Journal of Quality Engineering and Production Optimization
Vol. 1, No. 2, PP. 19-32,July – Dec. 2015

Coordinating a decentralized supply chain with a stochastic demand using quantity flexibility contract: a game-theoretic approach

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Abstract- *Supply chain includes two or more parties linked by flow of goods, information, and funds. In a decentralized system, supply chain members make decision regardless of their decision's effects on the performance of the other members and the entire supply chain. This is the key issue in supply chain management, that the mechanism should be developed in which different objectives should be aligned, and integrate their activities to optimize the entire system. Therefore, a coordination mechanism could be necessary to motivate members to achieve coordination. The contracts help the supply chain members to achieve coordination that will lead to improved supply chain performance. This paper analyzes a quantity-flexibility (QF) contract. The objective of this paper is to explore the applicability and benefits of the contracts, so to realize the importance of coordination by contracts, two cases have been studied. The first case is "no coordination" and the other case is "coordination with QF contract". Utilizing differential game theory, this paper formulates the optimal decisions of the supplier and the retailer in two different game scenarios: Nash equilibrium and cooperative game.It is expected that by designing the contracts as per the requirements of the supply chain members as well as the whole supply chain, supply chain performance can be improved.*

Keywords: *Decentralized supply chain, supply chain coordination, quantity flexibility contract, game theory*

I. INTRODUCTION

Flexibility is a term often used for companies that have to cope with uncertainty. (Tibben-Lembke, 2004) stated that supply chain flexibility has become more important as demand becomes more uncertain, and concluded that flexibility in supply chain contracts has proven to be a fertile area for research. A supply chain contract is a coordination mechanism that provides incentives to constituent actors so that the decentralized supply chain behaves like an integrated one and benefits from improved operational performance (Wang, 2010). Returns of product from customers to retailers are a common feature of competitive markets. Some consumers return products that perform unsatisfactorily while others return products that function satisfactorily for other reasons, such as not meeting expectations or tastes. The volume of returns in North America is significant and growing. (Chen, 2011) stated that ‘‘returned goods are estimated to exceed $100 billion per year in United States and in many categories, the number of returns is growing at better than 50% a year.’’ Therefore, quantity flexibility and buy back contracts are very popular in real world. This paper analyzes a supply chain contract model called the quantity-flexibility (QF) contract. In the quantity flexibility contract version considered here, the supplier offers to buy back unsold units from the retailer at the wholesale price. However, there is an upper limit to the amount of unsold products to be returned by the retailer. (Pasternack , 1985) was

Manuscript Received: 01-Dec-2014 and revised: 10- June-2015 *ISSN: 2423-3781*
Accepted on: 15-Oct-2015

the first to analyze BB and QF contracts. This study shows that neither full returns with full buy back credit nor no returns is system optimal. Then, numerous papers on BB and QF contracts have been appeared. (Padmanabhan & Png, 1995) investigated advantages and disadvantages of some return policies as well as the motivations why the return policies were carried out. (Tsay,1999) studied a QF contract in a more complicated model, which has incorporated the issues of capacity planning, information updating, and supply chain coordination carried out. (Arshnider et al., 2009) mentioned the usage of quantity flexibility contract in different types of industries as shown in Table 1. Also, according to (Lovejoy, 1999), quantity flexibility contracts have been used by Toyota and by Nippon Otis, a manufacturer of elevator equipment. (Connors et al., 1995) mentioned that quantity flexibility contracts have been used by IBM.

Preliminary research on the buyback contract includes those of (Pasternak et al., 1998). Emmons and Gilbert (…) have studied a model of price dependent demand, in which buyback policies would benefit both the retailer and the manufacturer. The authors have showed while the wholesale price is in a certain range, the manufacturer chooses a positive buyback price over not offering a buyback price. (Mahajan, 2010) has used the same price dependent demand model of Emmons and Gilbert (…) and considered a revenue sharing contract. It was illustrated that in this state, there is a positive revenue sharing ratio that the manufacturer would prefer. Finally, (Wang, 2007) has considered a model by considering effort dependent demand with buy back contract. The study considered a supply chain with a risk-neutral manufacturer and a loss-averse retailer. A class of distribution free coordinating contracts was identified by combining gain/loss-sharing and BB contracts. (Gan et al., 2005) proposed a risk sharing contract to coordinate a supply chain with a downside-risk-averse retailer. Interestingly, the contract proposed here is also a composite contract based on a BB contract and a QF contract.

