A New Uncertain Modeling of Production Project Time and Cost Based on Atanassov Fuzzy Sets

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Abstract- Uncertainty plays a major role in any project evaluation and management process. One of the trickiest parts of any production project work is its cost and time forecasting. Since in the initial phases of production projects uncertainty is at its highest level, a reliable method of project scheduling and cash flow generation is vital to help the managers reach successful implementation of the project. In the recent years, some scholars have tried to address uncertainty of projects in time and cost by using basic uncertainty modeling tools such as fuzzy sets theory. In this paper, a new approach is introduced to model project cash flow under uncertain environments using Atanassov fuzzy sets or intuitionistic fuzzy sets (IFSs). The IFSs are presented to calculate project scheduling and cash flow generation. This modern approach enhances the ability of managers to use their intuition and lack of knowledge in their decision-makings. Moreover, unlike the recent studies in this area, this model uses a more sophisticated tool of uncertain modeling which is highly practical in real production project environments. Furthermore, a new effective IFS-ranking method is introduced. The methodology is exemplified by estimating the working capital requirements in an activity network. The proposed model could be useful for both project proposal evaluation during feasibility studies and for performing earned value analysis for project monitoring and control.

Keywords: Production projects, Atanassov fuzzy sets, Intuitionistic fuzzy project scheduling, Intuitionistic fuzzy cost flow.

I. INTRODUCTION

Project managers rely on accurate prediction of cash flow time series over the life cycle of the project to be in a position to predict potential problems and to create appropriate strategies to minimize the negative impacts of such on successful project implementation (Hwee & Tong, 2002). Accurately forecasting production and estimating its required reserves have been more urgent and important than ever before (Duong, 2011). Moreover, in production projects the products’ characteristics create production constraints in different aspects like scheduling and cash flows. Future position of production firms highly depends on research and development (R&D) activities. Their survival is imperiled by ever-growing complexity of technologies. The importance of R&D in highly industrialized economies is undeniable (Santamaría et al., 2010). To put differently, innovation is considered by many firms as a central requirement to their survival programs. New product development (NPD) is a highly effective way to provide production firms with a leading edge over rivals and provide new opportunities (Lawson et al., 2006). To consider R&D projects, production organizations have to identify new projects to make their costs lower, bring new products to their markets, and make their quality higher. Properly addressing risk and uncertainty in R&D environment is essential since knowledge of the proposed projects are often vague and uncertain. This uncertainty puts firm’s plans and strategies in danger. Future events and opportunities have undeniable impact on R&D decisions. Consequently, most of the information used in project decision-makings is at the best uncertain and at the worst condition very unreliable (Bhattacharyya, 2015). On the other hand, the job-shops or smaller firms simply cannot match the dominance or resources that the larger firms
enjoy, allowing them to be inflexible along their supply chains (Chen et al., 2014). Therefore, they cannot afford to be stuck in unexpected situations.

Project cash flow consists of complete record of all cash related events like expenses, insufficiency, loans and borrowings. Cash flow fluctuations increase the level of expected external financing costs and as a result, project faces high monetary costs. Cash flow forecasting is almost the main objective of project cash flow related researches. Project cash flow forecasting should be based on a method that is both effective and reliable. Consequently, an effective and reliable approach for forecasting cash flow is necessary (Jiang et al., 2011).

In the conventional project management approach, the stability of the project plan is regarded as a critical success factor. However, the ever-increasing level of complexity and uncertainty in today’s business environment has made it almost impossible to ignore the great need to a high level of adaptability to expected and unexpected changes (Caron & Comandulli, 2014). One of the main sources of complexity in any project related analysis and planning process is uncertainty. Uncertainty can be caused by different sources like information that due to nature of the project is unquantifiable, incomplete or even non-obtainable (Mousavi et al., 2013).

The main purpose of this paper is to propose a model of project cost and time planning that addresses project uncertainty by using an uncertainty-modeling tool that is capable of expressing knowledge as well as lack of knowledge. In other words, this paper proposes a model of project scheduling and a model of cost forecasting that is based on Atanassov fuzzy sets or intuitionistic fuzzy sets (IFSs). This approach enables the decision maker (DM) to express membership, non-membership and hesitation degrees when expressing data that is based on judgment and due to lack of enough historical data cannot be expressed by statistical tools.

Furthermore, the main characteristics of this IFS based project cost and time planning that distinguish it from existing studies in the literature are as follows: (1) introducing a new approach in modeling project time and cost uncertainty based on the IFS, (2) proposing a new project scheduling method based on the concept of uncertain critical path method (CPM) and Atanassov fuzzy sets, (3) modeling a novel approach of IFS ranking and ordering, and (4) proposing a new model of cash forecasting that uses IFS as an uncertainty modeling tool.

This paper is organized as follows: Section II reviews the existing literature on the project cost and time analysis. Some preliminary introductions to IFSs are provided in section III. Section IV presents a novel ranking method for IFS and introduces IF-project scheduling. In section V, the project cost and cash flow analysis model is proposed. Section VI includes a practical example in construction projects and its corresponding results and finally, Section VII provides the conclusion remarks of this paper.

II. LITERATURE REVIEW

Using S-curves to predict project cash flow is very popular among project managers since it is the simplest way of projection (Touran et al., 2004). Project costs and durations are two primary quantities of total relevant quantity that form the S-curve (Cioffi, 2005). The term “S-Curve” is used due to the shape of cost-time curve that is quite like letter “S” (Cooke & Jepson, 1979). Accuracy of any S-curve based method is determined by the effectiveness of assumptions of conditions representing the real-world situation of the project (Boussabaine & Kaka, 1998).

