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An effective predictive heuristic Model in single-machine systems under uncertainty

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Abstract *– This paper takes a predictive scheduling approach to deal with machine disruption and uncertain job processing times in single-machine systems. A two-dimensional scale is proposed based on robustness and stability. The expected total realized tardiness of jobs and the expected sum of absolute deviation between the planned and realized job completion times are respectively considered as robustness and stability measures. Considering the total tardiness as a robustness measure includes due dates, the customer satisfaction enhancement level is achievable. We propose a novel heuristic to deal with such an NP-hard problem. Computational results show the proposed method's superiority in satisfying customers and staff and increasing systems accountability, especially in large-size problems over the common methods in the literature.*

Keywords*– Machine breakdowns, Predictive heuristic, Robust and Stable Scheduling, Uncertain processing time.*

I. INTRODUCTION

Disruptions same as processing time variability, machine breakdowns, due dates uncertainty, etc., are common in the production environment and prevent the schedule from executing as planned. So, it is expected to be some deviations between the real and initial schedules. The comparison of the actual schedule with the initial one can be carried out via robustness and stability measures (Goren & Sabuncuoglu, 2009). Robustness refers to the deviation between the performance criteria, and stability is concerned with the deviation between the solutions of the real and the initial schedules Al-Hinai et al., 2011). Predictive, reactive, or Predictive-reactive strategies are performed to adjust the deviations resulted from the uncertainties (Mehta & Uzsoy, 1998). Future uncertainties are considered upon setting the initial schedule in the predictive, contrary to the reactive scheduling strategy (Goren, 2002). Here, a two-stage algorithm is proposed to generate a robust and stable schedule. In the first stage, robust scheduling is generated based on some theorems. Then predictive and reactive strategies are exploited to deal with machine breakdown. Dealing with uncertainty takes a long time in reactive strategies such as rescheduling methods, which can be avoided by employing predictive strategies (Fazayeli et al., 2016). Here we adopt a predictive strategy to hedge against breakdowns. Idle-time insertion is one of the standard strategies to adjust the effect of downtimes (e.g., see Goren & Sabuncuoglu, 2009; Mehta & Uzsoy, 1998). However, finding the correct position and amount of buffer times is a serious challenge to this method (Nouiri, et al., 2017).

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Here we propose a linear programming model to adjust the idle-time insertion method. Solving this model determines the amount and position of idle times. The main contributions of this paper are:

- A single-machine system under two sources of uncertainty is considered.
- A triple scale is proposed to meet the needs of the customer, producer, and staff simultaneously.
- A linear programming-based predictive heuristic is proposed to adjust the deviations from the uncertainty of the job processing times and machine breakdown.
- A surrogate measure is proposed to enhance solution robustness.
- A linear programming model is proposed to determine the proper position and the amount of the buffer times.

This article is organized as follows. In Section 2, the literature is reviewed. The problem is defined, and the solution method is proposed in Section 3. The computational results and conclusions of the article are presented in sections 4 and 5.

Ergo, in this paper, an exposer to machine failure has been done with the predictive approach.

II. LITERATUE REVIEW

Scheduling under uncertainty applies in various fields such as industry and medicine (e.g., Rastgar& Sahraeian, 2017), (Maghzi et al., 2020), (Mousavi et al., 2015)). Mehta and Uzsoy (1998) consider machine breakdown disruption in a single machine environment and propose an optimized surrogate measure heuristic (OSMH) method based on idle-time insertion to generate a stable schedule (Mehta and Uzsoy, 1998). O'Donovan et al. (1999) studied a single-machine scheduling problem under machine failure disruption with total tardiness as the performance measure and the sum of absolute completion time deviations from the initial schedule as a stability measure. Liu et al. (2007) proposed Genetic Algorithm to produce a robust and stable schedule in a single machine scheduling problem under machine failure disruption with total weighted tardiness as a performance measure.

