2}\left(w\right)$ is given by the following equation:

$$\mathrm{E}^{2}\left(w\right)=\frac{1}{2}e^{2\mu+2\sigma^{2}}+\frac{1}{2}e^{2\mu+2\sigma^{2}}\operatorname{erf}\left(\frac{\log\left(c\right)-\mu-2\sigma^{2}}{\sqrt{2}\sigma}\right)+\frac{\left(\frac{1}{2}e^{\mu+\frac{\sigma^{2}}{2}}-\frac{1}{2}e^{\mu+\frac{\sigma^{2}}{2}}\operatorname{erf}\left(\frac{\log\left(c\right)-\mu-\sigma^{2}}{\sqrt{2}\sigma}\right)\right)^{2}}{S(c)}\,. \tag{26}$$

At the end, the variance of *CEV*-weights is calculated by replacing the Equation (26) into $\mathrm{E}^{2}\left(w\right)$ and $e^{\mu+\frac{\sigma^{2}}{2}}$ into E*(w)* in Equation (24). After computing the mean and variance of *CEV*-weights, the proposed residual in Equation (21) is used in the EWMA and CUSUM control charts. As a result, the statistics of the EWMA and CUSUM control charts will be modified as follows, respectively:

$$Q_{i}=\min\{0,\lambda u_{i}+\left(1-\lambda\right)Q_{i-1}\}, \tag{27}$$

and

$$c_{i}^{-}=\max\left\{0,k-u_{i}+c_{i-1}^{-}\right\}, \tag{28}$$

where $Q_{0}=0$ and $c_{0}^{-}=0$. The control schemes signal out-of-control statuses when the statistics of CUSUM control chart, c_{i}^{-} and EWMA control chart, Q_{i} fall below the corresponding lower control limit shown by LCL_4 and LCL_5, respectively. The LCL_4 and LCL_5 are set to achieve the desirable in-control ARL by simulation.

IV. PERFORMANCE ANALYSIS

In this section, performances of the proposed control charts are analyzed and compared to determine which monitoring method is better for detecting decreasing step shifts in the mean of second quality characteristic or equivalently the location parameter of the lognormal distribution in terms of zero-state and steady-state ARL criteria. Notably that the simulation runs are performed to calculate ARLs with 10000 replicates. Also, the lower control limits for all the control charts are set through simulation to achieve the in-control ARL of nearly 200.

Finally, the results are represented in three subsections; in terms of zero-state ARL, steady-state ARL criteria and the last subsection confirms that the control charts are cause-selecting in the absence of censoring. All the control charts and studies are constructed with assuming these input parameters values: $\lambda=0.2$, $\beta_{0}=1$, $\beta_{1}=0.5$,

$X\sim N\left(3,2\right)$, $\sigma_{Y}=1$.

The simulation runs are performed by MATLAB 2011b. In simulation studies, X values are generated randomly from a normal distribution with mean 3 and standard deviation 2. The location parameter of lognormal distribution (μ_i) is calculated by using Equation (7). Then, the values of y_i s are generated. The simulations are performed in 10000 runs and LCLs are set to obtain the in-control ARL of nearly 200.

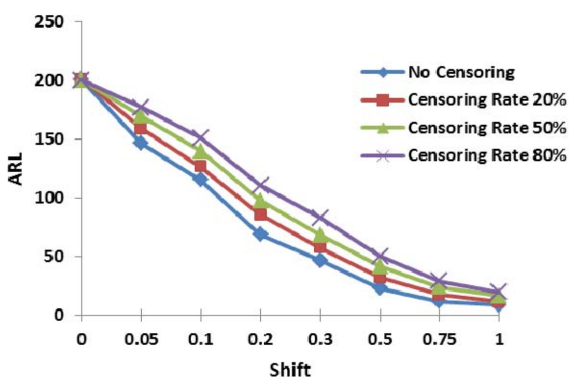
A. Performance Analysis in Terms of Zero-State ARL

The ARLs of the proposed control charts in terms of zero-state ARL are compared in this subsection. The assumption in this manner is that the shift occurs at the initial sample in the second stage of process. The performance of the proposed CUSUM and EWMA control charts is evaluated under the absence and presence of censoring. In the presence of censoring, three censoring rates of low (20%), moderate (50%) and high (80%) are considered. The results are summarized in Table 1.

