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Optimal pricing strategy and cooperation in the supply chain with one direct selling manufacturer and duopoly retailers

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Abstract *– corresponding to the advent of technology, manufacturers are taking into consideration novel means of selling their respective products directly to the customer. Thus, the present paper aims to find the optimal pricing strategy for a supply chain, including a single manufacturer and two retailers. In the proposed model, the manufacturer decides whether to consider the channel for his/her direct sales on the market. Three scenarios are considered: Lack of cooperation between retailers, cooperation between retailers with fixed-price sales, and cooperation between retailers with various sales prices. Stackelberg game is used to examine scenarios in which the manufacturer is a leader, and two retailers pose as followers. In case of no cooperation between retailers, the results demonstrate a higher rate of profit for the manufacturer, while the total profit of the retailers is greater in the event of cooperation, using different sale prices. Finally, a numerical example is provided in order to demonstrate the effectiveness of the aforementioned three scenarios in supply chain models.*

Keywords*– pricing strategy, game theory, cooperation, supply chain, direct sales.*

I. INTRODUCTION

Owing to the unleashed acceleration of recent technological advancements, the worldwide network is at last able to provide many manufacturers, who have for long distributed and delivered their products through traditional measures, with the opportunity to achieve direct sales to their intended end customers (Chiang et al., 2003). Therefore, it is aimed in the present paper to study a pricing problem of the dual supply chain (SC) with regard to a single manufacturer who produces a specific product and subsequently sells the product to a duo of separate retailers. The manufacturer decides to create an online channel to sell the product directly to the end customer. We assume that the manufacturer sets the wholesale and direct sale prices. Moreover, the two retailers choose their own sales prices, along with the quantities of their orders to the manufacturer.

The main objective of this paper is to examine the impacts and consequences, regarding various scenarios, including cooperation between retailers, or lack thereof, following the manufacturer's creation of a direct sale channel to reach the customer, outlining a two-echelon supply chain with two retailers. Our work basically expands the work of Xiao et al. (2014) and Huang et al. (2016). Originally, Xiao et al. (2014) considered a scenario comprising one manufacturer and one retailer in a supply chain, in which regard, the manufacturer sells the product through the retailer (indirect channel). Then, they studied whether the manufacturer finds an opportunity to create a direct (online) channel to sell its products to customers, and proceeded to answer the question of what would happen next. In addition, how extensive of a fraction

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of the market may be established by the manufacturer. However, as formerly stated, their work did not exceed a supply chain with one manufacturer and one retailer. The important difference between present paper and Xiao et al. (2014) is that we consider a manufacturer and two retailers. Later, Huang et al. (2016) took into account a manufacturer who supplies an identical product to two different retailers in the supply chain. They investigated several strategies under a variety of circumstances, including the manufacturer being dominant, the retailers being dominant, or neither, while also scrutinizing the cooperation between retailers in each strategy. Online sales by the manufacturer were, however, not taken into consideration in their paper. Consequently, the work at hand combines these two studies in a way that a manufacturer and two retailers are considered in the supply chain, further studying both direct and indirect sales approaches for the manufacturer alike. Retailers buy the product from the manufacturer by the wholesale price (w_i) and sell to customers using their own specified prices. We also contribute to the event of the manufacturer obtaining an opportunity to sell the product to the customers in a direct way. Additionally, investigation is implemented on the matter of cooperation in supply chain, considering the introduced scenarios; when the retailers do not cooperate with each other, and when they do. For the cooperative situation, we study two states as followed; they buy the product by the same wholesale price (w_c), while selling them for an equal price, or alternatively for different prices.

The remainder of this paper is organized as follows. In Section 2, the related researches in the literature are reviewed. In Section 3, we formulate the problems of determining optimal pricing for the manufacturer and the two retailers, investigated in the case of three scenarios. The numerical studies and the managerial insights are conducted and discussed in Section 4. Section 5 presents the conclusions of the paper, and some topics suggested for future research. We relegate all proofs to the Appendix.

II. LITERATURE REVIEW

Many manufacturers have redesigned their old channel structure by engaging in direct sales as a result of the emergence of e-commerce. Chiang et al. (2003) demonstrated the channel conflict of selling directly, as opposed to the traditional channel, in a manufacturer-retailer supply chain and investigated how optimal pricing policies could lessen the effect of double marginalization in a dual-channel SC. Boyaci (2005) studied a manufacturer-retailer supply chain, in which the manufacturer sells its product directly and through an independent retailer to customers. The author studied optimal stocking levels for each player, where products are substitutable, and demand at each channel is stochastic. Bernstein et al. (2008) analyzed the significance of the Internet as a sales channel for retailers and consumers. They considered the issue where a retailer runs both physical stores and an online store. Based on Yao & Liu (2005), they used Bertrand and Stackelberg models to look at price competition between traditional and direct sales channels. Under a single contract and a menu of contracts, Liu et al. (2010) defined joint production and pricing decisions for a SC with a manufacturer who sells products through the traditional channel and e-channel.

Huang et al. (2012) investigated pricing and production problem in a two-period manufacturer-retailer supply chain using a Stackelberg game under a centralized and decentralized supply chain when the demand was disrupted. Xu et al. (2013) considered a dual-supply chain, in which a single supplier supplies products to two different retailers: a physical retailer and an online retailer. The impact of price comparison services on pricing strategies was explored. Esmaeili et al. (2016) studied the customer's demand influenced by pricing and advertising in a supplier-manufacturer-retailer supply chain; they compared the decision models under a different power structure (Nash, Stackelberg and cooperative). Modak et al. (2016) has proposed a cooperative/non-cooperative closed-loop supply chain that contains a manufacturer and two retailers with the recycling facility considering Cournot and Collusion behaviors of duopolistic retailers When the product sell and collect by retailers. Ding et al. (2016) examined the pricing decisions of the dual-channel supply chain. As a leader, the manufacturer offers a product through a traditional channel to retailers and/or directly to customers. They used the Stackelberg game model under several operational strategies. Giri et al. (2017) presented a closed-loop supply chain that sells products through both the retailer and the e-channel at the same time. Hernant and Rosengren (2017) used a unique database consisting of a Swedish retailer that added a new online channel to its traditional offline stores. Customers' purchasing habits were studied before and after the establishment of an internet channel. Karray and Sigué (2018) considered a manufacturer–retailer supply chain such that the product is sold by the manufacturer through direct and indirect channels, but the traditional offline retailer decides to compete with manufacturer by creating online channel. They investigated how it effects on channel member's profits and strategies. Wang et al. (2018) investigated an e-channel decision problem for a manufacturing, in which the manufacturer chooses whether to complement its existing physical retail channel with a direct sales channel or a third-party delivery channel. They considered a manufacturer-retailer supply chain in their study. In a dual-channel supply chain, When a manufacturer sells through a traditional retailer and an online direct channel to the customer, Tian and Wu (2019) compared the decision models under alternative power structures and service strategies. Chen et al. (2019) developed a game theoretic model in which a firm sells a product in two periods: during the peak season, through a self-operated store, and during the off-season, through an online store. Fan et al. (2020) considered a decentralized dual supply chain when a product is sold by the manufacturer through the two competing online retailers to investigate the value of horizontal cooperation.

In a dual-channel supply chain, there has been a lot of research on the competition and price strategies between the manufacturer and the retailer. Tsay & Agrawal (2004), with regard to a dual-channel supply chain containing a manufacturer and a retailer, investigated the channel conflict and the coordination between members using game theory. Yang & Zhou (2006) considered the pricing and quantity decisions made by a manufacturer who distributes a product to two competing retailers in a two-echelon supply chain. They analyze the impact of three competing behaviors, Cournot, Collusion, and Stackelberg, on the manufacturer's and retailers' optimal decisions. Wu et al. (2009) investigated how two competing supply chains behaved in equilibrium when demand was unpredictable. Huang & Swaminathan (2009) focused on the optimal pricing methods for products supplied through two channels: the Internet and a traditional channel under Nash equilibrium. He et al. (2009) investigates channel coordination for a supplier-retailer supply chain with stochastic demand that is affected by both sales effort and retail pricing. For a competitive dual-retailer supply chain in which a supplier offers a credit period and a cash discount to retailers, Tsao (2011) addresses both the supplier's incentive decisions and the retailers' price and ordering decisions at the same time. Zhao and Wei (2014) explored how a retailer-manufacturer supply chain can be coordinated with a fuzzy demand that is influenced by both retail pricing and sales effort.