Projects involvement in uncertainties and complexities is notable and managers have to make decisions under uncertain environments (Mohagheghi et al., 2015). The complex nature of production project environment imposes a high degree of uncertainty in different aspects of project management including project cash flow analysis and project scheduling. In project environments, facing situations with few data, extreme values, emerging changes, uncertainties and uniqueness is very common. In fact, the aforementioned conditions are a part of project nature (San Cristóbal et al., 2015).

As a matter of fact, cash is the main bloodline of any organization. It is possible for a firm to survive without profit for some time, but without cash no firm can operate properly and all firms, no matter in which sector they are, risk facing bankruptcy after a while (Tangsucheeva & Prabhu, 2014). Several sources cause the financial risks, which highlights the need for a reliable cash flow assessing method that overcomes the existing difficulties by providing a comprehensive cash flow forecasting (Barbosa & Pimentel, 2001). Fuzzy sets theory has been widely applied as a tool to model uncertainty in different problems. Due to uncertain nature of projects and vague and unknown information of
projects, fuzzy sets theory has been applied as an appropriate tool to handle project uncertainty. (Kumar et al., 2000) introduced a model of working capital requirements prediction under fuzzy uncertainty. (Lam et al., 2001) proposed a fuzzy optimization model that finds the optimum cash flow using the minimum amount of resources. (Hsu, 2003) introduced statistical models based on S-curve to predict or evaluate construction project cash flow. (Blyth & Kaka, 2006) proposed a multiple linear regression model to predict S-curves of individual projects. They focused on standardizing the activities, and predicting the duration, cost and end dates of the activities. (Gormley & Meade, 2007) employed time series to introduce a model to predict expectations of further cash flows. (Jarrah et al., 2007) introduced a model based on a fourth degree polynomial regression analysis approach in order to forecast and predict cash flows and trends. (Khosrowshahi & Kaka, 2007) introduced a model based on the concept of decision support systems to manage cash flows of construction projects. (Cheng & Roy, 2009) introduced an evolutionary fuzzy decision model that applied S-curves to foresee project cash flow. (Maravas & Pantouvakis, 2012) developed a cash flow assessment model under fuzzy environment. (Rostami et al., 2013) introduced a fuzzy statistical expert system for cash flow analysis, which was designed to handle the uncertain environment of projects. (Chen et al., 2013) introduced a multivariate model in order to evaluate how key variables in project initiation and planning phases. (Ungureanu & Vernic, 2014) introduced a fuzzy cash flow model with applications in risk mitigation. (Cheng et al., 2015) proposed a model of cash flow prediction that was based on a hybrid artificial intelligence model. Their model was solely depending on data gathered from a specific project. In other words, the model was unable to address different sort of projects.

It is concluded from the above that several fuzzy techniques such as fuzzy averaging, fuzzy composition matrices, fuzzy reasoning, fuzzy optimization, fuzzy multi objective decision models, and neuro-fuzzy inference have been utilized to generate project cash flows. Despite this effort, all the mentioned studies were based on the concept of classical fuzzy sets theory, which has its own shortcomings. Although it is mainly accepted that uncertainty management is a vital requirement for effective project management, it can be discussed that it should get more sophisticated before obtaining practical results (Atkinson et al., 2006). One area of classical fuzzy sets theory inadequacies is where a DM is expected to give an exact opinion in a number in interval [0, 1]. As mentioned earlier in initial project phases the level of uncertainty is so high that using a crisp value to express the membership degree decreases the effectiveness of uncertainty modeling. Classical fuzzy sets are unable to express the expert’s degree of hesitation. Moreover, they only consider the membership degree and do not address non-membership degree. In other words, in today’s competitive business environment uncertainty needs to be fully addressed in any decision-making process. In production projects that often deal with new technological advancements and uncertain markets, uncertainty requires a sophisticated addressing tool. Since classical fuzzy sets lack the ability to fully express uncertain project elements, IFSs are applied in this paper. Using IFS gives the model the following advantages over the classical fuzzy based studies:

- IFS presents membership degree, non-membership degree and hesitancy degree by using three grades of membership function, respectively. Whereas the triangular and the trapezoidal fuzzy numbers lack this ability and each only can denote one crisp grade of membership in the unit interval [0,1] (Szmidt et al., 2014).
- Despite the similarities between IFS and interval-valued fuzzy set, a number of references in the literature on IFSs (around 1000 papers) suggest that many researchers find the advantages of IFSs over the equivalent interval-valued fuzzy sets (Atanassov, 2008); (Zhu & Liao, 2014).
- IFS unlike all of triangular fuzzy numbers, trapezoidal fuzzy numbers and interval-valued fuzzy numbers can reflect the “disagreement” of the DM in addition to the fuzziness of “agreement” (Zhu & Liao, 2014).

These sets despite their novelty have been successfully used in a wide range of real world problems. For instance, they were employed to select renewable energy technologies for electricity generation in Turkey (Boran et al., 2012). They were used in construction site layout planning (Ning et al., 2011). They were applied in supplier selection problems (Chai et al., 2012). They were employed to address software selection problems (Wang, 2012). Medical diagnosis were carried out by these sets (Szmidt & Kacprzyk, 2001); (Neog & Sut, 2011). Eventually, in project management one of its applications were presented by (Gerogiannis et al., 2011). They used IFSs in project evaluation and portfolio management.
III. PRELIMINARY KNOWLEDGE OF TRIANGULAR INTUTIONISTIC FUZZY SETS

In the following, in order to illustrate the basic concepts of the introduced model some basic concepts related to intuitionistic fuzzy sets are introduced.

