

Design of Economic Optimal Double Sampling Design with Zero Acceptance Numbers

Mohammad Saber FallahNezhad¹, Ahmad Ahmadi Yazdi²,

Parvin Abdollahi³ and Muhammad Aslam⁴

¹*Associate Professor of Industrial Engineering, Yazd University, Yazd, Iran*

²*Ph.D Student of Industrial Engineering, Isfahan University of Technology, Isfahan, Iran*

³*Department of Industrial Engineering, Yazd University, Yazd, Iran*

⁴*Department of Statistics, Faculty of Sciences, King Abdul Aziz University, Jeddah 21551, Saudi Arabia*

***Corresponding Author:** Mohammad Saber Fallah Nezhad (E-mail: Fallahnezhad@yazd.ac.ir)

Abstract- *In zero acceptance number sampling plans, the sample items of an incoming lot are inspected one by one. The proposed method in this research follows these rules: if the number of nonconforming items in the first sample is equal to zero, the lot is accepted but if the number of nonconforming items is equal to one, then second sample is taken and the policy of zero acceptance number would be applied for the second sample. In this paper, a mathematical model is developed to design single stage and double stage sampling plans. Proposed model can be used to determine the optimal tolerance limits and sample size. In addition, a sensitivity analysis is done to illustrate the effect of some important parameters on the objective function. The results show that the proposed two stage sampling plan has better performance than single stage sampling plan in terms of total loss function, sample size and robustness.*

Keywords: *Quality control, Acceptance sampling, Optimal design, Loss Function*

I. INTRODUCTION

An acceptance sampling plan is the overall scheme for accepting or rejecting a lot based on sample information. The acceptance plan identifies both the sample size and other criteria which are used to accept or reject the lot. Sampling plans can be classified as single, double, multiple or sequential plans. Acceptance sampling plan has importance in the area of quality control and it can be applied when its requirements fulfilled. For example, in single stage acceptance sampling plans, decision about a receiving lot is taken based on the results of inspection. If the number of nonconforming items are larger than the acceptance number (c) then the lot is rejected, otherwise the lot is accepted. Double sampling plans are the extension of single sampling plans. The double sampling plans are more efficient than the single sampling plans in terms of sample size. Double sampling plans are widely used when the inspectors cannot make decision based on the result of inspecting the first lot. The operation of double sampling plans can be found in (Aslam et al., 2009), (Aslam & Jun, 2010) and (Aslam et al., 2011) In sequential sampling plans, the inspectors continue the sampling until the decision about the lot made. Multiple sampling plans are the extension of the double sampling plans. This type of sampling plans is more efficient than the double sampling plans. However, sequential and multiple sampling plans are rarely used in practical environment because it may be very difficult to administrate them. However, if the incoming quality level is particularly good or particularly poor, double, multiple or sequential sampling plan will reach an acceptance or rejection decision faster, therefore the average sample size will reduce. Moreover, it may cause “psychological” disadvantage for producer when a single sampling plan is applied because there is no second chance for the rejected lots. In such situations, taking a second sample is preferable.

Acceptance sampling plan uses statistical methods to determine whether to accept or reject an incoming lot. Traditionally, two approaches are proposed for designing the acceptance sampling models. In the first approach, the sampling plan is designed based on two-point method. In this method, the designer specifies two points on the operating characteristic (OC) Curve. These two points define the acceptable and unacceptable quality levels for acceptance sampling. The two points also determine the risks associated with the acceptance/rejection decision. (Aslam et al., 2012), (Hailey, 1980), (Pearn & Wu, 2006).

In the second approach, the optimal acceptance sampling method is determined by minimizing the total loss function, which consist of the producer's loss and the consumer's loss (Niaki & Fallahnezhad, 2009), (Fallahnezhad & Hosseinasab, 2011), (Ferrell & Chhoker, 2002), (Moskowitz & Tang, 1992) and (Fallahnezhad et al., 2012).

Several approaches have been proposed for designing sampling plans in the literature. One approach is to design economically optimal sampling system. The other approach is to design a statistically optimal sampling system. Also, some studies have considered the combination of these two approaches. The proposed model can be categorized as an economic model for sampling system. This approach has been employed by many authors recently.

(Ferrell & Chhoker, 2002) presented an economical acceptance sampling plan. Their plan has 3 options: (1) they used continuous loss function. (2) Inspection error is considered in their sampling plan. (3) Their model can be used for designing near optimal sampling plan. They constructed graphs in order to make their model more understandable for practitioners. Moskowitz and Tang [10] presented a new acceptance sampling plan based on Taguchi loss function and Bayesian approach. (Fallahnezhad & Aslam, 2013) proposed an economical acceptance sampling plan based on Bayesian analysis. Wu et al. [13] proposed an optimization design of control charts based on Taguchi loss functions and random process shifts and they minimized overall mean value of Taguchi loss function by adjusting the sample size and control chart limits. (Elsayed & Chen, 1994) proposed an economic design of control chart. They used Taguchi continuous quadratic loss function. Their objective was to minimize the total quality cost and to determine the optimal parameters of control chart. (Kobayashi et al., 2003) used Taguchi quadratic loss function for economical operation of (\bar{x}, s) control chart. They considered sampling cost and the loss function in order to obtain total operation cost. (Arizono et al., 2003) presented variable sampling plan for normal distribution based on Taguchi loss function. (Fallahnezhad & Ahmadi Yazdi, 2015) proposed an optimization model for obtaining the optimal control tolerances and the corresponding critical acceptance and rejection thresholds based on the geometric distribution which minimizes the loss function for both producers and consumers.