Glock & Kim (2015) studied one manufacturer-multiple retailer's supply chain that competes on price and customer welfare. In addition, they considered a situation that the manufacturer merges with one of its retailers and additionally creates direct sales to the market. They selected wholesale price contracts with linear deterministic demand, where the retailers compete on price. Li et al. (2016) used a Stackelberg game modeled by the manufacturer in the centralized and decentralized models under a consistent pricing strategy to explore the pricing policies and greening strategies of a competitive dual-channel supply chain. A dynamic selling approach for a corporation with asymmetric information and product information in two periods was proposed by Dong et al. (2018). In each period, the firm has the option of using direct selling or agent selling. Pi et al. (2019) considered the pricing and service strategies with retailers' competition and cooperation in one manufacturer and two retailers supply chain, in which that manufacturer sells the product directly, although they added the level of service in-demand functions, they ignore the scenario in which retailers sell the product at the same price. Cai et al. (2020) looked at a supply chain arrangement in which one supplier delivers a basic product to two risk-averse retailers, with each retailer providing a warranty policy in an unpredictable market.

This paper investigates the effects of introducing a manufacturer who sells directly to customers and the two retailers' strategy behaviors towards the prices and profits of the supply chain members, under three scenarios: without cooperation, cooperation using the same sale price, and cooperation with different sale prices. Based on Table II, our work differs from the literature above, concerning two attributes: Firstly, we consider a situation that a manufacturer sells its product to two retailers, in addition to creating a direct sales channel, and secondly, three distinctive scenarios are put into examination, aimed at evaluating the effect of cooperation between these retailers, as shown in *Fig* 1.

Table I. Compares and contrasts this study with research that is similar to the current paper

		Game				*Channel*			
			Stackelberg-Leader						
Article	*The number of players*	*Nash*	*Retailer*	*Manufacturer*	*supplier*	*Direct*	*Indirect*	*competition*	*Coordination*
Huang, Ke, & Wang (2016)	3		✓	✓			✓	✓	✓
Boyaci (2005)	2	✓	1	1		✓	✓	✓	✓
Bernstein, Song, & Zheng (2008)	n	✓	n			✓	✓	✓	
Yao & Liu (2005)	2		1	✓		✓	✓	✓	
Huang, Yang, & Zhang (2012)	2		✓	✓		✓	✓	✓	
Chiang, Chhajed, & Hess, (2003)	2		1	✓		✓	✓	✓	
Xu, Liu, & Shen (2013)	3		2		1	✓	✓		
Ding, Dong, & Pan (2016)	2		1	✓		✓	✓	✓	
Wang, Leng, & Liang (2018)	2	✓	1	1		✓	✓		
Tsay & Agrawal (2004)	2		1	✓		✓	✓		✓
Yang & Zhou (2006)	3		✓	1		✓		✓	
Wu, Baron & Berman (2009)	2	✓	1	✓			✓	✓	✓
Huang & Swaminathan (2009)	2	✓	1	1		✓	✓		
Xiao, Choi, & Cheng(2014)	2		✓	✓		✓	✓	✓	
Glock & Kim (2015)	n		✓			✓	✓	✓	
Li, Zhu, Jiang, & Li (2016)	2		1	✓			✓	✓	
Giri, Chakraborty, & Maiti, (2017)	2	✓	✓	✓		✓	✓	✓	
Hernant, & Rosengren(2017)	1		1			✓	✓		
Karray& Sigué (2018)	2		1	✓		✓	✓	✓	✓
Pi, Fang, and Zhang (2019)	3	✓	2	1		✓	✓	✓	✓
Fan, Yin, and Liu (2020)	3		2	1		✓		✓	✓
Tian and Wu (2019)	2	✓	✓	✓		✓	✓	✓	
Current study	3		2	✓		✓	✓	✓	✓

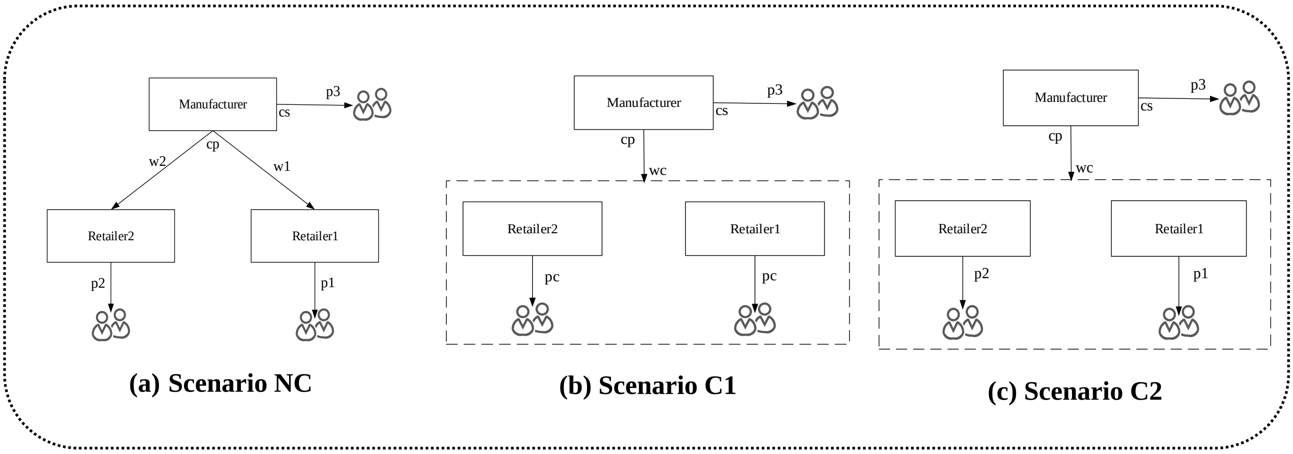


Fig. 1. A two-echelon supply chain problem including one manufacturer and a duopoly of retailers

III. PROBLEM DESCRIPTION

In this paper, a two-echelon supply chain problem including one manufacturer and a duopoly of retailers is considered, where the manufacturer produces a product, subsequently selling this product to the mentioned retailers, in accordance with a wholesale price, while also being able to establish direct sales channel to the end customers. The other side of the stated supply chain consists of two retailers that buy the product from the manufacturer and consequently sell it to the customers. The aim of the paper is to determine the optimal pricing strategy, with reference to the studied supply chain, both under the situation of our two retailers' cooperation, and lack thereof, alike.

In the present study, the Stackelberg game theory is applied between the supply chain members, in an embodiment that the manufacturer plays the role of the leader, leaving the retailers as followers. The competitive conditions and profits are analyzed and determined under two situations, first, where each retailer follows separately, and second when there is a cooperation between them in the supply chain.

Fig. 1 illustrates the aforementioned three scenarios considered in this paper. In the first scenario (*Fig.* 1 (a)), following the establishment of direct sales by the manufacturer, the retailers' trend will continue without any alteration, hence hinting at their lack of cooperation. The second scenario can be observed in *Fig.* 1 (b). In this scenario, the cooperation between two retailers is defined such that they both buy and sell at the same price. One of the reasons that this scenario was investigated is because it could take place in a deal when they agree to cooperate with each other to sell their products at the same price on the market that may be derived from the presence of the same power in the market. Another reason for setting the same price is to prevent mistrust that may occur to end consumers when shopping or referring to shop from these two retailers. Regarding the third scenario, similar cooperation is formed between the two retailers, only this time, the selling price of each retailer is set separately. Determining the different selling prices for each retailer was also considered a scenario because each of them could have different power in the market, and it was allowed to cooperate by determining the different selling prices.

The following notations are used in the paper to formulate the studied problem.