(Zadeh, 1965) proposed fuzzy sets to illustrate imprecise or vague information. Through the years fuzzy sets theory has proven itself as a useful tool to handle uncertain situations by denoting a degree to which a certain object belongs to a set. In real situations, an object can belong to a set to a certain degree, but there can be hesitations. In other words, when there is hesitation or uncertainty about the membership degree, fuzzy set theory has no way to imply that hesitation in the membership degrees (Zimmermann, 2001). IFS introduced by A (tanassov, 1983) is a possible solution to overcome this problem. IFS uses a degree of truth membership function \( u_\delta(x) \) and one of falsity membership function \( \nu_\delta(X) \) to represent lower bound \( (u_\delta(x)) \) and upper bound \( (1 - \nu_\delta(X)) \) such that \( u_\delta(x) + \nu_\delta(X) \leq 1 \). By complementing the membership degree with a non-membership degree that expresses how the element is not in the IFS, the interval \( [u_\delta(x), 1 - \nu_\delta(X)] \) can develop the fuzzy set of membership function. The hesitation or uncertainty of \( x \) can be measured for each \( x \) by the size of the interval \( \pi_\delta(x) = 1 - \nu_\delta(X) - u_\delta(x) \). If the \( \pi_\delta(x) \) is small, it represents more certainty about \( x \). As \( \pi_\delta(x) \) gets greater, it denotes more uncertainly about \( x \). Obviously, if \( u_\delta(x) = 1 - \nu_\delta(X) \) for all elements of the universe, the IFS becomes traditional fuzzy set (Shu et al., 2006).

From definition of triangular intuitionistic fuzzy set, four arithmetic operations for triangle vague sets are defined as follows (Shu et al., 2006). \( A \) and \( B \) are two IFSs, as depicted in Fig. 1 (Lee, 1998). If two intuitionistic fuzzy sets \( u_\delta(x) \neq u_\delta(x) \), and \( \nu_\delta(x) \neq \nu_\delta(x) \), then the arithmetic operations are defined as:

\[
A = \{(a'_1, b_1, c_1); u_A, (a_1, b_1, c_1); v_A)\}
\]
\[
B = \{(a'_2, b_2, c_2); u_B, (a_2, b_2, c_2); v_B)\}
\]
\[
A + B = \{(a'_1, b_1, c_1); u_A, (a_1, b_1, c_1); v_A)\} + \{(a'_2, b_2, c_2); u_B, (a_2, b_2, c_2); v_B)\}
= \{(a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(u_A, u_B), (a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(v_A, v_B)\}
\]
\[
A - B = \{(a'_1, b_1, c_1); u_A, (a_1, b_1, c_1); v_A)\} - \{(a'_2, b_2, c_2); u_B, (a_2, b_2, c_2); v_B)\}
= \{(a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(u_A, u_B), (a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(v_A, v_B)\}
\]
\[
A \times B = \{(a'_1, b_1, c_1); u_A, (a_1, b_1, c_1); v_A)\} \times \{(a'_2, b_2, c_2); u_B, (a_2, b_2, c_2); v_B)\}
= \{(a'_1 a_2 b_1 2 c_1 + c_2); \min(u_A, u_B), (a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(v_A, v_B)\}
\]
\[
A \div B = \{(a'_1, b_1, c_1); u_A, (a_1, b_1, c_1); v_A)\} \div \{(a'_2, b_2, c_2); u_B, (a_2, b_2, c_2); v_B)\}
= \{(a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(u_A, u_B), (a'_1 + a_2, b_1 + b_2, c_1 + c_2); \min(v_A, v_B)\}
\]

![Fig. 1. Triangular IFS A and B](image-url)
IV. THE INTRODUCED MODEL OF UNCERTAIN PRODUCTION PROJECT SCHEDULING

A. Proposed ranking method

In this paper, a distance-based similarity measure between two triangular intuitionistic fuzzy numbers is proposed, that is based on the concept of the model introduced by (Deng, 2014) for measuring the closeness between two IFNs. The step by step algorithm is introduced as follows:

1. Determine the triangular intuitionistic fuzzy positive ideal solution as
\[ \tilde{x}_{\text{max}} = ([x_{\text{max},1}, x_{\text{max},2}, x_{\text{max},3}], u_{\tilde{x}_{\text{max}}}, v_{\tilde{x}_{\text{max}}}) \] and the negative ideal solution as
\[ \tilde{x}_{\text{min}} = ([x_{\text{min},1}, x_{\text{min},2}, x_{\text{min},3}], u_{\tilde{x}_{\text{min}}}, v_{\tilde{x}_{\text{min}}}). \]

2. Determine the degree of similarity between each triangular intuitionistic fuzzy number
\[ \tilde{A}_i = ([a_{i,1}, a_{i,2}, a_{i,3}], u_{a_i}, v_{a_i}) \] (i = 1, 2, ..., n) and the positive triangular intuitionistic fuzzy ideal solution (\( \tilde{x}_{\text{max}} \)) by using the following (Liang et al., 2014):
\[
d_i^+(\tilde{A}_i, \tilde{x}_{\text{max}}) = \frac{1}{6} \left( (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_1 - (1 + u_{\tilde{x}_{\text{max}}} - v_{\tilde{x}_{\text{max}}})\tilde{x}_{\text{max},1} \right) \\
+ (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_2 - (1 + u_{\tilde{x}_{\text{max}}} - v_{\tilde{x}_{\text{max}}})\tilde{x}_{\text{max},2} \\
+ (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_3 - (1 + u_{\tilde{x}_{\text{max}}} - v_{\tilde{x}_{\text{max}}})\tilde{x}_{\text{max},3} \right)
\]

