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Multi-Period Multi-Product Distribution Network Redesign under Uncertainty

Mosayyeb Rahimi [1], Mansour Doodman [1], Ali Bozorgi- Amiri [1*]

[1] *School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

**** Corresponding Author:*** *Ali Bozorgi- Amiri (Email:alibozorgi@ut.ac.ir)*

Abstract *– Reviewing related studies to supply chain networks shows that various factors influence the supply chain network's optimal design. Demand changes in market zones, changes in customers' locations, various competitive conditions in different regions, network costs fluctuations, and different disruption pose risks to the supply chain network in terms of optimal condition. In such conditions, to help the network return to its optimality, the network needs to be redesigned. In this respect, this paper has formulated a multi-period multi-product mathematical model for redesigning the warehouses considering backorder shortage. In this regard, decisions such as the construction of new facilities, closing non-optimal facilities, increasing the active facilities' capacity, the optimal amount of order/shortage in each active warehouse in each period have been investigated. Due to the inherent uncertainty in real-world business, parameters related to cost and demand are considered uncertain. Then, to deal with uncertain parameters in the proposed model, Jimenez's possibilistic programming has been used. The obtained results show that redesigning the supply chain network by setting up new facilities/closing non-optimal facilities can significantly reduce inventory and transportation costs and shortages.*

Keywords*– Warehouse network redesign, Inventory management, Partial back-ordering, Facilities relocation, Possibilistic programming.*

I. INTRODUCTION

Redesigning the warehouse network is one of the essential parts of redesigning any supply chain's members. As a middle part of any chain, warehouses are of considerable importance among other echelons. The classic problem of warehouse location is about determining the optimal place between potential locations and new candidates. At the same time, warehouse network redesign is to address various issues. For example, which existing warehouses should remain active, which one is overcrowded and should be decommissioned, which warehouses should be transferred to other locations (under conditions in which customer service and goods delivery do not exceed a specified/acceptable time), which potential warehouses should be established, and finally which markets should be serviced by which warehouses.

In general, redesigning the warehouse network problem is the most focused current situation of facilities. Simultaneously, to reduce costs and save overhead costs without lowering service levels, it tries to integrate warehouses. Redesigning warehouse networks can be practical for military, organizational, industrial, and other vital warehouses and commercial and competitive companies that incur high costs in their supply chain network (Santoso et al.,2005).

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In most supply chains, some parameters like operating costs, raw material prices, demand/production capacity are considered uncertain factors. Uncertainty is an inherent characteristic of all supply chain decisions (strategic, technical, operational decisions) and affects the supply chain's final performance (Torabi et al.,2016). Accordingly, this issue should be formulated in the model, and then an appropriate solution methodology should be determined based on the type and level of the uncertainty.

The uncertainty in the amount of demand may lead to shortages and drive the customers unsatisfied. Therefore, shortages result in losing market share and thus reduce sales. Hence, considering shortages make the mathematical model closer to reality in problems such as how much customer demand is met at the time of demand, how much is fulfilled in later periods, how much demand is not met, and considered as lost sales (Melo & Nickel, 2014).

In this paper, a multi-period multi-product mathematical model is presented for redesigning warehouses considering partial back-ordering. As far as we reviewed the literature, no research has been provided a multi-period multi-product redesign for the supply chain network, considering decisions related to warehoused redesign and backorder shortage under uncertainty, simultaneously. Decisions related to warehouse redesign include locating new facilities, closing existing non-optimal facilities, expanding facilities' operating capacity at the strategic level, and assigning customers to warehouses. Moreover, considering the backorder shortage, decisions related to the amount of economic order quantity for each of the active warehouses in each period have been made simultaneously.

The rest of this paper is organized as follows: the literature about redesigning supply chain/warehouse networks is reviewed in Section 2. In Section 3, the problem is described, and the mathematical model is formulated. In Section 4, efficient possibilistic programming used to deal with uncertainties in the proposed model is addressed. In Section 5, a numerical example is presented, and the results of solving the model are examined. Finally, conclusions and suggestions for future research are presented in Section 6.

II. LITERATURE REVIEW

Melo et al. (2014) examined a multi-period multi-product problem with redesigning a supply chain in a network of suppliers, warehouses, and retailers. In this network, it was possible to transfer inventory between all facilities (Melo & Nickel,2014). Melo et al. (2011) developed an innovative two-phase solution to solve the proposed mathematical model. In the first phase, the actual values of the variables were approximated to be integers. In the second phase, an innovative approach was proposed to improve obtained/unjustified results. In a similar study, Analia et al. (2014) studied supply chain design and redesign in which the network could be designed in the first planning period, and in the next periods, the same network could be redesigned (Analia et al., 2014).

Cortinhal et al. (2015) presented a mathematical model for multi-level, multi-period, and multi-product supply chain network design. They made decisions such: locating new facilities and determining capacity levels for them, increasing the capacity of existing facilities/new-established facilities during the planning period, selecting suppliers, in addition to decisions related to purchasing/production, along with the possibility of outsourcing the production of final products (Cortinhal et al., 2015).

In 2015, Khatami et al. investigated reverse supply chain design and the forward supply chain simultaneously. They also redesigned the forward supply chain at the same time. Hammami and Farin (2014) investigated supply chain design on a global scale. In the current study, the transfer of production facilities to other countries was considered to benefit from lower production costs and tax incentives (Khatami et al., 2015).

Razmi et al. (2013) presented a bi-objective stochastic mathematical model to redesign a warehouse network. The first objective function aimed to minimize costs, including fixed costs of warehouse construction, distribution, relocation, and capacity development. The second objective was to maximize customer demand coverage percentage (Razmi et al., 2013). Peeters et al. (2001) proposed a mathematical model for constructing new facilities, reducing/increasing existing ones' capacity (Peeters et al., 2001).

Jena et al. (2015) presented mixed integer programming for a dynamic facility location problem. They developed the proposed mathematical model for situations under which facilities could be closed and be reopened again during the planning horizon; it also meant for the mode where the facility's capacity was increased/decreased from a period to the next (Jena et al., 2015). Jena et al. (2016) presented a multi-period multi-product mathematical model for facility location. Facilities can be relocated to new candidate places during the planning period.

In 2017, Feitó-Cespón et al. presented a mixed-integer nonlinear programming model for recyclable supply chain design under a sustainable approach. Decisions such as facility location, material flow design, and type of transportation selection have been investigated. Moreover, objectives related to economic, social, and environmental aspects have been considered integrated (Feitó-Cespón et al., 2017).

In 2017, Farahani et al. presented a location-inventory model considering a strategy to reduce the chain's disruption risk. They assumed that minor disruption could occur in a facility; therefore, they adopted the alternative product strategy to reduce disruption risk. Then, to solve the presented nonlinear model, they used the hybrid tabu search and neighborhood search algorithms (Farahani et al., 2017). Najafi et al. (2017) proposed a bi-objective model for designing a dynamic closed-loop supply chain under disruption risk of facility location. This paper's strategic decisions include locating distribution/recycling centers, allocating distribution centers to customers, and inventory-related decisions (Najafi et al., 2017).