It is assumed that the rejected lots are 100% inspected that means all items would be inspected. This concept is used in developing the objective functions where the cost of inspected items in the case of rejecting the lot involves both the producer loss and consumer loss.

The single-sampling plan is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size (n) and inspect each item. If the number of defects does not exceed a specified acceptance number (c), the consumer accepts the entire lot. This is the most common (and easiest) plan to use, although this plan is not the most efficient in terms of the average number of inspected items.

In double-sampling plan, after inspecting the first sample, there are three possibilities:

1. Accept the lot
2. Reject the lot
3. Take a second sample

In a double-sampling plan, management specifies two sample sizes (n_1 and n_2) and two acceptance numbers (c_1 and c_2). If the quality of the lot is very good or very bad, the consumer can make a decision to accept or reject the lot on the basis of the first sample, which is smaller than in the single-sampling plan. To use the plan, the consumer takes a random sample of size n_1 . If the number of nonconforming items is less than or equal to c_1 , the consumer accepts the lot. If the number of nonconforming items is greater than c_2 , the consumer rejects the lot. If the number of nonconforming items is between c_1 and c_2 , the consumer takes a second sample of size n_2 . If the combined number of nonconforming items in the two samples is less than or equal to c_2 , the consumer accepts the lot. Otherwise, it is rejected. Multiple-sampling plans are the extension of the double-sampling plans, where more than two samples are needed to make decision about accepting or rejecting a lot. The advantage of using multiple-sampling plans is the smaller sample sizes. A further refinement of the double-sampling plan is the sequential-sampling plan, in which the

consumer randomly selects items from the lot and inspects them one by one. Each time an item is inspected, a decision is made to (1) reject the lot, (2) accept the lot, or (3) continue sampling, based on the cumulative results so far. The analyst plots the total number of nonconforming against the cumulative sample size, and if the number of nonconforming is less than a certain acceptance number (c_1), the consumer accepts the lot. If the number is greater than another acceptance number (c_2), the consumer rejects the lot. If the number is somewhere between the two, another item is inspected.

In Electronic especially Hard Disk Drive (*HDD*) industry, the use of zero acceptance single sampling plans is widely adopted, particularly for a six sigma process where the quality of product is practically controlled under very low fraction defective level, i.e., in part per million basis.

Nowadays, the manufacturers are heading toward the implementation of lean production system, which is strongly compelling for the smaller lot sizing to eliminate unnecessary wastes/losses and to minimize the production cycle time. Nonetheless, the zero acceptance single sampling plans have been implemented as a protection to re-assure the quality of supplied product [18].

The zero acceptance number plans were originally designed and used to provide over all equal or greater consumer protection with less inspection than the corresponding MIL-STD-105 sampling plans. In addition to economic advantages, these plans are simple to use and administer. Because of these advantages and because greater emphasis is now being placed on zero defects and product liability prevention, these plans have found their place in many commercial industries, although they were originally developed for military products.

There is no specific sampling plan or procedure that can be considered the best suited for all applications. It is impractical to cite all of the applications in which these $c=0$ plans are used. Some examples are machines, formed, cast, powered metal, plastic and stamped parts; and electrical, electronic and mechanical components. They have found application in receiving inspection, in-process inspection and final inspection in many industries. Regardless of the products, wherever the potential for lot-by-lot sampling exists, the $c=0$ plans may be applicable.

We have designed a model to improve the performance of sampling designs with zero acceptance number which has many applications in the industrial environments.

The zero acceptance number single sampling plans have some advantageous over classical sampling plans. For example, it leads the customer to psychologically justify the quality level of their suppliers.

When it is used under two stages, it possess the minimum average total inspection if only a prescribed single point on the operating characteristics (*OC*) curve requirement must be achieved [19]. For small lot sizes, this will also help the manufacturer to minimize the average total inspection as well as the production lead time. To design the zero acceptance single-sampling plans, the sampling distribution of the observed defective must be taken into account, with respect to lot size for greater accuracy [20, 21].

Ferrell and Chhoker [9] proposed an economic single-sampling plan with inspection error. As mentioned above, the double-sampling plan is more efficient than the single acceptance sampling plan. By exploring the literature, it can be said, almost there is no study on designing the economic double (two-stage) acceptance sampling plan by considering inspection error. In this paper, an economic double (two -stage) sampling plan is designed. This model develops an economic model for the proposed sampling plan. The results of the proposed plan are compared with the other models of acceptance sampling plan, which were introduced in the previous studies.

