



Staff Assignment in Emergency Department Considering Service Time

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Abstract –This paper designs an optimization model for the emergency department of a hospital, considering related costs, nursing staff satisfaction, and waiting time for several diseases concerning the number of staff in each shift. This study's primary purpose is to minimize the related costs, maximize nursing staff satisfaction, and allocate nursing staff to working shifts in the emergency department. In the first stage, a simulation model is constructed based on the emergency department's status with ARENA 14 software. Then, the model is investigated under three different scenarios. In the second stage, mixed-integer programming is proposed to minimize the costs, nursing staff satisfaction and optimally allocate nurses to various shifts. Furthermore, the generalized center method is used to solve the model by converting the multi-objective model to a single-objective one. In Tabriz, Iran, Imam Reza hospital is considered our case study investigated by simulation and MIP models. Finally, the results of simulation and mathematical models demonstrate that six new nurses should be added to the emergency department.

Keywords–Staff assignment, Service time, Simulation, Mixed-integer programming, Generalized center method.

I. INTRODUCTION

Healthcare systems are of great significance among service systems (Cochran and Bharti, 2006). U.S budget for health ministry was approximately 1 trillion dollars that were 16.9% and 17.1% of its Gross domestic product in 2013 and 2014, respectively (W. H. Organization, 2013). It was also estimated that health service costs in the USA would increase to 4.4 trillion dollars in 2018 (Koizumi et al., 2005). The significance of healthcare systems is increasing in current societies. Nowadays, most governments aim is to allocate a noticeable amount of the annual budget to healthcare systems. The share of health and treatment costs from GDP is lower in developing countries, and it illustrates that the importance of human health in society increases with the level of development in countries. The share of health and treatment from GDP for thirteen developing countries in the Middle East and Africa was compared and analyzed in 2014 (White, 2015). The results showed that, on average, 68.5% of these countries' GDP is allocated to health care costs. In recent years, healthcare costs in Iran have been rising as one of the developing countries. According to World Health Organization (WHO), every Iranian citizen's per capita health costs in 2001 amounted to about \$ 422, approximately 3.6% of GDP (Ghaffari et al., 2008). According to WHO, a large portion of healthcare costs is paid by

the government. In 2014, the government spent about 41.20% of all spending on health care. In 2000, this figure was about 37% (9.6 percent of total government spending), and in 2007 it was about 46.8% (11.5 percent of total government spending). A comparison of these figures showed an increase in government expenditure in the healthcare industry.

However, given the government's limited resources and the increasing population, the health managers will soon face a shortage of resources. Therefore, the use of appropriate tools for the optimal utilization of resources seems necessary. Hospitals are significant healthcare sectors that account for more than 36% of government spending (Trybou et al., 2015). However, health systems suffer from other issues such as high congestion and costs. Carter et al. (1992) stated that "One of the main causes of inefficiency in the healthcare system is the residents, people working in the health system, are well aware of the work environment around them, but are unaware of the events occurring in the neighboring units. The physicians and nurses of the emergency unit or the operating room often do not know the employees' problems of the hospital's general wards or are reluctant to resolve the problems. Hospital staff does not value long-term benefits. Sometimes, thoughts such as " my job are more important "or" my problems are more than yours."

This issue is precisely where operation research experts can play an important role. The emergency department is one of the essential departments of the hospital. The high number and acute condition of referrals to the emergency department have made this part of the hospital one of the most crucial hospitals. Specialist staffs are the leading human resource of service in this department. Human resource specialists are the primary source of production and service in this department. Under no circumstances should this department be faced with a shortage of human resources, so optimizing the number of human resources in this department improves hospital productivity effectively. One of the most important indicators used in assessing emergency departments is the length of time for patients to receive diagnostic and therapeutic services (Jahantigh et al., 2017). Since one of this department's main requirements is to provide appropriate service and timely availability of qualified staff, especially the number of nurses, optimizing the number of staff in this department is of great importance. In literature, several methods such as statistics, work measurement, queuing models, and linear programming are used to optimize the hospital's number of staff. This study presents a simulation framework to evaluate three proposed emergency department scenarios with ARENA 14 software. A queuing network is considered for patient flow in the emergency department that plays a critical role in the hospital. After comparing scenarios with the emergency department's current status, we are proposed bi-objective MIP models to minimize costs, nursing staff satisfaction, and allocating staff optimally. Then the generalized center method is utilized to convert the multi-objective model into a single-objective model.

The rest of the paper is organized as follows; In section II, we review the literature. Next, a simulation model for the emergency department is developed, and a case study is done. In section IV, a mathematical model is presented for the problem of nursing staff allocation. Consequently, the conclusion part is stated in Section V.

II. LITERATURE

In this section, we have reviewed the literature related to the application of simulation, mathematical modeling, and queuing theory in healthcare systems, especially for the emergency department. Several surveys have been done for healthcare problems, especially staff scheduling and emergency. Gunal and Pidd (2010) reviewed discrete-event simulation models for the healthcare sector. (Paul et al. 2010) made a systematic review of emergency department simulation for the period of 1970 – 2006. Mielczarek and Uzialko-Mydlikowska (2012) presented a survey of a computer simulation model in the healthcare sector. Lim et al. (2012) implemented the wait time reduction strategies by mathematical modeling for queuing. Fanti and Ukovich (2014) presented a review of relevant approaches in discrete-event system models in healthcare sector management. Gul and Guneri (2015) surveyed some applications to model emergency department simulation during normal and disaster conditions. Saghafian et al. (2015) reviewed operation research methods used to optimize emergency department patient flow. Salleh et al. (2017) reviewed simulation modeling applications in the health sector. In this section, first, the studies on the emergency department simulation are

reviewed by us. Next, the studies on classification and planning in the emergency department are considered. Finally, studies related to queuing systems application in emergency department optimization are reviewed. A summary of studies related to healthcare studies in medical centers such as emergency departments in terms of utilized methods and focus is given in Table I.

