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A Mathematical Model for Unequal Area Stochastic Dynamic Facility Layout Problems

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Abstract *– The present research formulates unequal area stochastic dynamic facility layout problems for minimizing the total cost of material handling while considering their upper bound. In the proposed model, the area and shape of departments are considered to be stable during the implementation of an algorithm, and product demands are normally distributed with a known expected value and variance. Since these problems are NP-hard, thus particle swarm optimization (PSO) was employed, and a theoretical problem instance was presented to evaluate the efficiency and effectiveness of the algorithm. The findings confirmed the efficiency and validity of the proposed algorithm.*

Keywords*– Departments, Particle swarm optimization, Unequal area stochastic dynamic facility layout problems.*

I. INTRODUCTION

In facility layout problems (FLPs), the departments or facilities have to be arranged in the workspace to minimize the sum of the material handling costs. (Tompkins et al., 2018) stated that 20-50 % of the total cost of operation in a manufacturing environment belongs to material handling cost, which can be lowered to 10-30 % by the use of an effective facility rearrangement or arrangement. FLPs are classified into static FLPs (SFLPs) and uncertain FLPs. In SFLPs, there is a fixed material flow among the departments for a long time; however, in uncertain FLPs, the material flow changes over time. Uncertain FLPs are classified into stochastic FLPs (STFLPs), dynamic FLPs (DFLPs), and stochastic dynamic FLPs (STDFLPs).

The material flow in STFLPs among the departments is uncertain and stochastic while considering only one period (Derakhshan Asl et al., 2016). DFLPs are an extension of SFLPs in which the material flow changes in different periods; but, it is constant in each time period (Derakhshan Asl& Wong, 2017). Although all of these FLPs have been used in the marketplace, they cannot survive in today's competitive global marketplace because they cannot cope with the rapid product price and demand changes. As an alternative, STDFLPs encompass multi periods up to one year for material flow or product demand. This can provide stochastic, continuous, or uncertain material flow or product demand in each period (Moslemipour & Lee, 2012). They hold two categories, namely Equal-Area STDFLPs (EASTDFLPs), where all departments are the same in shape and area, and Unequal-Area STDFLPs (UASTDFLPs), where the area or shape of the departments are different, and the focus of the present study would be on the second category (i.e., UASTDFLPs).

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Depending on the state of the area and shape of departments fixed or non-fixed during the implementation of an algorithm, there are two types of UASTDFLPs. Making changes in the shape and area of departments is not always favored by companies (Yang & Peters, 1998). To this end, this paper aims to study UASTDFLPs with fixed areas and shapes among departments when implementing an algorithm. There are two types of problems in this regard: (1) UASTDFLPs having stochastic demands with a known expected value and variance in each period, and (2) UASTDFLPs having numerous scenarios and different probabilities for a material flow matrix in each period. While the sum of the probabilities is equal to 1 in each period, in the real world, most of the data for product demands are distributed normally for stochastic problems (Casella & Berger, 2002). To our knowledge, there is no research done on this subject. Hence, the study of UASTDFLPs with fixed areas and shapes of departments during the implementation of an algorithm seems warranted. Problem definition is given in Table I.

Since UAFLPs are NP-hard, exact methods are not able to solve them in a reasonable time (Mazinani et al., 2012). Thus, seeking an efficient algorithm is necessary. Particle swarm optimization algorithm (PSO), commonly used for solving combinatorial optimization problems, enjoys a very easy and simple implementation (Moslemipour et al., 2012). It also has an efficient global search method. Therefore, we used this meta-heuristic method to find better and efficient solutions.

TABLE I. PROBLEM DEFINITION

Studying UASTDFLPs
Departments have different shapes and areas
Departments have fixed areas and shapes during the implementation of an algorithm
Departments have free orientations

II. LITERATURE REVIEW

SFLPs were firstly formulated by (Koopmans & Beckmann, 1957). They minimized the total cost of material handling for two industrial problems with equal-size departments. Such problems were later developed by (Armour & Buffa, 1963).

STDFLPs were introduced by (Kouvelis & Kiran, 1991). Here there are numerous scenarios for a material flow matrix and different probabilities in each period, while the sum of probabilities is equal to 1. They used dynamic programming to solve the problems; however, very soon, they found that it was not suitable for large-size problems. (Palekaret al., 1992) formulated EASTDFLPs by applying quadratic integer programming, assuming that there are three levels for product demands: optimistic, most likely, and pessimistic. Dynamic programming was then developed to solve the problems. UASTDFLPs with fixed areas and shapes of departments were studied by (Yang & Peters, 1998). In addition, there were several scenarios for a material flow matrix with different probabilities in each period. They introduced the' Expected Flow Density' method for reducing UASTDFLPs to UADFLPs, and then proposed a heuristic method to solve the problems.

(Krishnan et al., 2006) and (Nayak 2007) studied EASTDFLPs with uncertain material flow among the departments in each period. They fitted a curve for the material flow between each pair of departments in each period and applied a genetic algorithm (GA) to solve the problems. EASTDFLPs with several scenarios for a material flow matrix with different probabilities in each period were investigated (Krishnan et al., 2008). (Moslemipour& Lee, 2012) developed EASTDFLPs with stochastic product demands having a known standard deviation and expected value in each period. Next, S.A. was applied to solve the problems. Later, (Lee et al., 2012) developed a hybrid ant colony optimization (HACO) and SA for problem-solving.