II NOTATIONS AND DEFINITIONS

We will use the following notations and definitions in the rest of the paper:

τ : Target of the quality characteristics

Δ : Half the specification width

$Cp(x)$: Producer's loss

$Cc(x)$: Consumer's loss

B : The cost spending by producer to repair or replace a rejected item

A : Coefficient of consumer loss function

N : Lot size

I : inspection cost per item

n_1 : sample size for first sampling stage

n_2 : sample size for second sampling stage

c : Specified acceptance threshold of nonconforming items in the second sampling stage

δ : Tolerance limit of quality characteristics

A. Proposed Model

This research focuses on economic design of sampling system thus the inspection cost and producer loss and consumer loss are explicitly considered in the model. The main concept considered here deals with the product design that we have determined the optimal value of tolerance for product quality inspection. The proposed model does not consider statistical measures like first type and second type errors because these risks are mostly considered in contracts between producer and consumer based on quality standards. Proposed model can be applied at the final inspection station in production lines where minimizing the cost is important.

It is assumed that the consumer's cost associated with a product is incurred when the quality characteristics fall within the specification limits, and the producer's loss to replace an item is incurred when the quality characteristics exceed the specification limits. A quadratic function is assumed to represent the consumer's cost when quality characteristics fall within the specification limits. The graphical solution to this problem is shown in Figure (1). The producer's loss to repair or replace an item, regardless of the values of the quality characteristics, is $\$B$. The consumer must spend $\$A$ to repair or replace the item if the quality characteristics exceed $\tau \pm \Delta$ where Δ is half the specification width and τ is the target of the quality characteristics (Ferrell & Chhoker, 2002). Therefore the probability of accepting an item, p_δ is determined as follows:

$$p_\delta = \int_{-\delta}^{\delta} f(x) dx. \quad (1)$$

$\tau \pm \Delta$ is specification limits that denote when the values of quality characteristics fall within these limits then the item is conforming but if we want to consider the consumer loss in the optimization then the tolerance limits change to $\tau \pm \delta$ because larger deviations from target value leads to increasing the consumer's loss. $\tau \pm \delta$ are the tolerance limits and similar to specification limits. They are applied for inspection process and when the values of quality characteristics fall within the tolerance limits, the item is conforming. This figure shows continuous quadratic function between the specifications, while the function passing through zero at the target. The intersection of loss functions for consumer and producer is inspection tolerances that minimize the total loss. Let $C_p(x)$ be the producer's loss (Equation(2)) and $C_c(x)$ be the consumer's loss function (Equation(3)).

$$C_p(x) = B, \quad (2)$$

$$C_c(x) = \frac{A}{\Delta^2} (x - \tau)^2. \quad (3)$$

The loss associated with one inspected item is determined as follows:

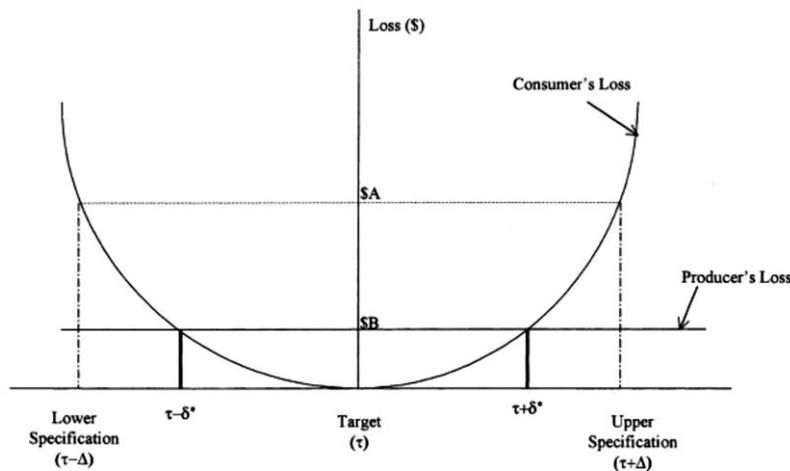


Fig. 1. Graphical quadratic function (Fallahnezhad & HosseiniNasab 2011)

$$K = I + \int_{-\infty}^{-\delta} Bf(x)dx + \int_{-\delta}^{\delta} A(x-\mu)^2 f(x)dx + \int_{\delta}^{\infty} Bf(x)dx, \tag{4}$$

where I is inspection cost per item. Also $\int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx$ is the cost of one item that has been accepted without inspection. (Ferrell & Chhoker, 2002) proposed the following model for designing a one stage-sampling (single-sampling plan) model. They assumed a sample size of n items is taken from the process and if the number of nonconforming item in this sample was more than zero then the lot is rejected otherwise it is accepted. Therefore, the loss model is determined as follows (Ferrell & Chhoker, 2002):

$$E(L_1) = n_1K + p(N-n_1) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx + (1-p)(N-n_1)K, \tag{5}$$

where n_1K is the expected loss of items in first sample, $p(N-n_1) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx$ is the expected loss of accepted items without inspection and $(1-p)(N-n_1)K$ is the expected loss of inspected items, $(N-n_1)K$ multiplied with probability of rejecting the lot (inspecting all items of the lot) $1-p$. Since the concept of zero acceptance number is utilized in sampling process thus p is determined as follows:

$$p = \sum_{j=0}^0 \binom{n}{j} p_{\delta}^j (1-p_{\delta})^{n-j} = (1-p_{\delta})^n. \tag{6}$$