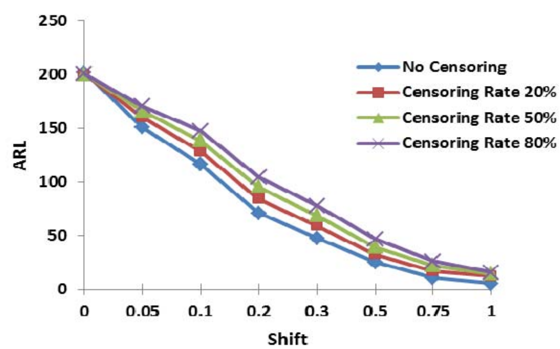
As shown in Table 1, the EWMA control chart performs better than the CUSUM control chart in small shifts in the absence of censoring and low censoring rate. In the large shifts, the results are different and the CUSUM control chart outperforms the EWMA control chart in detecting shifts. Also, the CUSUM control chart is better when the censoring rate equals to 50% and 80%. Presence of censoring mechanism increases the out-of-control ARL under different shifts and deteriorates the performance of the proposed control charts. As shown in Fig. (2), although censoring scenario can affect the performance of control charts and lead to increasing the corresponding out-of-control ARL values, using censoring scenario is unavoidable due to resource limitations.

TABLE I. Performance Comparison of the Proposed Control Charts Using Zero-State ARL

Control charts	**Censoring rates**	δ							
		0	**0.05**	**0.1**	**0.2**	**0.3**	**0.5**	**0.75**	**1**
EWMA	0	200.86	149.33	116.43	69.69	47.58	24.70	13.60	9.75
	20%	201.49	160.96	128.59	88.64	59.58	32.73	18.13	12.08
	50%	200.24	166.77	137.13	94.57	67.15	38.23	20.74	13.29
	80%	200.44	172.93	147.40	106.67	79.01	45.96	24.70	15.28
CUSUM	0	201.97	153.60	119.77	72.81	49.57	25.06	12.43	7.20
	20%	200.32	161.42	130.55	90.27	61.06	33.22	17.85	11.86
	50%	200.35	164.06	135.81	92.64	65.50	35.40	18.98	12.14
	80%	200.81	168.47	143.58	100.36	75.68	42.50	22.93	13.71



(a) EWMA control chart



(b) CUSUM control chart

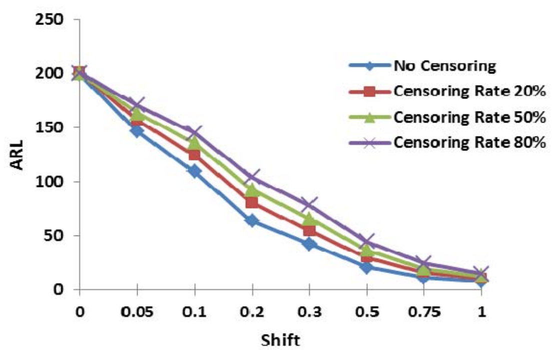
Fig. 2. Performance of the proposed control charts under censoring and no-censoring scenarios using zero-state ARL

B. Performance Analysis in Terms of Steady-State ARL

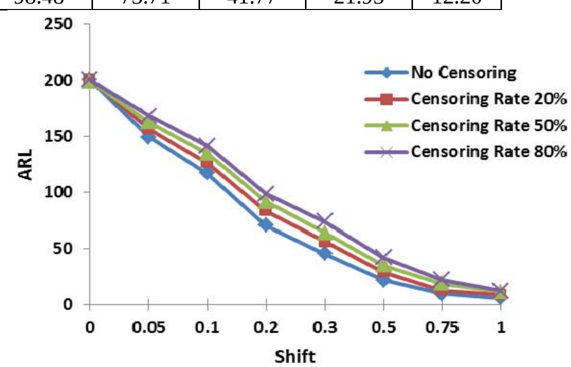
In this sub-section, performance of the proposed control charts is evaluated and compared by using the steady-state ARL criterion. In this case, the shift occurs after a specific time from the initial time the process is started. Notably that the steady-state ARL criterion is more applicable than zero-state ARL criterion. Because the process is usually in-control at the beginning and then goes to out-of-control state due to occurring an assignable cause. Simulation results are presenting the steady-state ARLs when the shifts are imposed after generating 100 in-control observations. The results are shown in Table II. As illustrated in Table 2, in the moderate and high censoring rates, the performance of the CUSUM control chart is better than the EWMA control chart. In the absence of censoring and low censoring, the results are different under large and small shifts. It means that the EWMA control chart performs better just in the small shifts and CUSUM control chart is better in the large shifts under this situation. As shown in Fig. (3), the performance of the proposed control charts deteriorates as the censoring rates increases.