(Vitayasak et al., 2017) introduced a new Modified Backtracking Search Algorithm (MBSA) to solve the stochastic DFLP with heterogeneous-sized resources. They minimized the combination of material flow and redesign costs. (Moslemipour & Lee, 2018) suggested a new hybrid algorithm using SA algorithm with a population of good initial solutions constructed through combining clonal selection, robust layout design, and ant colony approaches.

(Peng et al., 2018) employed Mont Carlo simulation to generate different scenarios randomly. Then they designed an improved adaptive genetic algorithm with a population initialization strategy and compared it with the PSO algorithm. (Tayal et al., 2020) proposed a three-stage methodology in which data envelopment analysis (DEA) is augmented with supervised and unsupervised machine learning (ML). A summary of the above researches is presented in Table II.

Accordingly, it seems that no study has so far been done in the field of UASTDFLPs having fixed shape and area of each department and stochastic product demand with a known expected value and variance in each period. Therefore, the present research aims to formulate this kind of problem under the following assumptions: (1) departments with square or rectangular shapes, (2) fixed area and shape of each department during the algorithm implementation and throughout the whole time horizon, (3) departments having free orientation (the length and width of each department can exchange in contrast to their original value), (4) departments having no overlap with each other, (5) having several periods for the product demand or material flow, (6) arrangement of all departments in a given area in each period, (7) using city block distance or rectilinear distance to determine the distance between departments, (8) independent product demands in each period, (9) normal distribution of each product demand with a known expected value and variance in each period, and (10) fixed product routing throughout the time horizon. Table III shows the assumptions of the model.

TABLE II. RESULTS OF LITERATURE REVIEW AND LITERATURE GAPS

Paper title	*Publication year*	*Authors' name*	*Solution method*	*Problem characteristic*
Assignment problems and the location of economic activities	1957	Koopmans & Beckmann	Game Theory and Price System	EASFLP
A heuristic algorithm and simulation approach to relative location of facilities	1963	Armour & Buffa	Computer Program Methodology	UASFLP
Single and multiple period layout models for automated manufacturing systems	1991	Kouvelis & Kiran	Dynamic Programming	EASTDFLP
Modeling uncertainties in plant layout problems	1992	Palekaret al.	Exact & Heuristic method	EASTDFLP
Flexible machine layout design for dynamic and uncertain production environments	1998	Yang & Peters	Heuristic Procedure	UADFLP
Dynamic From-Between Chart: a new tool for solving dynamic facility layout problems	2006	Krishnan et al.	Genetic Algorithm	EASTDFLP
Solutions to dynamic facility layout problems	2007	Nayak	Genetic Algorithm	EASTDFLP
Facility layout design for multiple production scenarios in a dynamic environment	2008	Krishnan et al.	Genetic Algorithm	EASTDFLP
Intelligent design of a dynamic machine layout in uncertain environment of flexible manufacturing systems	2012	Moslemipour & Lee	SA	EASTDFLP

Continue TABLE II. RESULTS OF LITERATURE REVIEW AND LITERATURE GAPS

Paper title	*Publication year*	*Authors' name*	*Solution method*	*Problem characteristic*
A novel hybrid ACO/SA approach to solve stochastic dynamic facility layout problem (SDFLP)	2012	Lee et al.	Hybrid ACO/SA	EASTDFLP
A tool for solving stochastic dynamic facility layout problems with stochastic demand using either a Genetic Algorithm or a modified Backtracking Search Algorithm	2017	Vitayasak et al.	Modified Backtracking Search Algorithms (MBSAs)	EASTDFLP
Solving stochastic dynamic facility layout problems using proposed hybrid AC-CS-SA meta-heuristic algorithm	2018	Moslemipour & Lee	Hybrid AC-CS-SA	EASTDFLP
An improved genetic algorithm based robust approach for a stochastic dynamic facility layout problem	2018	Peng et al.	Genetic Algorithm	EASTDFLP
Efficiency analysis for stochastic dynamic facility layout problem using meta-heuristic, data envelopment analysis, and machine learning	2020	Tayal et al.	Data Envelopment Analysis (DEA) & Machine Learning (ML)	EASTDFLP

TABLE III. THE MODEL ASSUMPTIONS

Assumptions	*Unequal Area Stochastic Dynamic Facility Layout Problem*
1	Departments with square or rectangular shapes
2	Fixed area and shape of each department during the algorithm implementation and throughout the whole time horizon
3	Departments having free orientation (the length and width of each department can exchange in contrast to their original value)
4	Departments having no overlap with each other
5	Having several periods for the product demand or material flow
6	Arrangement of all departments in a given area in each period
7	Using city block distance or rectilinear distance to determine the distance between departments
8	Independent product demands in each period
9	Normal distribution of each product demand with a known expected value and variance in each period
10	Fixed product routing throughout the time horizon

III. FORMULATION OF A MATHEMATICAL MODEL FOR UASTDFLPS

As mentioned earlier, in this research, we propose a mixed-integer nonlinear programming model for solving the problems. The following notations have been used to create the model.