3. Determine the degree of similarity between each triangular intuitionistic fuzzy number
\[ \tilde{A}_i = ([a_{i,1}, a_{i,2}, a_{i,3}], u_{a_i}, v_{a_i}) \] (i = 1, 2, ..., n) and the negative triangular intuitionistic fuzzy ideal solution (\( \tilde{x}_{\text{min}} \)) by using the following:
\[
d_i^+(\tilde{A}_i, \tilde{x}_{\text{min}}) = d_i^+(\tilde{A}_i, \tilde{x}_{\text{min}}) \]
\[
= \frac{1}{6} \left( (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_1 - (1 + u_{\tilde{x}_{\text{min}}} - v_{\tilde{x}_{\text{min}}})\tilde{x}_{\text{min},1} \right) \\
+ (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_2 - (1 + u_{\tilde{x}_{\text{min}}} - v_{\tilde{x}_{\text{min}}})\tilde{x}_{\text{min},2} \\
+ (1 + u_{\tilde{A}_i} - v_{\tilde{A}_i})a_3 - (1 + u_{\tilde{x}_{\text{min}}} - v_{\tilde{x}_{\text{min}}})\tilde{x}_{\text{min},3} \right)
\]

4. Calculate the overall performance index (\( P_i \)) of each triangular intuitionistic fuzzy number \( \tilde{A}_i \) (i = 1, 2, ..., n) by using the following:
\[
P_i = \frac{d_i^-}{d_i^- + d_i^+}, i = 1, 2, ..., n
\]

Step 5. Rank the triangular intuitionistic fuzzy number \( \tilde{A}_i \) (i = 1, 2, ..., n) in descending order of \( P_i \).
This method provides the DM with more control over the process. Selecting the triangular intuitionistic fuzzy positive ideal solution as \( \tilde{x}_{\text{max}} \) and the negative ideal solution as \( \tilde{x}_{\text{min}} \) is a step that can be done based on the nature of the project. To enhance the ability of model in ranking numbers with very different values a normalization step could also be added to the process. This approach, in addition to its simplicity, is effective which makes the process more practical for project environments.

B. Proposed project scheduling model

Applying project scheduling in modeling real-world projects requires a method that can model uncertainty and hesitations. Therefore, in this section an IF-project scheduling model is presented. IF-project scheduling can model
activities with uncertain durations by considering a degree of membership, non-membership and hesitation. By calculating early start and early finish time, forward pass can be obtained (Chanas & Kamburowski, 1981); (Mc Cahon & Lee, 1988); (Prade, 1979).

\[
\tilde{E}_{S}^{\text{start}} = (0)
\]

\[
\tilde{E}_S = \max (\tilde{E}_F)_{p \in P} = \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); u_{ef} \right\}, \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); v_{ef} \right\}
\]

\[
\tilde{d} = \left\{ \left( d_1, d_2, d_3 \right); u_d \right\}, \left\{ \left( d_1, d_2, d_3 \right); v_d \right\}
\]

\[
\tilde{E}_F = \tilde{E}_S + \tilde{d} = \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); u_{ef} \right\}, \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); v_{ef} \right\}
\]

\[
\left\{ \left( e_{f_1} + d_1, e_{f_2} + d_2, e_{f_3} + d_3 \right); \min (\mu_{ef}, \mu_d) \right\}, \left\{ \left( e_{f_1} + d_1, e_{f_2} + d_2, e_{f_3} + d_3 \right); \min (\nu_{ef}, \nu_d) \right\}
\]

where \( \tilde{E}_S \) is the IF-early start time, \( \tilde{E}_F \) is the IF-early finish time, \( P \) is the set of proceeding activities and \( \tilde{d} \) is the IF-activity duration.

V. THE PROPOSED COST FORECAST MODEL

Proposing IF-project scheduling will involve the impacts of activities durations vagueness in cost forecast and cash flow analysis. In order to get an insight of durations under optimistic and pessimistic situations, activities beginning in the earliest time and requiring the least duration (Min \( D_a \)) and activities beginning in the latest time and finishing at the longest duration (Max \( D_a \)) should be considered in cost forecasting calculations (Maravas & Pantouvakis, 2012). Duration calculation for activities with IF times and durations is presented. These durations for activities with early start \( \tilde{E}_S = \left\{ \left( e_{s_1}, e_{s_2}, e_{s_3} \right); u_{es} \right\}, \left\{ \left( e_{s_1}, e_{s_2}, e_{s_3} \right); v_{es} \right\} \) and early finish \( \tilde{E}_F = \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); u_{ef} \right\}, \left\{ \left( e_{f_1}, e_{f_2}, e_{f_3} \right); v_{ef} \right\} \) are presented as follows:

\[
\text{Min } D_a = \left[ \left( \min D_a \right); u_D \right], \left( \min D_a \right); (1 - v_D)
\]

\[
\text{Min } D_a = \left[ \left( \inf \tilde{E}_S \right); u_{es} \right], \left\{ \left( \inf \tilde{E}_S \right); v_{es} \right\}; \left( \min u_{es}, u_D \right)
\]

\[
= \frac{\alpha}{\alpha} \left( e_{s_2} - e_{s_1} \right) + \frac{\alpha}{u_{ef}} (e_{f_2} - e_{f_1}) + e_{f_1}
\]

\[
\text{Max } D_a = \left[ \left( \max D_a \right); u_D \right] \left( \max D_a \right); (1 - v_D)
\]

\[
\text{Max } D_a = \left[ \left( \inf \tilde{E}_S \right); u_{es} \right], \left\{ \left( \inf \tilde{E}_S \right); v_{es} \right\}; \left( \max u_{es}, u_D \right)
\]

\[
= \frac{\alpha}{\alpha} \left( e_{s_2} - e_{s_3} \right) + \frac{\alpha}{u_{ef}} (e_{f_2} - e_{f_3}) + e_{f_3}
\]

where \( \text{Min } D_a; u_D \) and \( \text{Min } D_a; (1 - v_D) \) represent \( \alpha \)-cut of upper and lower bounds of minimum duration in different levels of \( \alpha \). \( \text{Max } D_a; u_D \) and \( \text{Max } D_a; (1 - v_D) \) denote \( \alpha \)-cut of upper and lower bounds of maximum durations under different levels of \( \alpha \), respectively. \( \text{Sup} \) denotes supremum and \( \text{Inf} \) denotes infimum. It should be noted the trust degree of triangular intuitionistic fuzzy number is between \([u, 1 - v] \).}

The resulting IF-CPM not only enables the managers to express membership degree, non-membership degree and hesitation in calculations but also provides a more thorough understanding of the activities durations under different levels of knowledge by using the concept of \( \alpha \)-cuts.