A. Emergency department simulation

Discrete-event simulation models can analyze the patient flow in medical clinics and hospitals, related policies, and their influence on the healthcare system. Complex patient flows are expected for most emergency departments where patients' arrival is not estimable, and demand type is more than what was expected before (Fanoodi et al., 2019). Although the patient's arrival is unpredictable, the treatment arrangement can be controlled by clinical staff. Therefore, by changing the patient's flow and path, the patient's wait time may be reduced, and the rate of exploitation of the staff increased. Garcia et al. (1995) analyzed the emergency department's queuing system to minimize the wait time of patients. Emergency patients are usually prioritized according to the patient's degree of illness (the level of disease), and therefore, patients with less severity usually have more wait time than other patients to receive the service. They simulated the patient flow and found that a fast track lane in the emergency department decreases patients' time with low severity.

Table I. Summary of healthcare problems

Reference	Year	Method	Objective
Begen et al.	2012	Stochastic programming	Minimization of waiting time, idle time, and overtime
Balasubramanian et al.	2014	Markov Decision Process	Maximization of timely access to the resources
Anderson et al.	2015	Stochastic programming	Minimization of waiting time, idle time, and overtime
Choi and Banerjee	2016	Markov Decision Process	Minimization of timing costs such as delay and idle costs
Parizi and Ghate	2016	Markov Decision Process	Maximization of profit
Wiesche et al.	2017	Mixed-integer linear programming	Minimization of appointment slots
Deceuninck et al.	2018	Stochastic programming	Minimization of waiting time and idle time
Luo et al.	2018	Discrete event simulation	Effect of reservation policy, examination quality, waiting time
Ang et al.	2018	A mixed-integer sequential goal programming model	Nurse scheduling considering the level of performance
Jiang et al.	2019	Stochastic programming	Analysis of unpunctual arrival
Baia Medeiros et al.	2019	Discrete event simulation	Capacity planning
Swan et al.	2019	Discrete event simulation	Analysis of the emergency systems in terms of arrival rates and severity of patients
Schäfer et al.	2019	Decision support model	Patient-based allocation
Kang et al.	2019	Stochastic mixed-integer programming	Outdoor patients

Continue Table I. Summary of healthcare problems

Reference	Year	Method	Objective
Pourmadadkar et al.	2019	Decision support model	Service risk assessment in health centers
Behmanesh et al.	2020	Ant Colony Optimization	Scheduling
Hashemkhani Zolfani et al.	2020	Decision support model	Location selection of medical center
Khayat Rasoli et al.	2020	Markov Decision Process	Blood consumption policy
Delshad and Torkayesh	2020	Discrete event simulation	Service providers

Kirtland et al. (1995) examined eleven variables to optimize patient flow in the emergency department. Their findings showed that three variables could almost decrease the wait time by 38 minutes. They also developed a discrete-event simulation model for the emergency department in east Ontario for patients with a low disease level. Restricting access to primary care has led to increased use of emergency departments throughout the United States. A similar study has been done for evaluating the effects of the treatment process for patients and their wait time in the emergency department by (Mahapatra et al., 2003). Takakuwa and Shiozaki (2004) suggested a new simulation method for scheduling emergency operations to minimize patients' wait time. Sinreich and Marmor (2004) developed a simulation framework for the public emergency department considering the system's high flexibility and related parameters. Miller et al. (2004) presented a simulation model aimed at determining the optimal arrangement for the emergency department. One of the critical factors for the emergency department's more efficiency is the total time that the patient spends in an emergency department, called the length of stay. McGuire (1994) utilized MedModel for a hospital to reduce patients' length of stay in the emergency department. The results have shown that adding new staff in busy hours, establishing a waiting place for patients, and using physicians instead of resident officers would significantly reduce the emergency department's length of stay. El-Darzi et al. (1998) and Martin et al. (2003) simulated the patient flow to decrease the length of stay and maximize the patient's performance concerning the disease's acuteness. Edwards et al. (1994) showed simulation results for two medical clinics that sequential processing was suitable for the clinic with one queue, and parallel processing was suitable for clinics with short queues. Their findings indicated that patients' wait time could be reduced by up to 30% with semi-parallel processing. Johnson (1998) used a discrete-event simulation MedModel for evaluating the effects of wait time policy and physician performance on patient flow and parturition section in a hospital in Miami, U.S. This study led to an increase in the number of patients in the parturition section of the hospital. Miller et al. (2003) proposed a discrete-event simulation model for the emergency department of a hospital in the southeast of the U.S. to minimize the patients' length of stay. Samaha et al. (2003) described how discrete-event simulation works for decreasing the length of stay of patients in Cooper hospital. Their findings showed that length of stay is an issue more related to treatment rather than hospital capacities. Blasak et al. (2003) studied the length of stay in the emergency department. They used simulation to analyze the effects of the other sectors' processes on the emergency department. Ceglowski et al. (2007) used discrete-event simulation and data mining methods for studying the relationship between patient urgency, treatment, disposal, and the occurrence of queues in the emergency department. Santibáñez et al. (2009) built a simulation model to analyze the impact of operations, scheduling, resource allocation on patient wait time, clinic overtime, and resource utilization. Their findings showed that their method could have strongly reduced patient wait times. Gul and Guneri (2012) presented a discrete-event simulation model for the emergency department to reduce the average patient length of stay, improve patient throughput, and better utilize resources in the department. They investigated their model for a regional university hospital in Turkey. Weerawat et al. (2013) investigated the Thai public hospital using discrete-event stochastic simulation. They evaluated the high patient congestion, chronic wait time, and percent of patients that their demand is met on each day. They suggested that their simulation model can be

used as a decision support system for addressing hospital management issues. Konrad et al. (2013) built a discrete-event simulation model for processes in the emergency department. The validity of the model was tested for Saint Vincent Hospital in Worcester, US. Several scenarios considering patient flow and the number of nursing staff are proposed to investigate the split-flow process redesign. Kuo et al. (2016) proposed an integrated simulation-optimization approach for the emergency department to investigate the system changes shown via scenarios. They used a simulated annealing algorithm to solve their model. The impact of changes to the system was analyzed by ARENA software. The model was investigated for Hong Kong.