(Xie & Wei, 2009) proposed two models where consumer demand is determined by retail price and cooperative advertising efforts by channel members. They presented a bilateral monopoly model, in which one manufacturer sells through one retailer. They formulated the models in two different game scenarios: Stackelberg-manufacturer game and cooperative game. (Seyed esfahani, 2011) was closely related to (Xie & Wei, 2009)' study just mentioned, with a different demand-price functions. (Kim, 2011) analyzed a quantity flexibility contract between a customer and a supplier, and demonstrated the supplier's trade-off between the customer service level and the inventory risk whereas for the customer, the benefit keeps increasing and then remains constant as the flexibility rate increases. In general, the author stated that in a decentralized system, the quantity flexibility contract could provide an effective coordination mechanism for the supply chain. (Knoblich, 2015) evaluated the rolling horizon flexibility (RHF) contracts (a type of quantity flexibility contract used in the semiconductor industry to coordinate production and demand remains meager) to understand better how to set the quantity flexibility clauses in order to minimize inventory and maximize delivery performance (DP) and delivery reliability (DR), performance measures used by the supplier.

In this paper, a relatively general demand function is proposed. In addition, we investigate one cooperative and one non-cooperative game theoretic model. The major differences between this paper and two related studies mentioned above are as follows:

TABLE I. Usage of quantity flexibility contract

Type of industry	**Advantages to buyer**	**Advantages to supplier**
Computer manufacturers, electronic goods, auto industry	- The buyer's order quantity more in line with actual demand - It increases the profits of buyer -The supplier formally guarantees the buyer a specific safety cushion in excess of estimated requirements -It helps in sharing part of inventory and stock out cost burden with supplier -The buyer gets full protection on unsold but committed order quantity	-It reduces the over stock burden -It increases the profits of supplier -It improves the planning capability -The minimum purchase agreement by buyer shifts some of the demand risk downside

- Different demand-price and advertising-functions

- Using a contract (quantity flexibility contract) to coordinate supply chain members.

In this paper, the both cases have been studied to realize the importance of coordination by contracts; the models are solved in two solutions using game theory "cooperative and Nash equilibrium". Numerical results are presented to clearly show the effect of coordination on supply chain performance. Numerical results indicate that the contracts can increase profit of supply chain.

This paper is organized as following; in section 2, the model is formulated; in section 3, the solution for the model is presented; in section 4, numerical samples are solved, and finally, conclusions are presented in section 5.

II. MODEL ASSUMPTIONS AND NOTATIONS

We consider a single period, single product model with a supplier and a retailer. The retailer faces a random, advertising and price dependent demand. We consider the case where the selling season of this product is short. At the end of the selling season, the retailer can return the ratio of unsold unit to the supplier on wholesale price. The following notation will be used in the formulation:

Let p to be as the retail price, w as the wholesale price, c as the supplier's manufacturing cost, q as the retailer's order quantity and e as the advertising cost.

The unit cost and unit price should follow the following constraints because of the value addition at different stages of supply chain:

$c(\text{production cost}) \leq w(\text{wholesale price}) < p(\text{price of product})$

And the other notations are as follows:

D(p,e): Annual demand (depend upon price and advertising)

$E(\pi_s)$: Expected profit of supplier

$E(\pi_r)$: Expected profit of retailer

$E(\pi_{sc})$: Expected profit of whole supply chain $(E(\pi_r) + E(\pi_s))$

γ: Ratio of unsold unit that retailer returns to supplier at the end of period $(0 \leq \gamma \leq 1)$

The demand is stochastic and depends on price and advertising and considered the same for comparing two cases.

$$D(p,e) = \alpha - \beta p + \delta e + \varepsilon \quad (1)$$

α: The initial demand

$\beta$and δ: Sensitivity coefficients of demand to price and advertising (respectively)

ε: Random variable with uniform distribution on the interval $[a, b]$

III. MATHEMATICAL MODELING

The contracts help the supply chain members to achieve coordination which will lead to improve supply chain performance. To realize the importance of coordination by contracts, two cases have been studied. The first scenario is "no coordination" in which the S.C. members act independently, and the other case is "coordination with quantity flexibility contract". The various performance measures of the first scenario are compared with the second one.