Indexes

i, j	Indexes of department
t	Index of period
l	Index of product
L	Number of products
M	Number of departments
T	Number of periods

Parameters

W	Length of shop floor
H	Width of shop floor
w_i	Length of department i
h_i	Width of department$i$
d_{ijt}	City block distance or rectilinear distance from department i to department j in period$t$.
f_{ijt}	Frequency of material flow from department i to department j in period$t$.
f_{ijlt}	Frequency of material flow from department i to department j for product l in period$t$.
C_{ijt}	The cost per unit distance from department i to department j in period$t$.
A_{it}	Shifting cost for department i in period$t$.
$MHC(\pi)$	Material handling cost for a known layout (π)
D_{lt}	Demand for product l in period$t$.
$E(\,)$	Expected value of a parameter
$\sigma(\,)$	Standard deviation of a parameter
$p(\,)$	Probability of a parameter
$F(\,)$	Cumulative distribution function of a parameter

$Z_{1-\alpha}$ Standard normal Z value at confidence level $(1-\alpha)$.

(x_{i0}, y_{i0}) Center-coordinate of department i for the initial layout in the shop floor

$$r_{i0} = \begin{cases} 1 & \text{if the length and width of department } i \text{ for the initial layout exchange in contrast totheir original values} \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{ijlt} = \begin{cases} 1 & \text{if department } j \text{ appears immediately after department } i \text{ for the route of product } l \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

Variables

(x_{it}, y_{it}) Center-coordinate of department i in period$t$

$$r_{it} = \begin{cases} 1 & \text{if the length and width of department } i \text{ in period } t \text{ exchange in contrast to their original values} \\ 0 & \text{otherwise} \end{cases}$$

$$u_{it}^{x} = \begin{cases} 0 & if\, x_{it} = x_{i(t-1)} \\ 1 & otherwise \end{cases} \quad t = 1, 2, \dots, T$$

$$u_{it}^{y} = \begin{cases} 0 & if\, y_{it} = y_{i(t-1)} \\ 1 & otherwise \end{cases} \quad t = 1, 2, \dots, T$$

$$z_{it} = \begin{cases} 0 & if\, r_{it} = r_{i(t-1)} \text{ and } u_{it}^{x} = u_{it}^{y} = 0 \\ 1 & otherwise \end{cases} \quad t = 1, 2, \dots, T$$

In dynamic problems, Equation (1) is usually used as the objective function for minimizing the total cost of material handling among the departments and the total cost of shifting of departments among the consecutive periods, as follows:

$$minimize \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} C_{ijt}f_{ijt}\left(\left|x_{it}-x_{jt}\right|+\left|y_{it}-y_{jt}\right|\right)+\sum_{t=1}^{T}\sum_{i=1}^{M} A_{it}z_{it} \quad i \neq j \tag{1}$$

Equation (2) is employed to find out the frequency of material flow from department i to department j in each period for each product:

$$f_{ijlt} = \beta_{ijlt}D_{lt} \qquad i,j = 1,2,\dots,M \qquad l = 1,2,\dots,L \qquad t = 1,2,\dots,T \tag{2}$$

According to Assumption 9, each product demand is distributed normally with a known expected value and variance in each period. Then, Equations (3) and (4) can be written for determining the frequency of material flow among different departments, as below:

$$E\left(f_{ijlt}\right) = E\left(\beta_{ijlt}D_{lt}\right) = \beta_{ijlt}E(D_{lt}) \quad i,j = 1,\dots,M \quad l = 1,\dots,L \quad t = 1,\dots,T \tag{3}$$

$$\sigma^2\left(f_{ijlt}\right) = \sigma^2\left(\beta_{ijlt}D_{lt}\right) = \beta_{ijlt}^2\sigma^2(D_{lt}) \quad i,j = 1,\dots,M \qquad l = 1,\dots,L \qquad t = 1,\dots,T \tag{4}$$

The sum of material flow from department i to department j in each period can be obtained as follows:

$$f_{ijt} = \sum_{l=1}^{L} f_{ijlt} \qquad i,j = 1,2,\dots,M \quad t = 1,2,\dots,T \tag{5}$$

For this purpose, Equations (6) and (7) can be employed to obtain the expected value and variance of the total material flow from department i to department j in each period, respectively:

$$E\left(f_{ijt}\right) = \sum_{l=1}^{L} E\left(f_{ijlt}\right) = \sum_{l=1}^{L} \beta_{ijlt}E(D_{lt}) \quad i,j = 1,2,\dots,M \quad t = 1,2,\dots,T \tag{6}$$

$$\sigma^2\left(f_{ijt}\right) = \sum_{l=1}^{L} \sigma^2\left(f_{ijlt}\right) = \sum_{l=1}^{L} \beta_{ijlt}^2\sigma^2(D_{lt}) \qquad i,j = 1,2,\dots,M \quad t = 1,2,\dots,T \tag{7}$$

According to Equation (1), the total cost of material handling for a known layout, π , can be obtained as follows:

$$MHC(\pi) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} C_{ijt}f_{ijt}\left(\left|x_{it}-x_{jt}\right|+\left|y_{it}-y_{jt}\right|\right) \qquad i \neq j \tag{8}$$

The expected value and variance of the total cost of material handling in all periods for a known layout π, can be determined as follows:

$$E\big(MHC(\pi)\big) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} E(f_{ijt})C_{ijt}\big(|x_{it}-x_{jt}|+|y_{it}-y_{jt}|\big) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt}E(D_{lt})C_{ijt}\big(|x_{it}-x_{jt}|+|y_{it}-y_{jt}|\big)\ i \neq j \quad (9)$$

$$\sigma^2\big(MHC(\pi)\big) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} \sigma^2(f_{ijt})C_{ijt}^2\big(|x_{it}-x_{jt}|+|y_{it}-y_{jt}|\big)^2 = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt}^2\sigma^2(D_{lt})C_{ijt}^2\big(|x_{it}-x_{jt}|+|y_{it}-y_{jt}|\big)^2\ \ i \neq j \quad (10)$$