Fattahi et al. (2018), by considering price-dependent customer demand, presented a multi-level, multi-period, and multi-product stochastic mathematical model based on different scenarios for supply chain network redesign. They investigated decisions such as determining the location for establishing new production centers/warehouses, increasing the facilities' capacity, and production in each production center. They also used the Benders decomposition algorithm to solve the proposed model (Fattahi et al., 2018).

Razm et al. (2019) presented a multi-objective mathematical planning model for redesigning the biomass supply chain network. In their model, economic, social, and environmental goals were simultaneously considered. They used the ε-constraint method to solve the proposed multi-objective mathematical model. The redesign decisions addressed constructing new facilities, closing non-optimal facilities, and transferring facilities' capacity (Razm et al., 2019).

Mousazadeh et al. (2018) redesigned the health service supply chain network to develop a multi-period multi-objective three-level mathematical model. They considered decisions related to the construction of new facilities, closing/expanding active facilities' capacity over a period, and re-allocation demand points to facilities. In this study, to deal with uncertain parameters, a robust possibilistic programming approach has been used. Moreover, to obtain the efficient Pareto frontier, the new epsilon constraint method has been developed (Mousazadeh et al., 2018).

Jahani et al. (2018) presented a multi-period mathematical model for redesigning the supply chain network. The suggested supply chain network included manufacturers, distribution centers, and demand points, seeking to determine new distribution centers' locations to provide a new product in the market. In their study, uncertainty was stochastically assigned to demand parameters and product prices; Moreover, they conducted a case study in Australia (Jahani et al., 2018).

Razm et al. (2019) considered logistics-related decisions and financial matters by presenting a stochastic programming model for redesigning the biomass supply chain network. Decisions such as constructing new facilities and closing inactive facilities were assumed in their model. Their case study was conducted on the supply chain network in Iran and Armenia (Razm et al., 2019).

Gitinavard et al. (2019) proposed A multi-objective mathematical model to design a multi-echelon supply chain network by considering perishable products under uncertainty. The proposed model has been utilized for perishable products based on fuzzy customers' demands (Gitinavard et al.,2019).

Vakili et al. (2020) proposed a location-routing mathematical model by considering green, pickup, and delivery decisions to minimize the total cost. Due to the proposed model's uncertain nature, a robust stochastic programming approach was applied to deal with uncertain parameters (Vakili et al., 2020).

In a study by Mousazadeh et al. (2018), a multi-period multi-objective three-echelon model was developed to redesign health service supply chain networks. In this work, the uncertainty in parameters was handled with the help of a robust possibilistic programming method, and Pareto efficient solutions were obtained using a new ε-constraint method developed for this purpose (Mousazadeh et al., 2018).

Paz et al. (2015) modeled a four-echelon supply chain redesign with the possibility of intra-echelon relocation. The model was solved for stochastic demand using the standard average approximation method. A comparison between the results of deterministic and stochastic models demonstrated the importance of changes in demand (Paz et al., 2015). In a study by Jena et al. (2016), a multi-period multi-product model was developed for facility location over the planning horizon with the possibility of facility relocation (Jena et al., 2016).

Correia and Melo (2017) studied the problem of establishing new facilities over the planning horizon, closing the non-optimal facilities, and increasing active facilities' capacity. In this work, demand management was performed by dividing customers into two groups: (1) customers whose demand should be met immediately, and (2) customers whose demand can be back-ordered (Correia & Melo, 2017).

According to reviewed studies, the contributions of the current study are as follows:

- Introducing a newly redesigned model for multi-period multi-product distribution network
- Considering decisions related to establishing/closing facilities, inventory management, and capacity expansion in an integrated model
- Considering backorder shortage
- Developing efficient possibilistic programming to cope with existing uncertainties

III. PROBLEM DEFINITION

In this study, a mathematical model for warehouse location is presented to redesign warehouses in a three-level, multi-period, multi-product supply chain. The supply chain network includes manufacturers, candidate locations for establishing new/existing warehouses, and customers (Fig.(1)). During the planning horizon, existing warehouses could be closed, and a new warehouse can be opened at one of the candidate locations at different levels. If one of the active distribution centers was close during a period, it could not be reopened until the end of that period. Moreover, if a new warehouse candidate is established in a period, it cannot be close until the end of that period. Active warehouses can increase their capacity over periods. Different costs considered in this stage include the fixed cost of constructing a new warehouse, closing the existing plant, and increasing active plants' capacity in each period. By considering capacity constraints, each customer is assigned to one of the active warehouses in each period. Based on the demand allocated to the warehouse, the manufacturer receives the order of each warehouse. By considering the amount of backorder in each warehouse, the fixed value inventory policy (Q, r) is defined to determine the manufacturer's optimal order quantity. Therefore, in each period, a certain percentage of customer demand became unfulfilled as backorder shortages; thus, a percentage of shortage shall be met in the next periods. The costs considered in this stage are holding costs in warehouses, ordering costs for warehouses, transportation of products from the manufacturer to warehouses, fixed costs of warehouse establishment, the cost of closing existing warehouses, operating costs of active warehouses in each period, as well as shortage costs.

Due to the inherent uncertainty in the model's parameters, the parameters related to cost and customer demand are considered fuzzy, and the efficient method of Jimenez has been used to deal with the model's uncertainty.