In the second method, it is supposed that a lot with size N is received. The concept of zero acceptance number is utilized in two sampling stages. Suppose that first sample with size of n_1 items is inspected. For the received lot with N items, if the number of nonconforming items in the first stage of inspection was equal to zero then the lot is accepted but if one nonconforming item was found in the first stage of inspection then second sample size with n_2 items will be taken. If there were more than one nonconforming item in the first stage of sampling, then the lot would be rejected. Again, if the number of the nonconforming items in second sample was equal to zero, then the lot would be accepted otherwise the lot would be rejected. Therefore, the total loss function of the proposed method is determined as follows:

$$E(L_2) = n_1K + n_2p_2K + p_1(N-n_1) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx + p_2p_3(N-n_1-n_2) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx + (1-p_1-p_2)(N-n_1)K + p_2p_4(N-n_1-n_2)K, \tag{7}$$

where n_1K is the expected loss of inspected items in first sample and n_2p_2K is the expected loss of inspected items in the second sample. $p_1(N-n_1) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx$ is the expected loss of accepted items without inspection. $p_2p_3(N-n_1-n_2) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx$ is the expected loss of accepted items without inspection. $(N-n_1-n_2) \int_{-\infty}^{\infty} A(x-\tau)^2 f(x)dx$ multiplied with probability of taking the second sample, p_2 and probability of

accepting the lot in second sample, p_3 . $(1-p_1-p_2)(N-n_1)K$ is the expected value of accepting all remained items in the lot. $(N-n_1)K$ multiplied with probability of rejecting the lot in first sampling stage $(1-p_1-p_2)p_2p_4(N-n_1-n_2)K$ is the expected loss of inspecting all remained items in the lot. $(N-n_1-n_2)K$ multiplied with probability of taking the second sample, p_2 multiplied with the probability of rejecting the lot in second sampling stage, p_4 . Also p_1 denotes the acceptance probability in the first sampling stage and p_2 denotes the probability of taking the second sample (Equation (8)).

$$p_1 = \sum_{j=0}^0 \binom{n_1}{j} p_\delta^j (1-p_\delta)^{n_1-j} = (1-p_\delta)^{n_1},$$

$$p_2 = \sum_{j=1}^1 \binom{n_1}{j} p_\delta^j (1-p_\delta)^{n_1-j} = \binom{n_1}{1} p_\delta (1-p_\delta)^{n_1-1}. \quad (8)$$

Also p_3 denotes the acceptance probability in the second sampling stage and p_4 denotes the probability of rejecting the lot and inspecting all items in the lot (incurring the loss K for each item),

$$p_3 = \sum_{j=0}^0 \binom{n_2}{j} p_\delta^j (1-p_\delta)^{n_2-j} = (1-p_\delta)^{n_2},$$

$$p_4 = 1 - p_3. \quad (9)$$

Comparing the total loss of two sampling methods leads to following result:

$$E(L_1) - E(L_2) = \binom{n_1}{1} p_\delta (1-p_\delta)^{n_1-1} (1-p_\delta)^{n_1} (N-n_1-n_2)(K-A). \quad (10)$$

Thus the following decision making method is obtained:

If $K < A$, then single-sampling plan is preferred, otherwise, two stage sampling model (double-sampling plan) would be better. Please Note that all items are inspected after rejecting the lot and the objective function is designed based on rectified sampling. In the next section, the model is solved for some simulated cases.

We have not considered the statistical properties such as producer and consumer risks in the optimization model. Adding these risks as constraints in the model is possible where it is important to design an optimized economic - statistical sampling method which is suggested as a future research. The loss function of double-sampling plan is minimized separately. Also, the loss function of single-sampling plan is minimized separately. We have computed the difference between these two objective functions in order to find out which one is less and optimal. We have not minimized the difference between these two objective functions.

III NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate the performance of the proposed model. This numerical example shows how the proposed model can be applied to obtain the optimal values of parameters n_1, n_2, δ in order to minimize the total loss, including producer's loss and consumer's loss. In this example, the lot size is equal to 50000, and $\Delta=1$, $\tau=0$, $I=10$, $B=50$. The minimum total losses for two stages sampling plan and single stage sampling plan are obtained by solving optimization model with mentioned input parameters. The different combination of alternative values for n_1, n_2, δ are used together and their corresponding loss objective functions are determined. Since the search space is limited thus we used numerical simulation method to solve the proposed model. First, 104 sets of alternative values for n_1, n_2, δ are generated in logical intervals. Then, the proposed and classical models have been solved with these input values. To illustrate the performance and statistical advantages of the proposed sampling method, we calculated the average sample number (ASN) for each set of parameters.