Yang et al. (2002) showed the NP-completeness of the robust version of the sum of the completion time single machine scheduling problem under processing times uncertainty and proposed some heuristic methods. Goren and Sabuncuoglu (2009) studied a single-machine scheduling problem under machine breakdown disruption and processing time uncertainty and proposed branch-and-bound algorithm and some efficient theorems to produce robust schedules and stable schedules. Zhiqiang et al. (2015) studied a single machine scheduling problem under machine breakdown uncertainty and applied Genetic Algorithm to produce a robust and stable schedule. Rahmani (2017) applied Genetic Algorithm (GA) to produce a robust and stable schedule in a two-machine flow shop scheduling problem with machine breakdown and processing time uncertainty. Liao & Fu (2019) took into consideration the robustness of total completion time and the tardiness of production. They proposed a min-max regret criterion-based robust scheduling model to study the permutation flow-shop scheduling problem (PFSP) with interval production time. Niu et al. (2019) propose a distributionally robust optimization (DRO) model for single machine scheduling with uncertain processing times to find an optimal sequence that minimizes the expected worst-case total tardiness. Paprocka (2019) proposed a rescheduling and maintenance scheduling method based on probability theory. Abtahi et al. (2020) proposed a tri-component measure based on efficiency, robustness, and stability for a planning problem with uncertain processing time and machine failure through a predictive optimization method. Aissaoui, et al. (2020) addressed an Integrated Proactive Surgery Scheduling Problem while considering possible disruption of the emergency patient entry (new job arrival). To evaluate the performance of the proposed mixed-integer linear program, they conducted a Monte Carlo simulation intending to build stable and robust schedules that are less vulnerable to late starting or ending activities.

Abtahi et al. (2020) proposed SEPT-OSMH and LPOSMH heuristics to produce robust and stable schedules in a single machine scheduling problem under processing time uncertainty and machine breakdowns. Here we propose another heuristic method in a single machine scheduling problem under processing time uncertainty and machine breakdowns and compare its effectiveness with heuristics proposed in Abtahi et al. (2020), same as the LPOSMH method and others. The main differences of LPOSMH with the proposed heuristic method (PM) are:

- LPOSMH has no idea about the position of idle times.
- Idle-times inserts before all jobs in LPOSMH, but in the proposed heuristic, an idle-time may not allocate to all of the jobs.
- In LPOSMH, the quantity of the idle time of a job is depended on its expected processing time.
- Although in LPOSMH, the deterioration of the robustness measure is somehow controlled by an LP model, but this is done more accurately in the proposed method.

A. Gap analysis

Here are the results of reviewing related articles (See Table I). In the following, we have provided some suggestions to deal with these gaps in the research area.

- One source of uncertainty has often been considered.
- Except in some related studies, stability and robustness were considered separately.
- The robustness measure is often defined based on *Cmax,* which does not reflect the customers' needs.
- Dealing with uncertainty takes a long time in reactive strategies such as rescheduling methods, which can be avoided by employing predictive strategies (Fazayeli, 2016). Here we adopt a predictive strategy to hedge against breakdowns.
- Idle-time insertion is one of the standard strategies to adjust the effect of downtimes (e.g., see Mehta & Uzsoy, 1998; Goren, & Sabuncuoglu, 2009). However, finding the correct position and amount of buffer times is a serious challenge to this method (Nouiri et al., 2017). Here we propose a predictive heuristic method to adjust the idle-time insertion method. This linear programming-based predictive heuristic adjusts the deviations from the uncertainty of the job processing times and machine breakdown. Solving this model determines the amount and position of idle times. Moreover, a surrogate measure is proposed to enhance solution robustness in the objective function of the proposed linear programming model.
- A triple scale is proposed to meet the needs of the customer, producer, and staff simultaneously.

III. PROBLEM DEFINITION AND SOLUTION METHODS

In this section, we propose a heuristic method to the robust and stable single-machine problem under the uncertainty of job processing times and machine breakdowns.

The following hypotheses are considered here:

- Jobs are available at the beginning of the schedule.
- The machine has availability restrictions.
- Negative exponential is considered as the distribution of the time between two consecutive failures, and after each failure, a fixed repair time is added.
- The interrupted job will continue after machine repair.

Here we suppose that the processing time of job j follows the exponential distribution with rate λ_j, and the time between two consecutive breakdowns follows the exponential distribution with rate θ. Also, the expected total (realized) tardiness and the sum of absolute differences of the realized completion times are respectively the robustness and stability measures (*RM*, *SM*).