3.1. No coordination

In this case, there is no coordination between the retailer and supplier. The retailer determines his/her optimal order quantity and the supplier provides the order, and does not perform any efforts to encourage the retailer make any more orders.

3.1.1. Profit function of retailer

In this case, the retailer's revenue is derived from selling the product to customers, and his/her costs are expenses of advertising and expenses of buying the product.

$$\pi_r = \begin{cases} pD - wq - e a \le D < q \\ \\ pq - wq - e q \le D \le b \end{cases} \quad (2)$$

As mentioned in the assumptions, the demand function is considered to be dependent upon price and advertising; hence, "*D*" is replaced with equation (1) in the retailer's expected profit function.

$$E(\pi_r) = \int_a^q p(\alpha - \beta p + \delta e + x) f(x) d(x) + \int_q^b pq f(x) d(x) - wq - e \quad (3)$$

In this problem, "x" has uniform distribution in the interval [a, b]. Thus, in equation (3), $f(x)$ equals to $\frac{1}{b-a}$. If integration in equation (3) is done, the retailer's expected profit function will be obtained as the following.

$$E(\pi_r) = \frac{1}{b-a}\left[pq(b-q) + p\left[(\alpha - \beta p + \delta e)(q-a) + \frac{(q-a)^2}{2}\right]\right] - wq - e \quad (4)$$

3.1.2. Profit function of supplier

The supplier provides the retailer's order, so his/her revenue is earned from selling the product to the retailer. The equation is formulated as the following:

$$E(\pi_s) = q(w - c) \quad (5)$$

3.2 . Coordination with quantity flexibility contract

In this type of contract, the retailer returns the ratio (γ) of unsold unit to supplier at the end of period, and supplier charges wholesale price (w) for each unit. The profit equation can be formed by looking at the demand, whether the demand is between the lower limit and upper limit of quantity commitment or not. In case of "$D < (1-\gamma)q$", retailer returns "γq" to supplier and earns "w" for each unit, and in case of "$(1-\gamma)q \le D < q$", the unsold units at the end of period equals to"q-D" which is lower than "γq"; hence, retailer returns all of it to the supplier.

$$\pi_r = \begin{cases} pD + w\gamma q - wq - e a \le D < (1-\gamma)q \\ pD + w(q-D) - wq - e(1-\gamma)q \le D < q \\ pq - wq - e q \le D \le b \end{cases} \quad (6)$$

$$\pi_s = \begin{cases} wq - cq - w\gamma q a \le D < (1-\gamma)q \\ wq - w(q-D) - cq(1-\gamma)q \le D < q \\ wq - cq q \le D \le b \end{cases} \quad (7)$$

Just as in previous scenarios, Eq.1 is replaced with "*D*"; therefore, the expected profit function of retailer and supplier is formulated as per Eq. (8) and (9).

$$E(\pi_r) = \frac{1}{b-a}\left[pq(b-q) + p\left[(\alpha - \beta p + \delta e)(q-a) + \frac{(q-a)^2}{2}\right] + w\gamma q[(1-\gamma)q - a] + w\left[\gamma q^2 - \left[\gamma q(\alpha - \beta p + \delta e) + \frac{\gamma^2 q^2}{2}\right]\right]\right] - wq - e \tag{8}$$

$$E(\pi_s) = q(w-c) + \frac{w}{b-a}\left[-[\gamma q(1-\gamma)q - a] + \left[-\gamma q^2 + \left[\gamma q(\alpha - \beta p + \delta e) + \frac{\gamma^2 q^2}{2}\right]\right]\right] \tag{9}$$

IV. SOLUTION

In this section, two game-theoretic models based on non-cooperative games including Nash and a cooperative one is discussed. Solution methodology is described in Figure 1.