Project cost fluctuations are caused by uncertainty and vagueness of cost and duration. Correlation of lack of
knowledge and vagueness in time and cost can follow different patterns. In real production project environment, they are most of the times positively correlated. Therefore, the upper and lower bounds of the best and the worst conditions of cash distribution activity \( i \) with \( Cost_i = \{((c_{i1}, c_{i2}, c_{i3}); u_{ci}), ((c_{i1}, c_{i2}, c_{i3}); v_{ci})\} \) and duration of \( d_i = \{[(d_{i1}, d_{i2}, d_{i3}); u_{di}], [(d_{i1}, d_{i2}, d_{i3}); v_{di}]\} \) per unit of time \( t \) at level \( \alpha \) is introduced as the following.

\[
\begin{align*}
\min Cash Distribution_{t,a; u_{cd}} & = \frac{\inf cost_{t,a; u_{ci}}}{\sup Duration_{t,a; u_{ci}}} = \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3} \right)} \quad (20) \\
\min Cash Distribution_{t,a; (1 - v_{cd})} & = \frac{\inf cost_{t,a; (1 - v_{ci})}}{\sup Duration_{t,a; (1 - v_{ci})}} = \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (21) \\
\max Cash Distribution_{t,a; u_{cd}} & = \frac{\sup cost_{t,a; u_{ci}}}{\inf Duration_{t,a; u_{ci}}} = \frac{\alpha_c (c_{i2} - c_{i3}) + c_{i3}}{u_{ci} \left( d_{i2} - d_{i1} + d_{i1} \right)} \quad (22) \\
\max Cash Distribution_{t,a; (1 - v_{cd})} & = \frac{\sup cost_{t,a; (1 - v_{ci})}}{\inf Duration_{t,a; (1 - v_{ci})}} = \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (23)
\end{align*}
\]

In order to calculate the cost in the entire activity network, sum of direct cash distribution of all activities in each period can be obtained by the following.

\[
\begin{align*}
\min TC_{a; u_{tc}} & = \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3} \right)} \quad (24) \\
\min TC_{a; (1 - v_{tc})} & = \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (25) \\
\max TC_{a; u_{tc}} & = \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i3}) + c_{i3}}{u_{ci} \left( d_{i2} - d_{i1} + d_{i1} \right)} \quad (26) \\
\max TC_{a; (1 - v_{tc})} & = \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (27)
\end{align*}
\]

where \( TC \) is the total direct cash distribution of activities \((i = 0,1,2,..., n)\) being implemented in the \( t \) time period. Total cash flow of project can be calculated by using the following:

\[
\begin{align*}
\min CFT_{a; u_{cft}} & = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3} \right)} \quad (28) \\
\min CFT_{a; (1 - v_{cft})} & = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (29) \\
\max CFT_{a; u_{cft}} & = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i3}) + c_{i3}}{u_{ci} \left( d_{i2} - d_{i1} + d_{i1} \right)} \quad (30) \\
\max CFT_{a; (1 - v_{cft})} & = \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{\alpha_c (c_{i2} - c_{i1}) + c_{i1}'}{u_{ci} \left( d_{i2} - d_{i3} + d_{i3}' \right)} \quad (31)
\end{align*}
\]

The minimum and maximum values with their corresponding upper and lower bounds depict the limits of predicted
cost in the case of the best and the worst events. This method would enable the DM to have a thorough understanding of uncertain and vague data in different stages of the project life cycle and would help the manager take the proper decisions. Upper and lower bound of uncertainty in cash flow in different \( \alpha \) levels can be obtained by the following.

\[
CFU_\alpha = [CFU_{\alpha; u_{cfu}}, CFU_{\alpha; 1 - v_{cfu}}]
\]

\[
CFU_{\alpha; u_{cfu}} = \max CFT_{\alpha; u_{cf}} - \min CFT_{\alpha; u_{cf}} - \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{a_{ci}}{u_{ci}} (d_{i \rightarrow t} - d_{i \rightarrow t}) + d_{i \rightarrow t} - \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{a_{ci}}{u_{ci}} (d_{i \rightarrow t} - d_{i \rightarrow t}) + d_{i \rightarrow t}
\]

\[
CFU_{\alpha; (1 - v_{cfu})} = \max CFT_{\alpha; (1 - v_{cf})} - \min CFT_{\alpha; (1 - v_{cf})} - \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{a_{ci}}{(1 - v_{ci})} (d_{i \rightarrow t} - d_{i \rightarrow t}) + d_{i \rightarrow t} - \sum_{t=0}^{T} \sum_{i=1}^{n} \frac{a_{ci}}{(1 - v_{ci})} (d_{i \rightarrow t} - d_{i \rightarrow t}) + d_{i \rightarrow t}
\]

VI. APPLICATION EXAMPLE

In order to illustrate the proposed model, a network of main activities in a production project is presented and its cost and dates are analyzed by the proposed model. The activity network is displayed in Fig. 2, and the adopted information is reported in Table I regarding the vague and uncertain activities durations and costs.

