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Robust Optimal Desirability Approach for Multiple Responses Optimization with Multiple Productions Scenarios

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Abstract- *An optimal desirability function method is proposed to optimize multiple responses in multiple production scenarios, simultaneously. In dynamic environments, changes in production requirements in each condition create different production scenarios. Therefore, in multiple production scenarios like producing in several production lines with different technologies in a factory, various fitted response models are obtained for each response according to their related conditions. In order to consider uncertainty in these models, confidence interval of fitted responses has been defined in the proposed method. This method uses all values in the confidence region of model outputs to define the robustness measure. This method has been applied on the traditional desirability function of each scenario in order to get the best setting of controllable variables for all scenarios simultaneously. To achieve this, the Imperialist Competitive Algorithm has been used to find the robust optimal controllable factors setting. The reported results and analysis of the proposed method confirm efficiency of the proposed approach in a dynamic environment.*

Keywords: *Multiple production scenarios, Robustness, Desirability function, Uncertainty, Imperialist Competitive Algorithm*

I. INTRODUCTION

Different condition and changes in production requirements in each condition create different production scenarios. This situation is named Multiple Production Scenarios in this paper that has been illustrated in Fig. 1. Suppose that there are several production lines with different technologies in a factory. Each production line is defined as a production scenario which applies design variables identical to other scenarios. In this situation, adjustment of design variables should be considered simultaneously to get the best setting. Since, finding the best controllable factors according to one of desirability functions is not proper for other scenarios; therefore, setting of design variables in optimum mode, considering all scenarios during optimization causes some difficulties. In this study, the proposed method considers all of scenarios simultaneously.

If there are multiple responses drawn in production procedure, setting the optimum mode of design variables would require simultaneous consideration of all the responses. This is known as multi-response surface problem.

Desirability function approach is one of the creative ideas that have attracted wide attention (Derringer, 1980),(Derringer, 1994),(Kim & Lin 2000),(Jeong & Kim, 2009),(He, 2010). Rather not much effort has been done on the multi-response problem field that considers the optimality and robustness of the solution simultaneously.

Considering optimal and robust solution means to set the design variables in a manner that little unexpected change in these kinds of variables in optimal setting, does not make enormous changes in desirability level. Consequently, in such setting the uncertainty of the model will be minimized.

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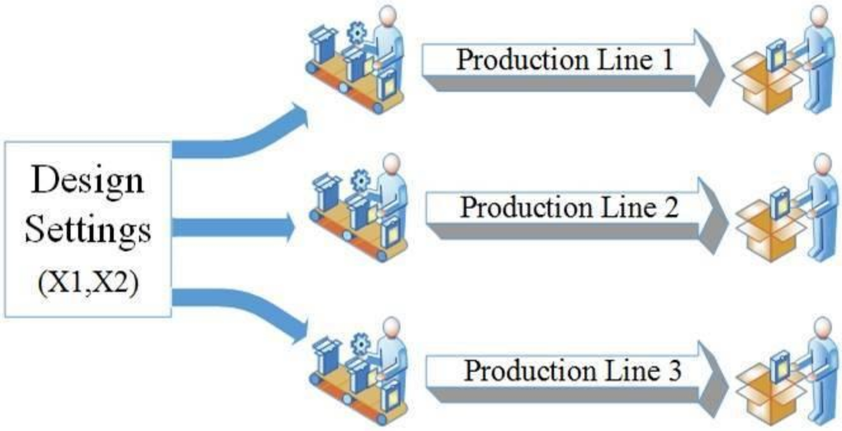

Fig. 1. Multi scenario production line with different technology in each line

Therefore, finding the stable setting is inevitable issue in design of experiment field. Considering the (*1-α*) confidence interval on the mean of response value (y) is an applicable method for applying this feature to desirability function. In statistical studies every point between the response value in lower bound ($y^L(x)$) and the response value in upper bound ($y^U(x)$) has the same value as the mean of response value in the confidence interval. However, different points in the mentioned interval have different desirability values.

For considering the desirability value of all points in the confidence interval, multi-objective optimization approaches to find the optimal robust setting are useful methods. In the most of papers, a linear desirability function is considered for each response and desirability functions are based on three response types, LTB (larger-the-better), STB (smaller-the-better) and NTB (nominal-the-best). However, in some cases the desirability function is nonlinear or the target values of the responses are not known. Hence, a desirability function should be assigned to each response to get the best setting of design variables. In dynamic environments, changes in production requirements in each condition create different production scenarios. In order to consider uncertainty in the models, confidence intervals of fitted responses should be defined. The confidence intervals for different response variables and scenarios are not the same. Therefore, the confidence intervals should be constructed such that overall desirability of all scenarios is maximized. In this study, we maximize the minimum, maximum and the total desirability of the confidence region in each scenario to get the robust overall desirability value.

Applying two aspects, interval confidence and considering scenarios all together, in the classical desirability function resulted in a robust optimal desirability function which is proposed in current study.