Subscript

i	Denotes the players, i=3 denotes the manufacturer and i=1,2 for retailers
m	Denotes the manufacturer

r Denotes the retailer

Parameters

σ Denotes the price-sensitivity of the product

γ Denotes the substitutability of the manufacturer and the retailers

c_s Cost of direct sales of each product for the manufacturer

c_p Cost of production

d_i The base demand for channel i, while i=3 represents the manufacturer

q_i The demand for player i in terms of direct sales

π_m The manufacturer's profit

π_{ri} The i^{th} retailer's profit

Decision variables

p_i Selling price for channel i, while i=3 represents the manufacturer

w_i Manufacturer's wholesale price of selling to retailer i

When a seller reduces the price, the demand will be increased. In demand functions, two types of customers are considered, switching customers and marginal customers are two types of customers studied (Anderson and Bao, 2010). Switching consumers will almost certainly purchase products from one of the competing retailers or online manufacturers, but they are price-sensitive and will look at the combination of seller's (retailers and manufacturer) non-price and price factors to make a purchase decision. On the other hand, marginal consumers will only buy products from one of the sellers if the price is below a certain level that is considered as a threshold. Both types of customers are included in the demand model.

In equations (1)-(3), the demand function with regard to both the retailers and the manufacturer is shown when they sell directly. Each of them has a base demand that is denoted by d_i, and the segment containing γ shows that how much each player can substitutability the other with a low or high price that he/she suggests.

$$q_1 = d_1 - \sigma p_1 + \gamma(-p_1 + p_2) + \gamma(-p_1 + p_3) \tag{1}$$

$$q_2 = d_2 + \gamma(p_1 - p_2) - \sigma p_2 + \gamma(-p_2 + p_3) \tag{2}$$

$$q_3 = d_3 + \gamma(p_1 - p_3) + \gamma(p_2 - p_3) - \sigma p_3 \tag{3}$$

This linear demand function was also used by Choi (1996), McGuire & Staelin (1983), Jeuland & Shugan (1988), Ingene & Parry (1995), Anderson & Bao (2010), and Wu et al. (2012), Choi (1991).

In order to calculate the expected profits of the manufacturer and the retailers, the sales of each player must be considered. For the manufacture, we have a production cost, cost of creating online selling and versus the revenue of online selling and wholesaling to retailers. However, each retailer sells its purchased product to the end customer. The equations (4)-(6) specify the profit function respectively for the manufacturer and both retailers, the former including two parts; the first representing direct sales profit, and a second part expressing the manufacturer's wholesale profit of selling to retailers. The retailers have their profits subtracted by the purchase costs.

$$\pi_m = (p_3 - c_s)(q_3) + (q_1)(w_1 - c_p) + (q_2)(w_2 - c_p) \tag{4}$$

$$\pi_{r1} = p_1(q_1) - (q_1)w_1 \tag{5}$$

$$\pi_{r2} = p_2(q_2) - (q_2)w_2 \tag{6}$$

In subsection A, we study the scenario in which no cooperation occurs between the two retailers in the mentioned supply chain. In subsections B and C, we consider two strategies when declaring the price announced by the manufacturer (due to the leader role of the manufacturer in this Stackelberg game); both retailers cooperate with each other with the same sale price and different sale prices, respectively.

Throughout the next three sections, these scenarios are described in more detail, and also the required equations are presented.

A. Non-cooperation between the two retailers

In this scenario, the retailers do not express any reactions following the establishment of direct sales by the manufacturer. However, no cooperation occurs between the two of them. In this regard, the term "NC" is used to identify such a scenario. A mix of Stackelberg and Nash game theories is applied to model the mentioned scenario. A Stackelberg game is used between the manufacturer as a leader and retailers as followers, while a Nash game is applied between the two retailers. In the first step, the manufacturer as a leader determines the direct sale price and the wholesale price of the product in order to earn the most profit, and the retailers try to increase their own profits in a competitive market when they are not willing to cooperate. The objective function of this case is shown in Eq. (7). This function comprises the profits of selling the product minus the purchase costs for the retailers and the profit of sales via the direct channel plus the wholesale manner for the manufacturer.

$$\begin{cases} \max_w \pi_m^{NC} = (p_3 - c_s)(q_3) + (q_1)(w_1 - c_p) + (q_2)(w_2 - c_p) \\ \qquad where(p_1.p_2)\ solve \\ \qquad \begin{cases} \max_{p1} = \pi_{r1}^{NC} = p_1(q_1) - (q_1)w_1 \\ \max_{p2} = \pi_{r2}^{NC} = p_2(q_2) - (q_2)w_2 \end{cases} \end{cases} \tag{7}$$

First derivatives of each retailers' profit function with decision variables *p1* and *p2*, respectively shown in $\frac{\partial \pi_{r1}^{NC}}{\partial p1} = 0$ and $\frac{\partial \pi_{r2}^{NC}}{\partial p2} = 0$, providing the optimal values for decision variables.

$$\frac{\partial \pi_{r1}^{NC}}{\partial p_1} = d_1 - \sigma p_1 + \gamma(-p_1 + p_2) + \gamma(-p_1 + p_3) + (-2\gamma - \sigma)(p_1 - w_1) = 0$$

$$\frac{\partial \pi_{r2}^{NC}}{\partial p_2} = d_2 + \gamma(p_1 - p_2) - \sigma p_2 + \gamma(-p_2 + p_3) + (-2\gamma - \sigma)(p_2 - w_2) = 0$$

The second derivative of the decision variable of each retailer p_i is $\frac{\partial^2 \pi_{ri}^{NC}}{\partial p_i^2} = -4\gamma - 2\sigma \leq 0$ for *i*=1, 2, which means the retailer's profit function is concave.

$$p_1 = -\frac{-4\gamma d_1 - 2\sigma d_1 - \gamma d_2 - 5\gamma^2 p_3 - 2\gamma\sigma p_3 - 10\gamma^2 \mathrm{w} - 9\gamma\sigma \mathrm{w} - 2\sigma^2 w}{15\gamma^2 + 16\gamma\sigma + 4\sigma^2} \tag{8}$$

$$p_2 = -\frac{-\gamma d_1 - 4\gamma d_2 - 2\sigma d_2 - 5\gamma^2 p_3 - 2\gamma\sigma p_3 - 10\gamma^2 \mathrm{w} - 9\gamma\sigma \mathrm{w} - 2\sigma^2 \mathrm{w}}{15\gamma^2 + 16\gamma\sigma + 4\sigma^2} \tag{9}$$

Subsequent to the calculation of the optimum values for p1 and p2, Eq. (8) and (9) are substituted into the manufacturer's profit function, hence enabling p_3, $w_1$and w_2 to be determined.

Theorem 1. The profit function of the manufacturer in the case of the non-cooperative scenario is concave and also has a unique optimal solution.

Proof. See Appendix A.

Once the equations $\frac{\partial \pi_m^{NC}}{\partial p3} = 0$, $\frac{\partial \pi_m^{NC}}{\partial w_1} = 0$, and $\frac{\partial \pi_m^{NC}}{\partial w_2} = 0$ have been solved, the values of p_3, $w_1$and w_2 can be calculated as follows:

$$w_1^{NC} = \frac{\sigma(3\gamma + \sigma)c_p + (\gamma + \sigma)d_1 + \gamma(d_2 + d_3)}{2\sigma(3\gamma + \sigma)} \tag{10}$$

$$w_2^{NC} = \frac{\sigma(3\gamma + \sigma)c_p + \gamma d_1 + \gamma d_2 + \sigma d_2 + \gamma d_3}{2\sigma(3\gamma + \sigma)} \tag{11}$$

$$p_3^{NC} = \frac{\sigma(3\gamma + \sigma)c_s + \gamma d_1 + \gamma d_2 + \gamma d_3 + \sigma d_3}{2\sigma(3\gamma + \sigma)} \tag{12}$$

Thus, as can be observed in equations (13) - (15), the profit function for each player can be determined when the retailers choose their sale price independently.