4.1. Nash game

When the supplier and the retailer have the same decision power, they determine their strategies independently and simultaneously. This situation is called as a Nash game and the solution to this structure is the Nash equilibrium (Seyed esfahani et al., 2010). The results of solving the model for three scenarios are as below.

4.1.1. No coordination

Solution methodology is as follows:

I. The amount of "q" is obtained by differentiating the profit function of retailer

II. "q" is replaced in profit function, then "p" & "e" are obtained by differentiating the profit function

III. The decision variables of supplier are obtained by letting the decision variables of retailer from Nash, equal to cooperative.

The results of solving the model are as follows:

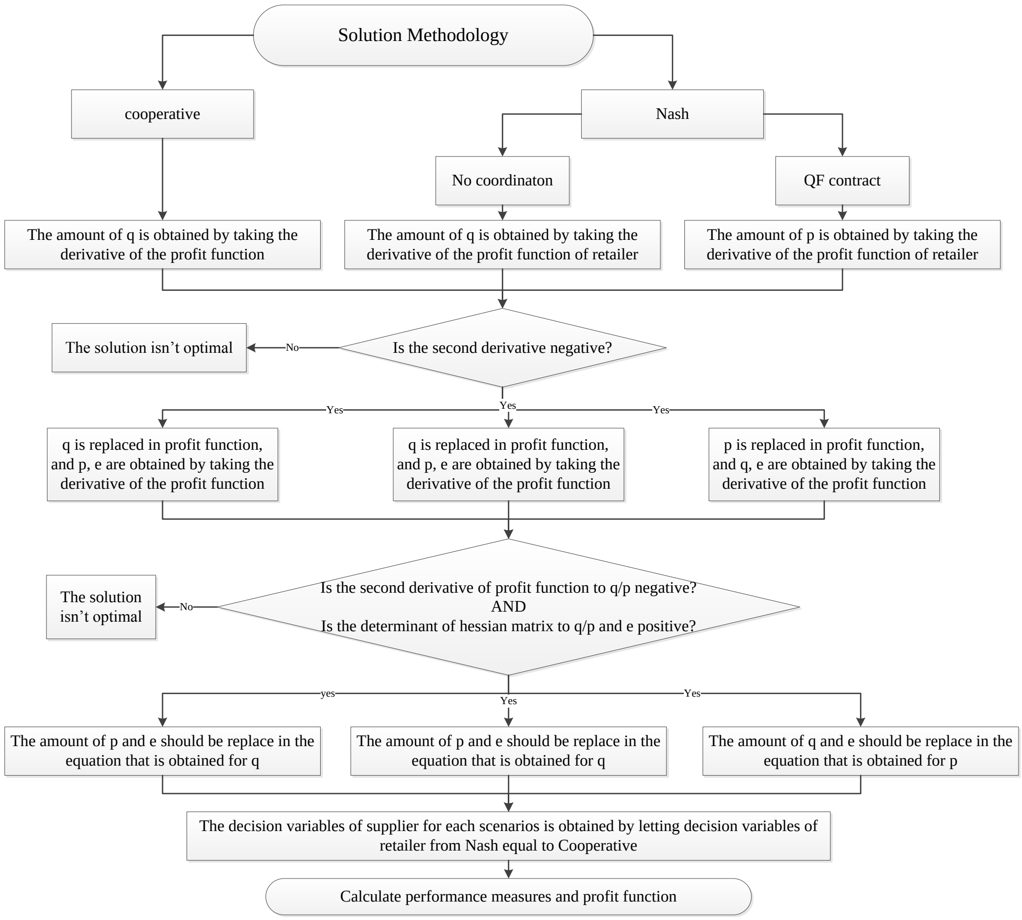

Fig. 1. Solution Methodology

$$p^* = \frac{\sqrt{2\delta(\delta a - \beta)(2w\delta + 1)(a - b)}}{2\delta(\delta a - \beta)} \tag{10}$$

$$\begin{aligned} e^* = \frac{-1}{2\delta\sqrt{\delta(\delta a - \beta)(2w\delta + 1)(a - b)}} \Big[\Big(&2a^2\delta^2 w + 2a^2\delta - 2a\delta^2 wb - 4\delta\beta wa - 2ba\delta - 3a\beta \\ &+ 2a\sqrt{2\delta(\delta a - \beta)(2w\delta + 1)(a - b)} + 4wb\beta\delta \\ &- b\sqrt{2\delta(\delta a - \beta)(2w\delta + 1)(a - b)} - \alpha\sqrt{2\delta(\delta a - \beta)(2w\delta + 1)(a - b)} \\ &+ 3\beta b\Big)\sqrt{2}\Big] \end{aligned} \tag{11}$$