It is worth noting that $UB(\pi, 1-\alpha)$ is the upper bound of the total cost of material handling for a known layout π, in all periods in which the confidence level is equal to $(1-\alpha)$. Thus, this research can minimize the mentioned upper bound for layout π, $(UB(\pi, 1-\alpha))$, instead of minimizing it for layout π, $(MHC(\pi))$ as follows:

$$p\big(MHC(\pi) \leq UB(\pi, 1-\alpha)\big) = 1-\alpha \quad (11)$$

Equation (11) can be standardized as below:

$$p\left(\frac{MHC(\pi)-E\big(MHC(\pi)\big)}{\sigma\big(MHC(\pi)\big)} \leq \frac{UB(\pi,1-\alpha)-E(MHC(\pi))}{\sigma(MHC(\pi))}\right) = 1-\alpha \quad (12)$$

Assuming Z to be equal to $\frac{MHC(\pi)-E(MHC(\pi))}{\sigma(MHC(\pi))}$, Equation (13) can replace Equation (12) as follows:

$$p\left(Z \leq \frac{UB(\pi,1-\alpha)-E(MHC(\pi))}{\sigma(MHC(\pi))}\right) = 1-\alpha \quad (13)$$

Since Z is normally distributed with an expected value and variance equal to zero and one, respectively, then:

$$F(Z_{1-\alpha}) = 1-\alpha \quad (14)$$

Equation (15) can be formulated by combining Equations (13) and (14).

$$Z_{1-\alpha} = \frac{UB(\pi, 1-\alpha) - E(MHC(\pi))}{\sigma(MHC(\pi))} \tag{15}$$

Based on Equations (15), (9), and (10), we can write the following single equation:

$$UB(\pi, 1-\alpha) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt} E(D_{lt}) C_{ijt}\left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}|\right) + Z_{1-\alpha}\left(\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt}^2 \sigma^2(D_{lt}) C_{ijt}^2 \left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}|\right)^2\right)^{\frac{1}{2}} \quad i \neq j \tag{16}$$

Assuming the coordinate of the bottom left is equal to $(0, 0)$, a mixed-integer nonlinear programming model (Equations (17) to (22)) can be suggested for UASTDFLPs with fixed areas and shapes of departments under the previously mentioned assumptions. In addition, the product demand is stochastic, with a known expected value and variance in each period.

$$minimize \quad \sum_{t=1}^{T}\sum_{i=1}^{M} A_{it} z_{it} + \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt} E(D_{lt}) C_{ijt}\left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}|\right) + Z_{1-\alpha}\left(\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt}^2 \sigma^2(D_{lt}) C_{ijt}^2 \left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}|\right)^2\right)^{\frac{1}{2}} \quad i \neq j \tag{17}$$

$$\frac{w_i}{2}(1 - r_{it}) + \frac{h_i}{2} r_{it} \leq x_{it} \leq W - \left(\frac{w_i}{2}(1 - r_{it}) + \frac{h_i}{2} r_{it}\right) \quad i = 1, \dots, M \quad t = 1, \dots, T \tag{18}$$

s.t.

$$\frac{h_i}{2}(1 - r_{it}) + \frac{w_i}{2} r_{it} \leq y_{it} \leq H - \left(\frac{h_i}{2}(1 - r_{it}) + \frac{w_i}{2} r_{it}\right) \quad i = 1, \dots, M \quad t = 1, \dots, T \tag{19}$$

$$|x_{it} - x_{jt}| + |y_{it} - y_{jt}| \geq \left(\frac{w_i}{2}(1 - r_{it}) + \frac{h_i}{2} r_{it} + \frac{w_j}{2}(1 - r_{jt}) + \frac{h_j}{2} r_{jt}\right) + \left(\frac{h_i}{2}(1 - r_{it}) + \frac{w_i}{2} r_{it} + \frac{h_j}{2}(1 - r_{jt}) + \frac{w_j}{2} r_{jt}\right) \quad i, j = 1, \dots, M \quad i \neq j \quad t = 1, \dots, T \tag{20}$$

$$u_{it}^{x} + u_{it}^{y} + \left|r_{i(t-1)} - r_{it}\right| \le 3z_{it} \qquad i = 1, \dots, M \qquad t = 1, \dots, T \tag{21}$$

$$x_{it}, y_{it}, x_{jt}, y_{jt} \ge 0 \quad r_{it}, r_{jt}, u_{it}^{x}, u_{it}^{y}, z_{it} \in \{0,1\} \qquad i, j = 1, \dots, M \quad t = 1, \dots, T \tag{22}$$

According to this formulation, Equation (17) deals with the objective function of UASTDFLPs. Equations (18) and (19) imply that departments must be on the shop floor along the $x$and y axes, respectively. Using Equation (20), we can prevent overlapping of department pairs. Equation (21) displays the possible change in the position of a department in consecutive periods. Finally, Equation (22) shows the proposed mathematical model's variables.