The rest of this paper is organized as follows: In Section II, a brief literature of desirability functions is given and the necessity of finding robust setting is discussed. Section III provides a brief introduction of the robust optimal desirability used to create a new desirability function. Then, the proposed solution algorithm based on the Imperialist Competitive Algorithm, which is suitable to find the best and stable settings of factors in multi production scenarios, is explained. A numerical example is studied in Section IV to validate the proposed model. Finally, conclusions and remarks are presented in Section V.

II. LITERATURE REVIEW

Desirability function is an approach that converts each estimated response value ($\hat{y}$) into individual desirability which are then aggregated into a combined function (D). This combined desirability function D is defined as the weighted geometric mean of all individual desirability values. If there are several independent responses in the model, the desirability functions introduced by (Derringer & Suich, 1980) can be applied to evaluate the efficiency of these responses. They proposed individual desirability functions based on three response types, LTB (larger-the-better), STB (smaller-the-better) and NTB (nominal-the-best).

(Kim & Lin, 2000) proposed a method to maximize the overall minimal level of satisfaction with respect to all the responses. (Ch'ng et al., 2005) proposed a method that minimizes the difference between the mean and the target desirability values of each response to get the optimal variable settings under the assumption of normality for error variances . (Wu, 2004) introduced a new approach that variance and correlations among the model responses have been applied in individual and combined desirability function. In this approach, authors did not mention the uncertainty and robustness of the optimal solution.

In another study, (Ribardo & Allen, 2003) used a desirability-based method that combined effect of the mean and dispersion of responses in desirability function to get the better function. If multi-response optimization problems grow in either the number of design variables or the number of responses, classical optimization algorithms may not succeed to locate the optimal settings. As a consequence, (Ortiz et al., 2004) proposed a new approach that applies a genetic algorithm (GA) combined with an unconstrained desirability function which makes the algorithm capable to distinguish between far-from and nearly feasible solutions. In following, (Lee & Kim, 2007) proposed a new version of desirability function for multi-response model referred to as expected desirability function. This function is based on the probability distribution of the predicted response variables. In some situations, using perception of customers can cause negative or zero values for individual and combined desirability values. (Das & Sengupta, 2010) defined a negative exponential transformation on desirability function so that the negative and zero values can be used in desirability.

According to (Costa et al., 2011), most of the recent studies in optimization does not consider the quality of predictions. Therefore, the most attention has been paid to the individual or average desirability and the robustness of the optimal solution has been ignored. But, (He et al., 2010) proposed a robustness desirability function distinguished from the optimization desirability function and also proposed an overall desirability function approach, which make balance between robustness and optimization for multiple response problems. In the other words, they presented a strategy to deal with robustness and optimization simultaneously for multiple responses. In following, (He et al., 2012) introduced a new approach to calculate the robust desirability function by introducing the confidence interval in order to consider the uncertainty of the model design variables. In fact, they substituted the confidence interval method with weighting method to maximize minimum desirability in confidence interval. Authors applied GA to get the optimal and robust setting by maximizing the minimum desirability. A summarized literature on desirability function is given in TABLE 1. The last row of this table shows the purpose of this study.

TABLE I. A summarized literature review on the desirability function

Authors (year)	**Main contribution**
Derringer and Suich (1980)	Maximizing the weighted geometric mean to optimize several response variables
Kim and Lin (2000)	A modeling approach to optimize a multi response system to maximize the degree of overall satisfaction with respect to all the responses
Ribardo and Allen (2003)	Proposing a desirability function to weigh together multiple objectives which accounts for the combined effect of the mean and the dispersion of quality characteristics
Ortiz et al. (2004)	Proposing a genetic algorithm for the simultaneous optimization of multiple responses
Ch'ng et al. (2005)	Proposing a method to solve desirability function in non-differentiable points
Wu (2004)	Presenting a method to optimize correlated multiple quality characteristics by using percentage of quality loss reduction
Lee and Kim (2007)	Proposing an expected desirability function which is the average of the classical desirability values based on the probability distribution of the predicted response variable
He et al. (2010)	Individual desirability functions that account for quality of predictions and a composite desirability function that makes balance between robustness and bias
Das and Sengupta (2010)	the negative exponential transformation used to modify desirability function in order to use the negative and zero values in desirability functions
He et al. (2012)	robust desirability function method for multi-response surface optimization considering uncertainty in the model by using confidence interval
Proposed method	Robust optimal desirability function by considering the maximum, minimum and the total amount of desirability function in confidence interval for multiple production scenarios to get the overall desirability value

Following the mentioned methods by (He et al., 2010, 2012) in the proposed method of this paper, the robust optimal setting is achieved as a result of a multi-objective approach through using Imperialist Competitive algorithm. Due to the possibility of being several production lines with different technologies in factories, we extend this approach to multiple production scenarios. Therefore, in this approach a multiple responses with multiple production scenarios is considered. In the classic desirability function methods, only the maximum value of desirability had been considered. However, in this paper the robust desirability function has been modified and the robust optimal desirability value is resulted by maximizing the minimum, maximum and the total desirability in the confidence interval to find the robust optimal setting of controllable factors.