$$\pi_{r1}^{NC} = \frac{(2\gamma + \sigma)(-(5\gamma^2 + 7\gamma\sigma + 2\sigma^2)c_p + \gamma(5\gamma + 2\sigma)c_s + 4\gamma d_1 + 2\sigma d_1 + \gamma d_2)^2}{4(3\gamma + 2\sigma)^2(5\gamma + 2\sigma)^2} \tag{13}$$

$$\pi_{r2}^{NC} = \frac{(2\gamma + \sigma)(-(5\gamma^2 + 7\gamma\sigma + 2\sigma^2)c_p + \gamma(5\gamma + 2\sigma)c_s + \gamma d_1 + 4\gamma d_2 + 2\sigma d_2)^2}{4(3\gamma + 2\sigma)^2(5\gamma + 2\sigma)^2} \tag{14}$$

$$\pi_m^{NC} = \frac{\begin{pmatrix} 2\sigma(30\gamma^4 + 67\gamma^3\sigma + 52\gamma^2\sigma^2 + 17\gamma\sigma^3 + 2\sigma^4)c_p^2 + \sigma(60\gamma^4 + 149\gamma^3\sigma + 115\gamma^2\sigma^2 + 36\gamma\sigma^3 + 4\sigma^4)c_s^2 + 15\gamma^3 d_1^2 + 19\gamma^2\sigma d_1^2 \\ +10\gamma\sigma^2 d_1^2 + 2\sigma^3 d_1^2 + 30\gamma^3 d_1 d_2 + 26\gamma^2\sigma d_1 d_2 + 6\gamma\sigma^2 d_1 d_2 + 15\gamma^3 d_2^2 + 19\gamma^2\sigma d_2^2 + 10\gamma\sigma^2 d_2^2 + 2\sigma^3 d_2^2 + 30\gamma^3 d_1 d_3 + 32\gamma^2\sigma d_1 d_3 \\ -2\sigma(30\gamma^3 + 37\gamma^2\sigma + 15\gamma\sigma^2 + 2\sigma^3)c_p(2\gamma c_s + d_1 + d_2) + 8\gamma\sigma^2 d_1 d_3 + 30\gamma^3 d_2 d_3 + 32\gamma^2\sigma d_2 d_3 + 8\gamma\sigma^2 d_2 d_3 \\ +15\gamma^3 d_3^2 + 31\gamma^2\sigma d_3^2 + 20\gamma\sigma^2 d_3^2 + 4\sigma^3 d_3^2 - 2\sigma(15\gamma^2 + 11\gamma\sigma + 2\sigma^2)c_s(\gamma d_1 + \gamma d_2 + (3\gamma + 2\sigma)d_3) \end{pmatrix}}{4\sigma(3\gamma + \sigma)(3\gamma + 2\sigma)(5\gamma + 2\sigma)} \tag{15}$$

B. Cooperation between the two retailers with the same sale price

In the previous section, we considered a situation featuring no cooperation in the supply chain, while in the present scenario, cooperation between our two retailers is scrutinized. Correspondingly, here we consider that the retailers agree to sell the products for the same price in the market. We use the term"C1" to call this scenario. The manufacturer plays the leader role in Stackelberg's game theory. Now, the objective function of this case is shown in Eq. (16). This function includes the profit of selling the product minus the purchase cost for both retailers, and the profit of selling the product through the direct channel, and also the wholesale means for the manufacturer.

$$\begin{cases} \max_w \pi_m^{C1} = (p_3 - c_s)(q_3) + (q_1 + q_2)(w_c - c_p) \\ \quad where\ p_1.p_2\ \ solve \\ \max_{(pc)} = \pi_{r1+r2}^{C1} = (p_c - w_c)(q_1 + q_2) = (p_c - w_c)(q_1 + q_2) \end{cases} \tag{16}$$

The first derivative of the cooperative retailers' profits function is shown in $\frac{\partial \pi_{r1+r2}^{C1}}{\partial p_c} = 0$. This derivative provides the optimal value for the decision variable.

$$\frac{\partial \pi_{r1+r2}^{C1}}{\partial p_c} = d_1 + d_2 + 2\gamma(p_3 - p_c) - 2\sigma p_c + 2(-\gamma - \sigma)(p_c - w_c) = 0$$

The second derivative of the cooperative retailers' profits function is $\frac{\partial^2 \pi_{r1+r2}^{C1}}{\partial p_c^2} = -4(\gamma + \sigma) \leq 0$, which means the retailers' profit function is concave.

Therefore, the result can be achieved as Eq. (17).

$$p_c^{C1} = \frac{d_1 + d_2 + 2\gamma p_3 + 2\gamma w_c + 2\sigma w_c}{4(\gamma + \sigma)} \tag{17}$$

Theorem 2. The profit function of the manufacturer in cooperative mode is concave and also has a unique optimal solution.

Proof. See Appendix B.

Following the solution of equations $\frac{\partial\, \pi_m^{C1}}{\partial p3} = 0$ and $\frac{\partial\, \pi_m^{C1}}{\partial w_c} = 0$, the values of p_3 and w_c may be calculated as follows:

$$w_c^{C1} = \frac{2\sigma(3\gamma+\sigma)c_p + (2\gamma+\sigma)d_1 + 2\gamma d_2 + \sigma d_2 + 2\gamma d_3}{4\sigma(3\gamma+\sigma)} \tag{18}$$

$$p_3^{C1} = \frac{\sigma(3\gamma+\sigma)c_s + \gamma d_1 + \gamma d_2 + \gamma d_3 + \sigma d_3}{2\sigma(3\gamma+\sigma)} \tag{19}$$

With the calculation of all variables, now we are able to obtain the profit functions for the manufacturer and the sum of retailers' profits in equation (20) - (21), in case of the retailers choosing Scenario C1.

$$\pi_{r1+r2}^{C1} = \frac{(-2(\gamma+\sigma)c_p + 2\gamma c_s + d_1 + d_2)^2}{32(\gamma+\sigma)} \tag{20}$$

$$\pi_m^{C1} = \frac{\begin{pmatrix} 4\sigma(\gamma+\sigma)^2(3\gamma+\sigma)c_p^2 + 4\sigma(3\gamma^3 + 10\gamma^2\sigma + 6\gamma\sigma^2 + \sigma^3)c_s^2 + 4\gamma^2 d_1^2 + 3\gamma\sigma d_1^2 + \sigma^2 d_1^2 + 8\gamma^2 d_1 d_2 \\ +6\gamma\sigma d_1 d_2 + 2\sigma^2 d_1 d_2 + 4\gamma^2 d_2^2 + 3\gamma\sigma d_2^2 + \sigma^2 d_2^2 - 4\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p(2\gamma c_s + d_1 + d_2) \\ +8\gamma^2 d_1 d_3 + 8\gamma\sigma d_1 d_3 + 8\gamma^2 d_2 d_3 + 8\gamma\sigma d_2 d_3 + 4\gamma^2 d_3^2 + 8\gamma\sigma d_3^2 + 4\sigma^2 d_3^2 - 4\sigma(3\gamma+\sigma)c_s(\gamma d_1 + \gamma d_2 + 2(\gamma+\sigma)d_3 \end{pmatrix}}{16\sigma(\gamma+\sigma)(3\gamma+\sigma)} \tag{21}$$

C. Cooperation between the two retailers with different sale prices

Previously, we studied a situation highlighting cooperation between the retailers, who would correspondingly agree to sell the product at the same price on the market. As an alternative scenario, here they come to reach an agreement on cooperation while selling their products separately. The term "C2" is used to designate the present scenario. Same as before, the manufacturer plays the leader role in Stackelberg game theory. Now, the objective function of this case appears as shown in Eq. (22). This function includes the profit of product sales, deducted by the purchase cost in the case of the retailers, and the profit of product sales via the direct channel and through the wholesale approach for the manufacturer.

$$\begin{cases} \max_{c_1} \pi_m^{C2} = (p_3 - c_s)(q_3) + (q_1 + q_2)(w_c - c_p) \\ \qquad\qquad where\ p_c\ solve \\ \max_{(p_1.p_2)} = \pi_{r1+r2}^{C2} = p_2(q_2) + p_1(q_1) - (q_1 + q_2)w_c \end{cases} \tag{22}$$

First derivatives of the cooperative retailers' profit functions with decision variables pi are indicated as $\frac{\partial \pi_{r1+r2}^{C2}}{\partial p_1} = 0$ and $\frac{\partial \pi_{r1+r2}^{C2}}{\partial p_2} = 0$. These equations provide the optimal values for decision variables.

$$\frac{\partial \pi_{r1+r2}^{C2}}{\partial p_1} = d_1 - 2(2\gamma+\sigma)p_1 + 2\gamma p_2 + \gamma p_3 + \gamma w_c + \sigma w_c = 0$$

$$\frac{\partial \pi_{r1+r2}^{C2}}{\partial p_2} = d_2 + 2\gamma p_1 - 4\gamma p_2 - 2\sigma p_2 + \gamma p_3 + \gamma w_c + \sigma w_c = 0$$