![Sample activity network](image)

**TABLE I. Activity network data**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Predecessors</th>
<th>IF-Duration (days)</th>
<th>IF-cost (k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>{([3,4,5];0.6),([1,4,7];0.3)}</td>
<td>{([7,10,13];0.6),([5,10,16];0.3)}</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>{([4,5,6];0.6),([2,5,8];0.3)}</td>
<td>{([11,15,18];0.6),([8,15,22];0.3)}</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>{([6,8,10];0.6),([2,8,14];0.3)}</td>
<td>{([30,35,40];0.6),([25,35,45];0.3)}</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>{([3,4,5];0.6),([1,4,7];0.3)}</td>
<td>{([12,15,17];0.6),([10,8,21];0.3)}</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
<td>{([9,12,15];0.6),([3,12,21];0.3)}</td>
<td>{([16,20,26];0.6),([14,20,36];0.3)}</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>{([15,20,25];0.6),([5,20,35];0.3)}</td>
<td>{([38,40,44];0.6),([32,40,50];0.3)}</td>
</tr>
<tr>
<td>G</td>
<td>D</td>
<td>{([5,7,9];0.6),([1,7,13];0.3)}</td>
<td>{([27,30,34];0.6),([22,30,38];0.3)}</td>
</tr>
<tr>
<td>H</td>
<td>E,F,G</td>
<td>{([8,11,14];0.6),([2,11,20];0.3)}</td>
<td>{([6,10,14];0.6),([4,10,18];0.3)}</td>
</tr>
</tbody>
</table>

A. Computational results

Early start time and early finish time under vague environment is calculated by applying IF-project scheduling. Table II displays the corresponding results. For the purpose of illustration, the calculation for activity C is displayed in...
the following.

\[ \hat{E}_C = \hat{E}_S + \hat{a}_C = \{(3,4,5); (0,6)\}, \{(1,4,7); (0,3)\} + \{(6,8,10); (0,6)\}, \{(2,8,14); (0,3)\} \]

\[ = \{(9,12,15); (0,6)\}, \{(3,12,21); (0,3)\} \]  

Cash distributions of activities under different levels of uncertainty are calculated, and the results are displayed in Tables III. For the purpose of illustration, the calculations for activity A are presented in the following.

\[
\min \text{Cash Distribution}_{A,t,0.3; 0.6} = \frac{0.3}{0.6} (10 - 7) + 7 = 2.4
\]

\[
\min \text{Cash Distribution}_{A,t,0.3; (1 - 0.3)} = \frac{0.3}{0.7} (10 - 5) + 5 = 0.86
\]

\[
\max \text{Cash Distribution}_{A,t,0.3; 0.6} = \frac{0.3}{0.6} (10 - 13) + 13 = 3.28
\]

\[
\max \text{Cash Distribution}_{A,t,0.3; (1 - 0.3)} = \frac{0.3}{0.7} (10 - 16) + 16 = 5.7
\]

TABLE II. IF-project scheduling (days)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Early start</th>
<th>Early finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>{(0,0,0); (0,0,0); (0,0,0)}</td>
<td>{(3,4,5); (1,4,7); (0,3)}</td>
</tr>
<tr>
<td>B</td>
<td>{(3,4,5); (1,4,7); (0,3)}</td>
<td>{(7,9,11); (3,9,15); (0,3)}</td>
</tr>
<tr>
<td>C</td>
<td>{(3,4,5); (1,4,7); (0,3)}</td>
<td>{(9,12,15); (3,12,21); (0,3)}</td>
</tr>
<tr>
<td>D</td>
<td>{(3,4,5); (0,6); (1,4,7); (0,3)}</td>
<td>{(6,8,10); (2,8,14); (0,3)}</td>
</tr>
<tr>
<td>E</td>
<td>{(7,9,11); (3,9,15); (0,3)}</td>
<td>{(16,21,26); (6,21,36); (0,3)}</td>
</tr>
<tr>
<td>F</td>
<td>{(9,12,15); (3,12,21); (0,3)}</td>
<td>{(24,32,40); (8,32,56); (0,3)}</td>
</tr>
<tr>
<td>G</td>
<td>{(6,8,10); (2,8,14); (0,3)}</td>
<td>{(15,20,25); (13,21,28); (0,3)}</td>
</tr>
<tr>
<td>H</td>
<td>{(24,32,40); (0,6); (8,32,56); (0,3)}</td>
<td>{(32,43,54); (10,43,76); (0,3)}</td>
</tr>
</tbody>
</table>

TABLE III. Cash distribution \(\frac{k\$/day}{\text{day}}\)