III. PROPOSED METHOD

In this section, the proposed method has been presented. In this model it is assumed that a factory has several production lines with different technologies. As a consequence, each scenario has various design variables which should be set at the near optimal level to optimize the desirability function. The following notations are used in the rest of this paper:

Parameters:

C Vector of design variables within the experimental region.

Y Response variable.

$\hat{\sigma}^2$ Estimated error variance by using the replicates at the center

L Number of responses in each scenario.

K Number of design variables.

X The data matrix.

α Coefficient of total desirability .

w_i Importance parameter for *i*th response.

N Number of experimental runs.

P Number of model parameters which should be estimated.

Variables:

$\hat{y}(x)$ Proper approximation for response variables.

d_i The desirability of *i*th response variable.

$y_L(x)$ Lower bound in confidence interval.

$y_U(x)$ Upper bound in confidence interval.

η_{ij} The confidence interval for *i*th response in *j*th scenario.

$\mathrm{D}_{Rij}^{min}(x)$ Minimum desirability value in confidence interval for *i*th response in *j*th scenario.

$\mathrm{D}_{Rij}^{max}(x)$ Maximum desirability value in confidence interval for *i*th response in *j*th scenario.

$\mathrm{D}_{Rij}^{total}(x)$ Total desirability of confidence interval for *i*th response in *j*th scenario.

f_{d_i} Desirability function of the *i*th response.

If the model consists of several independent responses, there should be a method to trade-off between these responses. To achieve this goal, the desirability function method proposed by (Derringer & Suich,1980) and (Derringer, 1994) can be applied. The desirability function involves transformation of each estimated response variable $\hat{y}_i(x)$ to a desirability value d_i, that d_i varies over the interval [0 1]. The value of d_i increases as the desirability of *i*th response variable increases. According to this method, the design variables are chosen to maximize the following equation:

$$\mathrm{D}(d_1[\hat{y}_1(x)], d_2[\hat{y}_2(x)], \dots, d_l[\hat{y}_l(x)] = (\prod_{i=1}^{l} d_i[\hat{y}_i(x)]^{w_i})^{1/\sum w_i}, \quad (1)$$

Since constructing a highly accurate model is too costly, facing unexpected noise in experiments is inevitable. Hence, it is necessary to consider uncertainties in Derringer's method to get more reliable solutions. In order to deal with uncertainties, itis needed to apply robustness approach to Derringer traditional method by using the (*1-α*) confidence region for output responses.

(He et al.,2012)explained how to construct (*1-α*) confidence region on the response y(x) using the following equation:

$$[y_L(x), y_U(x)]=[\hat{y}(x) \pm t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 z(x)'(C'C)^{-1}z(x)}, \quad (2)$$

$$z(x) = (1, x_1, \dots, x_k, x_1^2, \dots, x_k^2, x_1x_2, \dots, x_{k-1}x_k)', \quad (3)$$

where $t_{\alpha/2,n-p}$ is the $(1-\alpha/2)$ quantile for the *t* distribution with (*n-p*) degrees of freedom. Since it is supposed that there are *l* independent responses, the confidence region should simultaneously be constructed for all of them.

Here we suppose there are some different scenarios and each one includes some independent responses. Desirability of each response is defined by a certain function in a confidence interval calculated by Equation (2). Then, instead of Equation (1) that only takes worst case into account we consider three measures for desirability of each response. First two measures are the worst and the best desirability values and the third one is total desirability of related function in the calculated confidence interval.

$$\mathrm{D}_{Rij}^{min}(x) = \min_{\eta_{ij}}\{f_{d_i}(y_{ij}) \,|\eta_{ij} \in [y_{ij}^L(x), y_{ij}^U(x)]\}, \quad (4)$$

$$\mathrm{D}_{Rij}^{max}(x) = \max_{\eta_{ij}}\{f_{d_i}(y_{ij}) \,|\eta_{ij} \in [y_{ij}^L(x), y_{ij}^U(x)]\}, \quad (5)$$

$$\mathrm{D}_{Rij}^{total}(x) = \int_{y_{ij}^U(x)}^{y_{ij}^L(x)} f_{d_i}(y_{ij})\,dy_{ij}. \quad (6)$$

Now we consider each of these triple parameters as a goal and a goal programming is needed to be constructed and solved. Therefore, we apply the maximum value of each term of the objective function in total objective function which means that the goal of $\mathrm{D}_{Rij}^{min}(x)$ and $\mathrm{D}_{Rij}^{max}(x)$ is value of 1 where the objective function has been forced to choose the point with nearer value to 1. And the last term due to its negative sign should be maximized. In the other words, the key point is to consider all scenarios in the same time as depicted in FIG 2.