Second derivatives of the decision variables, pointing to the retailers' profits, p_i are specified as $\frac{\partial^2 \pi_{r1+r2}^{C2}}{\partial p_i^2} = -4\gamma - 2\sigma \leq 0$ for i=1, 2, revealing the retailers' profit functions to be concave.

$$p_1^{C2} = \frac{(2\gamma + \sigma)d_1 + \gamma d_2 + (3\gamma + \sigma)(\gamma p_3 + (\gamma + \sigma)w_c)}{2(3\gamma^2 + 4\gamma\sigma + \sigma^2)} \tag{23}$$

$$p_2^{C2} = \frac{\gamma d_1 + (2\gamma + \sigma)d_2 + (3\gamma + \sigma)(\gamma p_3 + (\gamma + \sigma)w_c)}{2(3\gamma^2 + 4\gamma\sigma + \sigma^2)} \tag{24}$$

However, variables w_c and p_3 happen to be figuring in the equations above. Thus, we proceed to substitute these amounts in the manufacturer's profit function by solving $\frac{\partial \pi_m^{C2}}{\partial p_3^{C2}} = 0$ and $\frac{\partial \pi_m^{C2}}{\partial w_c^{C2}} = 0$ to calculate the variables.

$$w_c^{C2} = \frac{2\sigma(3\gamma + \sigma)c_p + (2\gamma + \sigma)d_1 + 2\gamma d_2 + \sigma d_2 + 2\gamma d_3}{4\sigma(3\gamma + \sigma)} \tag{25}$$

$$p_3^{C2} = \frac{\sigma(3\gamma + \sigma)c_s + \gamma d_1 + \gamma d_2 + \gamma d_3 + \sigma d_3}{2\sigma(3\gamma + \sigma)} \tag{26}$$

Theorem 3. The profit function of the manufacturer in Scenario C2 is concave while also featuring a unique optimal solution.

Proof. See Appendix C.

Following the calculation of all decision variables, the resulting functions for the manufacturer's profit and the sum of the retailers' profits may be ultimately determined as demonstrated in Eqs. (27) - (28), with the retailers choosing Scenario C2.

$$\pi_{r1+r2}^{C2} = \frac{\begin{pmatrix} 4(\gamma + \sigma)^2(3\gamma + \sigma)c_p^2 + 4\gamma^2(3\gamma + \sigma)c_s^2 + 7\gamma d_1^2 + 5\sigma d_1^2 - 2\gamma d_1 d_2 - 6\sigma d_1 d_2 + 7\gamma d_2^2 \\ +5\sigma d_2^2 + 4\gamma(3\gamma + \sigma)c_s(d_1 + d_2) - 4(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p(2\gamma c_s + d_1 + d_2) \end{pmatrix}}{32(\gamma + \sigma)(3\gamma + \sigma)} \tag{27}$$

$$\pi_m^{C2} = \frac{\begin{pmatrix} 4\sigma(\gamma + \sigma)^2(3\gamma + \sigma)c_p^2 + 4\sigma(3\gamma^3 + 10\gamma^2\sigma + 6\gamma\sigma^2 + \sigma^3)c_s^2 + 4\gamma^2 d_1^2 + 3\gamma\sigma d_1^2 + \sigma^2 d_1^2 + 8\gamma^2 d_1 d_2 \\ +6\gamma\sigma d_1 d_2 + 2\sigma^2 d_1 d_2 + 4\gamma^2 d_2^2 + 3\gamma\sigma d_2^2 + \sigma^2 d_2^2 - 4\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p(2\gamma c_s + d_1 + d_2) + 8\gamma^2 d_1 d_3 \\ +8\gamma\sigma d_1 d_3 + 8\gamma^2 d_2 d_3 + 8\gamma\sigma d_2 d_3 + 4\gamma^2 d_3^2 + 8\gamma\sigma d_3^2 + 4\sigma^2 d_3^2 - 4\sigma(3\gamma + \sigma)c_s(\gamma d_1 + \gamma d_2 + 2(\gamma + \sigma)d_3) \end{pmatrix}}{16\sigma(\gamma + \sigma)(3\gamma + \sigma)} \tag{28}$$

Proposition 1: The sum of the retailers' profits in Scenario C2 is always higher than in Scenario C1 ($\pi^{C2}_{r1+r2} \geq \pi^{C1}_{r1+r2}$).

Proof: $\pi^{C2}_{r1+r2} - \pi^{C1}_{r1+r2} = \frac{(d_1-d_2)^2}{8(3\gamma+\sigma)} \geq 0$.

According to the above-mentioned proposition, we can conclude that in the case of cooperation between our two retailers, given that the base demand is different for each retailer ($d_1 \neq d_2$), Scenario C2 is always superior to Scenario C1. In other words, in the case of cooperation between retailers, it is preferable for each retailer to sell the product at a different price on the market. If the base demand was to be equal for the two retailers ($d_1 = d_2$), the outputs of both Scenarios would be the same.

Proposition 2: The selling price of the retailers in Scenario C2 is equal to the average selling price of retailers in Scenario C1 ($p^{C1}_c = \frac{p^{C2}_1 + p^{C2}_2}{2}$).

Proof: See Appendix D.

Proposition 2 notes that If two retailers decide to sell the product at the same price, that optimal selling price is the average selling price in C2 scenario. According to Proposition 1, it can be concluded that setting a unique selling price can eliminate part of the market demand, which will practically reduce the profit of both retailers in this type of cooperation.

IV. NUMERICAL ANALYSIS

In this section, some numerical examples are used to analyze the effects of the retailers' both cooperative and non-cooperative approaches, given that a manufacturer joins the market, selling its product directly to customers. And further, to investigate the retailers' behaviors, with reference to three distinctive scenarios, aimed to outline the circumstances that they occur to choose the best strategy thereunder. Let $\sigma = 0.4$, $\gamma = 0.6$,$c_p = 2$, $c_s = 5.5$, and the base demand parameters are $d_1 = 3$ $d_2 = 5$, $d_3 = 1$.

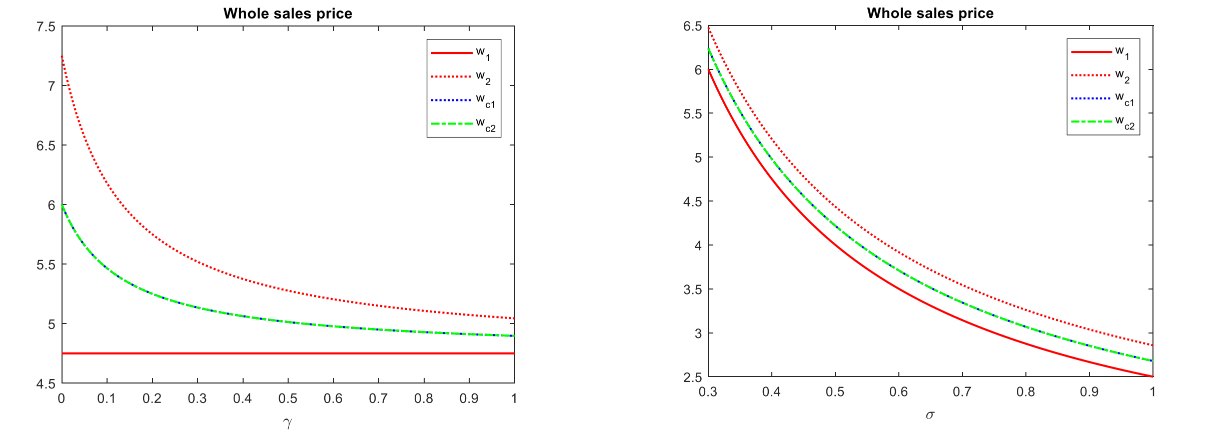
The wholesale price at which the product is sold by the manufacturer to retailers in all scenarios plays an important role in choosing the optimal strategy. Hence, the sensitivity of this variable will be examined further. All the data and computer program code (written by Wolfram Mathematica 11) are available upon request from the authors.

A. Sensitivity analysis

In this subsection, sensitivity analyses on the parameters of the model are described.