<table>
<thead>
<tr>
<th>Activity</th>
<th>((1 - \nu_{cfu}))</th>
<th>(u)</th>
<th>Max cash distribution</th>
<th>Maravas and Pantouvakis method (2012)</th>
<th>((1 - \nu_{cfu}))</th>
<th>(u)</th>
<th>Min cash distribution</th>
<th>Maravas and Pantouvakis method (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>3.73</td>
<td>3</td>
<td>3.47</td>
<td>1.76</td>
<td>2.08</td>
<td>1.78</td>
<td>2.08</td>
<td>1.78</td>
</tr>
<tr>
<td>C</td>
<td>6.02</td>
<td>5</td>
<td>5.5</td>
<td>3.31</td>
<td>3.85</td>
<td>3.47</td>
<td>3.85</td>
<td>3.47</td>
</tr>
<tr>
<td>D</td>
<td>3.73</td>
<td>4.27</td>
<td>4.76</td>
<td>1.76</td>
<td>3.23</td>
<td>2.86</td>
<td>3.23</td>
<td>2.86</td>
</tr>
<tr>
<td>E</td>
<td>2.61</td>
<td>2</td>
<td>2.3</td>
<td>1.25</td>
<td>1.44</td>
<td>1.27</td>
<td>1.44</td>
<td>1.27</td>
</tr>
<tr>
<td>F</td>
<td>2.73</td>
<td>2.25</td>
<td>2.49</td>
<td>1.55</td>
<td>1.82</td>
<td>1.68</td>
<td>1.82</td>
<td>1.68</td>
</tr>
<tr>
<td>G</td>
<td>2.6</td>
<td>2.85</td>
<td>3.17</td>
<td>2.09</td>
<td>2.23</td>
<td>2.04</td>
<td>2.23</td>
<td>2.04</td>
</tr>
<tr>
<td>H</td>
<td>1.46</td>
<td>1.13</td>
<td>1.34</td>
<td>0.61</td>
<td>0.72</td>
<td>0.59</td>
<td>0.72</td>
<td>0.59</td>
</tr>
</tbody>
</table>

The results presented in Table III show the upper and lower bounds of cash distribution of each activity under different levels of uncertainty. In order to verify the proposed model, illustrative example was solved with the recent method proposed by (Maravas & Pantouvakis, 2012). Applying the existing model yielded close results to the proposed model while it lacked some advantages of the proposed model under uncertainty. The difference is that while applying
IFSs, the model provided a better understanding of the situation by showing results based on membership and non-membership degrees. For instance, maximum cash distribution of activity A under classical fuzzy sets is 3.47 units whereas the IFS gives a result between 3 and 3.75. Furthermore, the ability of the IFS in expressing hesitation and non-membership degree, and consequently better expressing uncertainty should be added to the advantages of the introduced model.

Moreover, to give a better understanding of costs in different periods of the projects, shortest and longest activity durations should be calculated. Table IV displays maximum and minimum activity durations under different levels of uncertainty. For the purpose of illustration, the calculations for activity B are presented in the following.

\[
\text{Min } D_{0.3} : 0.6 = \left[ \frac{0.3}{0.6} (4 - 3) + 3, \frac{0.3}{0.6} (9 - 7) + 7 \right] = [3.5, 8] \tag{40}
\]

\[
\text{Min } D_{0.3} : (1 - 0.3) = \left[ \frac{0.3}{0.7} (4 - 1) + 1, \frac{0.3}{0.7} (9 - 3) + 3 \right] = [2.3, 5.6] \tag{41}
\]

\[
\text{Max } D_{0.3} : 0.6 = \left[ \frac{0.3}{0.6} (4 - 5) + 5, \frac{0.3}{0.6} (9 - 11) + 11 \right] = [5.5, 12] \tag{42}
\]

\[
\text{Min } D_{0.3} : (1 - 0.3) = \left[ \frac{0.3}{0.7} (4 - 7) + 7, \frac{0.3}{0.7} (9 - 15) + 15 \right] = [8.3, 17.6] \tag{43}
\]

In order to verify the proposed model, the activity duration calculations were done with classical triangular fuzzy numbers. Similarly, the comparison has provided the close results. The proposed model that is based on IFSs, presented a better understanding of the condition by displaying results based on a better method of uncertainty modeling. For example, minimum activity duration of activity A under classical fuzzy set is [0, 3.5] days whereas IFS gives results between [0, 3.8] and [0, 2.7]. This means that the introduced model helps the managers avoid unpleasant surprises by providing a better image of the activity.

### B. Discussion of results

\(\alpha\)-cut limits the degree of fuzziness and measures robustness of predictions. A higher levels of \(\alpha\) denotes a higher confidence in the parameters (Li & Vincent, 1995). The risk level increases from “none” to “high” as the \(\alpha\)-cut moves from 1 to 0. To demonstrate this risk analysis approach, calculations for 3 different \(\alpha\)-levels of 0.1, 0.3 and 0.5 for \(u\) and 3 different \(\alpha\)-levels of 0.2, 0.4 and 0.6 for \(1 - \nu_{ceu}\) were made. Table V displays maximum and minimum cash distribution of each activity. The results show different amounts for different levels of knowledge and risk. Furthermore, the same approach is carried out for activity duration. Finally, Table VI displays maximum activity duration and Table VII shows minimum activity duration.