Table I. Reviewing and categorizing the related articles

	DEAL WITH	UNCERTAINTY				THE MAIN				
SOLVING METHOD	PREDICTIVE -REACTIVE	REACTIVE	PREDICTIVE	STABILITY	ROBUSTNESS	OBJECTIVE FUNCTION	THE KIND OF UNCERTAINTY	RESOURCE	YEAR	NO
B&B						Cmax	job processing times	Kouvelis, P. et al.	2000	1
B&B						Cmax	job processing times	Balasubramanian, J., & Grossmann, I.	2002	2
Meta heuristic method						Cmax	machine breakdown	Liu, L. et al.	2007	3
Analytical method						Cmax	job processing times machine breakdown	Sabuncuoglu, I., & Goren, S	2009	4
Meta heuristic method						Cmax	normal job processing times	Liu, Q	2011	5
Meta heuristic method						Cmax	arrivals of a new job	Rahmani and Heydari	2014	6
Meta heuristic method						Cmax	machine breakdown	Zhiqiang Lu et al.	2015	7
Meta heuristic method						Cmax	job processing times machine breakdown	Rahmani. D	2017	8
Meta heuristic method						Cmax	machine breakdown	Shen, J., & Zhu, Y.	2017	9
Meta heuristic method						Cmax	machine breakdown	Nouiri, M	2017	10
A hybrid variable neighborhood search						Cmax	arrivals of a new job	Liu, L.	2019	11
Meta heuristic method						Cmax	normal job processing times	Ma, S et al.	2019	12
Meta heuristic method						Cmax	machine breakdown	Sajadi, S. M. et al.	2019	13
E-learning						Cmax	machine breakdown	Yang, Y., et al.	2020	14
Robust optimization method						Total tardiness	job processing times machine breakdown	Abtahi,z. et al.	2020	15
Analytical & heuristic methods						Total completion time	job processing times machine breakdown	Abtahi,z. et al.	2020	16

Table II. INDICES AND PARAMETERS

i, j	Job index, $i, j = 1, 2, \ldots, n$
r	The expected value of repair time after each breakdown
U	Uptimes; The time between two consecutive machine breakdowns (which follows an exponential distribution with rate θ
d_j	Due date of job j
λ_j	The exponential distribution rate of generation initial processing time of job j on the machine
$E(P_j)$	The expected value of processing time of job j
pr_j	The probability of machine breakdown during the processing of job j

ADT_j	The average idle time of job *j*; Average time required to return the machine to an operational mode in the event of a machine failure during the processing of job j
S_j	the predictive planned start time of the job *j*
C_j	the predictive planned completion time of job j
T_j	the predictive planned tardiness of job j
T_j^r	the realized tardiness of job *j*
AT_j	the idle-time of job j
AT_{kj}	the sum of idle times between the jobs k and j

According to the classification of Graham, a robust and stable single-machine scheduling problem under machine disruption and job processing time uncertainty is represented as:

$$1\left|p_j \sim \exp(\lambda_j);brkdwn:U \sim \exp(\theta)\right|\alpha.RM + (1-\alpha).SM \tag{1}$$

Here, the expected total tardiness is taken as a primary objective. The problem $1//\sum_j T_j$ is known to be *NP*-hard even if deterministic job processing times are considered, and no machine breakdowns occur (Briskorn et al., 2011). Assuming Erlang Distribution processing times of jobs, the Tabu search algorithm proposed to handle the single machine stable total weighted tardiness problem (Bożejko et al., 2017). Goren and Sabuncuoglu (2009) analytically proved the optimality of *SEPT* for single machine expected total tardiness problem when the job processing times follow the exponential distribution with rate λ_j . According to a theorem of them, the optimal sequence for Eq. (2) is acquired via Shortest Expected Processing Time (*SEPT*) rule.

$$1|\, p_j \sim \exp\left(\lambda_j\right); d_j = d \,|\, E\left(\sum_j T_j^r\right) \tag{2}$$

To handle the underdiscution problem, heuristic methods are proposed based on the above and the idea of *OSMH* (Optimized Surrogate Measure Heuristic Predictive*)*. *OSMH* was proposed to minimize the maximum lateness in the job shop environment under machine breakdown disruption (Mehta & Uzsoy, 1998). *OSMH* is performed in two stages; a predictive schedule is produced to minimize the primary objective without considering breakdowns first, then the idle times are inserted into the schedule to enhance the stability without considering the effects on the primary objective. O'Donovan (1999) modified *OSMH* to minimize the total tardiness in a single machine scheduling under uncertainty of machine breakdowns proposing *ATC* in the first stage of OSMH. A modified two-stage GA was proposed to obtain a robust and stable schedule in a single machine problem under machine breakdown disruption (Liu, 2007). We propose a predictive heuristic in two stages to solve the under-discussion problem. First, the initial robust schedule is generated without considering breakdowns. Then, idle times are inserted to improve schedule stability. Here, we modify the challenges of the idle-time insertion method through a linear programming model. Then the proposed method is compared with an effective LP-based heuristic to the under discussion problem called LPOSMH.