Fig. 2 shows the impact of the price sensitivity of the product (γ) and the substitutability of the retailers and manufacturer (σ) on the wholesale price of the product. As it is obvious from *Fig.* 2, the wholesale prices in the two cooperative scenarios (C1 and C2) are equal. In *Fig.* 2 (a), with the increase of γ, noticeable is the decrease in the wholesale prices in all scenarios. Likewise, in the scenario NC, w_2 is more sensitive to the variation of γ than w_1. The fact that w_2 is greater than w_1 has its foundations laid in the base demand of each retailer, implying that the manufacturer sells his/her product at a higher price to the retailer who is in possession of a larger share of the market in the first scenario. As can be seen in *Fig.* 2 (b), the overall change in the wholesale prices in all studied scenarios is a decreasing trend compared to the increase of σ.

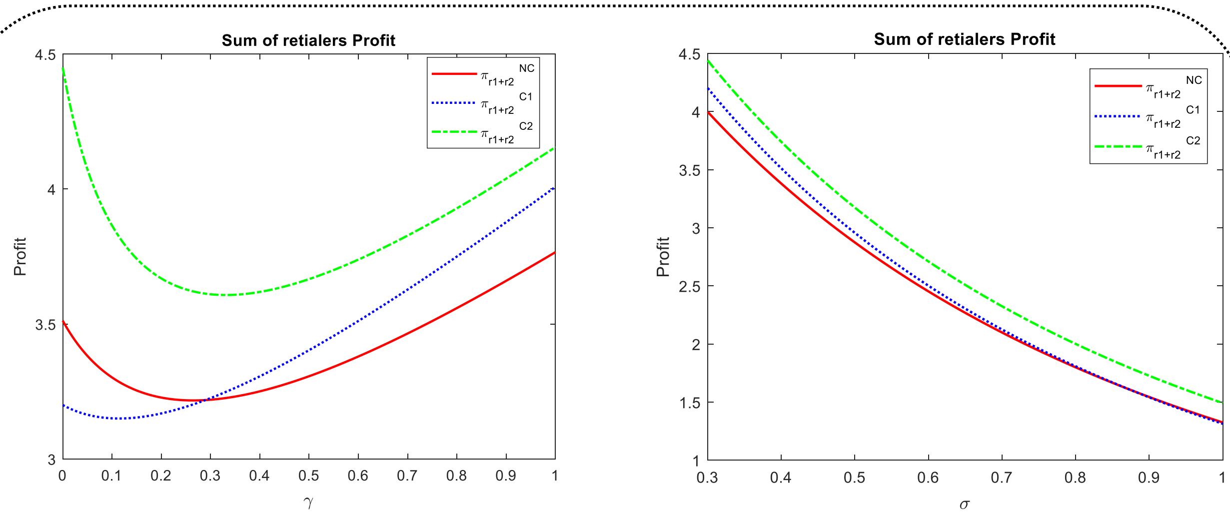
The total profit of retailers in the three scenarios can help the retailers behave rationally in the market, although ignoring the significance of the price sensitivity of the product (γ) and the substitutability of the retailers and manufacturer (σ) is not an option. Hence, the sensitivity of the total profit of retailers will be examined further.



(a) Impact of γ

(b) Impact of σ

Fig. 2. The impact of γ and σ on the wholesale price in three scenarios



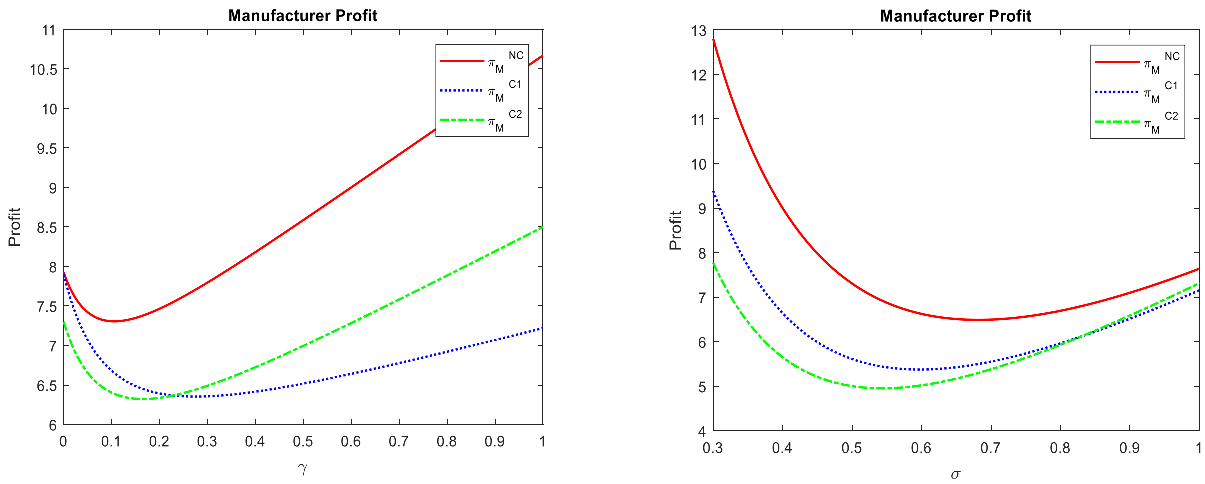
(a) Impact of γ

(b) Impact of σ

Fig. 3. The impact of γ and σ on the sum of retailers' profits in the scenarios.

Fig. 3 shows the impact of γ and σ on the total profit of both retailers. *Fig.* 3(a) shows the effects of γ on the total profit of retailers regarding all mentioned scenarios. The equality of profits in Scenario NC and Scenario C1 is realized at $\gamma \approx 0.289$. A comparison of these two scenarios reveals that the retailers try to cooperate for $\gamma \geq 0.289$, and this cooperation will not occur for $\gamma \leq 0.289$. As it is apparent from *Fig.* 3, Scenario C2 rates are superior to the other scenarios, performance-wise. As demonstrated in *Fig.* 3(b), by increasing the value of σ, a decreasing trend is imposed on the total profit of retailers in all three scenarios.

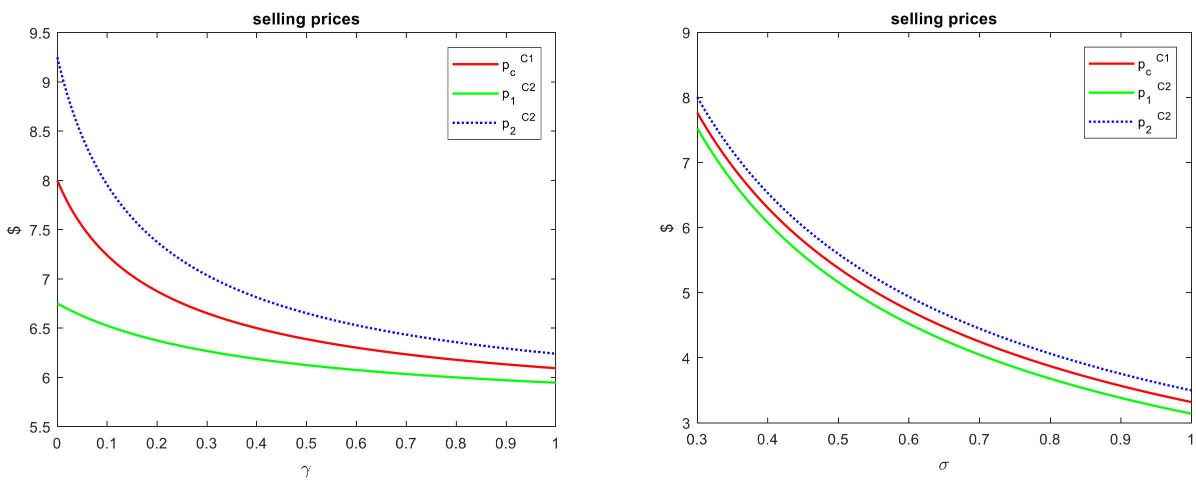
The manufacturer, as a leader, tries to maximize profit, which will inevitably depend on the choice of scenarios in the market by the retailers. Accordingly, the impacts of γ and σ and the sensitivity of the manufacturer's profit will be examined.



(a) Impact of γ **(b) Impact of σ**

Fig. 4. The impact of γ and σ on the manufacturer's profit in three scenarios.