Haghinejad et al. (2015) used ARENA software to model an emergency department of a hospital in Iran to determine the number of waiting patients and the wait time for services. The results showed that an essential factor influencing wait time is the number of beds in the emergency department. Wang et al. (2015) utilized a simulation model to find an optimal emergency department layout and staff assignment. They tried to optimize the staff assignments, minimize the waiting time, and maximize the service level. The model's findings showed that the average wait time decreased by 50%, and the service level increased by nearly 80%. Azadeh et al. (2016) used a simulation model for an emergency department by modeling human error divided into repeated venipuncture, unsafe transportation, and sampling errors. They developed seventy scenarios and evaluated each of them by data envelopment analysis. Expense cost and the number of nursing staff are considered the model's parameters, and patient duration and queue length are derived from the model as results. The model was investigated for a hospital in Iran. Cimellaro et al. (2017) proposed discrete-time simulation models to investigate patients' waiting time in the emergency department to assess the emergency department's resilience. Liu et al. (2017) proposed a hybrid methodology including an agent-based simulation model and an optimization model for emergency department analysis where there was a lack of input data. The model focused on assessing the model parameters and selecting the more important one for the emergency department analysis. Baia Medeiros et al. (2019) studied the emergency department for mental health and addiction services in capacity planning using a discrete event simulation model. In another study, Vanbrabant et al. (2019) presented a framework to assess the importance of the quality of input data used in simulation models for emergency departments. Swan et al. (2019) utilized a discrete event simulation model to analyze the emergency department design using a pod system and a unit-based design in terms of arrival rates and patients' severity. Barni et al. (2019) presented an in situ simulation model for treating anaphylaxis in a pediatric emergency department. The study was focused on showing whether in situ simulation training may increase the epinephrine use and check whether there is a modification in the number of children before and after training. McKinley et al. (2020) developed a discrete event simulation model to assess the impact of patient flow to decrease antibiotic delivery for children diagnosed with cancer for a case study from January 2016 until June 2017 In New York City, USA. Waiting time and length of stay are two main parameters used in the simulation model.

B. Classification and Planning

The medical community requires high-quality staff to deliver medical services. Staff classification and planning are essential factors in designing a healthcare delivery system. Besides, set-off among clinical staff is not enough to meet the demand; similarly, low clinical staff utilization can significantly impact economic viability. Discrete-event simulation plays a crucial role in discussing the inherent issues of the set-off. The following researches are conducted on the classification and planning of medical staff with discrete-event simulation. Badri and Hollingsworth (1993) analyzed different scenarios' impact on scheduling, staff planning, and patient demand for hospitals in the UAE. Kalfehn and Owens (1987) studied the relationship between patient flow and staff numbers for the emergency department. Weng and Houshmand (1999) simulated an outpatient clinic process to determine the optimal number of staff that would maximize patients' performance and minimize patient flow.

Baesler et al. (2003) used a discrete-event simulation model for forecasting the length of stay of patients in the emergency department. They tried to estimate the maximum demand increment with the minimum number of human resources. They investigated their model for a case in Chile. Rico et al. (2007) used ARENA software and OptQuest heuristic optimization software to minimize the patient wait time in the queue and maximize patient flow during a

pandemic influenza outbreak. Their findings resulted in a 37% decrease in the number of patients waiting in the queue. Ahmed and Alkhamis (2009) proposed an integrated simulation – optimization model to develop a decision support system for the operations of an emergency department in Kuwait. Their methodology aimed to determine the optimal number of doctors, lab technicians, nurses required to maximize the patient output and decrease patient time in the hospital by considering budget restrictions. The simulation model's obtained results demonstrated a noticeable increase in patient output and decreased wait time. Rau et al. (2013) built a discrete-event simulation model for strategic capacity planning for outpatient physical therapy in Taiwan by investigating patient mixes' dynamics with treatment plans and estimating the capacity. Results disclosed that increasing staff in the emergency department would decrease the patient wait time. Kazemi et al. (2013) did an empirical investigation in Iran to measure service quality in the hospital. Exploratory factor analysis, conformity factor analysis, and structural equation modeling were used to investigate patient satisfaction in hospitals concerning service quality. Results showed a strong relationship among all factors with each other. Specifically, service quality has an undeniable influence on patient satisfaction. Eskandari et al. (2011) utilized integrated simulation and multi-criteria decision-making methods to find the best scenario for decreasing wait time. They studied the patient flow for a case study in Tehran, Iran. Their results showed an average 42% reduction in wait time. Choy and Cheong (2012) introduced a staff preference framework by considering the nurses' psychological needs. A greedy double swap heuristic was used to solve the model. Wang et al. (2013) proposed a model to examine the patient or workflow in an emergency department when resources such as patients, doctors, and nurses are available.