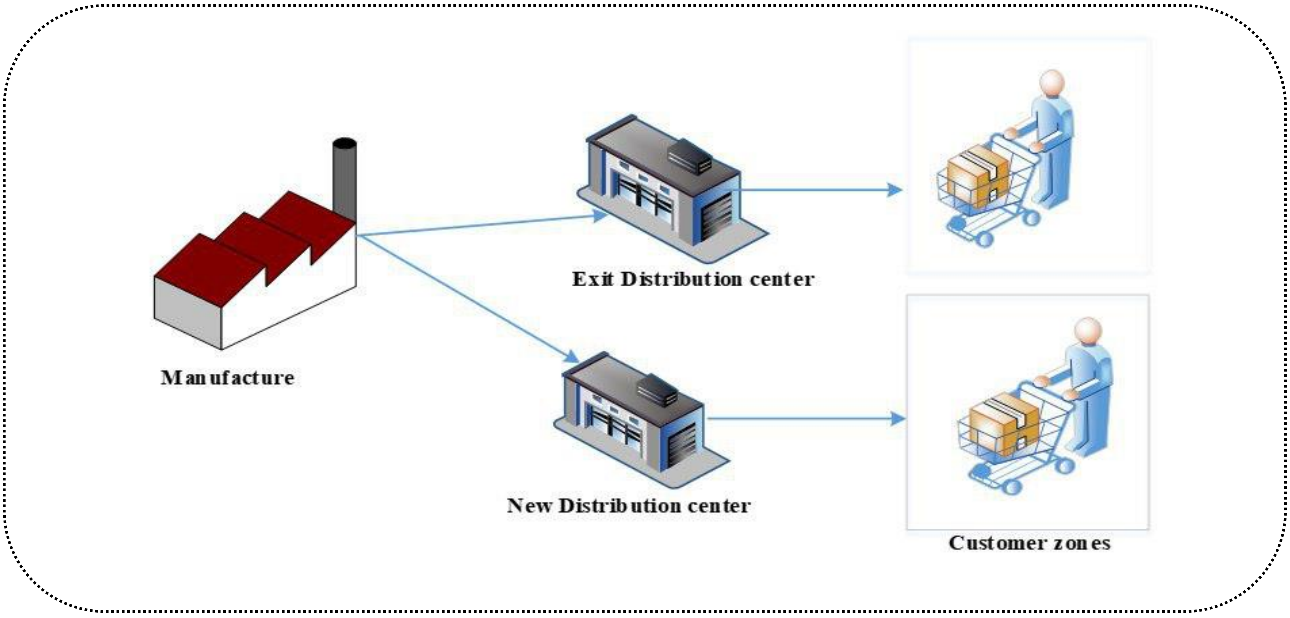

Fig. 1. Warehouse network

Sets and indexes and the parameters and decision variables are introduced to present the mathematical model. The optimization model for the problem of redesigning the supply chain network is formulated.

Sets

$w \in w^e \bigcup w^n = \{1,2,...,w\}$	The set of all active warehouses and candidate location for establishing
$w^e = \{1,2,...,w^e\}$	The set of active warehouses at the beginning of the period
$w^n = \{1,2,...,w^n\}$	The set of candidate location for establishing of new warehouses
$k_w = \{1,2,...,k_w\}$	The set of warehouse capacity level
$C = \{1,2,...,c\}$	The set of customers (demand zones)
$P = \{1,2,...,p\}$	The set of final products
$T = \{1,2,...,t\}$	The set of periods
$T_L \in T = \{1,2,...,t_l\}$	The set of strategic periods in which new facilities can be established, existing facilities can be closed, or facility capacity could be increased.
$U = \{1,2,...,u\}$	Reinforcement level

Parameters

fc_{tj}^{u} : The fixed cost of establishing a new facility at location $j \in w^{n}$ at level $u \in U$ in period $t \in T_l$

sc_{tj} : The fixed cost of closing the existing facility at location $j \in w^{e}$ in period $t \in T_l$

ic_{tjk} : The cost of installing capacity at level $k \in k_w$ at location $j \in w$ in period $t \in T_l$

oc_{tj} : The cost of facility operation at location $j \in w$ in period $t \in T$

sf_{tj} : Saving cost on closing facility $j \in w$ in period $t \in T_l$

fo_{wtp} : The fixed cost of shipping product $p \in P$ from supplier to warehouse $w \in w^{e} \cup w^{n}$ in period $t \in T$

g_{wtp} : The fixed cost of ordering product $p \in P$ from the supplier for warehouse $w \in w^{e} \cup w^{n}$ in period $t \in T$

h_{wp}^{t} : Holding cost a unit of product $p \in P$ in warehouse $w \in w^{e} \cup w^{n}$ in period $t \in T$

a_{twp} : The variable cost of shipping product $p \in P$ from supplier to warehouse $w \in w^{e} \cup w^{n}$ in period $t \in T$

Q_{j}^{e} : The capacity of existing facilities $j \in w^{e}$ at the beginning of the period $t \in T$

l_{wt}^{p} : The delivery time of product $p \in P$ in warehouse $w \in w^{e} \cup w^{n}$ in period $t \in T$

$\sigma_{cp}^{2,t}$: The variation of customer demand $c \in C$ for product $p \in P$ in period $t \in T$

μ_{cpt} : Average customer demand $c \in C$ for product $p \in P$ in period $t \in T$ with scenario

β : The weight of inventory costs

θ : The weight of transportation costs

ρ_{tp} : The percentage of backorder shortage for product $p \in P$ in period $t \in T$

π_{0} : The fixed cost of each backorder shortage unit

π_{tp} : The cost of backorder shortage per unit of product $p \in P$ in period $t \in T$

ω_{0} : The fixed cost of lost sale shortage per unit of product

ω_{tp} : The fixed cost of lost sale per unit of product $p \in P$ in period $t \in T$

ξ_{0} : The average cost of shortage independent of time

ξ_{tp} : Average shortage cost in period $t \in T$

Variables

$Y_{t,j}^{nu}$ equal to 1 if new facility at location $j \in w^n$ is established at the beginning of period $t \in T_L$ at strength level $u \in U$; otherwise, 0.

$Y_{t,j}^{e}$ equal to 1 if the existing facility at location $j \in w^n$ was closing at the beginning of period

$t \in T_L$ otherwise 0.

Z_{wcrtp} equal to 1 if customer $c \in C$ is assigned to warehouse $w \in w^e \cup w^n$ in period $t \in T_L$ to purchase the product; otherwise 0.

U_{tjk} equal to 1 if capacity level $k \in k_w$ is allocated to the facility in location $j \in w$ in period $t \in T_L$; otherwise 0.

A_{tj}^{n} facility's capacity level in location $j \in w^e$ in period $t \in T$

A_{tj}^{e} facility's capacity level in location $j \in w^n$ in period $t \in T$

Mathematical model

$$Min = \sum_{t \in T_l} \sum_{j \in w^n} \sum_{u} fc_{tj}^{u} y_{tj}^{nu} + \sum_{t \in T_L} \sum_{j \in w^e} sc_{tj} Y_{tj}^{e} + \sum_{t \in T_L} \sum_{j \in w} \sum_{k \in k_j} ic_{tjk} U_{tjk} - \sum_{j \in w^e} sf_{tj} (\sum_{t \in T_L} Y_{tj}^{e}) +$$
$$+ \left(\begin{array}{l} \sum_{w} \sum_{p} \sum_{t} \sum_{c} \left(\sqrt{2(\theta fo_{wtp} + \beta g_{wtp}) \mu_{ctps} Z_{wctps} \theta h_{wtp}} - \frac{\theta h_{wtp} (\xi_0 \mu_{ctp} Z_{wctp})^2}{\theta h_{wtp} + \xi_{tp}} \sqrt{\frac{\xi_{tp}}{\theta h_{wtp} + \xi_{tp}}} \right) + \\ \theta \sum_{w} \sum_{p} \sum_{t} \sum_{c} \left(\frac{\theta h_{wtp} \xi_0 \mu_{ctps} Z_{wctps}}{h_{wtp} + \xi_{tp}} \right) \end{array} \right)$$
$$+ \left(\theta \sum_{w} \sum_{c} \sum_{t} \sum_{p} z_{wctp} \mu_{ctp} a_{wtp} + \beta \sum_{w} \sum_{p} \sum_{t} h_{wtp} z_{\alpha} \sqrt{L_{wt}^{p} (\sum_{c} \delta_{cp}^{t} Z_{wctp}) \sum_{w} \sum_{c} \sum_{t} \sum_{p} z_{wctp} \mu_{ctp} a_{wtp}} + \sum_{r} \sum_{h} \sum_{t} \sum_{w} rh_{twr} cr_{tw} \right) \qquad (1)$$