As it is mentioned former, we are looking for maximizing the minimum, maximum and total desirability of each response in the related confidence interval simultaneously by using ICA. Each of them is considered as an objective, so the integrated objective function can be formulated as follows:

$$D = Min \sum_{j=1}^{J}\sum_{i=1}^{l} w_i[(1-\mathrm{D}_{Rij}^{min}(x)) + \left(1-\mathrm{D}_{Rij}^{max}(x)\right) - \alpha\left(\mathrm{D}_{Rij}^{total}(x)\right)], \quad (7)$$

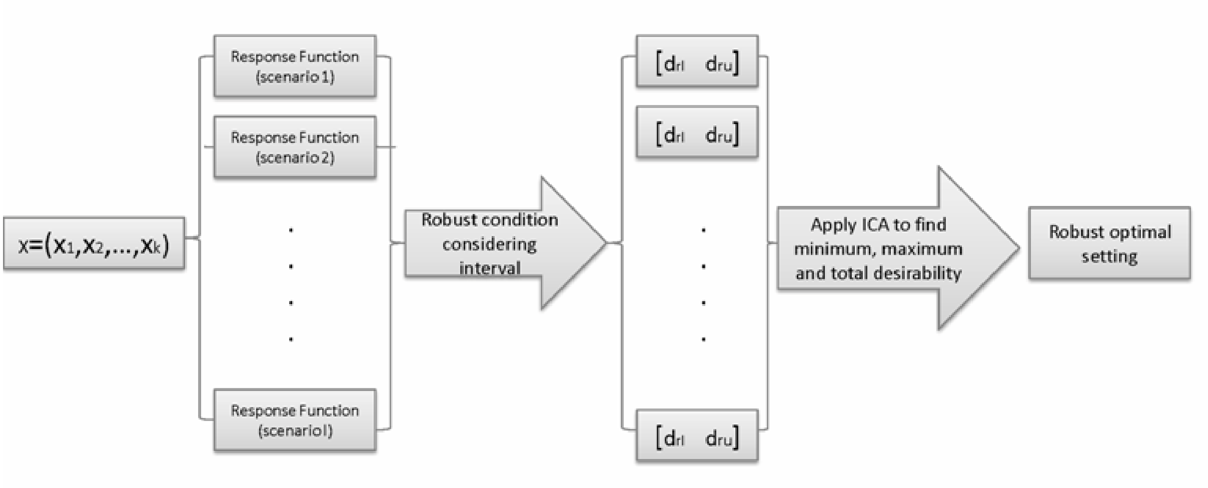

Fig. 2. Flow chart of the proposed robust optimal desirability approach

Adding the robustness feature into classical desirability function method makes the problem more complex in computation that increases the computational time (Heet al., 2012). Moreover, the presented problem includes simultaneous optimization of multiple responses in a multi scenario production, which increases the complexity of the problem. In recent years, various population based optimization method such as Tabu search, genetic algorithm (GA), particle swarm optimization (PSO), differential evolution (DE), Simulated annealing (SA) and ICA have been applied to solve multi objective optimization problems. The performance of ICA has been consistently reinstated by successful application in many different industrial engineering areas (Ghasemi, 2014). Among these algorithms, GA is one of the algorithms which is widely used in recent papers in different fields of study. (Bashiri, 2013) shows that in multi response optimization, ICA has better performance in comparison with GA. Measures related to the extracted non dominated solution in their research indicated that ICA is more efficient compared to GA.

To overcome this difficulty, we employ ICA to the procedure of finding overall desirability by using the minimum, maximum and the total desirability in confidence region. ICA is a new evolutionary algorithm for optimization which is inspired by imperialistic competition. Like the all other evolutionary algorithms, this one starts with an initial population which is called country. In this paper, ICA algorithm starts with 380 initial solutions that are called countries in this algorithm. Initial imperialists are chosen by random value of robust optimal function for each country. The competition among these empires is the base of this evolutionary algorithm. During this competition, weak empires collapse and powerful ones take possession of their colonies. Imperialistic competition hopefully converges to a state in which there exist only one empire and its colonies are in the same position and have the same cost as the imperialist.

The pseudo code of ICA based on (Atashpaz, 2007) is as follows:

1- Selecting random points to create initialized empires

2- Make colonies similar to their related imperialist

3- Exchange the position of colony and the imperialist under the special condition

4- Computing the total cost of empires

5- Give the weakest colony of weakest empire and give it to nearest empire

6- Omit the weak empires

7- If there is just one empire, stop, if not go to 2.

IV. NUMERICAL EXAMPLE

In this section, we use hypothetical production scenarios. In this example, the operator is interested in finding the optimal and robust setting of design variables for three lines of a company under three different production scenarios. Therefore, we shall find the global optimal and robust point to set the design variables.