TABLE II. Performance Comparison of the Proposed Control Charts Using Steady-State ARL

Control charts	Censoring rates	δ							
		0	0.05	0.1	0.2	0.3	0.5	0.75	1
EWMA	0	199.19	146.35	109.20	63.55	42.46	20.75	11.11	7.78
	20%	200.88	157.75	127.38	83.46	56.45	29.87	15.94	10.50
	50%	200.03	163.29	136.07	92.46	65.53	35.65	19.30	12.43
	80%	200.29	170.80	144.95	104.05	78.28	44.40	23.97	14.52
CUSUM	0	200.82	150.29	118.18	71.41	46.11	22.47	10.44	6.64
	20%	201.36	159.81	129.76	86.16	59.24	30.72	14.57	9.64
	50%	199.78	162.91	134.79	91.65	63.83	33.93	17.72	11.29
	80%	200.84	167.62	140.66	98.48	73.71	41.77	21.95	12.20



(a) EWMA control chart



(b) CUSUM control chart

Fig. 3. Performance of the proposed control charts for the censoring and no-censoring scenarios using steady-state ARL

C. Performance Analysis under Shift in the mean of the first stage quality characteristic

In this sub-section, the proposed cause selecting control charts (CSC) under the absence of censoring is applied in order to eliminate the effect of any shifts in the mean of the quality characteristic in the first stage on the ARLs of the CSC control charts. For this purpose, the performance of the proposed control charts is studied under decreasing shifts of $\mathrm{E}(X)-\gamma\sigma_X$ in the mean of quality characteristic *X*, in the first stage. Finally, the results are shown in Table III. As given in this table, the ARL values of both EWMA and CUSUM control charts do not change and this confirms that the designed control charts are cause selecting under the absence of censoring.

TABLE III. The ARLs of the CSC Control Charts Under Shift in the Mean of the First Stage Quality Characteristic

γ	EWMA	CUSUM
0	200.86	201.97
0.5	199.71	202.29
1	202.51	199.08
1.5	199.03	201.28
2	202.75	199.67

V. CONCLUSION AND FUTURE RESEARCHES

This paper considered a two-stage process with reliability data following lognormal distribution in the second stage and a quality characteristic in the first stage. The accelerated failure time (AFT) model was applied to relate the quality characteristics of both stages. To eliminate the effect of the quality characteristic in the first stage on the lognormal reliability data in the second stage, the residuals were computed and monitored by the EWMA and CUSUM control charts. In addition, due to the applicability of censoring in the reliability data, a new statistic based on the residuals are designed and monitored by using the EWMA and CUSUM control charts as well. The proposed control charts are compared by using zero-state and steady-state ARL criteria through simulation studies. The results show that the out-of-control steady-state ARL values are less than the out-of-control zero-state ARL values. Moreover, Censoring scenario leads to increasing the out-of-control ARL values of the proposed control charts. In addition, the EWMA control chart is better than the CUSUM control chart in the absence of censoring or low censoring rate when the step shifts are small. In other situations, the CUSUM control chart is better than the EWMA control chart to detect decreasing shifts. These results are the same under both zero-state and steady-state ARL criteria. As a future research, the proposed control charts of this paper can be extended for the multi-stage processes where the stages are more than two and the quality characteristics of previous stages describe the reliability of products. Moreover, developing monitoring procedures in the multi-stage processes when each stage has several correlated reliability based quality characteristics can be a fruitful area for the future research.

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REFERENCES

Asadzadeh, Sh., & Aghaie, A. (2012). Improving the product reliability in multistage manufacturing and service operations. *Quality and Reliability Engineering International*, 28, 397–407.

Asadzadeh, Sh., Aghaie, A., & Niaki, S.T.A. (2013). AFT regression-adjusted monitoring of reliability data in cascade processes. *Quality and Quantity*, 47, 3349-3362.

Asadzadeh, Sh., Aghaie, A., & Shahriari,H. (2014). Using frailty models to account for heterogeneity in multistage manufacturing and service process. *Quality and Quantity*,.48, 593-604.

Batson, R.G., Jeong, Y.,& Fonseca, D.J. Ray P.S. (2006). Control charts for monitoring field failure data. *Quality and Reliability Engineering International*, 22, 733–755.

Elsayed, A.E. (2012). Reliability Engineering. New York: Wiley.

Gautschi, W. (1970). Efficient computation of the complex error function. *SIAM Journal on Numerical Analysis*,7, 187-198.