$$\sum_{t \in T_L} \sum_{u} Y_{tj}^{un} \le 1 \qquad \forall j \in w^n \qquad (2)$$

$$\sum_{u} Y_{tj}^{un} \le \sum_{k \in k_j} U_{tjk} \le \sum_{\tau=1, \tau \in T_L}^{t} \sum_{u} Y_{\tau j}^{un} \qquad \forall t \in T, j \in w^n \qquad (3)$$

$$\sum_{k \in k_j} U_{tjk} \le 1 - \sum_{\tau=1, \tau \in T_L}^{|T|} Y_{\tau j}^{e} \qquad \forall t \in T_L, j \in w^e \qquad (4)$$

$$A_{tj}^{n} = A_{(t-1)j}^{n} + \sum_{k \in k_j} Q_{jk} u_{tjk} \qquad \forall j \in w^n, t \in T \qquad (5)$$

$$A_{tj}^{e} = \sum_{k \in k_j} Q_{jk} u_{tjk} + A_{(t-1)j}^{e} \qquad \forall j \in w^e, t \in T \qquad (6)$$

$$\sum_{j \in w} z_{jcpt} = 1 \qquad \forall\, p \in P, c \in C, t \in T \tag{7}$$

$$z_{jcpt} \le \sum_{\tau=1, \tau \in T_L}^{t} \sum_{h} Y_{\tau jh}^{n} \qquad \forall j \in w^{n}, p \in P, t \in T \tag{8}$$

$$Z_{jcpt} \le 1 - \sum_{\tau=1, \tau \in T_L}^{|T|} \sum_{h} Y_{\tau j}^{e} \qquad \forall j \in w^{e}, p \in P, t \in T \tag{9}$$

$$\sum_{c} \sum_{p} z_{jcpt} \mu_{ctp} \le A_{tj}^{n} \qquad \forall j \in w^{n}, p \in P, t \in T \tag{10}$$

$$\sum_{c} \sum_{p} z_{jcpt} \mu_{ctp} \le A_{tj}^{e} \qquad \forall j \in w^{n}, p \in P, t \in T \tag{11}$$

$$Y_{t,j}^{n,u}, Y_{t,j}^{e}, Z_{jcpt}, U_{tjk} \in (o,1) \qquad \forall j \in w, p \in P, t \in T, u \in U \tag{12}$$

$$A_{tj}^{e}, A_{tj}^{n} \ge 0 \qquad \forall j \in w, p \in P, t \in T \tag{13}$$

Equation (1) indicates the objective function of the model, which includes minimizing fixed costs of ordering, transportation, the variable cost of ordering, holding costs in warehouses, shortage costs, fixed cost of establishing new facilities, fixed cost of closing existing facilities, cost of increasing facility capacity, operating cost of active facilities in each period. Equation (2) defines that if a facility became active in a period, it could not be closed until that period. Equation (3) is related to increasing the capacity of new facilities in each period and creating capacity in new facilities established. According to this constraint, the facility capacity can be increased if the facility was active during that period. Moreover, if a facility was activated in any period, one of the capacity levels must be allocated. Equation (4) indicates that existing facilities' capacity can be increased if that facility remained active until the end of that period. Equations (5) and (6) are related to capacity programming for existing facilities and active established facilities in each period. Facility capacity in each period is equal to the amount of capacity added in the intended period and the amount of capacity transferred from the previous period. Equation (7) defines that each demand zone in each period for each product must be allocated precisely to one of the active facilities. Equations (8) and (9) indicate that a customer can be assigned to a facility in a period if the facility was active during that period. Equations (10) and (11) are related to facility capacity constraints in each period. Equations (12) and (13) indicate the constraints of signs associated with decision variables.

IV. APPLIED POSSIBILISTIC PROGRAMMING

Fuzzy mathematical programming is classified into two categories: (1) flexible programming and (2) possibilistic programming; flexible programming approaches are modeled by fuzzy sets based on preference, and in cases where there are objective values for objective function and constraints, is used. Possibilistic programming approaches are used in cases related to data loss or lack of knowledge about the problem's exact amount of input parameters. It should be noted that uncertain parameters can be modeled by appropriate possibilistic functions such as triangular or trapezoidal, based on insufficient available data/knowledge and decision-makers (Torabi et al., 2016). Since the dynamic supply chain's network design parameters are ambiguous and uncertain, the proposed model belongs to the possibilistic programming category. Due to the dynamic nature of businesses, the instability of trends, and the lack of sufficient data on model parameters, it is very difficult or impossible in some cases to determine the exact amount of parameters (Torabi et al., 2016, Journal et al. 2007, Gitinavard & Akbarpour Shirazi, 2018, Gitinavard & Zarandi, 2016). Accordingly, this matter's importance has led many researchers to focus on developing practical possibilistic programming approaches to overcome the problem. In this study, to deal with uncertainties in cost parameters and the level of customers' demand, Jimenez's possibilistic programming method has been used [24] due to the high efficiency. Jimenez et al. proposed a fuzzy method that was programmed based on the expected value and the expected interval. Due to computation efficiency and simplicity, a triangular fuzzy distribution method has been used to deal with the model's uncertain parameters. Suppose $\tilde{\zeta}$ is a triangular fuzzy number, the membership function of this fuzzy number $\mu_{\tilde{\zeta}}(x)$ is defined as follows:

$$\mu_{\tilde{\zeta}}(x)=\begin{cases} f_{\zeta}(x)=\dfrac{x-\zeta^{p}}{\zeta^{m}-\zeta^{p}} & if\ \zeta^{p}\le x\le \zeta^{m} \\ 1 & ifx=\zeta^{m} \\ g_{\zeta}(x)=\dfrac{\zeta^{o}-x}{\zeta^{o}-\zeta^{m}} & if\ \zeta^{m}\le x\le \zeta^{0} \\ 0 & ifx\le \zeta^{p} orx\ge \zeta^{0} \end{cases} \tag{14}$$

The expected interval (EI) and the expected value (EV) of the triangular fuzzy number are calculated based on equations (15) and (16):

$$EI(\tilde{\zeta})=\left[E_1^{\zeta}\ E_2^{\zeta}\right]=\left[\int_0^1 f_{\zeta}^{-1}(x)dx\ \ \int_o^1 g_{\zeta}^{-1}(x)dx\right]=\left[\frac{1}{2}(\zeta^{p}+\zeta^{m})\ \ \frac{1}{2}(\zeta^{o}+\zeta^{m})\right] \tag{15}$$

$$EV(\tilde{\zeta})=\frac{E_1^{\zeta}+E_1^{\zeta}}{2}=\frac{\zeta^{p}+2\zeta^{m}+\zeta^{o}}{4} \tag{16}$$