Malmir et al. (2017) also integrated simulation and a multi-criteria decision-making technique to build a decision support system so that a potential patient can diagnose their disease in its early stage and no wait time in hospitals to get diagnosed of potential disease is needed. Chalgham et al. (2019) proposed an integrated multi-criteria decision-making method using AHP, ELECTRE, TOPSIS, and PROMETHEE methods to evaluate emergency department overcrowding and inpatient boarding problem. The proposed decision-making tool was investigated for a real case in Tunisia. Lim et al. (2016) studied nursing staff assignment and scheduling problems for a multi-objective model. By considering nursing staff specialty, the first objective function aimed at allocating nursing staff to surgical operations. The second objective function was to generate lunch break allocations for the nursing staff allocated in surgery operations by the first objective function. The column generation algorithm was used as a solution approach to solve the model. The model was investigated for a case in Texas, the U.S. Based on the obtained results, the used algorithm can easily set the daily schedules for 100 nurses. Hamasha and Rumbe (2017) used the Markov decision-making process to find the optimal assignments of resources in emergency departments. Findings would greatly contribute to the manager's decision regarding resource allocation and cost minimization. Andersen et al. (2019) proposed an integer programming and Markov chain model for a time-dependent patient flow problem. They proposed a matheuristic to solve both problems to optimality. Gul et al. (2020) developed a hybrid framework using artificial neural networks and a discrete event simulation model to estimate casualties in the emergency department. The simulation model was then used to analyze the effects of patient demand during a natural disruption. The proposed methodology was investigated for a real case study for Istanbul, Turkey. Tsai et al. (2020) used a discrete-event simulation model for the emergency department's capacity planning for daily inpatient bed allocation.

C. Queue Theory Model

The theory of queues is a branch of applied possibilities used in many fields such as computer science, reliability, telecommunications, and health systems (Medhi, 2002). Hospital's mission is beyond the provision of specialized and clinical services, so planning for the provision of health services and health promotion is one of the organization's key responsibilities (Smith et al., 2006). Queue systems have been used since 1904 in various areas of healthcare. When deciding to allocate resources (financial and human, etc.) or designing new services, queuing models have been used to determine the appropriate number of personnel, equipment, and beds. Timely access is one of the qualitative factors of health and. Therefore, health institutions focus on reducing latency. Also, queuing system analysis is a valuable tool for more efficient use of resources and reduced lag. Queuing theory, in addition to its old concept, has much more recent health applications. Queuing modeling helps us determine dense levels (patient congestion) or determine the required capacity at the highest efficiency level (concerning the limitations of specialist human resources, etc.). Another

application of queuing theory in the field of healthcare is the performance of pharmacies. One of the pharmacies' efficiency indicators is the wait time for the patient, which is one of the pharmacy managers' goals to minimize. Also, the long wait time for patients in the queue increases the disease's severity, economic and social costs. Pharmacy management can use these analysis results in future decisions (Bahadori et al., 2014). Using a sampling method, Takagi et al. (2014) provided a new hospital emergency service model. They defined the service time (based on hours, days, and months) into three high, moderate, and low referral rates, resulting in a 30-minute reduction in treatment time. They also found that to reduce the incidence of 0.8% of patients who left the hospital without receiving any service, they should reduce the hospitalization time by 10%. Takagi et al. (2017) constructed a queuing network to model obstetric patients' flow for a hospital in Japan. They aimed at estimating the probability distribution for the number of patients in each working shift at night. Alternatively, the model can be utilized for the capacity planning of the hospital. Fitzgerald et al. (2017) presented a queue-based Monte Carlo Analysis to facilitate the decision-making process for fast-track implementation of an emergency department. Waiting time and nursing resource demand were two main parameters used in this study for patient queue flow in low-acuity patients. Aziati and Hamdan (2018) proposed an integrated queuing theory model and a simulation model to study patient flow in outpatient departments like the emergency department. ARENA software was used to build the model. Waiting time, arrival rate, and service rate were essential parameters used in the simulations.

Carment et al. (2018) used the Markov-modulated flow queue model for the inpatient boarding in emergency departments. Congestion and resource capacity were the two main parameters considered in this study. The proposed model investigated for a real case, and the results showed that boarding reduction policies are working better when they are focused on onboarding time. Benevento et al. (2019) developed a queue-based process mining tool for busy time prediction in the emergency department. This model's utilization enables decision-makers in the health sector to get more information about patient-flow and congestion levels related to each activity. Antunes et al. (2019) employed a hybrid framework using simulation mode, optimization model, and process mining tools to improve patients' queue performance referred to the emergency department. Hou and Zhao (2020) used Markov queue models to predict the waiting times for multi-class patients to increase the patient's satisfaction by decreasing the waiting time in the emergency departments. A numerical example was generated using simulation models to show the proposed methodology's applicability in real life.

In many cases, decision-makers in the health sector, especially in the emergency department, are medical staff rather than human resource specialists; using planning tools can be very useful in optimizing the required human resources and how they are distributed. For several decades, researchers have used various statistics, job measurement, queuing models, and linear programming to solve workforce planning problems. Over the past two decades, using simulation models for planning as a decision-making tool has expanded considerably in healthcare areas. One of the main reasons for this is the increasing complexity of health systems. Concerning our knowledge, for the first time in the literature, we constructed a simulation model for patient flow to minimize the patients waiting time under different sectors of the emergency department, considering the different diseases and their service times. As the main novelty of this work, we proposed a multi-objective mixed integer programming model to optimize the emergency department's required staff and assign them to the right shifts to increase their satisfaction. In the first objective, the model tries to minimize the related costs in the emergency department. However, the model aims to assign the nurses to work shifts efficiently and, therefore, maximize nursing staff satisfaction.

III. PROBLEM DESCRIPTION

The information from the queuing system, including arrival rates (λ), rate of service (μ), number of nurses, and the number of sectors in the emergency department, are needed to simulate the emergency department. A case study of Imam Reza Hospital in Tabriz, Iran, is requested in coordination with the university. Data for our study is obtained from Imam Reza hospital. For this purpose, by using the arrival and departure date of the emergency department in 2014, the arrival rate and its distribution function are tried to find. The emergency department specialists are then asked to enlist

standard services for patients, service times, and percent of patients provided with each service to determine the service rate. After the computational process in the simulation model, a multi-objective model is formulated to minimize the costs and maximize nurses' satisfaction by an optimal assignment to the corresponding sectors. The steps of the methodology are presented in Figure 1.