The impact of parameters γ and σ on the manufacturer's profit are depicted in *Fig.* 4. As can be perceived from this figure, the manufacturer's profit is higher in Scenario NC. In other words, cooperation between retailers can reduce the manufacturer's profit. Moreover, this cooperation increases the total profit of retailers, as shown in *Fig.* 3. In *Fig.* 4(a), a divergence trend can be observed with regard to the manufacturer's profit generated by different scenarios with the increase of γ. Scenarios C1 and C2 behave similarly at $\gamma \approx 0.225$. The benefit of the manufacturer is greater in Scenario C2 at $\gamma \geq 0.225$ and in Scenario C1 at $\gamma \leq 0.225$. *Fig.* 4(b) reveals a convergence trend in the manufacturer's profit along with the increase in the value of σ. For σ≈0.825, the amount of profit for the manufacturer in the two cooperative scenarios is approximately equal.



(a) Impact of γ **(b) Impact of σ**

Fig. 5. The impact of γ and σ on the selling price of retailers in C1 and C2 scenarios

Fig. 5 shows the impact of γ and σ on the selling price of retailers in C1 and C2 scenarios. In *Fig.* 5 (a), with the increase of γ, the decrease in the selling prices of retailers in. The fact that p_2^{c2} is greater than the other selling price it's laid in the base demand of each retailer, implying that the retailer who is in possession of a larger share of the market sells the product at a higher price to the customer. As can be seen in *Fig.* 5 (b), the overall change in the selling price in C1 and C2 scenarios is a decreasing trend compared to the increase of σ. As can be seen in *Fig.* 5, p_c^{C1} is between p_1^{c2} and p_2^{c2}, which implies that according to proposition 2, the selling price of the retailers in Scenario C2 is equal to the average of the selling price of retailers in Scenario C1.

Table II illustrates the behaviors of other variables, including sale prices and wholesale prices, based on the model parameters. An increase in σ will cause a reduction to the entirety of both in all three scenarios. That is to say, if the sensitivity of the customers to the price increases, the manufacturer and retailers will proceed to lower down their prices. On the other hand, corresponding to an increase in γ, a drop in retailer sales prices and an increase in manufacturer selling prices are bound to be witnessed in the market. In other words, the rise in the replacement rate urges the manufacturer to sell to the retailers at a higher wholesale price. By an escalation of production costs, all sale prices are rising as expected. Also, increasing c_s provides an opportunity for the retailers to sell their products at a higher price. In addition, the manufacturer's cost for creating a direct channel to the market does not affect the wholesale price. As Table II proposes, the manufacturer's sales price, regardless of the retailers' intentions on whether to cooperate, are identical in all three scenarios it because we consider the leader role for the manufacturer.

Table II. The impact of parameters on sale and wholesale prices.

		Non-Cooperative (NC)					***Cooperative (C1)***			***Cooperative (C2)***			
		p_1	p_2	p_3	w_1	w_2	p_c	p_3	w_c	p_1	p_2	p_3	w_c
σ	0.50	4.7857	5.4705	5.3152	4.0000	4.4348	5.3765	5.3152	4.2174	5.1591	5.5939	5.3152	4.2174
	0.45	5.1681	5.8689	5.6389	4.3333	4.7778	5.7937	5.6389	4.5556	5.5714	6.0159	5.6389	4.5556
	0.40	5.6377	6.3554	6.0455	4.7500	5.2045	6.3023	6.0455	4.9773	6.0750	6.5295	6.0455	4.9773
	0.35	6.2306	6.9660	6.5706	5.2857	5.7508	6.9393	6.5706	5.5183	6.7068	7.1719	6.5706	5.5183
	0.30	7.0069	7.7609	7.2738	6.0000	6.4762	7.7659	7.2738	6.2381	7.5278	8.0040	7.2738	6.2381
γ	0.70	5.6079	6.2404	6.1000	4.7500	5.15	6.2341	6.1000	4.9500	6.0341	6.4341	6.1000	4.9500
	0.65	5.622	6.2944	6.0745	4.7500	5.1755	6.2663	6.0745	4.9628	6.0536	6.4791	6.0745	4.9628
	0.60	5.6377	6.3554	6.0455	4.7500	5.2045	6.3023	6.0455	4.9773	6.0750	6.5295	6.0455	4.9773
	0.55	5.6551	6.4246	6.0122	4.7500	5.2378	6.3426	6.0122	4.9939	6.0987	6.5865	6.0122	4.9939
	0.50	5.6746	6.5039	5.9737	4.7500	5.2763	6.3882	5.9737	5.0132	6.1250	6.6513	5.9737	5.0132
c_p	2.50	5.7915	6.5092	6.0455	5.0000	5.4545	6.4273	6.0455	5.2273	6.200	6.6545	6.0455	5.2273
	2.25	5.7146	6.4323	6.0455	4.8750	5.3295	6.3648	6.0455	5.1023	6.1375	6.5920	6.0455	5.1023
	2.00	5.6377	6.3554	6.0455	4.7500	5.2045	6.3023	6.0455	4.9773	6.0750	6.5295	6.0455	4.9773
	1.75	5.5607	6.2784	6.0455	4.6250	5.0795	6.2398	6.0455	4.8523	6.0125	6.4670	6.0455	4.8523
	1.50	5.4838	6.2015	6.0455	4.5000	4.9545	6.1773	6.0455	4.7273	5.9500	6.4045	6.0455	4.7273
c_s	6.5	5.7530	6.4707	6.5455	4.7500	5.2045	6.4523	6.5455	4.9773	6.2250	6.6795	6.5455	4.9773
	6.0	5.6953	6.413	6.2955	4.7500	5.2045	6.3773	6.2955	4.9773	6.1500	6.6045	6.2955	4.9773
	5.5	5.6377	6.3554	6.0455	4.7500	5.2045	6.3023	6.0455	4.9773	6.0750	6.5295	6.0455	4.9773
	5.0	5.5800	6.2977	5.7955	4.7500	5.2045	6.2273	5.7955	4.9773	6.0000	6.4545	5.7955	4.9773
	4.5	5.5223	6.2400	5.5455	4.7500	5.2045	6.1523	5.5455	4.9773	5.9250	6.3795	5.5455	4.9773

B. Managerial implications

This study provides theoretical evidence for a common occurrence in the business world, which is the reaction of traditional offline retailers to compete with a manufacturer that decided to expand online operations. Moreover, we analyze three scenarios when the retailers try to react to the manufacturer's online selling to the end customer. This study suggests the following managerial insights.

- Making the best decision for retailers, such as cooperation in the presence of online manufacturers, is a strategic necessity if they want to keep increasing their profitability in the market. Nonetheless, because of the online expansions of the manufacturer, in the context studied in this paper, the retailers must be cautious and try to cooperate with each other. They should set their pricing strategies and avoid setting price convergence, but after a good understanding of market conditions, they must use price indiscrimination to gain more profit.
- On the other hand, the maximum of manufacturer’s profit happens when we have no cooperation between retailers, but when they cooperate. If substantiality of players (y) is high (close to one) or less (close to zero), the scenario C1 and scenario C2 respectively have a low and high profit for the manufacturer.
- When the retailers cooperate, In order to capture the maximum market capacity, they should try to determine the selling price of the product independently, in which case the retailers will make more profits.

V. CONCLUSIONS AND FUTURE RESEARCH

With the ever-evolving advancements and technologies dawning on the horizon at every breath, a novel tendency for the manufacturers to sell their products directly to the customers, in addition to the retailers, is just on the brink of emergence. Thus for the purpose of investigating this matter, we have explored a model, visualizing a manufacturer that sells products to two retailers for variable prices, while at the same time taking into consideration the decision of the manufacturer to enter the market, as well as creating a channel to sell directly. We considered three scenarios in this research. In the first scenario, the retailers repeated the same procedure without reaction. In the second, the retailers began to cooperate with each other to buy their required products at a fixed price and sell it at a constant price to customers. And lastly, a scenario of the two retailers still cooperating but selling the products at different prices. In solving these scenarios, we used the Stackelberg game theory, in which the manufacturer is a leader, and two retailers are followers. Studying these scenarios, the following results are achieved. The total profit of retailers in Scenario C2 is higher than the other two scenarios. In other words, after the cooperation, retailers will have to buy the same product at the same price from the manufacturer and sell it at a different price to customers. The manufacturer’s profit in Scenario NC is higher than the other scenarios; that is, cooperation between the retailers reduces the manufacturer’s profits.