Moreover, for a pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree to which $\tilde{a}$ is greater than $\tilde{b}$ is calculated based on equation (16):

$$\mu_M(x)=(\tilde{a},\tilde{b})=\begin{cases} 1 & ifE_1^{a}>E_2^{b} \\ \dfrac{E_2^{a}-E_1^{b}}{E_2^{a}-E_1^{b}-(E_1^{a}-E_1^{a})} & ifE_1^{a}\in\left[E_1^{a}-E_2^{b}\ E_2^{a}-E_1^{b}\right] \\ 0 & ifE_2^{a}<E_1^{b} \end{cases} \tag{17}$$

$\mu_M\left(\tilde{a},\tilde{b}\right)\ge\alpha$ shows in α degree, $\tilde{a}$ is greater than/equal to $\tilde{b}$, and it is defined as $\tilde{a}\ge_{\alpha}\tilde{b}$. Besides, for the pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, where $\tilde{a}$ is equal to $\tilde{b}$ it can be said: $\tilde{a}\ge_{\frac{\alpha}{2}}\tilde{b}, \tilde{a}\le_{\frac{\alpha}{2}}\tilde{b}$

By defining the following fuzzy mathematical programming model, all the parameters are considered fuzzy:

$$\begin{aligned} &\min z=\tilde{c}^{t}x \\ &\tilde{a}_i x\ge \tilde{b}_i x \quad i=1,\dots,1 \\ &\tilde{a}_i x=\tilde{b}_i x \quad i=l+1,\dots,m \\ &x\ge o \end{aligned} \tag{18}$$

According to Jimenez's method, it can be said:

$$\frac{E_2^{a_i x} - E_2^{b_i}}{E_2^{a_i x} - E_1^{b_i} - (E_1^{a_i x} - E_2^{b_i})} \geq \alpha \quad i = 1,\ldots,l \tag{19}$$

$$\frac{\alpha}{2} \leq \frac{E_2^{a_i x} - E_2^{b_i}}{E_2^{a_i x} - E_1^{b_i} - (E_1^{a_i x} - E_2^{b_i})} \leq 1 - \frac{\alpha}{2} \quad i = l+1,\ldots,m \tag{20}$$

According to equations (18), (19), and (20), it can be said:

$$[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]\text{x} \geq (1-\alpha)E_1^{b_i} + \alpha E_2^{b_i} \qquad i = 1,\ldots,l \tag{21}$$

$$[(1-\frac{\alpha}{2})E_1^{a_i} + \frac{\alpha}{2}E_2^{a_i}]\text{x} \leq (1-\frac{\alpha}{2})E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i} \quad i = 1+l,\ldots,m \tag{22}$$

$$[(1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}]\text{x} \geq (1-\frac{\alpha}{2})E_1^{b_i} + \frac{\alpha}{2}E_2^{b_i} \quad i = l+1,\ldots,m \tag{23}$$

$$\begin{aligned}
&\min EV(\tilde{c})x \\
&[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]\text{x} \geq (1-\alpha)E_1^{b_i} + \alpha E_2^{b_i} \qquad i = 1,\ldots,l \\
&[(1-\frac{\alpha}{2})E_1^{a_i} + \frac{\alpha}{2}E_2^{a_i}]\text{x} \leq (1-\frac{\alpha}{2})E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i} \quad i = 1+1,\ldots,\text{m} \\
&[(1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}]\text{x} \geq (1-\frac{\alpha}{2})E_1^{b_i} + \frac{\alpha}{2}E_2^{b_i} \quad i = 1+1,\ldots,\text{m}
\end{aligned} \tag{24}$$

Now, according to the Jimenez method, it is possible to convert the multi-period multi-product possibilistic model provided for redesigning the supply chain network, as follows:

$$\begin{aligned}
Min = &\sum_{t \in T_l}\sum_{j \in w^n}\sum_{u}(\frac{FC_{tj}^{up} + 2FC_{tj}^{um} + FC_{tj}^{uo}}{4})y_{tj}^{nu} + \sum_{t \in T_L}\sum_{j \in w^e}(\frac{SC_{tj}^{p} + 2SC_{tj}^{m} + SC_{tj}^{o}}{4})Y_{tj}^{e} \\
&-\sum_{j \in w^e}(\frac{SC_{tj}^{p} + 2SC_{tj}^{m} + SC_{tj}^{o}}{4})(\sum_{t \in T_L} Y_{tj}^{e}) + +\sum_{t \in T_L}\sum_{j \in w}\sum_{k \in k_j}(\frac{IC_{tjk}^{p} + 2IC_{tjk}^{m} + IC_{tjk}^{o}}{4})U_{tjk} \\
&+\left(\begin{aligned}\sum_{w}\sum_{p}\sum_{t}&\sum_{c}(\sqrt{2(\theta fo_{wtp} + \beta g_{wtp})((\mu_{ctps}^{o} + 2\mu_{ctp}^{m} + \mu_{ctp}^{p})/4)Z_{wctp}\theta h_{wtp} - \frac{\theta h_{wtp}(\xi_0((\mu_{ctp}^{o} + 2\mu_{ctps}^{m} + \mu_{ctp}^{p})/4)Z_{wctp})^2}{\theta h_{wtp} + \xi_{tp}}}\sqrt{\frac{\xi_{tp}}{\theta h_{wtp} + \xi_{tp}}}) \\
&+\theta\sum_{w}\sum_{p}\sum_{t}\sum_{c}(\frac{\theta h_{wtp}\xi_0((\mu_{ctp}^{o} + 2\mu_{ctp}^{m} + \mu_{ctp}^{p})/4)Z_{wctp}}{h_{wtp} + \xi_{tp}})\end{aligned}\right) \\
&+\left(\begin{aligned}&\theta\sum_{w}\sum_{c}\sum_{t}\sum_{p} z_{wctp}((\mu_{ctp}^{o} + 2\mu_{ctp}^{m} + \mu_{ctp}^{p})/4)a_{wtp} + \\
&\beta\sum_{w}\sum_{p}\sum_{t} h_{wtp} z_{\alpha}\sqrt{L_{wt}^{p}(\sum_{c}\delta_{cp}^{t} Z_{wctp})\sum_{w}\sum_{c}\sum_{t}\sum_{p} z_{wctp}((\mu_{ctp}^{o} + 2\mu_{ctp}^{m} + \mu_{ctp}^{p})/4)a_{wtp}}\end{aligned}\right)
\end{aligned} \tag{25}$$

$$\sum_c \sum_p z_{jcpt} \left((1-\alpha/2)(\frac{\mu^p_{ctp}+\mu^m_{ctp}}{2}) + (\alpha/2)(\frac{\mu^o_{ctp}+\mu^m_{ctp}}{2}) \right) \le A^n_{tj} \qquad \forall j \in w^n, p \in P, t \in T \tag{26}$$

$$\sum_c \sum_p z_{jcpt} \left((1-\alpha/2)(\frac{\mu^p_{ctp}+\mu^m_{ctp}}{2}) + (\alpha/2)(\frac{\mu^o_{ctp}+\mu^m_{ctp}}{2}) \right) \le A^e_{tj} \qquad \forall j \in w^e, p \in P, t \in T \tag{27}$$

Other Constraints (13), (12), (9), (2) (28)

V. NUMERICAL PROBLEM-SOLVING RESULTS

In this section, the proposed multi-period mathematical model's efficiency to redesign warehouse networks has been examined. Required data for this problem's parameter is generated using the uniform distribution. Table I shows the critical parameters needed to solve the problem.