IV. THE SOLUTION METHOD

To address such a problem, a two-step predictive method has been developed. Following Mehta and Uzsoy (1998), the processing time uncertainty is considered as the only source of uncertainty in the first stage. At this stage, based on the proceeding theorem of optimality of SEPT for the expected sum of realized tardiness, a robust schedule is produced. The breakdown effect is then predictively adjusted, and appropriate amounts of idle times are determined through a linear programming model to form a robust and stable schedule.

A. The proposed predictive procedure

Stage 1: Generating a robust solution.

Here, we consider $RM = \sum_{j=1}^{n} T_j^r$ as a robustness measure. A robust partial solution is generated based on the proceeding theorem of optimality of SEPT for the expected sum of realized tardiness.

Stage 2: producing the planned predictive schedule.

Step 2.1. Generating the proper idle times.

The proper position and quantity of idle-time of jobs obtained through solving a Linear Programming model (Eqs. (6)- (16)) applying CPLEX 12.6 to improve the stability of the schedule.

Step 2.2. Modification of the partial schedule.

Modify the partial schedule resulting from stage 1 to produce the planned predictive robust and stable schedule.

Stage 3: Producing the actual schedule.

Step 3.1. Random breakdown generation.

Step 3.2. Reaction to breakdowns.

Implement righting shift rescheduling policy on the robust partial schedule obtained from *stage1* to get the actual scheduling once breakdown occurrence.

Stag 4. Robustness and stability calculation.

Calculate the robustness and stability measures via Eqs. (17) and (18).

B. Generating the proper idle-times

Improving the stability level of the schedule will deteriorate its robustness level Rahmani (2017). Therefore the amount of the idle-times should be determined in such a way that it does not cause the robustness level to deteriorate. The idle-time of job *j* (ADT_j) acquires from Eq. (3) Pinedo (2016).

$$ADT_j = r.\mathrm{E}(P_j) \Big/ MTBF \tag{3}$$

In the objective function of the proposed model, robustness and stability are optimized simultaneously. In other words, in addition to minimizing the expected total tardiness of jobs, the instability of scheduling is also minimized. The probability of a machine breakdown is included in the definition of instability. The idle-time allocated to each job in the $OSMH$ method is obtained from Eq. (3). This way of allocating idle times leads to the deterioration of the schedule robustness. In the proposed model, instead of allocating idle-time to each job, a share of the total idle-time of job j to k is allocated to them in the schedule. The probability of a machine breakdown determines this share. The probability of machine breakdown during the procession of job j is obtained from Eq. (4).

$$pr_j = 1 - \exp(E(P_j)/\theta) \tag{4}$$

Let ADT_{kj} as the number of idle-times from job k to job j. It is determined as close as possible to ADT_j. So, the measure of instability is defined as Eq. (5).

$$ISM = \sum_{j=1}^{n}\sum_{k=2}^{j-1} pr_j \max\left\{ADT_j - AT_{kj}, 0\right\} \tag{5}$$

Also, $RM = \sum_{j=1}^{n} T_j$ is a robustness measure. The LP model is:

$$\min z = \alpha \sum_{k=1}^{n} T_j + (1\text{-}\alpha)\sum_{j=1}^{n}\sum_{k=2}^{j-1} pr_j \max\left\{ADT_j - AT_{kj}, 0\right\} \tag{6}$$

$$s.t.\ S_k = S_{k-1} + E(P_{k-1}) + AT_k \qquad \forall k \ge 2 \tag{7}$$

$$C_k = S_k + E(P_k) \qquad \forall k \tag{8}$$

$$AT_{kj} = \sum_{l=k+1}^{j} AT_l \qquad \forall j \ge 2, \forall k < j \tag{9}$$