Since statistical measures like risks are not included in the optimization model thus analyzing power of sampling system is not needed but we have obtained risks at AQL and LQL points to see the behavior of the proposed sampling plan. The results have been summarized in Table (1). According to Table (1), it is observed that the risk of producer ($1-Pa(AQL)$) and the risk of consumer ($Pa(LQL)$) in the proposed two-stage method is less than classical one stage method in most of the cases.

It can be seen that the optimal sampling design in two stages method is $n_1=4$, $n_2=13$, $\delta=4.2$ and its minimum loss is

equal to 1857344. It can be seen that the optimal sampling design in single stage method is $n=6, \delta=4.2$ and its minimum loss is equal to 1859660. The value of objective function in two stages sampling model is less than single stage sampling model. This result was expected because $K=37.34 > A=36$ in this system. Also, ASN in two stages sampling method is 6 and it is equal to sample size of single stage method. Also, producer and consumer risks in two stages sampling model is 0.010 and 0.02, respectively where the values of these risk in single stage method are 0.04 and 0.05, respectively that denote the better performance of the proposed method considering risk values.

We have compared the proposed double-sampling plan with classical single-sampling plan and we have done a sensitivity analysis to compare the model performance under different scenarios of parameters selection. We have not considered the *ATI* objective function in the model because we have developed an economic model for minimizing total loss of sampling problem, so average total inspection (*ATI*) is not considered in optimization model. The advantages of the proposed model rather than the existing classical ones is to help decision maker to select the optimal sampling parameters in the case that zero acceptance number policy is employed in order to decrease the total loss for both producer and consumer. Employing Markov chain is not needed in this case because we have only two stages for decision making.

TABLE I: The value of cost function for alternative values of n_1, n_2, δ in the proposed plan and the value of cost function for alternative values of n, δ in single stage sampling plan

No.	Two stages sampling plan							No.	Single stage sampling plan				
	n_1	n_2	ASN	δ	TOTAL	$I-P_a(AQL)$	$P_a(LQL)$		n	δ	TOTAL	$I-P_a(AQL)$	$P_a(LQL)$
1	3	11	14	1	2591667	0.03	0.05	1	7	1	2591664	0.04	0.02
2	9	13	11	1.2	2514176	0.05	0.04	2	7	1.2	2514391	0.03	0.03
3	3	13	11	1.4	2439532	0.04	0.03	3	7	1.4	2439510	0.04	0.08
4	4	12	13	1.6	2367466	0.04	0.03	4	7	1.6	2367414	0.02	0.07
5	3	12	4	1.8	2298591	0.04	0.03	5	7	1.8	2298494	0.06	0.02
6	3	9	6	2	2217475	0.04	0.05	6	7	2	2233141	0.05	0.07
7	7	14	19	2.2	2172051	0.03	0.05	7	7	2.2	2171744	0.07	0.03
8	3	7	5	2.4	2112036	0.02	0.04	8	7	2.4	2114698	0.08	0.07
9	7	9	7	2.6	2059190	0.03	0.01	9	7	2.6	2062399	0.04	0.03
10	3	9	11	2.8	2016165	0.01	0.05	10	7	2.8	2015253	0.06	0.08
11	5	14	9	3	1974670	0.01	0.05	11	7	3	1973671	0.02	0.08
12	6	10	11	3.2	1938065	0.03	0.10	12	7	3.2	1938066	0.07	0.04
13	3	8	7	3.4	1904513	0.02	0.04	13	7	3.4	1908844	0.08	0.12
14	7	13	14	3.6	1881900	0.01	0.02	14	7	3.6	1886383	0.06	0.09
15	4	10	12	3.8	1872067	0.03	0.02	15	7	3.8	1870996	0.09	0.07
16	5	8	8	4	1862808	0.01	0.02	16	7	4	1862873	0.08	0.04
17	5	7	8	4.2	1861693	0.04	0.05	17	7	4.2	1862004	0.05	0.09
18	6	14	14	4.4	1851188	0.02	0.02	18	7	4.4	1868047	0.06	0.05
19	7	14	12	4.6	1882929	0.04	0.06	19	7	4.6	1880169	0.02	0.04
20	7	13	8	4.8	1887367	0.01	0.03	20	7	4.8	1896808	0.03	0.12
21	6	8	12	5	1920637	0.02	0.02	21	7	5	1915374	0.02	0.10
22	8	15	16	1	2591666	0.02	0.02	22	8	1	2591666	0.07	0.07
23	4	9	6	1.2	2514176	0.02	0.04	23	8	1.2	2514398	0.03	0.11
24	7	17	21	1.4	2439533	0.02	0.06	24	8	1.4	2439528	0.03	0.07
25	7	15	14	1.6	2366717	0.01	0.03	25	8	1.6	2367453	0.04	0.02
26	6	11	8	1.8	2298597	0.03	0.04	26	8	1.8	2298568	0.09	0.09
27	4	13	10	2	2231583	0.01	0.03	27	8	2	2233269	0.03	0.00
28	3	6	4	2.2	2169650	0.05	0.05	28	8	2.2	2171949	0.04	0.08
29	3	14	10	2.4	2114698	0.06	0.03	29	8	2.4	2115000	0.00	0.04