Table I: The values of numerical example parameters

fc^u_{tj}	$uniform(100,120)$	Q^e_j	$uniform(200,300)$
sc_{tj}	$uniform(90,100)$	μ_{cpt}	$uniform(120,130)$
ic_{tjk}	$uniform(20,30)$	l^p_{wt}	$uniform(5,12)$
oc_{tj}	$uniform(10,15)$	$\sigma^{2,t}_{cp}$	$uniform(10,30)$
sf_{tj}	$uniform(10,15)$	β	$uniform(.01,.3)$
fo_{wtp}	$uniform(8,12)$	θ, B	$uniform(.01,.3)$
g_{wtp}	$uniform(5,8)$	ξ_0	$uniform(.3,.6)$
h^t_{wp}	$uniform(6,9)$	ξ_{tp}	$uniform(.2,.5)$
a_{twp}	$uniform(10,15)$		

The numerical example provided in different periods, Figs. (2) and (3) show how to establish new facilities and close non-optimal existing facilities. According to fig. (2) and fig. (3), it can be seen that a new facility with (customer) number 3 will be established in the first period, a new facility with number 5 will be established in the third period, and the existing facility will be close with number 1 in the third period. Moreover, Table II shows the new facility's programming capacity, in which numbers 1 and 0 indicate establishing of the new facility and the closing of the existing one, respectively. New facilities number 3 and 5 have been established at capacity levels 2 and 1, respectively.