IV. SOLVING A SMALL-SIZED PROBLEM USING GAMS SOFTWARE

To verify and validate the proposed model, a small-sized problem instance was solved by Gams win 32 24.1.3 software. It includes three departments, two periods and three products. The length and width of the shop floor, as well as the shifting cost of each department, are all equal to 20. The other required information of problem instance were achieved from (Moslemipour& Lee, 2012) and (Yang & Peters, 1998) (Tables IV and V). The results of solving the model by Gams win 32, 24.1.3 software are given in Table VI. The objective function value (total cost) was equal to 406703.8698.

TABLE IV. VARIANCE AND EXPECTED VALUE OF THE PRODUCT DEMANDS AND ROUTES OF PRODUCTS

Product name	***Period 1***		***Period 2***		***Routes of products in all periods based on the number of department***
	Variance	***Mean***	***Variance***	***Mean***	
A	1893	7623	2318	9120	1→3→2
B	1573	2067	2578	4347	3→2→1
C	1283	8965	2251	2358	2→1→3

TABLE V. CENTER COORDINATES OF DEPARTMENTS FOR THE INITIAL LAYOUT

Departments	1	2	3
Center-coordinate of departments along the axis for the initial layout (x_{i0})	6	23	6.5
Center-coordinate of departments along the axis for the initial layout (y_{i0})	16.5	22.5	22
Orientation of departments in comparison with their original orientation for the initial layout (r_{i0})	1	0	1
Original length (w_i) of each department	5	7	6
Original width (h_i) of each department	4	5	4

Table VI. COMPUTATIONAL RESULTS OF SOLVING THE MODEL BY GAMS SOFTWARE

Initial layout and period	*Center-coordinate of departments along the axis(x_{it})*			*Center-coordinate of departments along the axis(y_{it})*			*Orientation of departments in comparison with their original orientation (r_{it})*		
	1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*
Initial layout	6.0000	23.0000	6.5000	16.5000	22.5000	22.0000	1	0	1
Period1	7.0901	11.5901	3.0901	6.5301	6.5301	6.5301	1	1	1
Period2	17.0288	8.5288	13.0288	9.7180	9.7180	9.7180	1	1	1

V. PSO METHOD FOR SOLVING UASTDFLPS

In PSO, which was initially introduced by (Kennedy & Eberhart, 1995), to find an optimal solution, the particles' positions changed. PSO is a population-based stochastic optimization method. The position of each particle in each generation can be obtained by the following equations (Engel Brecht, 2019):

$$v^k(g+1) = wv^k(g) + c_1r_1\left(x^{k,best}(g) - x^k(g)\right) + c_2r_2\left(x^{gbest}(g) - x^k(g)\right) \quad (23)$$

$$x^k(g+1) = x^k(g) + v^k(g+1) \quad (24)$$

Where k is the index of particle,g is the index of generation, $v^k(g)$ is the velocity of particle k at generation g, $x^k(g)$ is the position of particle k at generation g,$x^{g,best}(g)$ is the best global position at generation g, $x^{k,best}(g)$ is the best personal position of particle k at generation g,r_1 and r_2 are uniform random variables between zero and one (r_1 and $r_2 \sim U(0,1)$),w is the inertia weight coefficient (0.4 - 0.9), and c_1 and c_2 are acceleration coefficients of the best personal and global solutions, respectively. The values ofc_1 and c_2 must be between 0 and 2.

A. Solution representation for UASTDFLPs

We used continuous solution representation to represent the solutions for UASTDFLPs. This is the first effort that employs this type of solution representation in the field of STDFLPs. To arrange the departments for an UASTDFLP, we must first determine the center-coordinates of departments along x and $y$axes in a shop floor and their orientations in comparison with their original orientations in all periods. In other words, we have to calculate the values of x_{it} , y_{it} and r_{it} ($i = 1, 2, \dots, M$and$t = 1, 2, \dots, T$) first. It is worth noting that continuous algorithms usually work with variable values between zero and one. Hence, a shadow of the center-coordinates of departments along the $x$axis and $y$axis and a shadow of their orientations in comparison with their original orientations in each period is used to represent a solution. Shadow representation is adapted to encode the solutions based on Equations (25), (26), and (27).

Therefore, we need three parts to represent a solution for a UASTDFLP. The first and second parts represent the shadow of the departments' center-coordinates along the $x$and y axes in each period, respectively. The third part is a shadow of the departments' orientations in comparison with their original orientations in each period. $\hat{x}_{it}$, $\hat{y}_{it}$ and $\hat{r}_{it}$ are employed to indicate the shadow of the center-coordinate of department i along the x and$y$ axes, as well as the shadow of its orientation in comparison with its original orientation in period$t$, respectively. In the solution representation,$\hat{x}_{it}$,$\hat{y}_{it}$and $\hat{r}_{it}$ ($i = 1, 2, \dots, M$and$t = 1, 2, \dots, T$) are numbers from zero to one.