Eventually, we should replace Eqs. (10) and (11) in the amount obtained for q:

$$q^* = -\frac{\sqrt{2}\,(\delta a - \beta)\left[\frac{-\sqrt{2\delta(\delta a - \beta)(2w\delta + 1)(a - b)}}{2(\delta a - \beta)} + b - a\right]}{\sqrt{\delta(\delta a - \beta)(2w\delta + 1)(a - b)}} \tag{12}$$

4.1.2. Coordination with Quantity flexibility contract

The retailer's decision variables can be calculated as following:
First, differentiate the retailer's expected profit function in terms of "p"; then, replace "p" in Eq. (8) and consequently calculate "q" and "e".

$$\begin{cases} M = -6w\gamma\delta^2 + 6w^2\gamma^2\delta^2 \\ J = 2wb\delta^2 + b\delta - \delta a - 12w\gamma^2 a\delta^2 - 2wa\delta^2 + 17a\delta^2 w\gamma - 2w\gamma b\delta^2 \\ K = -16w\gamma a^2\delta^2 + 4abw\gamma\delta^2 + 2a^2\delta + 4wa^2\delta^2 - 4wba\delta^2 - 2ab\delta + 6wa^2\gamma^2\delta^2 \\ L = -2\beta b^2 - 2\beta a^2 - 2w\gamma a\beta\delta b - 3a^2 b\delta - 2wa^3\delta^2 + 4\beta ba + 2ab^2\delta + 3w\gamma a^3\delta^2 + 2wba^2\delta^2 \\ \qquad + a^3\delta + 2w\gamma\delta\beta a^2 \end{cases} \tag{13}$$

The value of "q" is obtained by solving the following equations.

$$q^* \rightarrow Mq^3 + Jq^2 + Kq + L = 0 \tag{14}$$

$$p^* = \frac{w\gamma q\delta + b - a}{\delta(q - a)} \tag{15}$$

$$e^* = \frac{\delta q^2 + 2w\gamma q\beta\delta + 2qa\delta - 2qb\delta - a^2\delta - 2\alpha q\delta - 4\beta a + 2\alpha a\delta + 4\beta b}{2(q - a)\delta^2} \tag{16}$$

4.2. Cooperative game

In the previous sub-section, a non-cooperative game was discussed. Now, we model the supplier–retailer relationship as a cooperative game in which both channel members agree to cooperate and maximize the profit of the whole system.

The cooperation expected profit function is the same as Eq. (17)

$$\pi_{sc} = q(w - c) - wq - e + \frac{1}{b - a}\left[pq(b - q) + p\left[(\alpha - \beta p + \delta e)(q - a) + \frac{(q - a)^2}{2}\right]\right] \tag{17}$$

$$q^* = -\frac{\left[b - a - \frac{a\sqrt{2\delta(a\delta - \beta)(2c\delta + 1)(a - b)}}{2(a\delta - \beta)}\right]\sqrt{2}(a\delta - \beta)}{\sqrt{\delta(a\delta - \beta)(2c\delta + 1)(a - b)}} \tag{18}$$

$$p^* = \frac{\sqrt{2\delta(a\delta - \beta)(2c\delta + 1)(a - b)}}{2\delta(a\delta - \beta)} \tag{19}$$

$$\begin{aligned} e^* = \frac{-1}{2\delta\sqrt{\delta(a\delta - \beta)(2c\delta + 1)(a - b)}} \Big[\Big(& 2a^2\delta^2 c + 2a^2\delta - 2acb\delta^2 - 2ba\delta - 4\delta\beta ca \\ & + 2a\sqrt{2\delta(a\delta - \beta)(2c\delta + 1)(a - b)} - 3\beta a + 4\beta\delta cb \\ & - b\sqrt{2\delta(a\delta - \beta)(2c\delta + 1)(a - b)} + 3\beta b \\ & - \alpha\sqrt{2\delta(a\delta - \beta)(2c\delta + 1)(a - b)}\Big)\sqrt{2}\Big] \end{aligned} \tag{20}$$

Proof for the optimality of the obtained solution is presented in the Appendix.