TABLE II. Experimental design and response values of the numerical example

	Natural variables		**Coded variables**		**Responses**								
					Scenario 1			**Scenario 2**			**Scenario 3**		
	ζ_1	ζ_2	x_1	x_2	y_1	y_2	y_3	y'_1	y'_2	y'_3	y''_1	y''_2	y''_3
1	80	170	-1	-1	76.5	62	2940	79.1	68.6	3378.3	84.9	68.5	3377.2
2	90	170	1	-1	78	66	3680	81.7	69.7	3374.5	79.9	70.9	3368.4
3	80	180	-1	1	77	60	3470	74.3	63.1	3379.7	79.7	63.1	3379.6
4	90	180	1	1	79.5	59	3890	76.9	71.5	3383.5	83.9	66.3	3370.8
5	77.93	175	-1.414	0	75.6	71	3020	76.2	64.8	3379	82.4	65.2	3380.2
6	92.07	175	1.414	0	78.4	68	3360	79.8	71.6	3379	81.9	69.2	3367.8
7	85	167.9	0	-1.414	77	57	3150	81.4	69.6	3375.3	82.6	70.7	3372.3
8	85	182.0	0	1.414	78.5	58	3630	74.6	66.9	3382.7	81.7	63.7	3375.7
9	85	175	0	0	79.9	72	3480	72.4	68.2	3401.3	82.1	67.2	3374
10	85	175	0	0	80.3	69	3200	74.8	72	3394.9	79.1	53.8	3400
11	85	175	0	0	80	68	3410	82.3	66.3	3420.3	81.3	60.3	3480
12	85	175	0	0	79.7	70	3290	78	69.8	3900.2	78.4	68.2	2900
13	85	175	0	0	79.8	71	3500	79.7	70.1	3379	79.1	67.7	3900

This empirical example for three production scenarios and three response variables in each scenario which is extracted from (Montgomery,2005) with some modifications for our problem. The experimenter wants to determine the optimal and stable condition that optimizes the response variables of all the scenarios at the same time.

$[y_1, y_2, y_3]$, $[y'_1, y'_2, y'_3]$ and $[y''_1, y''_2, y''_3]$ are the response variables of the first, the second and the third scenarios, respectively. Two controllable variables influence these responses: their coded variables are shown by (x_1) and (x_2). x_1 and x_2 are controllable variables that should be fixed for all scenarios and different values of them result in different desirability values. Prior experiment reveals that the optimum is likely to be reached between 80 and 90 min for x_1 and between 170 and 180 °F for x_2. The experimental design and their correspondence response values have been reported in TABLE 2.

The fitted models of coded variable for all scenarios are as follows which have been achieved by analysis of CCD (Central Composite Design) experimental design:

First scenario:

$\hat{y}_1$(x)=79.94+0.995x_1+0.515x_2−1.376x_1^2−1.001x_2^2+0.25x_1x_2 ($\hat{\sigma}_1$=0.266).
$\hat{y}_2$(x)=70−0.155x_1−0.948x_2−0.688x_1^2−6.688x_2^2−1.250 x_1x_2 ($\hat{\sigma}_1$=2.275).
$\hat{y}_3$(x)=3386.15+205.1x_1+177.35x_2 ($\hat{\sigma}_1 = 165.63$).

Second scenario:

$\hat{y}'_1$(x)=77.444+1.299x_1−2.43x_2+0.27804x_1^2−0.27804 x_2^2 ($\hat{\sigma}'_1$=2.765).
$\hat{y}'_2$(x)=69.284+2.4001x_1−0.94988x_2−0.52708x_1^2−0.52708x_2^2+1.85x_1x_2 ($\hat{\sigma}'_1$=1.63).

$\hat{y}'_3$(x)=3499.2−2.6083x_2−60.087x_1^2−60.087x_2^2 +1.9x_1x_2 ($\hat{\sigma}'_1$=169.93).

Third scenario:

$\hat{y}''_1$(x)=80.22−0.20007x_1−0.2999x_2+1.05924$x_1{}^2$+1.0592$x_2{}^2$+2.3x_1x_2 ($\hat{\sigma}''_1$=2.296).
$\hat{y}''_2$(x)=63.434+1.4001x_1−2.5x_2 +1.8833$x_1{}^2$+1.8833$x_2{}^2$+0.2x_1x_2 ($\hat{\sigma}''_1$=4.74).

$\hat{y}''_3$(x)=3410.8−4.3924x_1+1.2011x_2−18.403$x_1{}^2$−18.403 $x_2{}^2$($\hat{\sigma}''_1$=251.618).

Previous studies like (Jeong, 2003) assumed that the desirability function is linear; while, in some cases the desirability function can be nonlinear and as a predefined function, hence, in this study we assume that each response has different desirability function. Suppose that the desirability function of each response is as follows:
$f_1(y) = -0.012346{y_1}^2 + 2y_1 - 80.2$,

$f_2(y) = -0.0044444{y_2}^2 + 0.57778y_2 - 17.78$,

$f_3(y) = -2.7778 \times 10^{-6}{y_3}^2 + 0.018889y_3 - 30.781$,

Also, it is assumed that the weights of responses are $w_1 = 0.3$, $w_2 = 0.4$ and $w_3 = 0.3$, respectively.