30	6	11	12	2.6	2062817	0.03	0.02	30	8	2.6	2062817	0.03	0.10
31	3	6	4	2.8	2016165	0.06	0.05	31	8	2.8	2015796	0.03	0.04
32	8	14	17	3	1974338	0.03	0.03	32	8	3	1974338	0.06	0.02
33	5	11	15	3.2	1938846	0.03	0.06	33	8	3.2	1938848	0.01	0.08
34	3	13	8	3.4	1898984	0.03	0.03	34	8	3.4	1909725	0.02	0.08
35	5	12	9	3.6	1884736	0.04	0.03	35	8	3.6	1887356	0.02	0.07
36	6	14	6	3.8	1873185	0.04	0.03	36	8	3.8	1872080	0.03	0.07
37	6	14	11	4	1844612	0.04	0.05	37	8	4	1864148	0.07	0.01
38	4	17	5	4.2	1841142	0.06	0.02	38	8	4.2	1863638	0.06	0.05
39	4	19	8	4.4	1871954	0.03	0.06	39	8	4.4	1870335	0.03	-0.06
40	6	10	13	4.6	1879755	0.01	0.04	40	8	4.6	1883549	0.03	0.06
41	5	10	8	4.8	1864467	0.04	0.04	41	8	4.8	1901846	0.06	0.00
42	4	14	11	5	1899777	0.02	0.03	42	8	5	1922678	0.03	0.04
43	7	10	8	1	2591664	0.04	0.02	43	9	1	2591667	0.01	0.02
44	6	13	13	1.2	2513275	0.06	0.02	44	9	1.2	2514400	0.02	0.11
45	5	15	11	1.4	2431506	0.01	0.03	45	9	1.4	2439532	0.00	0.07
46	7	17	15	1.6	2367466	0.01	0.04	46	9	1.6	2367463	0.09	0.01
47	9	14	15	1.8	2285233	0.02	0.02	47	9	1.8	2298591	0.11	0.14
48	5	13	8	2	2233269	0.06	0.05	48	9	2	2233312	0.07	0.14
49	7	13	16	2.2	2153935	0.03	0.02	49	9	2.2	2172024	0.06	0.06
50	9	19	23	2.4	2115120	0.05	0.03	50	9	2.4	2115120	0.06	0.10
51	7	15	16	2.6	2063074	0.05	0.02	51	9	2.6	2062997	0.08	0.04
52	5	14	19	2.8	2014071	0.06	0.05	52	9	2.8	2016048	0.05	0.10
53	4	12	8	3	1969639	0.03	0.02	53	9	3	1974670	0.02	0.02
54	9	12	15	3.2	1938842	0.04	0.03	54	9	3.2	1939263	0.08	0.01
55	9	16	17	3.4	1910503	0.03	0.05	55	9	3.4	1910222	0.10	0.04
56	5	14	14	3.6	1888283	0.03	0.03	56	9	3.6	1887937	0.06	0.04
57	8	15	19	3.8	1866464	0.05	0.03	57	9	3.8	1872764	0.11	0.05
58	5	10	5	4	1857903	0.04	0.02	58	9	4	1864994	0.01	0.07
59	4	14	13	4.2	1861898	0.01	0.01	59	9	4.2	1864777	0.03	0.11
60	4	13	12	4.4	1867942	0.01	0.08	60	9	4.4	1872006	0.01	0.10
61	9	18	18	4.6	1842268	0.05	0.05	61	9	4.6	1886130	0.03	0.03
62	6	16	17	4.8	1856221	0.02	0.03	62	9	4.8	1905860	0.08	0.08
63	9	16	21	5	1892069	0.02	0.06	63	9	5	1928740	0.03	0.08
64	3	11	7	1	2591664	0.05	0.06	64	10	1	2591667	0.02	0.08
65	4	6	8	1.2	2514355	0.02	0.05	65	10	1.2	2514400	0.02	0.09
66	3	10	11	1.4	2439433	0.05	0.02	66	10	1.4	2439533	0.06	0.04
67	6	16	21	1.6	2364643	0.03	0.07	67	10	1.6	2367466	0.04	0.07
68	8	14	13	1.8	2298591	0.02	0.07	68	10	1.8	2298597	0.02	0.06
69	9	14	14	2	2232752	0.04	0.02	69	10	2	2233326	0.05	0.09
70	6	9	14	2.2	2171949	0.02	0.02	70	10	2.2	2172051	0.08	0.08
71	6	10	15	2.4	2113940	0.01	0.01	71	10	2.4	2115168	0.03	0.04
72	4	11	8	2.6	2062399	0.05	0.05	72	10	2.6	2063074	0.05	0.04
73	7	16	11	2.8	2015796	0.01	0.02	73	10	2.8	2016165	0.10	0.10
74	6	16	10	3	1974337	0.06	0.04	74	10	3	1974836	0.03	0.07
75	7	12	17	3.2	1916445	0.02	0.06	75	10	3.2	1939483	0.03	0.08
76	8	10	15	3.4	1907280	0.03	0.03	76	10	3.4	1910503	0.06	0.09