$$T_k = \max\{C_k - d_k, 0\} \tag{10}$$

$$S_1 = 0 \tag{11}$$

$$AT_1 = 0 \tag{12}$$

$$S_i \ge 0 \tag{13}$$

$$C_i \ge 0 \tag{14}$$

$$AT_j \ge 0 \tag{15}$$

$$AT_{kj} \ge 0 \qquad \forall j, k < j \tag{16}$$

Constraint (6) shows the equality of the planned start time of job k to the sum of the planned start time, the start time of its immediately before the job (job), its expected processing time, and its idle time. Constraints (7) to (10) compute the total tardiness of job k. Constraint (8) gives the completion time of job k. Constraint (10) computes the tardiness for job k. Constraint (9) calculates the sum of idle-times between job k, j. Constraint (11) ensures that the start time of the first job is zero. Constraint (12) shows that there is no idle time before the first job. Constraints (13)-(15) respectively emphasize the positivity of the start, completion, and idle-time of job k. Also, constraint (16) indicates the positivity of the idle times between jobs k, j.

V. COMPUTATIONAL RESULTS

A. Data generation

Data generation is done according to Abtahi et al. (2020). The number of jobs is 10, 30, 50, 70, 90. Job processing times follow from exponential distributions with rates $\lambda_1, \lambda_2, ...\lambda_i ...\lambda_n, \lambda_j \in Uniform[0.1,1]$. For each combination, 100 instances are generated, increasing the number of tests to a total of 500. There is a common due date for all jobs, equal to five times the maximum expected processing time of jobs. The time between two consecutive failures follows from an exponential distribution with a mean $\theta E(P_j) = \theta\lambda_j$ where $\theta = \{2, 5, 10\}$. The repair times follow from a uniform distribution, i.e. $r \in [\beta_1 E(P_j), \beta_2 E(P_j)] = [\beta_1\lambda_j, \beta_2\lambda_j]$. The unit considered for the job processing times (minute, hour, day, and so on) is the same as the unit considered for the common due date, the time between breakdowns, and the breakdowns' duration. The machine availabilities for B_1, B_2, B_3, B_4, B_5 and B_6 are respectively 97.1%, 94.3%, 87%, 87%, 76.9%, and 57%, calculated through the binomial approximation (Table III). So there are 500 samples exposed to 6 types of failures, that is, 3,000 compounds.

Table III. TYPE OF MACHINE BREAKDOWN*

Type of machine breakdown B_i	*The mean time between breakdowns* $\theta E[p_j]$	*Breakdown durations* $uniform[\beta_1 E[p_j], \beta_2 E[p_j]]$	*Machine availability (%)* $A = \theta/(\theta+\mu)$
B_1	10	$(\beta_1, \beta_2) = (0.1, 0.5)$	0.97
B_2	5	$(\beta_1, \beta_2) = (0.1, 0.5)$	0.94
B_3	2	$(\beta_1, \beta_2) = (0.1, 0.5)$	0.869
B_4	10	$(\beta_1, \beta_2) = (1, 2)$	0.869
B_5	5	$(\beta_1, \beta_2) = (1, 2)$	0.769
B_6	2	$(\beta_1, \beta_2) = (1, 2)$	0.57

*Abtahi et al. (Abtahi, 2020)

B. Robustness, stability, and the objective function

The robustness measure (RM) calculates from Eq. (17), where $\sum T_j^r$ is the total tardiness of the actual schedule.

$$RM = \sum T_j^r \tag{17}$$

The stability measure (*SM*) is stated as an absolute deviation of job completion times, where C_j^r, and C_j are the completion time of job *j* in the actual, and the predictive schedules (Eq. (18)).

$$SM = \sum_{j=1}^{n} \left| C_j^r - C_j \right| \tag{18}$$

$$Z = \alpha . RM + (1-\alpha) . SM \tag{19}$$

The objective function acquires from (Eq. (19)), where $\beta(=1-\alpha)$ indicates the robustness importance degree, and $\beta(=1-\alpha)$ indicates the stability importance degree. The proposed method (*PM*) and *LPOSMH* were compared through the value of objective functions (see Table IV). Calculations show the average *RM*, *SM*, and *Z* for different B_i when the number of jobs is 70.