As we know, when the condition of a decentralized supply chain is closer to integration channel, it means supply chain members act more coordinated. Thus in this paper, we suppose the retailer's decision variables coincide with the decision variables of integrated channel. Hence, the supplier's decision variables for each scenario is obtained by letting the retailer's decision variables from Nash game equal to cooperative game as following:

$$\gamma^* = \frac{-1}{2\beta w\delta(w-4)}\Bigg(\sqrt{2}\Big(5\delta w\sqrt{\beta\delta(2\delta+1)} + \beta w\delta\sqrt{2} + 5w\sqrt{\beta\delta(2\delta+1)} - \Big(\beta\delta w\Big(50w\delta^3 + 125w\delta^2 - 40\delta w\sqrt{2\beta\delta(2\delta+1)} + 100w\delta + 2w\delta\beta + 15w\sqrt{2\beta\delta(2\delta+1)} + 25w + 40\delta\sqrt{2\beta\delta(2\delta+1)} - 20\sqrt{2\beta\delta(2\delta+1)} + 10\delta w^2\sqrt{2\beta\delta(2\delta+1)}\Big)\Big)^{\frac{1}{2}}\Big)\Bigg) \quad (21)$$

IV. NUMERICAL EXAMPLE AND ANALYSIS

First, we describe the experimental parameters used in the numerical analysis for the model. Then, we analyze these results and get better view over the contract properties for industrial practice.

Tables 1 and 2 show the improvements provided by contract in a supply chain performance. As indicated in the table, the improvements for both of the supplier and retailer can be considerable. For example, when "β" is 15 and "δ" is 10, for the case of "No coordination" the whole supply chain profit is 21.7208 and for the other scenario it equals to 37.07. So, obviously the expected profit of the whole supply chain increases when contracts are used. For example in the second case, the expected profit of the whole supply chain increases by 41.4%.

The experimental parameters used are as following. First, consider the parameters of "α"(the primary demand) and "β" (consumer's price-sensitivity). In the related literature, Yao (2008) considered three states for "α" and "β":

$$\begin{cases} \alpha = 20, \beta = 0.01, 0.1, 1.5, 2 \\ \alpha = 200, \beta = 5, 15, 25, 35, 45 \\ \alpha = 2000, \beta = 1, 10, 100, 200 \end{cases}$$

He proved that in case of "$\frac{\alpha}{\beta} < 15$", the results are difficult to interpret and if 200 units are added up to the upper bound of "$\frac{\alpha}{\beta}$", it will lead to very high retail price and very small order quantity. In this paper, we select the results calculated by letting "α = 200" and "β = 5, 15, 25, 35, 45" to analyze the performance of the different scenarios. The consumer's price-sensitivity (δ) is considered to be "10, 20, 30". In addition, we let the production cost as "c=1$/unit", the wholesale price as "w=2 $/unit" and the decision variable as "$\varepsilon \sim U[0,100]$". The results used to conduct our analysis have been tabulated in Table 2.

Let observe contracts behavior by comparing the results in Table 2. The supplier's profit and retailer's profit in the scenarios which the contract is considered is generally larger than the corresponding profit in the case of "no coordination". It means that both the retailer and the supplier have earned more profit than what they would earn in a "no coordination" case.

For each of scenarios, by increasing of "β", retailer's expected profit (π_r) decreases; on the other hand, by increasing of "δ", both "π_s" and "π_r" decrease, this means if the customer's sensitivity to advertising increases, retailer and supplier will achieve lower profit. By increasing "β", supplier's profit increases.