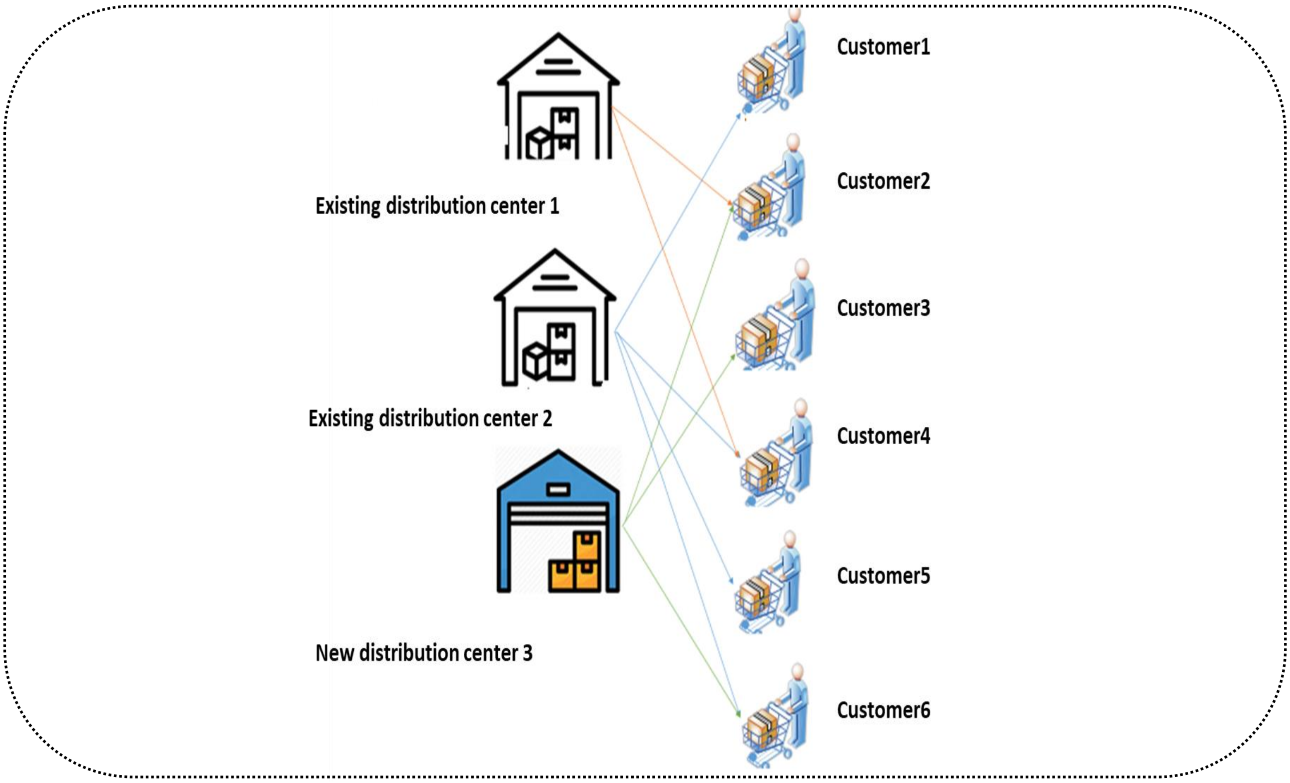

Fig. 2. Redesigned chain network in the first and second periods

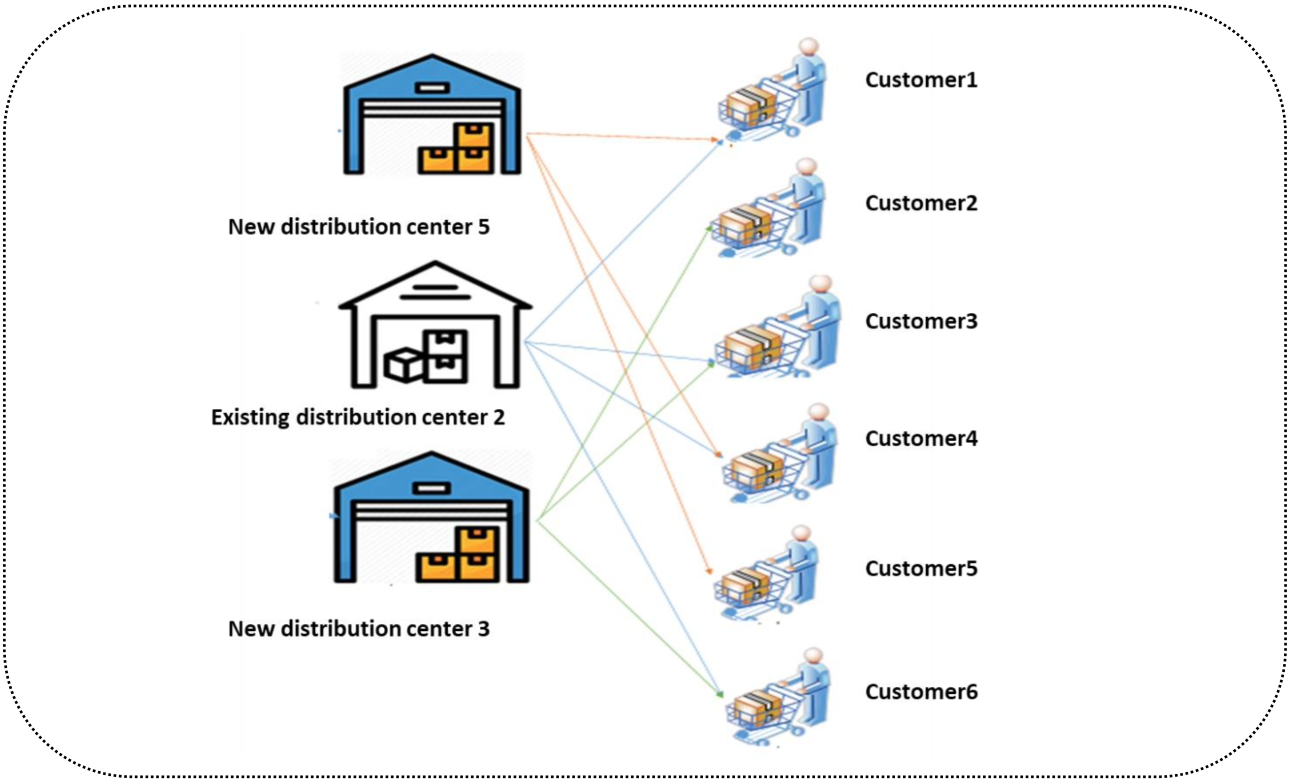

Fig.3.Redesigned chain network in the third and fourth periods

Table II. The new facility construction program

Distribution Center / *Strength Level*	*Strategic Periods*	
	1	*3*
3/1	1	0
5/1	0	1

- ***Investigating total changes in supply chain costs***

Figure (4) shows the changes in the objective function (including the cost of constructing the facility, cost of closing the active facility, cost of the increasing facility, shortage cost, and transportation cost) compared to decision-makers' level of satisfaction. It can be seen that as the level of decision-makers satisfaction increases, the total cost of the supply chain network increases as well; the rate of these changes from 0.1 to 0.5 is very significant, while from 0.5 to 0.9 changes in the objective function shown small fluctuations. At lower decision-maker satisfaction levels (0.1 and 0.3), the model attempts to respond to different regions' demand by increasing active facilities' capacity in each period. At a level of 0.5 satisfaction, the number of established facilities has increased, which causes a significant jump in costs. Moreover, at the satisfaction levels 0.7 and 0.9, the model attempts to respond to customer demand by increasing the facility's capacity without establishing any additional facility compared to those active facilities at 0.5 satisfaction level. Fig. (5) and fig. (6) show the cost function changes compared to the weight changes in inventory and transportation costs. As the share of inventory and transportation costs increases, the objective function's total value becomes worse. Fig. (7) shows the share of inventory costs, variable transportation costs, fixed costs of establishing new facilities/closing non-optimal facilities, the cost of increasing the facility's capacity, and savings on closing the facility at the satisfaction level of 0.5.

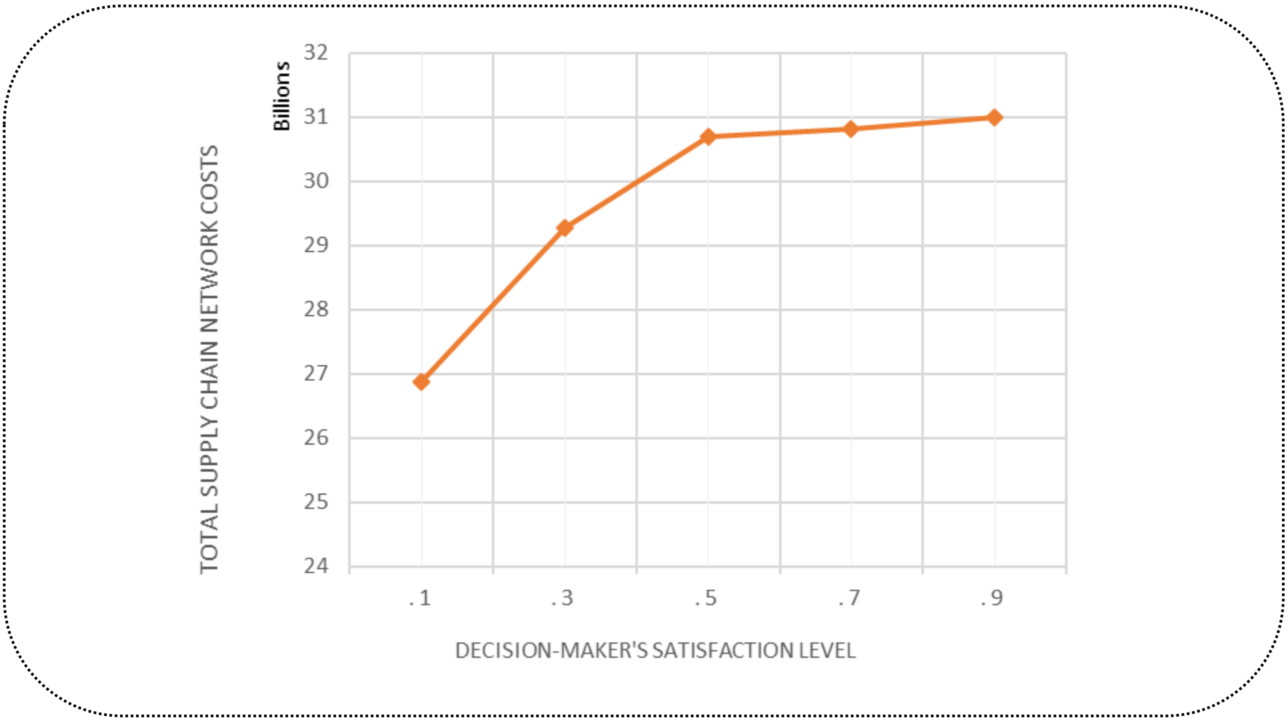

Fig. 4. Changes in costs of supply chain network/decision-makers satisfaction level