TABLE IV. THE COMPARISON OF THE OBJECTIVE FUNCTION OF PM AND LPOSMH

		PM			***LPOSMH***		
n	(α, β)	*RM*	*SM*	***Z***	*RM*	*SM*	***Z***
70	(0, 1)	2690.73	260.22	260.22	2761.92	679.2225	679.2225
70	(0.1,0.9)	2688.385	260.9475	503.6913	2773.68	678.805	888.2925
70	(0.2,0.8)	2696.75	261.2625	748.36	2726.64	680.4625	1089.698
70	(0.3,0.7)	2698.395	264.3375	994.5548	2723.84	680.5675	1293.549
70	(0.4,0.6)	2692.34	260.465	1233.215	2748.48	679.66	1507.188
70	(0.5,0.5)	2683.905	257.6225	1470.764	2800	678.375	1739.188
70	(0.6,0.4)	2694.545	264.6875	1722.602	2751.28	679.95	1922.748
70	(0.7,0.3)	2697.52	261.405	1966.686	2724.4	680.5	2111.23
70	(0.8,0.2)	2707.04	263.39	2218.31	2661.12	682.2125	2265.339
70	(0.9,0.1)	2704.59	260.3775	2460.169	2667.84	681.745	2469.231
70	(1, 0)	2704.31	263.55	2704.31	2689.512	681.85	2689.512

Here, the performance of the predictive LPOSMH is compared with the proposed heuristic. In the LPOSMH method, the robust schedule is obtained based on the optimality of SEPT for a robust measure, and an LP-based model is performed to control the amount of the idle-times (see (Abtahi et al., 2020) for more details). The main differences of LPOSMH with the proposed heuristic method (PM) are:

- LPOSMH has no idea about the position of idle times.
- Idle-times inserts before all jobs in LPOSMH, but in the proposed heuristic, an idle-time may not allocate to all of the jobs.
- In LPOSMH, the quantity of the idle time of a job is depended on its expected processing time.
- Although in LPOSMH, the deterioration of the robustness measure is somehow controlled by an LP model, but this is done more accurately in the proposed method.

C. Sensitivity Analysis

The results are reported for a different number of jobs and parameters. *ZLPOSMH* and *ZPM* indicate the objective function of LPOSMH and the proposed method.

The effect of machine availability. It can be concluded from computations that:

- LPOSMH outperforms the proposed method (PM) for B1, B2 (see Fig. (3) and Fig. (4)).
- At the higher level of availability, PM is preferred and offers a more stable schedule (see Fig. (4) and Fig. (5)).
- In the case of a higher availability level, the stability is more improved (see Fig. (2) and Fig. (5)).

That is, in the case of machine wear and high failure rate and repair time, the performance of LPOSMH is better.

The effect of the number of jobs. It can be concluded from computations that:

- As the number of jobs increases, the effect of stability is more evident, and the ratio of the objective function LPOSMH to PM is greater (see Fig. (5)).
- The effect of availability on the stability and the objective function is greater than the effect of the job numbers.

In the comparison of Fig. (5) and Fig. (6), the ratio of the objective function of LPOSMH to PM is about 25 in Fig. (5) and 2.5 in Fig. (6).

Comparison of LPOSMH with PM. It can be concluded from computations that:

- PM produces a more stable schedule than LPOSMH (see Fig. (2), Fig (5), and Fig (6)).
- The difference in the performance of the two methods stems from the difference in the level of stability. In other words, the effect of the difference in robustness is much less than the difference in stability.

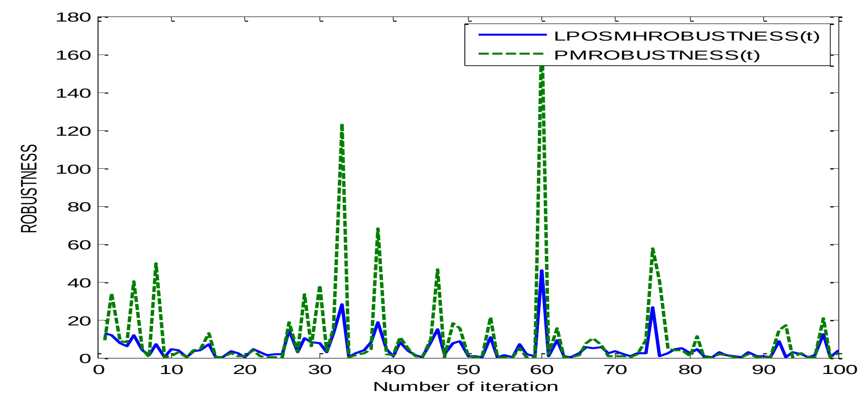

Fig. 1. Comparison of the robustness of *LPOSMH* and *PM* for 10 jobs, B_1 , and alpha=0.5