Results Analysis by the Solution Approach

First and second parts of the objective function minimize the undesirability of $\mathrm{D}_{Rij}^{min}(x)$ and $\mathrm{D}_{Rij}^{max}(x)$ in confidence interval. The last part of the objective function maximizes$\mathrm{D}_{Rij}^{total}(x)$.

The computation results have been shown in Table 3. The results show that choosing x_1=0.89118 and x_2=−0.95254 leads to the best overall desirability considering all scenarios with objective function value of 0.27512.

Also, the process of improving objective function and position of the best empire in ICA has been displayed in Fig. 3. . We applied Taguchi design to tune ICA . For ICA , the experiments involves varying number of decades, number of colonies and number of initial imperialists. We tested each parameter at three levels of (30, 40, 50) for number of decades, (280,330,380) for number of colonies and (6, 10, 15) for number of initial Imperialists. Finally, based on the obtained results from Taguchi design, number of decades, colonies and initial imperialists are set to 50, 380 and 6, respectively.

TABLE III. The best maximum, minimum and total desirability of scenarios in related interval

	Scenario 1			Scenario 2			Scenario 3		
	Resp.1	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**
$\mathrm{D}_{Rij}^{max}(x)$	0.7178	1	0.67	0.8	0.9589	0.67	0.8	1	0.67
$\mathrm{D}_{Rij}^{min}(x)$	0.6755	0.9662	0.5746	0.6503	0.8014	0.5673	0.6580	0.5193	0.4082
$\mathrm{D}_{Rij}^{total}(x)$	0.4149	5.039	236.46	4.6929	3.2453	241.96	3.9264	8.9366	334.8

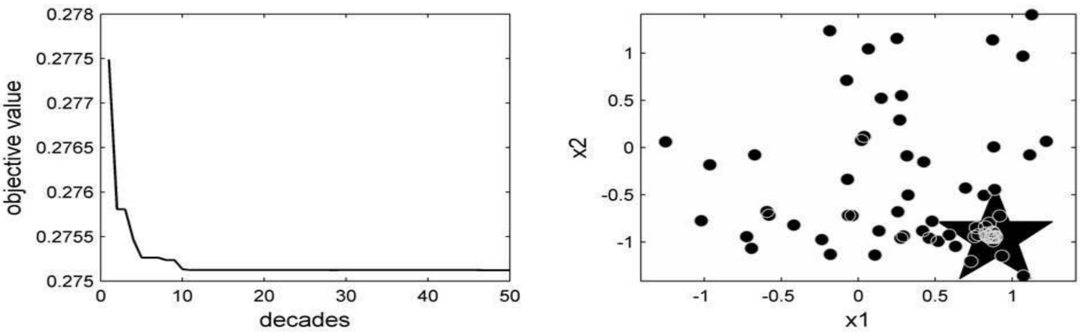

Fig. 3. Process of improving objective function and position of the best empire in ICA

TABLE IV. The best maximum, minimum and total desirability of scenarios in related interval using the optimum point of the third scenario. An '*' indicates the scenario that is optimized.

	Scenario 1			**Scenario 2**			**Scenario3***		
	Resp.1	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**
$D_{Rij}^{max}(x)$	0.5257	1	0.67	0.7319	1	0.67	0.8	1	0.67
$D_{Rij}^{min}(x)$	0.4396	0.8579	0.1850	0	0.9344	0.5002	0.6907	0.7774	0.3364
$D_{Rij}^{min}(x)$	0.3333	5.6265	219.97	2.2438	4.1389	274.32	4.5423	11.547	371.87

In order to show that it is necessary to solve the problem considering the whole scenarios in the same time, three problems have been solved. In each problem based on the former example, we just consider one of the scenarios and try to find its optimum solution . Then, the maximum, minimum and total desirability of other two scenarios have been computed using the calculated solution. Results of solving problem by considering the optimized settings of third scenario have been illustrated in Table 4.

Results indicate that desirability of some of responses in optimized scenario has increased but it causes significant decreasing in desirability of other scenarios (Fig. 4,Fig. 5). Since desirability has increased in some responses and has decreased in the others, it is necessary to compare the total objective value of these three problems in condition that all the scenarios are concerned in the same time. Table 5 shows the overall objective values of cases which have optimized one of the scenarios. In each column, the best setting related to one scenario is obtained and the objective value of the related scenario is reported in the second row. In the last row, the overall objective value of all three scenarios is reported by using the obtained setting. Obviously, using the design variables settings obtained from optimization of only one scenario has weak performance in the other two scenarios; that is why the values of the third row are greater than the second one. It shows the importance of considering optimization of all the scenarios in the same time.