The orientations of departments in contrast to their original orientations and the center-coordinates of departments along the x and y axes for each period in a shop floor can be calculated as follows, respectively.

$$r_{it} = \begin{cases} 0 & if\ 0 \le \hat{r}_{it} < 0.5 \\ 1 & if\ 0.5 < \hat{r}_{it} \le 1 \end{cases} \quad i = 1,2,\dots,M \; t = 1,2,\dots,T \tag{25}$$

$$x_{it} = Xmin_{it} + (Xmax_{it} - Xmin_{it})\hat{x}_{it} \rightarrow \begin{cases} Xmin_{it} = \frac{w_i}{2}(1 - r_{it}) + \frac{h_i}{2} r_{it} \\ Xmax_{it} = W - \left(\frac{w_i}{2}(1 - r_{it}) + \frac{h_i}{2} r_{it}\right) \end{cases} \quad i = 1,\dots,M \; t = 1,\dots,T \tag{26}$$

$$y_{it} = Ymin_{it} + (Ymax_{it} - Ymin_{it})\hat{y}_{it} \rightarrow \begin{cases} Ymin_{it} = \frac{h_i}{2}(1 - r_{it}) + \frac{w_i}{2} r_{it} \\ Ymax_{it} = H - \left(\frac{h_i}{2}(1 - r_{it}) + \frac{w_i}{2} r_{it}\right) \end{cases} \quad i = 1,\dots,M \; t = 1,\dots,T \tag{27}$$

Where $(Xmax_{it}, Ymax_{it})$ and $(Xmin_{it}, Ymin_{it})$ are the maximum and minimum values of the center-coordinates of department i in period t, respectively.

As mentioned earlier, in this research, the first type of continuous solution representation was employed for solving the problems. This solution representation is applied when a problem is solved by PSO, which uses a matrix with T rows and $3 \times M$ columns.

As an example, this type of solution representation is shown for a problem with four departments and three periods in Fig.1. It is clear from the figure that the values of cells in row 1 and column 2, row 2 and column 7, and row 3 and column 12 indicates a shadow of the center-coordinate of department 2 along the x axis in period 1 ($\hat{x}_{21}$), a shadow of the center-coordinate of department 3 along the y axis in period 2 ($\hat{y}_{32}$), and a shadow of the orientation of department 4 in period 3 ($\hat{r}_{43}$) respectively.

	Shadow of the center-coordinates of departments along the x axis				Shadow of the center-coordinates of departments along the y axis				Shadow of the orientations of departments			
Period 1	0.8460	0.3351	0.3703	0.7934	0.4583	0.4481	0.8919	0.8483	0.5122	0.2673	0.2183	0.7566
Period 2	0.2752	0.1935	0.7420	0.5708	0.2735	0.7607	0.7236	0.2235	0.5924	0.2180	0.8488	0.8108
Period 3	0.2805	0.8373	0.7651	0.2292	0.5673	0.8461	0.3434	0.8444	0.2868	0.2301	0.2440	0.1922

Fig.1.The first type of solution representation

In the proposed algorithm, the objective function is calculated below:

$$\text{Minimize} \sum_{t=1}^{T}\sum_{i=1}^{M} A_{it} z_{it} + \left(\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt} E(D_{lt}) C_{ijt} \left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}| \right) + Z_{1-\alpha} \left(\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{L} \beta_{ijlt}^{2} \sigma^{2}(D_{lt}) C_{ijt}^{2} \left(|x_{it} - x_{jt}| + |y_{it} - y_{jt}| \right)^{2} \right)^{\frac{1}{2}} \right) (1 + pv) \quad i \neq j \tag{28}$$

Where p is a coefficient with a value of 1200 that is set after five replications of running the proposed algorithms and v is the total average violation. In the above equation, v is used to avoid the overlapping of departments. The value of the total violation,v, among the departments can be obtained using Equation (29) as follows:

$$v = \frac{1}{T(M^2 - M)} \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} v_{ijt} \; i \neq j \tag{29}$$

Where, v_{ijt} is the violation between departments i and j in period t, which can be calculated as follows:

$$v_{ijt} = \min(vx_{ijt}, vy_{ijt}) \; t = 1,2,\dots,T i, j = 1,2,\dots,M i \neq j \tag{30}$$

Where,vy_{ijt} and $vx_{ijt}$are the violations between department i and department j in period t in the direction of the$y$and $x$axes, respectively.

VI. NUMERICAL EXPERIMENTS

Based on the literature review, there is no problem instance in the field of UASTDFLPs with normally distributed product demands and known expected value and variance.

The theoretical problem instance, called STDFLP-II, includes10 products, 5 periods, and 12 machines. The variance and expected value of each product demand and product routes for STDFLP-II were selected from (Moslemipour& Lee, 2012) (Tables VII and VIII, respectively).

The other data for STDFLP-II, which were taken from (Yang & Peters, 1998), are described as follows. The departments should be placed on a 60×60 shop floor for each period. According to (Yang & Peters, 1998), some departments'corner-coordinates were negative for the initial layout of STDFLP-II. So, each department's center-coordinate was summed with 6 and 14 in the x and y axes, respectively. The center-coordinates of departments and their orientations in comparison with their original orientations are provided in Table IX for the initial layout of the problem. Table X presents the shifting cost of departments separately for this problem.