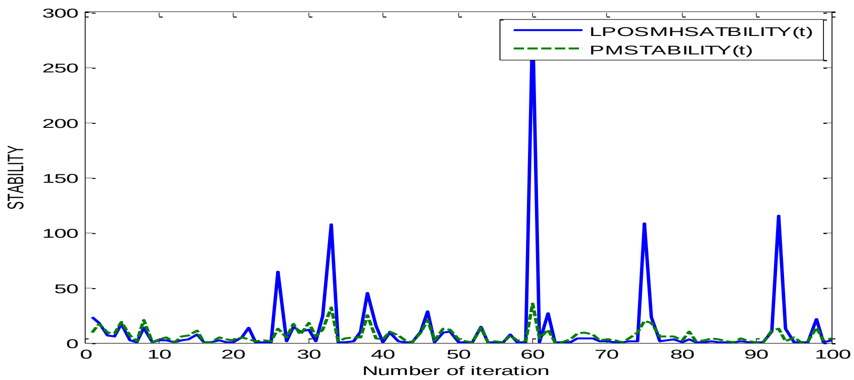

Fig. 2. Comparison of the stability of *LPOSMH* and *PM* for 10 jobs, B_1 , and alpha=0.5

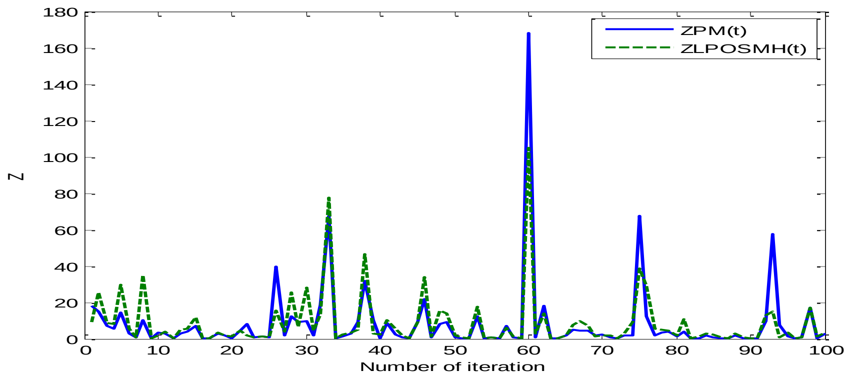

Fig. 3. Comparison of the objective function of *LPOSMH* and *PM* for 10 jobs, B_1 , alpha=0.5

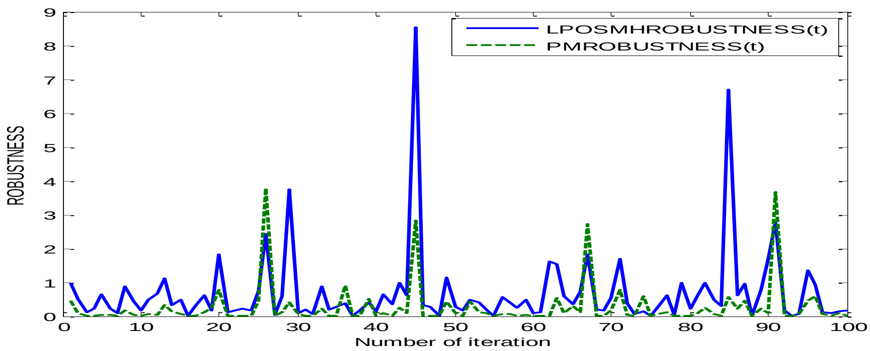

Fig. 4. Comparison of the robustness of *LPOSMH* and *PM* for 10 jobs, B_6 , and alpha=0.5.

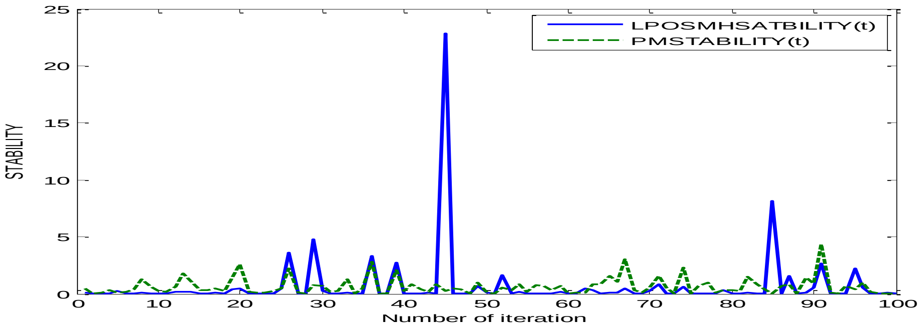

Fig. 5. Comparison of the stability of *LPOSMH* and *PM* for 10 jobs, B_6 , and alpha=0.5.