Jearkpaporn, D., Montgomery, D.C., Runger, G.C., & Borror, C.M. (2003). Process monitoring for correlated gamma distributed data using generalized-linear model-based control charts. *Quality and Reliability Engineering International*, 19, 477–491.

Jearkpaporn, D., Montgomery, D.C., Runger, G.C.,& Borror, C.M. (2005). Model-based process monitoring using robust generalized linear models. *Quality and Reliability Engineering International*, 28, 309-328.

Jearkpaporn, D., Borror, C.M., Runger, G.C.,& Montgomery, D.C. (2007). Process monitoring for mean shifts for multiple stage processes. *International Journal of Production Research*, 45, 5547–5570.

Jin, M., Li, Y., & Tsung, F. (2010). Chart allocation strategy for serial-parallel multistage manufacturing processes. *IIE Transactions*, 42, 577–588.

Khoo, M.B.C., & Xie, M. (2009). A study of time-between-events control chart for the monitoring of regularly maintained systems. *Quality and Reliability Engineering International*, 25, 805–819.

Lawless, J.F. (2003). Statistical Models and Methods for Lifetime Data. New York: Wiley.

Li, Z., Zhou, S., Sievenpiper, C., &Choubey, S. (2012). Statistical Monitoring of Time-to-Failure Data Using Rank Tests. *Quality and Reliability Engineering International*, 28, 321–333.

Meeker, W.Q., &Escobar, L.A. (1998). Statistical Methods for Reliability Data. New York: Wiley.

Montgomery, D.C. (2005). Introduction to Statistical Quality Control. 5th edition, John Wiley & Sons.

Nichols, M.D., & Padgett,W.J. (2006). A bootstrap control chart for weibull percentile. *Quality and Reliability Engineering Internationa*l, 22, 141–151.

Pascaul, F., & Li, S. (2011). Monitoring the Weibull Shape Parameter by Control Charts for the Sample Range of Type II Censored Data. *Quality and Reliability Engineering International*, 28, 233–246.

Sego, L.H., Reynolds Jr, M.R., & Woodall, W.H. (2009). Risk-adjusted monitoring of survival times. *Statistics in Medicine*, 28, 1386–1401.

Skinner, K.R., Montgomery, D.C., & Runger, G.C., (2003). Process monitoring for multiple count data using generalized linear model-based control charts. *International Journal of Production Research*, 41, 167–1180.

Skinner, K.R., Montgomery, D.C., & Runger, G.C., (2004). Generalized-linear model-based control charts for discrete semiconductor process data. *Quality and Reliability Engineering International*, 20, 777–786.

Steiner, S.H., MacKay, R.J., (2000). "Monitoring processes with highly censored data", *Journal of Quality Technology*, 32, 199–208

Steiner, S.H., & MacKay,R.J. (2001a). Detecting changes in the mean from censored lifetime data. *Frontiers in Statistical Quality Control*, 6, 275–289.

Steiner, S.H., & MacKay, R.J. (2001b). Monitoring processes with data censored owing to competing risks by using exponentially weighted moving average control charts. *Applied Statistics*, 50, 293–302.

Surucu, B., & Sazak, H.S. (2009).Monitoring reliability for a three-parameter weibull distribution. *Reliability Engineering & System Safety*, 94, 503–508.

Tsung, F., Li, Y., & Jin, M. (2008). Statistical process control for multistage manufacturing and service operations: a review and some extensions. *International Journal of Services Operations and Informatics*, 3, 191–204.

Wade, M.R., & Woodall, W.H. (1993). A review and analysis of cause-selecting control charts. *Journal of Quality Technology*, 25, 161–169.

Xie, M., Goh, T.N., & Ranjan, P. (2002). Some effective control chart procedures for reliability monitoring. *Reliability Engineering and System Safety*, 77, 143–150.

Yang, C.M. (2006). Optimal processes management for over-adjusted process with dependent steps on bank industry. *Quality & Quantity*, 40, 697–719.

Zhang, G.X. (1984). A New Type of Control Charts and Theory of Diagnosis with Control Charts. World Quality Congress Transactions. Milwaukee, WI: American Society for Quality Control, 3, 175–185.

Zhang, H.Y., Shamsuzzaman, M., Xie, M., & Goh, T.N. (2011). Design and application of exponential chart for monitoring time-between-events data under random process shifts. *The International Journal of Advanced Manufacturing Technology*, 57, 849–857.

Zhang, L., & Chen, G. (2004). EWMA charts for monitoring the mean of censored weibull lifetimes. *Journal of Quality Technology*, 36, 321–328.