TABLE VII. VARIANCE AND EXPECTED VALUE OF THE PRODUCT DEMANDS FOR STDFLP-II

Product	*Period 1*		*Period 2*		*Period 3*		*Period 4*		*Period 5*	
	Variance	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*
A	1073	6220	1118	5656	2584	3764	2629	6503	1553	6503
B	2824	2565	2442	8863	1609	6636	1344	9101	2940	7589
C	1893	7623	2318	9120	1372	3543	1668	4554	1388	5948
D	1573	2067	2578	4347	2986	2646	2262	9746	1812	8496
E	1283	8965	2251	2358	1909	2720	1898	7540	1663	8085
F	2892	8736	1190	9998	1045	7804	1417	3677	2998	2066
G	1373	6823	2493	8104	2062	6861	2499	4910	1856	8772
H	1030	6088	2751	9696	1195	3116	1052	9253	2355	8257
I	1641	6907	2087	7493	1854	4458	1384	5141	2676	6664
J	2316	4093	1447	5496	1177	1606	2648	7172	1236	5258

TABLEVIII.ROUTES OF PRODUCTS IN ALL PERIODS FOR STDFLP-II

Product	*Routes of products*	*Product*	*Routes of products*	*Product*	*Routes of products*
A	5→3→10→9→11	B	11→10→3→9→5	C	1→12→8
D	12→8→1	E	8→1→12	F	7→2→6
G	2→4→7→6	H	6→7→4→2	I	2→6
J	5→10→3				

TABLE IX.CENTER COORDINATES OF DEPARTMENTS IN THE INITIAL LAYOUT FOR STDFLP-II

Department	1	2	3	4	5	6	7	8	9	10	11	12
Center-coordinate of the department along the axis for the initial layout (x_{i0})	6	11.5	17.5	22	11.5	22	11.5	23	17	17.5	17.5	6.5
Center-coordinate of the department along the axis for the initial layout (y_{i0})	16.5	27.5	16	18	22	27.5	14	22.5	22	32.5	27.5	22
Orientation of department in comparison with its original orientation for the initial layout (r_{i0})	1	0	1	0	0	1	1	0	1	0	0	1

TABLE X.SHIFTING COST OF DEPARTMENTS FOR STDFLP-II

Department	1	2	3	4	5	6	7	8	9	10	11	12
Shifting cost of department	50	50	50	50	50	50	50	50	50	50	50	50

The original length and width of departments for STDFLP-II are given in table XI. The cost per unit distance was assumed to be $1.0.

TABLE XI.LENGTH AND WIDTH OF DEPARTMENTS IN ALL PERIODS FOR STDFLP-II

Department	1	2	3	4	5	6	7	8	9	10	11	12
Original length (w_i)	5	7	6	4	6	5	10	7	6	5	5	6
Original width (h_i)	4	5	5	4	6	4	7	5	5	5	5	4

Each PSO has five parameters as follows:$(MaxIt, nPop, c_1, c_2, w)$. $MaxIt$ is the maximum number of generations, $nPop$ is the number of particles, c_1 is the acceleration coefficient of the best personal solution, c_2 is the acceleration coefficient of the best global solution, and w is the inertia weight coefficient. We run the proposed PSO 20 times for STDFLP-II at each confidence level with different parametric values. The results revealed the following values for $MaxIt, nPop, c_1, c_2$, and w: 600 or 650, 500 or 700, 0.5 or 0.6, 1.0 or 1.2, and 0.4 or 0.5, respectively. Overall, 32 situations were concluded for the STDFLP-IIparameters at each confidence level. As a result, 20 situations were selected randomly, and the modified PSO was tested with these parametric values. Parameter setting for STDFLP-II at each confidence level is reported in Table XII.

TABLEXII.SETTING OF PARAMETER FOR THE MODIFIED PSO

Problem	***Confidence level ($1-\alpha$)***	$MaxIt$	$nPop$	c_1	c_2	w
STDFLP-II	0.85	600	500	0.5	1.0	0.4
	0.95	650	700	0.6	1.2	0.5

VII.SENSITIVITY ANALYSIS

Tables XIIIand XIV present the PSO results for solving STDFLP-II at two confidence levels ($1-\alpha = 0.85$,0.95), respectively. Each problem was solved 10 times using the modified algorithm. Material handling cost, the sum of the shifting cost, total cost, and mean of the total cost for 10 replications as well as run time for the best solution at two confidence levels ($1-\alpha = 0.85$,0.95) are shown in Tables XIII and XIV, respectively.

VIII.CONCLUTION

UASTDFLPs having normally distributed product demands with a known expected value and variance in each period were studied for the first time in the present research. Since the investigated problems are very complex and NP-hard with stubborn behavior, PSO was used to create solutions for them. The findings revealed the efficiency and effectiveness of the PSO method. For future work, this research can be extended in many ways, including:1) Using other types of distributions for product demands, 2) Adding the output and input for departments, and 3) Applying other meta-heuristic algorithms to solve the problems.

TABLE XIII. SENSITIVITY ANALYSIS OF THE MODIFIED PSO FOR STDFLP-II($1-\alpha=0.85$)

Run	*Material handling cost*	*Shifting cost*	*Total cost*	*Run time for the best solution (second)*
1	5477779.3805875	600	5478379.3805875	13259
2	5457130.8847875	600	5457730.8847875	13259
3	5487890.2694875	600	5488490.2694875	13259
4	5447019.9958875	600	5447619.9958875	13259
5	5386924.2021	600	5387524.2021	13259
6	5498001.4906875	600	5498601.4906875	13259
7	5436908.7746875	600	5437508.7746875	13259
8	5508112.6018875	600	5508712.6018875	13259
9	5426797.6634875	600	5427397.6634875	13259
10	5569044.4884	600	5569644.4884	13259
Mean of the total cost for 10 replications: 5470160.9752				

TABLE XIV. SENSITIVITY ANALYSIS OF THE MODIFIED PSO FOR STDFLP-II (1-α=0.95)