A. DATA

Arrival rate (λ). The survey data includes the arrival and departure data of all individuals in 2014 to the hospital's emergency department, which the Hospital Informatics Department collects. The total number of patients referring to the emergency department in 2014 was 84,968 people. Because there were no significant differences between the number of people referring to the emergency department in each month of the year, we calculated this parameter for each month using a simple averaging, which resulted in 7,080 people every month. The arrival of patients is considered the Poisson process with a rate of (λ).

Service rate (μ). According to reports of emergency department staff, all incoming patients' types are examined, and then their service type is determined to obtain average service time. Each part of the 5-section emergency department has some standard services and some unique services. For the calculations' accuracy, the questionnaires are designed for each part separately and provided to the nursing staff. Each section is described below.

Triage section. The emergency department triage part of Imam Reza Hospital (Tabriz) is located at the entrance of this section, and in each shift, a nurse serves as a triage nurse in this section. The triage nurse categorizes patients based on the Emergency Severity Index (ESI) and refers them to different emergency department parts. Table I shows the service time of each group of patients with different ESI and percent of referred patients. (C = CPR, T = Trauma, I = Internal, O = Outpatient) The service time in the triage section (μ_{te}) for our case is calculated with the following equation:

$$\mu_{te} = \sum_i (\mu_i * a) = 3.6 \quad (1)$$

Table II. Service time in the Triage section

No.	ESI level	Service time	Percent of patients at this level	Referred to
1	1	Less than 15 seconds	20%	C
2	2	Less than 30 seconds	10%	C/T/I
3	3	4 minutes	50%	C/T/I
4	4	7 minutes	10%	O
5	5	8 minutes	10%	O

CPR section. As indicated in Table II, patients with ESI 1 and some of ESI 2 & ESI 3 should be referred to CPR (Cardiopulmonary resuscitation) section. Patients in CPR are those who are evaluated with high emergency severity. A list of essential services is determined with senior nurses' collaboration and then is provided to nurses to obtain service time in this section. The services and service demand percent of patients that are obtained by questionnaires are shown in Table III.

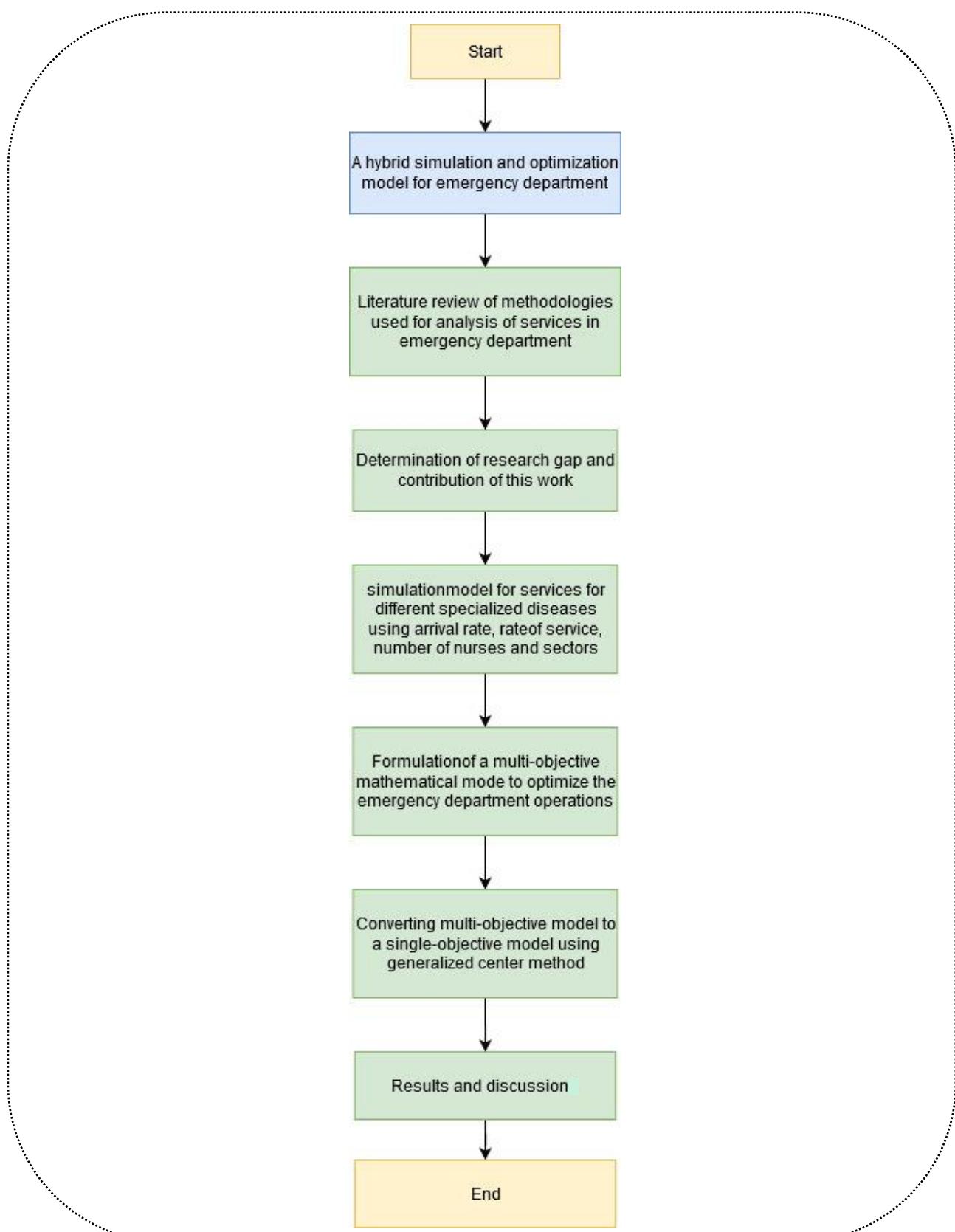


Fig. 1. Flowchart of the proposed methodology

The service time in CPR (μ_c) for our case is calculated with the following equation:

$$\mu_c = \sum_i (\mu_i * \alpha) = 33.92 \quad (2)$$

Trauma section. There is no fixed number of beds in this section; however, it can accommodate 11 beds. This section accommodates a large number of patients, and it has two nurses. Table VI is shown the indicate service rate in the trauma section.