77	4	9	8	3.6	1877179	0.06	0.05	77	10	3.6	1888284	0.01	0.12
78	9	14	21	3.8	1866516	0.03	0.02	78	10	3.8	1873196	0.01	0.02
79	4	12	8	4	1860869	0.02	0.04	79	10	4	1865556	0.04	0.11
80	4	14	7	4.2	1855842	0.05	0.02	80	10	4.2	1865571	0.04	0.16
81	6	8	6	4.4	1869544	0.02	0.04	81	10	4.4	1873227	0.03	0.04
82	5	9	13	4.6	1857755	0.02	0.07	82	10	4.6	1888101	0.07	0.01
83	7	14	14	4.8	1876103	0.07	0.02	83	10	4.8	1909059	0.01	0.04
84	7	13	7	5	1866495	0.06	0.05	84	10	5	1933771	0.06	0.08
85	4	6	5	1	2591664	0.02	0.09	85	6	1	2591650	0.05	0.04
86	4	13	16	1.2	2514400	0.04	0.06	86	6	1.2	2514355	0.04	0.03
87	6	16	18	1.4	2439533	0.03	0.03	87	6	1.4	2439433	0.08	0.06
88	6	15	13	1.6	2366717	0.03	0.01	88	6	1.6	2367267	0.07	0.09
89	4	13	6	1.8	2298245	0.03	0.05	89	6	1.8	2298245	0.10	0.06
90	5	11	11	2	2217434	0.02	0.06	90	6	2	2232753	0.08	0.05
91	3	14	11	2.2	2171184	0.04	0.06	91	6	2.2	2171184	0.07	0.02
92	5	11	6	2.4	2115120	0.02	0.03	92	6	2.4	2113940	0.07	0.06
93	4	14	17	2.6	2063074	0.03	0.05	93	6	2.6	2061433	0.08	0.04
94	5	17	14	2.8	2006152	0.04	0.03	94	6	2.8	2014086	0.01	0.15
95	3	12	5	3	1974336	0.03	0.04	95	6	3	1972331	0.04	0.04
96	5	12	12	3.2	1933817	0.04	0.04	96	6	3.2	1936594	0.06	0.14
97	4	17	19	3.4	1908833	0.02	0.03	97	6	3.4	1907283	0.02	0.08
98	6	9	12	3.6	1887355	0.05	0.05	98	6	3.6	1884756	0.02	0.10
99	6	14	13	3.8	1873196	0.04	0.05	99	6	3.8	1869276	0.07	0.04
100	6	11	17	4	1857956	0.03	0.04	100	6	4	1860954	0.05	0.12
101	4	13	6	4.2	1843997	0.01	0.02	101	6	4.2	1859660	0.04	0.05
102	76	12	15	4.4	1859357	0.05	0.06	102	6	4.4	1864914	0.05	0.04
103	6	6	6	4.6	1873398	0.05	0.01	103	6	4.6	1875742	0.04	0.07
104	7	16	9	4.8	1895342	0.01	0.04	104	6	4.8	1890484	0.05	0.11

IV SENSITIVITY ANALYSIS

In this section, we examine the effects of some important input parameters like coefficient of consumer loss function (A), producer’s cost to repair or replace a rejected item (B) and lot size (N) on the objective function (See Figures (2-4))

Figure 2 shows the variation of the objective function with respect to the lot size. It is clear that objective function increases by increasing the lot size. This means that it is better to provide a small value of lot size for lot acceptance model in order to decrease the expected loss for each item in quality inspection plan.

Also, a sensitivity analysis is performed in order to investigate the effect of producer’s cost to repair or replace a rejected item (B) on the objective function. According to Figure 3, it is clear that the objective function increases by increasing the value of B with fewer slope rather than Figure 2. Finally, Figure 2 shows that total loss function increases considerably by increasing the lot size.