This research can be extended in several ways. In the present paper, we discussed the cooperation between retailers, as a manufacturer creates a direct channel to end customers, while it should also be put to notice that competition also has a significant impact on the structure. Correspondingly, it would be of interest to examine the case of vertical and horizontal competition between the manufacturer and two retailers. Furthermore, we laid focus on the additional direct channel that the manufacturer creates when the demand is deterministic. Prospectively, development and focus could be dedicated to the impact of stochastic demand on the structure of behavior. Finally, considering the risk-averse retailers under uncertain market demand is another interesting topic.

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Appendix A

Proof of Theorem 1

We calculate the first order derivative of the profit function with respect to choice factors in order to establish that the manufacturer's profit function in the non-cooperative scenario, as stated in Eq. (7), is concave and also has an optimum solution. As a result, the Hessian matrix is established:

$$H_A=\begin{bmatrix}\frac{\partial^2\pi_m}{\partial p_3^2} & \frac{\partial^2\pi_m}{\partial p_3\partial w_1} & \frac{\partial^2\pi_m}{\partial p_3\partial w_2}\\ \frac{\partial^2\pi_m}{\partial p_3\partial w_1} & \frac{\partial^2\pi_m}{\partial w_1^2} & \frac{\partial^2\pi_m}{\partial w_2\partial w_1}\\ \frac{\partial^2\pi_m}{\partial p_3\partial w_2} & \frac{\partial^2\pi_m}{\partial w_2\partial w_1} & \frac{\partial^2\pi_m}{\partial w_2^2}\end{bmatrix}=\begin{bmatrix}-\frac{2(4\gamma^2+7\gamma\sigma+2\sigma^2)}{3\gamma+2\sigma} & \frac{2\gamma(2\gamma+\sigma)}{3\gamma+2\sigma} & \frac{2\gamma(2\gamma+\sigma)}{3\gamma+2\sigma}\\ \frac{2\gamma(2\gamma+\sigma)}{3\gamma+2\sigma} & -\frac{28\gamma^3+46\gamma^2\sigma+24\gamma\sigma^2+4\sigma^3}{15\gamma^2+16\gamma\sigma+4\sigma^2} & \frac{2\gamma(2\gamma+\sigma)^2}{(3\gamma+2\sigma)(5\gamma+2\sigma)}\\ \frac{2\gamma(2\gamma+\sigma)}{3\gamma+2\sigma} & \frac{2\gamma(2\gamma+\sigma)^2}{(3\gamma+2\sigma)(5\gamma+2\sigma)} & -\frac{28\gamma^3+46\gamma^2\sigma+24\gamma\sigma^2+4\sigma^3}{15\gamma^2+16\gamma\sigma+4\sigma^2}\end{bmatrix}$$

$$H_1=-\frac{2(4\gamma^2+7\gamma\sigma+2\sigma^2)}{3\gamma+2\sigma}\le 0$$

$$H_2=\frac{8(6\gamma^4+23\gamma^3\sigma+22\gamma^2\sigma^2+8\gamma\sigma^3+\sigma^4)}{(3\gamma+2\sigma)(5\gamma+2\sigma)}\ge 0$$

$$H_3=-\frac{8\sigma(6\gamma^2+5\gamma\sigma+\sigma^2)^2}{(3\gamma+2\sigma)(5\gamma+2\sigma)}\le 0$$

Thus, we have $H_1\le 0$, $H_2\ge 0$ and $H_3\le 0$ i.e., the Hessian matrix is negative definite and hence, the profit function of the manufacturer in the non-cooperative scenario is concave.

Appendix B

Proof of Theorem 2

We compute the first order derivative of the profit function with respect to decision variables to show that the manufacturer's profit function in Scenario C1, as indicated in Eq. (16), is concave and has an optimal solution. As a result, we've got the Hessian matrix:

$$H_B=\begin{bmatrix}\frac{\partial^2\pi_m}{\partial p_3^2} & \frac{\partial^2\pi_m}{\partial p_3\partial w_c}\\ \frac{\partial^2\pi_m}{\partial p_3\partial w_c} & \frac{\partial^2\pi_m}{\partial w_c^2}\end{bmatrix}=\begin{bmatrix}-\frac{2(\gamma^2+3\gamma\sigma+\sigma^2)}{\gamma+\sigma} & 2\gamma\\ 2\gamma & -2(\gamma+\sigma)\end{bmatrix}$$

$$H_1=-\frac{2(\gamma^2+3\gamma\sigma+\sigma^2)}{\gamma+\sigma}\le 0$$

$$H_2=4\sigma(3\gamma+\sigma)\ge 0$$

Thus, we have $H_1 \leq 0$ and $H_2 \geq 0$ i.e. the Hessian matrix is negative definite and hence, the profit function of the manufacturer in Scenario C1 is concave.

Appendix C

Proof of Theorem 3

To demonstrate that the manufacturer's profit function is correct, in Scenario C2 as shown in Eq. (22) is concave and also has an optimal solution, we derive the first order derivative of the profit function with respect to decision variables. Therefore, we have the Hessian matrix:

$$H_B = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial p_3^2} & \frac{\partial^2 \pi_m}{\partial p_3 \partial w_c} \\ \frac{\partial^2 \pi_m}{\partial p_3 \partial w_c} & \frac{\partial^2 \pi_m}{\partial w_c^2} \end{bmatrix} = \begin{bmatrix} -\frac{2(\gamma^2 + 3\gamma\sigma + \sigma^2)}{\gamma + \sigma} & 2\gamma \\ 2\gamma & -2(\gamma + \sigma) \end{bmatrix}$$

$$H_1 = -\frac{2(\gamma^2 + 3\gamma\sigma + \sigma^2)}{\gamma + \sigma} \leq 0$$

$$H_2 = 4\sigma(3\gamma + \sigma) \geq 0$$

Thus, we have $H_1 \leq 0$ and $H_2 \geq 0$ i.e. the Hessian matrix is negative definite and hence, the profit function of the manufacturer in Scenario C1 is concave.

Appendix D

Proof of Preposition 2

With replacing the $w_c^{C1}$and $p_3^{C1}$of Eqs.(18,19) in Eq. (17) we can get the value of p_c^{C1} based on the parameters of model as below

$$p_c^{C1} = \frac{2\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p + 2\gamma\sigma(3\gamma + \sigma)c_s + 4\gamma^2 d_1 + 9\gamma\sigma d_1 + 3\sigma^2 d_1 + 4\gamma^2 d_2 + 9\gamma\sigma d_2 + 3\sigma^2 d_2 + 4\gamma^2 d_3 + 4\gamma\sigma d_3}{8\sigma(\gamma + \sigma)(3\gamma + \sigma)}$$

Likewise, With replacing the $w_c^{C2}$and $p_3^{C3}$of Eqs.(25,26) in Eqs. (23,24) we can get the value of p_1^{C2} and p_2^{C2} based on the parameters of model below:

$$p_1^{C2} = \frac{2\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p + 2\gamma\sigma(3\gamma + \sigma)c_s + 4\gamma^2 d_1 + 11\gamma\sigma d_1 + 5\sigma^2 d_1 + 4\gamma^2 d_2 + 7\gamma\sigma d_2 + \sigma^2 d_2 + 4\gamma^2 d_3 + 4\gamma\sigma d_3}{8\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)}$$

$$p_2^{C2} = \frac{2\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p + 2\gamma\sigma(3\gamma + \sigma)c_s + 4\gamma^2 d_1 + 7\gamma\sigma d_1 + \sigma^2 d_1 + 4\gamma^2 d_2 + 11\gamma\sigma d_2 + 5\sigma^2 d_2 + 4\gamma^2 d_3 + 4\gamma\sigma d_3}{8\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)}$$

By obtaining the average of selling prices of retailers in C2 scenario, It can be seen that this value is equal to the selling price of retailers in C1 scenario.

$$\frac{p_1^{C2} + p_2^{C2}}{2} = \frac{2\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)c_p + 2\gamma\sigma(3\gamma + \sigma)c_s + 4\gamma^2 d_1 + 9\gamma\sigma d_1 + 3\sigma^2 d_1 + 4\gamma^2 d_2 + 9\gamma\sigma d_2 + 3\sigma^2 d_2 + 4\gamma^2 d_3 + 4\gamma\sigma d_3}{8\sigma(3\gamma^2 + 4\gamma\sigma + \sigma^2)}$$