Another issue is to show that why the total desirability of each response in the confidence interval has been used in the computation. Large value of total desirability in an interval shows that by slight change of the estimated response, probability of significant decrease in maximum and minimum desirability reduces. While the desirability problem is solved regardless of total desirability of points existed in the calculated interval, maximum and minimum of desirability may improve but total desirability of interval decreases. It amplifies probability of having less desirability by a slight change in estimated response. To show the mentioned point, our example is calculated regardless of total desirability in the interval.

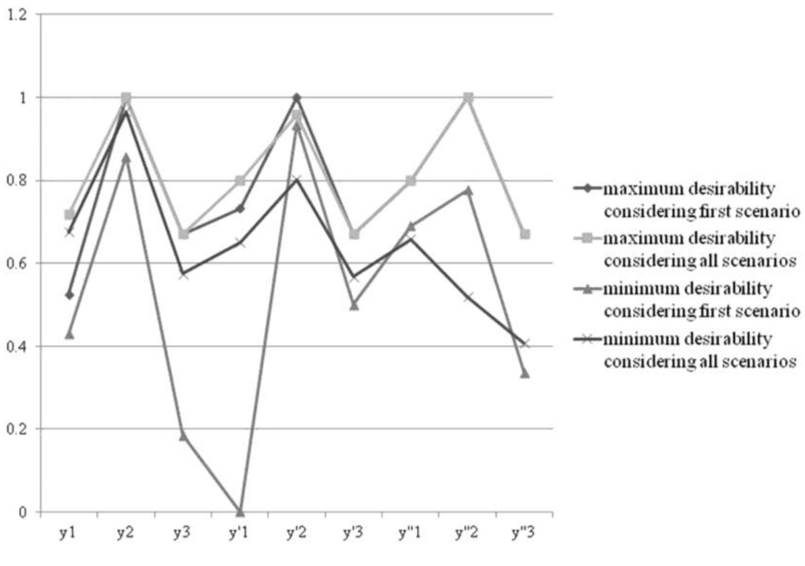

Fig. 4. Comparison of maximum and minimum desirability between considering all scenarios and considering the third scenario

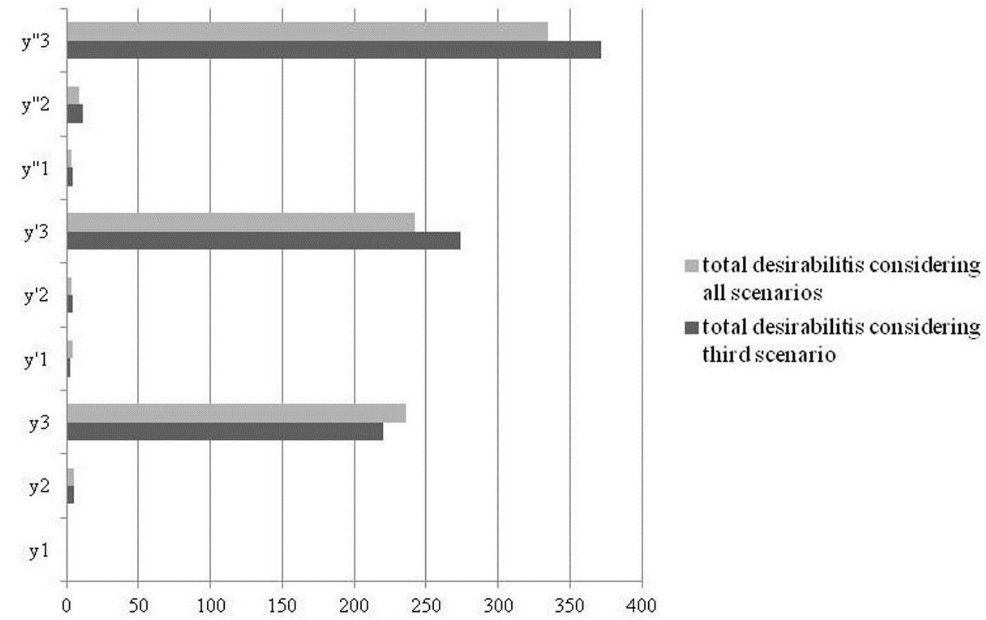

Fig. 5. Comparison of total desirability between considering all scenarios and considering the third scenario

TABLE V. Objective values of optimizing one scenario and total objective values using the optimum point of optimizing one scenario. An '*' indicates the scenario that have been optimized.

	Considering scenario 1*	**Considering** scenario 2*	**Considering** scenario 3*
Best point	x_1 =1.0185 $x_2 =$ −1.0074	x_1 =−0.36613 x_2 = −1.4132	x_1 =−1.3484 x_2 =0.49137
Overall objective value of considered scenario	0.047528	0.074369	−0.047864
Overall objective value	0.31427	1.06	0.64855

Then by using optimum points, the total desirability in each interval is computed. Table 6 shows the related results. It is clear that the total desirability is significantly less than the case which total desirability is considered in the objective (Fig. 6).

TABLE VI. Computing the maximum and minimum of desirability regardless of total desirability and then calculating the total desirability in the related interval.