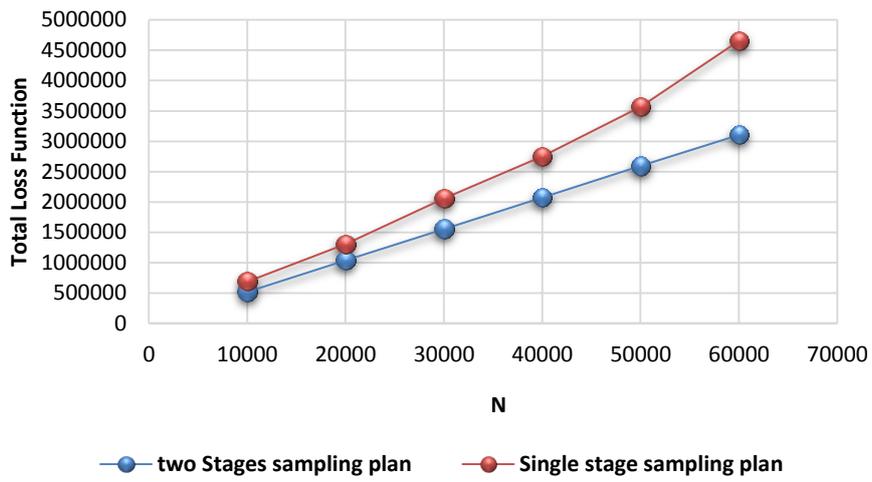


Fig. 2. N vs. Objective function

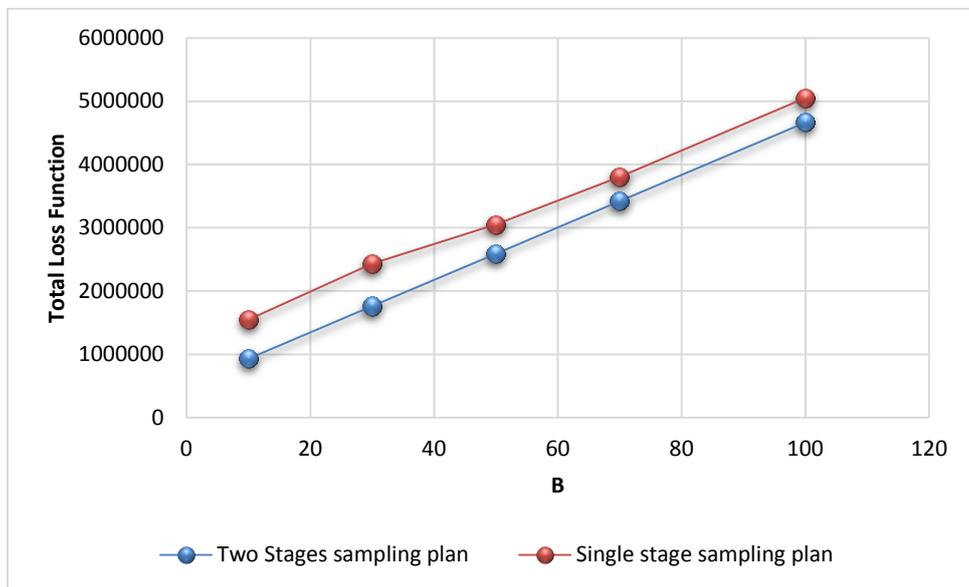


Fig. 3. B vs. Objective function

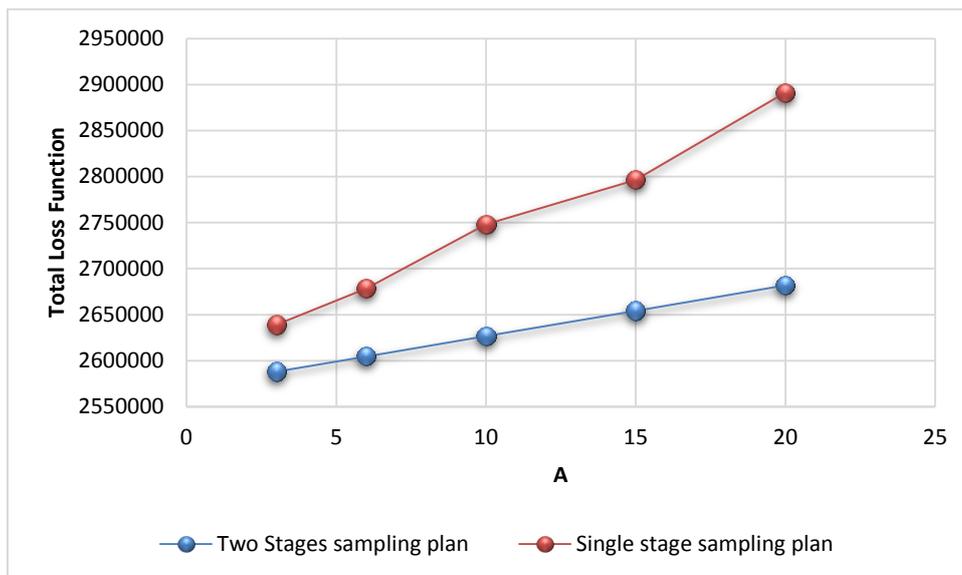


Fig. 4. A vs. Objective function

V CONCLUSION AND FUTURE RESEARCHES

In this paper, a comparison study is performed between single stage sampling model and double-sampling model based on loss objective function for plans with zero acceptance number. Proposed method provides the protection for both producer and consumer by minimizing the summation of loss for each one. The proposed plan can be extended to multiple and sequential sampling plans as a future research. Also, the proposed sampling plan can be easily extended to find the optimal values of acceptance numbers and sample sizes.

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