<table>
<thead>
<tr>
<th>Activity</th>
<th>((1 - \nu_{ceu}))</th>
<th>(u)</th>
<th>Maravas and Pantouvakis method (2012)</th>
<th>((1 - \nu_{ceu}))</th>
<th>(u)</th>
<th>Maravas and Pantouvakis method (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[0.5, 3]</td>
<td>[0.4, 2]</td>
<td>[0.4, 5]</td>
<td>[0.2, 7]</td>
<td>[0.3, 8]</td>
<td>[0.3, 5]</td>
</tr>
<tr>
<td>B</td>
<td>[5.3, 11.6]</td>
<td>[4.2, 9.3]</td>
<td>[4.5, 10]</td>
<td>[2.7, 6.4]</td>
<td>[3.8, 8.7]</td>
<td>[3.5, 8]</td>
</tr>
<tr>
<td>C</td>
<td>[5.3, 15.9]</td>
<td>[4.2, 12.5]</td>
<td>[4.5, 13.5]</td>
<td>[2.7, 8.1]</td>
<td>[3.8, 11.5]</td>
<td>[3.5, 10.5]</td>
</tr>
<tr>
<td>D</td>
<td>[5.3, 10.6]</td>
<td>[4.2, 8.3]</td>
<td>[4.5, 9]</td>
<td>[2.7, 5.4]</td>
<td>[3.8, 7.7]</td>
<td>[3.5, 7]</td>
</tr>
<tr>
<td>E</td>
<td>[11.6, 27.4]</td>
<td>[9.3, 21.8]</td>
<td>[10.23.5]</td>
<td>[6.4, 14.6]</td>
<td>[8.7, 20.2]</td>
<td>[8.18.5]</td>
</tr>
<tr>
<td>F</td>
<td>[15.9, 42.3]</td>
<td>[12.5, 33.3]</td>
<td>[13.5, 36]</td>
<td>[8.1, 21.7]</td>
<td>[11.5, 30.7]</td>
<td>[10.5, 28]</td>
</tr>
<tr>
<td>G</td>
<td>[10.6, 24]</td>
<td>[8.3, 20.8]</td>
<td>[9, 22.5]</td>
<td>[5.4, 17.6]</td>
<td>[7.7, 19.2]</td>
<td>[7.17.5]</td>
</tr>
<tr>
<td>H</td>
<td>[42.3, 57.1]</td>
<td>[33.3, 44.8]</td>
<td>[36.48.5]</td>
<td>[21.7, 28.9]</td>
<td>[30.7, 41.2]</td>
<td>[28.37.5]</td>
</tr>
</tbody>
</table>
TABLE V. Cash distribution under different levels of risk

<table>
<thead>
<tr>
<th>Activity</th>
<th>Max cash distribution</th>
<th>Min cash distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1 - v cf_u)</td>
<td>u</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 - v cf_u)</td>
</tr>
<tr>
<td>A</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>B</td>
<td>8.65</td>
<td>5.78</td>
</tr>
<tr>
<td>C</td>
<td>15.25</td>
<td>8.91</td>
</tr>
<tr>
<td>D</td>
<td>13.4</td>
<td>6.75</td>
</tr>
<tr>
<td>E</td>
<td>7.87</td>
<td>4.25</td>
</tr>
<tr>
<td>F</td>
<td>6.8</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>3.27</td>
<td>2.92</td>
</tr>
<tr>
<td>H</td>
<td>5.13</td>
<td>2.49</td>
</tr>
</tbody>
</table>

TABLE VI. Maximum activity duration under different levels of risk

<table>
<thead>
<tr>
<th>Activity</th>
<th>Max u</th>
<th>Max (1 - v cf_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[0.4,8]</td>
<td>[0.4,5]</td>
</tr>
<tr>
<td>B</td>
<td>[4.8,10.7]</td>
<td>[4.5,10]</td>
</tr>
<tr>
<td>C</td>
<td>[4.8,14.5]</td>
<td>[4.5,13.5]</td>
</tr>
<tr>
<td>D</td>
<td>[4.8,9.7]</td>
<td>[4.5,9]</td>
</tr>
<tr>
<td>E</td>
<td>[10.7,25.2]</td>
<td>[10,23.5]</td>
</tr>
<tr>
<td>F</td>
<td>[14.5,38.7]</td>
<td>[13,53,6]</td>
</tr>
<tr>
<td>G</td>
<td>[9.7,24.2]</td>
<td>[9,22.5]</td>
</tr>
<tr>
<td>H</td>
<td>[38.7,52.2]</td>
<td>[36,48.5]</td>
</tr>
</tbody>
</table>

TABLE VII. Minimum activity duration under different levels of risk

<table>
<thead>
<tr>
<th>Activity</th>
<th>Min u</th>
<th>Min (1 - v cf_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[0.3,2]</td>
<td>[0.3,5]</td>
</tr>
<tr>
<td>B</td>
<td>[3.2,7,3]</td>
<td>[3.5,8]</td>
</tr>
<tr>
<td>C</td>
<td>[3.2,9,5]</td>
<td>[3.5,10,5]</td>
</tr>
<tr>
<td>D</td>
<td>[3.2,6.3]</td>
<td>[3.5,7]</td>
</tr>
<tr>
<td>E</td>
<td>[7.3,16.8]</td>
<td>[8,18.5]</td>
</tr>
<tr>
<td>F</td>
<td>[9.5,25.3]</td>
<td>[10,5,28]</td>
</tr>
<tr>
<td>G</td>
<td>[6.3,15,8]</td>
<td>[7,17,5]</td>
</tr>
<tr>
<td>H</td>
<td>[25,3,3,3,8]</td>
<td>[28,37,5]</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

In this paper, a new approach in production project cash flow generation for activities with uncertain and vague duration and cost is proposed. This novel cost-forecasting model is based on IF-CPM that is proposed in this paper. This model gives DMs a comprehensive and thorough insight of project cost in different stages of project life cycle. This comprehensive insight improves the DM’s knowledge of vagueness and lack of knowledge and as a result, upper and lower bounds of the required resources in different stages of project implementation are identified. Consequently, this
model helps avoid unpleasant surprises in the worst-case scenarios. Applying IFSs enables the method to model uncertainty, vagueness and hesitation with more flexibility in addition to adding all the advantages of IFSs over other fuzzy sets. Using IFSs makes the model more suitable for projects like new product development (NPD) and research and development (R&D), in which the information is vague, unclear and with hesitation. Since the introduced model applies a sophisticated uncertainty modeling technique, it could be beneficial in feasibility study in addition to project implementation stage. To put differently, the results could provide reliable inputs in project evaluation methods such as net present value. For the purpose of illustration, the proposed model is applied in a practical example. In the example, uncertain early start times and early finish times are calculated under vague environment. Cash distribution of activities under different levels of uncertainty is also provided to demonstrate different cost conditions under different risk levels. Applying this method as an evaluation tool in earned value analysis could be a promising research direction.

REFERENCES