Table III. Service time in CPR section

No.	Service	Service time μ_i (min)	Service demand percent (α)
1	Iv line	1	100%
2	Serum therapy	1	100%
3	ECG	2	99%
4	O ₂ therapy	1	99%
5	Reporting	7	100%
6	Intubation	12	40%
7	Heart resuscitation	35	35%
8	Nebulizers	3	15%
9	Medicinal injections	3	15%
10	BS check	1	100%
11	Monitoring	1	100%
12	Oximetry pulse	1	100%
13	Tests	1	100%

Table IV. Service time in the Trauma section

No.	Service	Service time μ_i (min)	Service demand percent (α)
1	Iv line	1	85%
2	Serum therapy	1	85%
3	ECG	3	10%
4	Monitoring	1	3%
5	Reporting	5	100%
6	Tests	1	50%
7	Glucometry	2	10%

The service time in the trauma section (μ_c) for our case is calculated with the following equation:

$$\mu_c = \sum_i (\mu_i * \alpha) = 7.73 \quad (3)$$

Internal section. Patients with neural, kidney, liver problems and shortness of breath are referred to this section. This section accommodates about 18 beds, but the number of beds can be increased due to demand. In our case, it has five nurses. The service rate in the internal section is shown in Table V. The service time in the Internal section (μ_c) for our case is calculated with the following equation:

$$\mu_c = \sum_i (\mu_i * \alpha) = 13.87 \quad (4)$$

Table V. Service time in the Internal section

No.	Service	Service time μ_t (min)	Service demand percent (α)
1	Iv line	2	100%
2	ECG	2	90%
3	Serum therapy	1	100%
4	Medicinal injections	1	50%
5	Reporting	5	100%
6	Tests	2	99%
7	BS check	1	99%
8	Monitoring	1	10%
9	O ₂ therapy	1	50%

Outpatient section. There is a bed in this section, and it has a nurse. Patients with cold, fever, and stomachache are referred to this section. Table VI illustrates the service time in this section as follows.

Table VI. Service time in the Outpatient section

No.	Service	Service time μ_t (min)	Service demand percent (α)
1	Iv line	2	100%
2	Medicinal injections	1	50%
3	Serum therapy	1	100%
4	Tests	2	99%
5	Visiting	1	100%
6	Others	3	100%

The service time in the Internal section (μ_c) for our case is calculated with the following equation:

$$\mu_c = \sum_i (\mu_i * \alpha) = 9.48 \quad (5)$$

B. Arena simulation

Arena software is one of the powerful simulation software that can simulate discrete-event models. Discrete-event is the system's performance on sequential events in time, and each event happens in a moment. It shows a state change in the system. In our case study, Imam Reza hospital is considered a discrete-event system, and the Arena software is used for the simulation. However, the simulation model is presented. A proper analysis of the system's actual status can be obtained by using a simulation model, and a more efficient system can be achieved by applying some changes to the actual status. For this purpose, it is necessary to compare the emergency system's actual status with different scenarios and achieve the most optimal status with the lowest wait time, costs, and highest job satisfaction. It is needed to find places that can be optimized to design new scenarios. For this, the necessary information from the model and specialists in the emergency department is derived. Scenarios are elaborated below.

Scenario 1 (actual status): The emergency department currently provides services to patients by having 64 nurses and 11 nurses per shift. The data are used as input for the Arena software simulation model within one week, and the results are represented in Table VII. Due to the hospital's regulations and the impossibility of changing the number of triage nurses, this section is excluded from the simulation model's calculations. For validation of our simulated model, the emergency section specialists are asked to evaluate the results.

Table VII. Results of scenario 1

Parameters	Internal	Trauma	Outpatient	CPR	Total
Number of nurses	4	2	1	3	10
Average wait time	15.23	14.79	14.73	14.82	59.57
Average queue length	55.29	54.05	30	34.42	173.76

Scenario 2: Nurses in the emergency department are declared a shortage in the number of staff in Trauma, Internal, and CPR sections. So, due to that, the number of staff in our model is increased. The obtained results demonstrate Table VIII.

Table VIII. Results of scenario 2

Parameters	Internal	Trauma	Outpatient	CPR	Total
Number of nurses	6	4	1	5	16
Average wait time	0.4	0.35	0.36	0.38	1.49
Average queue length	1.37	1.23	0.59	0.75	3.94

Table XI. Results of scenario 3

Parameters	Internal	Trauma	Outpatient	CPR	Total
Number of nurses	1	2	1	2	6
Average wait time	42.47	44.60	45.6	44.6	177.27
Average queue length	133.43	148.79	90.06	98.55	470.83

Scenario 3: In this scenario, the minimum number of nursing staff for each section in the emergency department is considered. However, based on the division of the tasks in some emergency department sections, it is impossible to allocate one nurse to a section. The results are represented in Table XI.