	Scenario 1			**Scenario 2**			**Scenario 3**		
	Resp.1	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**
$\mathrm{D}_{\mathrm{Rij}}^{\mathrm{min}}(x)$	0.7662	0.9818	0.6668	0.7957	0.9560	0.67	0.8	1	0.67
$D_{Rij}^{min}(x)$	0.7493	0.8798	0.4750	0.5554	0.8693	0.5578	0.7682	0.9512	0.5787
$D_{Rij}^{min}(x)$	0.2816	2.9817	137.82	2.7277	2.0867	151.47	2.5317	6.5133	225.4

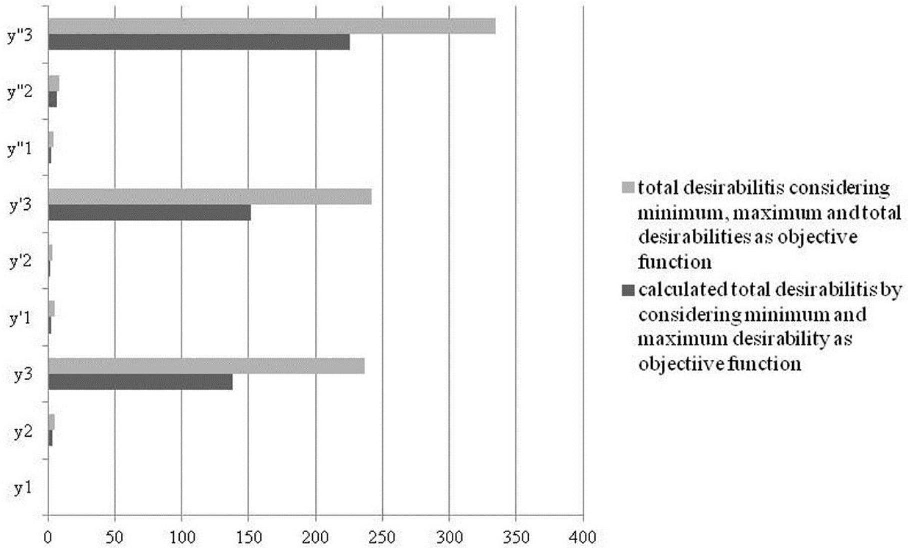

Fig. 6. Comparison between total desirability when it is considered as objective function or not

TABLE VII. Comparison between robustness of traditional desirability method and the proposed method

	Traditional method			**Proposed method**		
	Scenario 1			**Scenario 1**		
	Resp.1	**Resp.2**	**Resp.3**	**Resp.1**	**Resp.2**	**Resp.3**
$D_{Rij}^{min}(x)$	0.7884	0.9945	0.644	0.6946	1	0.67
$D_{Rij}^{min}(x)$	0.7757	0.8969	0.2970	0.6392	0.9498	0.5226
$D_{Rij}^{min}(x)$	0.3393	3.5454	136.19	0.4589	5.7966	268.2

From another point of view, we could use traditional desirability function method to maximize the overall desirability. However, using the traditional method (Heet al., 2012) causes the solution not to be robust. Let's consider the first scenario of our numerical example. By using traditional method we can get the maximized solution by setting $\mathbf{x_1 = 0.9262987}$and $\boldsymbol{x_1 = 0.3116185}$. But, the confidence region constructed using this point is not robust.

As indicated in Table 7, however, maximum of desirability is not less than the value computed using the the proposed method, the minimum and total desirability values in confidence region are significantly lower. In comparison to the traditional method, results show that considering the total desirability value of confidence interval leads to more robust design variable setting. This method uses the confidence interval to decrease the sensitivity of desirability function to slight changes in response variables. Therefore, by selecting the confidence interval that contains greater total desirability value for all points includes in the interval.

V. CONCLUSION AND FUTURE RESEARCH

One of the popular methods in optimization of multiple responses is the desirability function approach. The overall desirability is maximized in the multiple responses. In some cases, the multiple responses are as the same as different responses in different production lines while some controllable factors affect them directly. In this paper, a new approach was proposed for tackling with optimization of controllable factors in a process with multiple production scenarios. The numerical analysis confirmed that the proposed approach is more efficient than classical approaches which consider responses separately

An optimal robust desirability function method that contains model estimations uncertainty was proposed for multi response optimization. This method was a new version of classical desirability function that simultaneously gives the robust optimal setting. This approach added robustness to the desirability function by maximizing the minimum, maximum and average of desirability in a dynamic confidence interval to get the robust and optimal setting at the same time. The numerical analysis confirmed the efficiency of the proposed method to give the more stable and reliable solutions.

The solutions of this method had the uncertainty of setting the design variables in optimal condition. Therefore, few changes in these variables cannot make immense changes in desirability value.

In this paper, response variables are considered to be uncorrelated. If the model contains correlated responses, it would change the desirability function. Considering the correlated responses and handling the effect of their correlations in the desirability function can be mentioned as a future direction of this study.

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