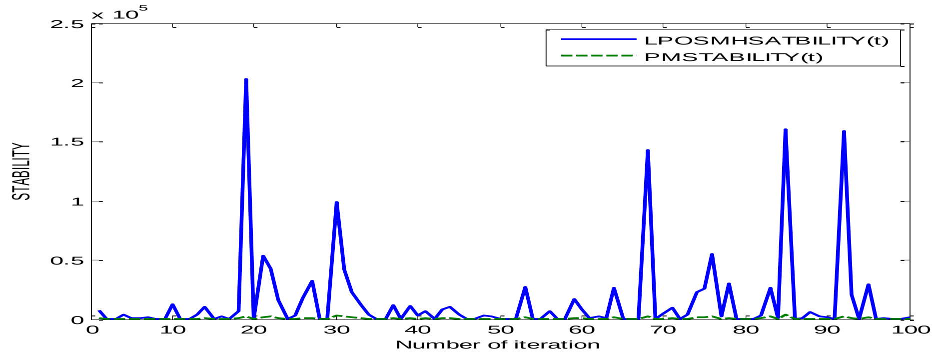

Fig. 6. Comparison of the stability of *LPOSMH* and *PM* for 50 jobs, B_1 , alpha=0.5

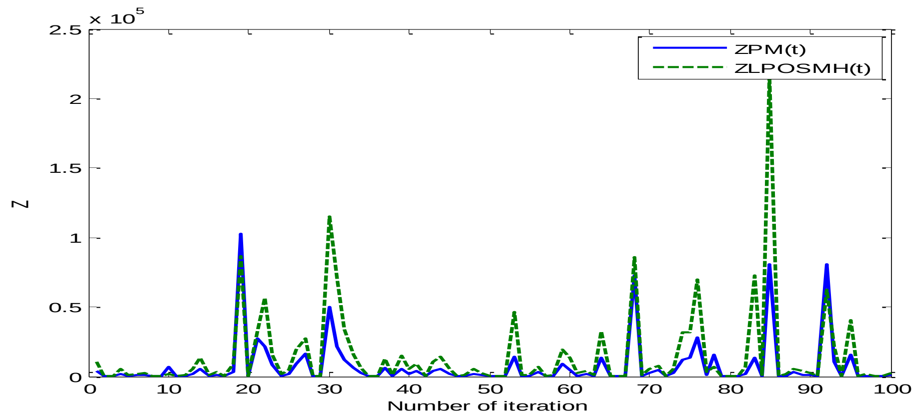

Fig. 7. Comparison of the objective function of *LPOSMH* and *PM* for 50 jobs, B_1 , alpha=0.5

The superiority of *LPOSMH* is approved for the same problem over the reactive and hybrid methods in Abtahi et al. (2020). So, we can conclude that the proposed method has a higher performance than these reactive and hybrid methods.

VI. CONCLUSION

In this paper, a single machine scheduling problem under uncertainty of job processing times and machine breakdown is presented. The problem is modeled as a bi-objective problem of robustness and stability. In the first stage of the proposed two-stage predictive heuristic, a robust schedule is generated considering the uncertainty of job processing times. This robust partial schedule is produced based on the optimality of *SEPT* for the expected total tardiness of jobs. In the second stage, the weaknesses of the idle-time insertion method are justified. A linear programming model is proposed in this stage to enhance the stability and control the deterioration of the schedule robustness. The proposed heuristic is compared with LPOSMH. Since previously the superiority of LPOSMH approved for the same problem over the reaction and hybrid methods, we can conclude that the proposed method has a higher performance than these methods, too. When the machines are not yet very worn (in higher availability modes), the advantage of the proposed heuristic is more pronounced. Also, by increasing the number of jobs, the proposed heuristic has shown its advantage by producing more robust schedules.

In future researches, other systems, such as flow shop systems, can be considered. Moreover, the method can be generalized to include the breakdowns of more than one machine. Another possibility for extending this work is the consideration of other distributions for machine failure. Consequently, maintenance inclusion is also suggested.

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