IV. MODEL FORMULATION

After doing the simulation using the data provided by Imam Reza hospital, we can move on to address the nurse scheduling and assignment problem. In this section, the assignment problem for nurses in the emergency department is formulated by taking costs, times, and constraints into account. A bi-objective mixed-integer linear programming is proposed to minimize the service cost and maximize nurses' satisfaction by finding the best assignment strategy. In this model formulation, notations are considered as follows:

Notations

Sets

N	Set of total nurses
S	Set of total shifts
T	Set of total days
i	Index for nurse, $i = 1, 2, \dots, I$ $i \in N$
s	Index for shift, $s = 1, 2, 3$ $s \in S$
t	Time horizon, $t = 1, 2, \dots, 7$ $t \in T$
j	Sections of emergency department, $j = 1, 2, \dots, 5$

Parameters

D_j	The needed number of nurses in section j
C_{ist}	Fixed working cost of nurse i in shift s on day t

C'_{ist} Overtime working cost of nurse i in shift s on day t

U The maximum working hour of a nurse in a week

L The minimum working hour of a nurse in a week

Variables

Y_{ist} Nurse i satisfaction level of allocation to shift s on day t

X_{ijst} 1, If nurse i is allocated to section j in shift s on day t; otherwise, 0

To provide a suitable model for this section, we need a better understanding of nursing staff allocation rules and constraints. Therefore, we first learned from the head nurse of the section about the governing rules in this section, and then the rules of nursing productivity are checked. By comparing these rules with the enacted laws in the Imam Reza Hospital emergency department in Tabriz, it is concluded that all the cases are by the approved laws. Model is formulated considering in mind some assumptions. Assumptions used in this problem are as follows:

- Nurses have to provide services for 44 hours per week.
- The total possible regular and overtime working hours is 66 hours per week.
- Hospitals determine shifts. In our case, there are three working shifts as follows:
Morning shift: 7:30 – 14 (8 working hours), evening shift: 13:15 – 20 (8 working hours), night shift: 19:15 – 8 (16 working hours)
- Eleven nurses work in each shift.
- Each nurse can work for a maximum of 16 hours a day.
- Nurses working night shifts are free to rest for the next day.
- Nurses receive 100,000 IRR per working hour and receive 58,000 IRR per overtime working hour.

Based on the presented sets, parameters, and decision variables defined above, nurses' allocation problems could be formulated.

$$\text{Min } Z_1 = \sum_i \sum_s \sum_t 66 C'_{ist} * 44 C_{ist} * 100000 + (k * 58000) \quad (6)$$

$$\text{Max } Z_2 = \sum_i \sum_j \sum_s \sum_t Y_{ist} X_{ijst} \quad (7)$$

s.t:

$$\sum_j X_{ijst} = 1 \quad \forall i, s, t \quad (8)$$

$$\sum_j X_{ijst} = D_j \quad \forall i, s, t \quad (9)$$

$$X_{ijst} + \sum_{s=1}^S X_{ijs(t+1)} \leq 1 \quad \forall i, j, t \quad (10)$$

$$44 \leq \sum_j \sum_{t=1}^7 (8X_{ij1t} + 8X_{ij2t} + 16X_{ij3t}) \leq 66 \quad \forall i \quad (11)$$

$$\sum_j (8X_{ij1t} + 8X_{ij2t} + 16X_{ij3t}) \leq 16 \quad \forall i, t \quad (12)$$

$$k = \sum_i \sum_j \sum_t (8X_{ij1t} + 8X_{ij2t} + 16X_{ij3t}) - 64 * 44 \quad (13)$$

$$0 \leq Y_{ist} \leq 1 \quad (14)$$

$$X_{ijst} \in \{0, 1\} \quad (15)$$

The first objective function (6) aims to minimize monthly and overtime costs related to nurses. This equation aims to minimize the fixed working and overtime working costs of the nurses. In the second part, nurses' revenue is added concerning the shifts that they are working. The second objective function (7) aims to maximize the nursing staff satisfaction level by optimizing the allocation. It leads to an increase in the service's quality and a reduction in services' processing time. Constraint (8) shows that a nurse cannot work in more than one section in each shift. This equation also shows that each nurse must be allocated to a task in a shift. Constraint (9) guarantees that the number of nurses used in the emergency department's section each day is equal with the D_j . Constraint (10) presents that a night shift working nurse should rest the next day. So, he or she cannot be allocated for the next day. Constraint (11) guarantees that a nurse's working time is between the minimum and maximum working hours. Constraint (12) indicates that the maximum working hour in a day is 16 hours. Constraint (13) calculates the K , which is used in the objective function (1). Constraint (14) guarantees that the satisfaction level is between 0 and 1. Constraint (15) defines the variable.

A. Generalized Center Method

In this part, we aim to convert the multi-objective model to a single-objective model. For this purpose, we can use some methods such as the Generalized Center Method or ε -Constraint method to come up with a single-objective model. The Generalized Center Method is decided to use, which is a relatively newer method compared to the ε -Constraint method. Cheng and Li (1999) introduced the generalized center method (GCM) for solving the multi-objective optimization models by calculating the centers of a sequence of level sets. Implementation steps of this method are as follows:

- 1) Converting all objective functions type to maximization functions
- 2) Solving each objective function without considering other objective functions and finding f^* and x^* values.
- 3) Minimization of total deviations is done by equation (16).

$$\text{Min } \frac{f_1^* - f_1}{f_1^*} + \frac{f_2^* - f_2}{f_2^*} + \dots \quad (16)$$

B. Computational Results

In this part, the results that we obtained from the proposed methodology are proposed. First, a real-life case in the emergency department of Imam Reza hospital in Tabriz, Iran, is the city's biggest emergency department. Furthermore, GAMS 24.9.1 software is implemented to solve the model with Imam Reza hospital data on a personal laptop with Core™ i7 – 4500U 1.80 Hz and 8 GB RAM. Our model's two new scenarios are developed to compare the actual status with different scenarios to find the most optimal scenario. Scenario 1 shows the current status of the Imam Reza hospital that has 11 nurses in each shift. In scenario 2, the number of nurses is increased to 17, and ran the model with the same parameters in scenario 1. In scenario 3, the lowest number of nurses is considered for the emergency department so that in each shift, only seven nurses can work. We have also considered that nurses' overtime work is an average of 10 hours per week. The results for the three scenarios are represented in Table X. Also, the average wait times are 78.32, 1.49, and 177.27 for scenarios 1, 2, and 3, respectively.