Run	*Material handling cost*	*Shifting cost*	*Total cost*	*Run time for the best solution (second)*
1	5628048.2830875	3000	5631048.2830875	15647
2	5607399.7872875	3000	5610399.7872875	15647
3	5638159.1719875	3000	5641159.1719875	15647
4	5570288.8983875	3000	5600288.8983875	15647
5	5585973.3494	3000	5588973.3494	15647
6	5648270.3931875	3000	5651270.3931875	15647
7	5587177.6771875	3000	5590177.67771875	15647
8	5658381.5043875	3000	5661381.5043875	15647
9	5577066.5659875	3000	5580066.5659875	15647
10	5673435.9491	3000	5676435.9491	15647
Mean of the total cost for 10 replications: 5623120.158				

REFERENCES

Armour, G.C., & Buffa, E.S., (1963). A heuristic algorithm and simulation approach to relative location of facilities.*Management Science*9(2):294-309.

Casella, G., & Berger, R.L., (2002). Statistical inference. Thomson, India.

Derakhshan Asl, A., Wong, K.Y., &Tiwari, M.K., (2016). Unequal-area stochastic facility layout problems: solutions using improved covariance matrix adaptation evolution strategy, particle swarm optimisation, and genetic algorithm.*International Journal of Production Research* 54(3):799-823.

Derakhshan Asl, A., & Wong, K.Y., (2017). Solving unequal-area static and dynamic facility layout problems using modified particle swarm optimization.*Journal of Intelligent Manufacturing* 28(6):1317-1336.

Engel Brecht, A.P., (2019). Computational intelligence: an introduction. John Wiley & Sons, Chichester.

Kennedy, J., & Eberhart, R., (1995). Particle Swarm Optimization. In: *Proceedings of IEEE International Conference on Neural Networks*, pp. 1942-1948.

Koopmans, T.C., & Beckmann, M., (1957). Assignment problems and the location of economic activities. Econometrica: *Journal of the Econometric Society* 25(1):53-76.

Kouvelis, P., & Kiran, A.S., (1991). Single and multiple period layout models for automated manufacturing systems.*European Journal of Operational Research*52(3):300-314.

Krishnan, K.K., Cheraghi, S.H., & Nayak, C.N., (2006). Dynamic From-Between Chart: a new tool for solving dynamic facility layout problems. *International Journal of Industrial and Systems Engineering* 1(1-2):182-200.

Krishnan, K.K., Cheraghi, S.H., & Nayak, C.N., (2008). Facility layout design for multiple production scenarios in a dynamic environment. *International Journal of Industrial and Systems Engineering* 3(2):105-133.

Lee, T.S., Moslemipour, G., Ting, T.O., & Rilling, D., (2012). A Novel Hybrid ACO/SA Approach to Solve Stochastic Dynamic Facility Layout Problem (SDFLP). *Emerging Intelligent Computing Technology and Applications* 304:100-108.

Mazinani, M., Abedzadeh, M., &Mohebali, N., (2012).Dynamic facility layout problem based on flexible bay structure and solving by genetic algorithm.*The International Journal of Advanced Manufacturing Technology*65(5-8):929-943.

Moslemipour, G., & Lee, T., (2012).Intelligent design of a dynamic machine layout in uncertain environment of flexible manufacturing systems.*Journal of Intelligent Manufacturing*23(5):1-12.

Moslemipour, G., Lee, T.S., &Rilling, D., (2012). A review of intelligent approaches for designing dynamic and robust layouts in flexible manufacturing systems.*The International Journal of Advanced Manufacturing Technology* 60(1-4):11-27.

Moslemipour, G., Lee, T. S., &Loong, Y. T. (2018).Solving stochastic dynamic facility layout problems using proposed hybrid AC-CS-SA meta-heuristic algorithm. *International Journal of Industrial and Systems Engineering*, *28*(1), 1-31.

Nayak, C.N., (2007). Solutions to dynamic facility layout problems: Development of Dynamic From Between Chart (DFBC) and its applications to continuous layout modeling. Dissertation, Wichita State University.

Palekar, U.S., Batta, R., Bosch, R.M., &Elhence, S., (1992).Modeling uncertainties in plant layout problems.*European Journal of Operational Research*63(2):347-359.

Peng, Y., Zeng, T., Fan, L., Han, Y., & Xia, B. (2018). An Improved Genetic Algorithm Based Robust Approach for Stochastic Dynamic Facility Layout Problem. *Discrete Dynamics in Nature and Society*, *2018*.

Tayal, A., Kose, U., Solanki, A., Nayyar, A., & Saucedo, J. A. M. (2020).Efficiency analysis for stochastic dynamic facility layout problem using meta-heuristic, data envelopment analysis and machine learning.*Computational Intelligence*, 36(1), 172-202.

Tompkins, J.A., White, J.A., Bozer, Y.A., & Tanchoco, J.M.A., (2018). *Facilities Planning*. Wiley, New York.

Yang, T., & Peters, B.A., (1998).Flexible machine layout design for dynamic and uncertain production environments. *European Journal of Operational Research* 108(1):49-64.

Vitayasak, S., Pongcharoen, P., & Hicks, C. (2017).A tool for solving stochastic dynamic facility layout problems with stochastic demand using either a Genetic Algorithm or modified Backtracking Search Algorithm. *International Journal of Production Economics*, *190*, 146-157.