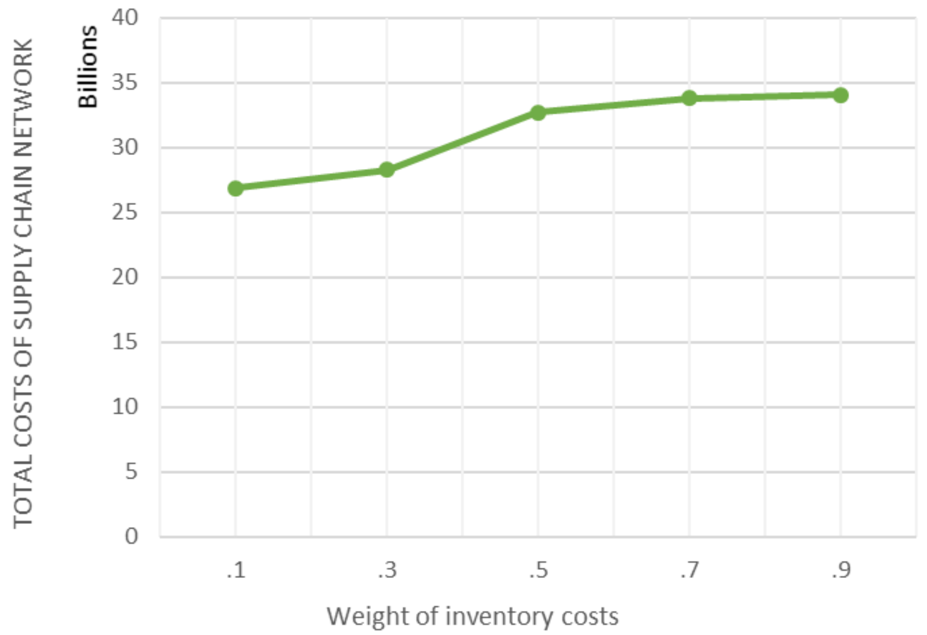

Fig. 5. Changes in the supply chain costs/weight of inventory costs

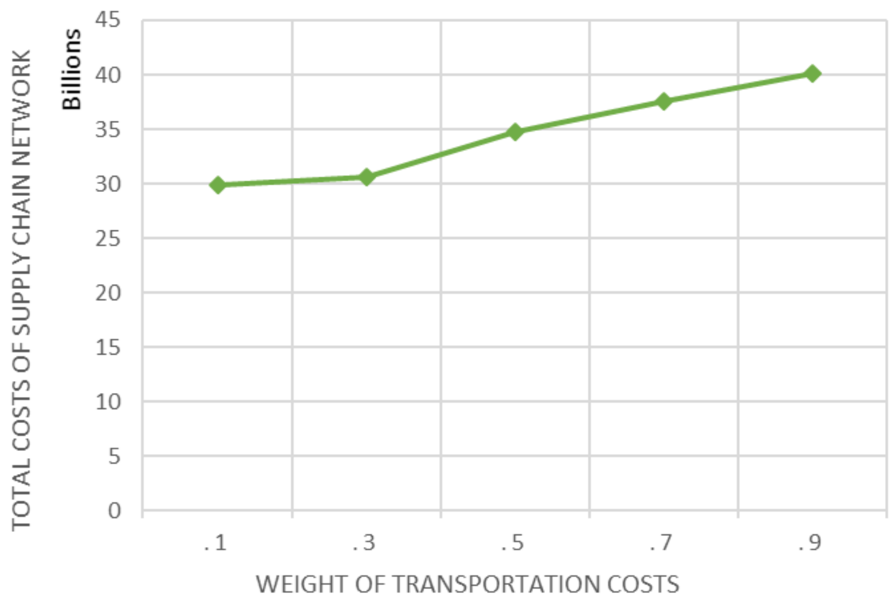

Fig. 6. Changes in the supply chain costs/weight of transportation costs

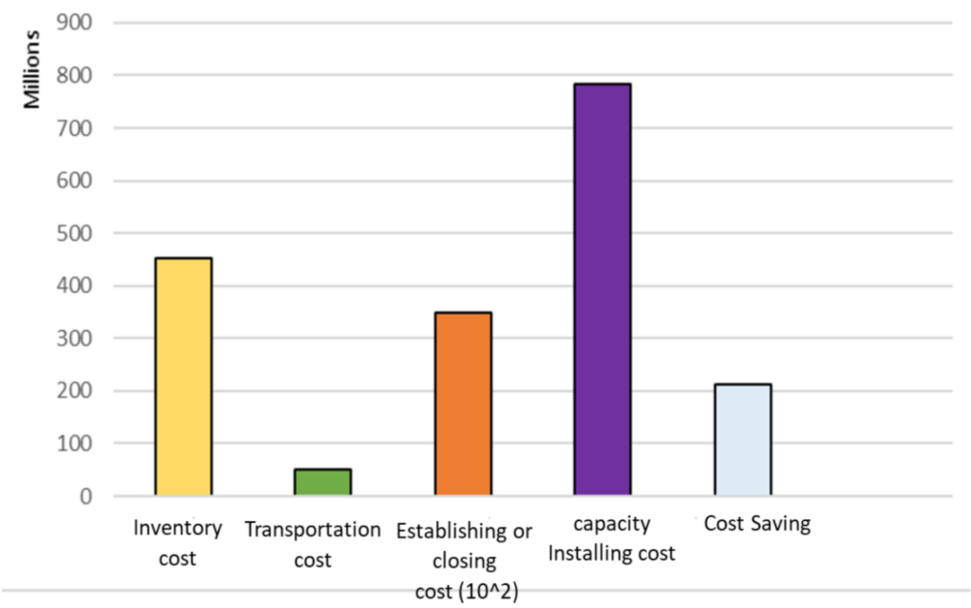

Fig.7. The different costs of the supply chain network

- ***Investigating the effect of supply chain redesign on shortage quantity***

Table III shows the ratio of each product shortage to the total demand for the product in all demand zones in each period (total demand/shortage). It should be noted that redesigning the supply chain has a significant impact on reducing the shortage rate. Because redesigning the supply chain has established new distribution centers at candidate points, it increased the capacity of active facilities in some areas.

Table III. The comparison of product shortage ratio in the existing and redesigned supply chain (by %)

Period / *Product*		*1st period*	*2nd period*	*3rd period*	*4th period*
Existing supply chain	1st product	28/39	25/76	26/17	26/14
	2nd product	28/63	24/43	26/56	25/34
	3rd product	27/56	27/62	27/83	28/02
	4th product	25/71	26/19	25/81	25/68
Redesigned supply chain	1st product	19/51	17/77	20/93	16/55
	2nd product	19/68	16/85	21/25	16/26
	3rd product	20/21	18/41	22/27	17/75
	4th product	18/85	29/12	26/43	23/82

VI. CONCLUSION AND RECOMMENDATIONS

In this paper, to redesign the warehouse network, a multi-period multi-product mathematical model has been presented considering partial backorder shortage. Based on the conducted review, no studies have been conducted on the redesign of multi-period, multi-product supply chain networks, considering decisions related to warehouse redesign and partial backorder shortage simultaneously. Decisions related to supply chain network redesign include: locating new facilities at different strength levels, closing existing non-optimal facilities, expanding facilities operating in strategic periods, and allocating customers to warehouses. Besides, decisions related to the amount of optimal economic order quantity for each of the active warehouses in each period, considering backorder shortage, have been investigated simultaneously. The uncertainty in cost and demand parameters has been considered due to the uncertain nature of the proposed model parameters. Jimenez's possibilistic programming method has been used to deal with the stochastic model.

The results demonstrate that (1) although redesigning a supply chain imposes the fixed cost of establishing new facilities and closing down the non-optimal facilities, it can significantly reduce the inventory and transportation costs. (2) Using possible programming, we can control the uncertainties in the supply chain by using decision-maker satisfaction levels (α). The higher the α level, the higher the cost of the supply chain. Therefore, managers and decision-makers have to decide whether demands should be met by expanding the existing facilities' capacity or constructing new facilities subject to supply chain budget constraints.

This paper's recommendations for future research include solving the proposed mathematical model using metaheuristic algorithms and problem modeling by considering decisions related to routing and using other uncertainty methods to deal with existing uncertainties.

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