Table X. The results of the mathematical model under scenarios

Objective function values	Scenario 1		Scenario 2		Scenario 3	
	Z₁	Z₂	Z₁	Z₂	Z₁	Z₂
Objective functions values separately	22607200	197	33913600	291.4	21493600	124.5
Objective functions values by GCM	22607200	196.8	33913600	298.3	21493600	126.5

It is necessary to make a broad investigation and determine the importance of objective functions and wait time values to make the right decision to choose the most optimal scenario considering both objective functions and wait times. For this, the analytic hierarchy process (AHP) is used to make the best decision. In this problem, scenarios are considered alternatives and objective functions values separately, objective functions values by GCM, average waiting time, and nurse staff are considered criteria. The results obtained from AHP for scenarios are 0.16, 0.7, and 0.34, which shows scenario 2 is the best scenario for the emergency department in Imam Reza hospital. In this scenario, we declared that we face a shortage in the number of nurse staff in the emergency department. So, our model is solved based on the assumptions made in this scenario. Our model's only goal is to allocate nurses optimally to the emergency department sections for three shifts in a week. Due to extensive computational results obtained from GAMS software, one nurse's results in scenario two are represented as the best scenario in Table XI. Results for this case are also validated using MICROSOFT EXCEL. As represented in Table XI, nurses' assignment to different emergency department sections is determined for three shifts on each day of the week.

C. Discussion

Our model is investigated for Imam Reza hospital in Tabriz, Iran, to verify the proposed methodology's feasibility and applicability in real cases. Imam Reza hospital is one of the biggest hospitals in the northwest of Iran, with 800 beds for patients. The emergency department is the most significant section in Tabriz city with a high capacity for outpatients. Therefore, this hospital is selected to show the applicability of the proposed method in this study. Several experts are asked from the hospital to provide us the data needed for this study to make the model and corresponding results more reliable and logical. Realistic data will give us more reliable and robust insights that can be considered within the emergency department of Imam Reza hospital. One of the main reasons for the high wait time for patients in the hospital is the lack of adequate specialist staff. Traditional and unscientific methods used in hospitals for allocating nurses have caused lower nursing staff satisfaction and costs.

Table XI. The results of the allocation problem for nursing staff

Days	Shifts	Nurses									
		1	2	3	4	5	6	7	8	9	10
Saturday	Morning			C		C		T.R.		In	
	Evening		T.E.					In		T.R.	
	Night	In			T.R.		C		C		
Sunday	Morning			In				T.R.		C	
	Evening		In	In							T.R.
	Night										
Monday	Morning								O	In	
	Evening	T.R.	C	C				In		O	C
	Night					C					
Tuesday	Morning				C				C		
	Evening	C			In	TE		TR			In
	Night		T.R.				In				
Wednesday	Morning				In	TR	TR	C			
	Evening			In							
	Night		In	In					C		In
Thursday	Morning	C				C					
	Evening			T.E.	In	In			TR	TR	
	Night						TR				
Friday	Morning				C				C	In	
	Evening		C	T.R.		C	In				In
	Night	C						In			

First, the emergency department's simulation is implemented by the Arena 14 software to solve these problems, using accurate information from the hospital informatics department and direct observation and interviews and questionnaires. In addition to examining the actual and existing status, several different scenarios in this application are developed, and then the queue length and wait time for each one are calculated.

Moreover, a multi-objective programming model is presented for finding the most optimal values for satisfaction and costs related to nursing staff under different scenarios. In the subsequent step, GCM is used to solve the

mathematical model. According to information obtained from the current emergency, the nursing staff is dissatisfied with the low number of staff members and claimed that 11 in total is not enough for the referring patients. The model's outputs are illustrated that this claim is correct, and in order to prevent the generation of queues, specifically in critical parts such as CPR, trauma, and internal, two other nurses should be added to each of these sections, and the total number should reach 17. In this case, queue length and wait time are decreased noticeably. The values of the allocation problem's objective functions show that the satisfaction of nursing staff increases when six new nurses are added to the emergency department. Currently, the allocation and scheduling problem of nursing staff is done manually and through traditional methods, which, in addition to being time-consuming, can lead to low job satisfaction. Recent studies in the literature have developed different mathematical optimization models that tackle nurse scheduling problems considering specific constraints.

In comparison to other recent studies, we are not only solved an optimization model for the whole emergency department and the nurses. Most of the studies have ignored several specialized sections within the emergency department so that nurses cannot be randomly assigned in the emergency department within different shifts. This issue is why this vital characteristic of the emergency department is considered in our methodology. Looking at Table XI, we can see nurses' allocation to different shifts on different days in diverse specialized sections. Indeed, this will increase the emergency department's efficiency by increasing nurses' satisfaction since they would be allocated to a specific section that they have great potential and skill. On the other hand, specialized nurses in different sections would significantly affect the service rate so that patients would be administrated correctly.

V. CONCLUSIONS

Healthcare systems are one of the vital systems that always need to be optimized. It is needless to say that ignorance of this system will result in drastic consequences. Decision-making systems are essential to such systems where managers and staff in the hospitals need to use tools that can be reliable since a wrong decision can lead to costly consequences. In this study, a decision-making tool is presented by us for representing a simulation and MIP model with an aim at optimizing emergency department scheduling and assignment by minimizing waiting time, determining the number of nursing staff, and allocating them to different sections of the emergency department which are used for specific diseases. For future studies, our study can be extended in some ways. One may consider other staff working in the hospital's different sections and analyze their performance using queuing networks. Next to human resources, technical resources are critical for emergency departments so that a study can focus on this problem. Most of the time, patients are transferred from the emergency department to other hospitals to receive service from more specialized staff. One study may construct simulation and queuing models for patient flow from the emergency department to others. One may extend this study by adding other aspects to the mathematical model though considering different objective functions. Moreover, solution approaches like benders decomposition, meta-heuristics, and matheuristic algorithms such as tabu search, simulated annealing, can be used to solve the model with a large data set.

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