TABLE II. Optimal solutions of two cases

No coordination								Coordination with quantity flexibility					
β	δ	p^*	q^*	e^*	$E(\pi_r)$	$E(\pi_s)$	$E(\pi_{sc})$	w'^*	p^*	q^*	e^*	$E(\pi_r)$	$E(\pi_s)$	$E(\pi_{sc})$
5	10	6.4031	1.56	23.5	23.6	1.562	25.1586	0.47	5.59	2.15	25.48	25.43	10.667	36.092
	20	6.36	0.79	11.8	11.93	0.79	12.7232	0.48	5.53	1.09	12.64	12.61	5.5748	18.188
	30	6.35	0.52	7.87	7.735	0.52	8.25501	0.49	5.51	0.73	8.401	8.387	3.7745	12.161
15	10	3.7	2.71	18.8	19.01	2.71	21.7208	0.46	3.6	3.74	21.41	21.11	15.959	37.07
	20	3.67	1.36	9.45	9.455	1.36	10.8154	0.48	3.59	1.9	10.56	10.47	8.3619	18.829
	30	3.67	0.91	6.32	6.357	0.91	7.26747	0.49	3.59	1.27	7.003	6.958	5.6659	12.624
25	10	2.86	3.49	15.5	15.62	3.49	19.1145	0.46	2.98	4.84	18.24	17.86	18.129	35.987
	20	2.85	1.76	7.84	7.935	1.76	9.69505	0.48	2.99	2.46	8.957	8.841	9.5087	18.35
	30	2.84	1.17	5.25	5.233	1.17	6.40319	0.48	2.99	1.65	5.929	5.874	6.4447	12.319
35	10	2.42	4.13	12.9	13.05	4.13	17.175	0.45	2.64	5.74	15.43	15.05	18.909	33.96
	20	2.41	2.08	6.53	6.615	2.08	8.69469	0.47	2.67	2.91	7.548	7.433	9.9186	17.352
	30	2.4	1.39	4.38	4.406	1.39	5.79608	0.48	2.67	1.95	4.988	4.934	6.7222	11.656
45	10	2.13	4.69	10.6	10.77	4.69	15.4588	0.45	2.43	6.52	12.85	12.51	18.822	31.332
	20	2.12	2.36	5.4	5.461	2.36	7.82096	0.47	2.46	3.3	6.255	6.153	9.864	16.017
	30	2.117	1.57	3.62	3.649	1.575	5.22374	0.48	2.47	2.21	4.126	4.078	6.6825	10.761

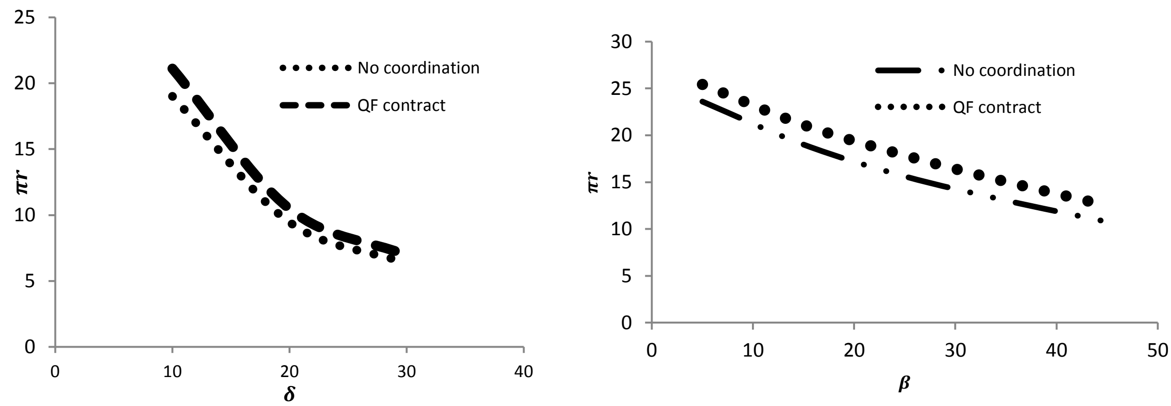

Fig. 2. The retailer's expected profit function in terms of "β" and "δ"

In Fig. 2, it can be seen that the expected profit function in terms of "β" and "δ"for retailer is descending, which means if "β" and "δ" rise, the expected profit function of retailer will decrease. However, the sensitivity of functions to "δ" is a bit more than "β".

The diagrams in Fig. 3 indicate the sensitivity of the supplier's expected profit function in terms of "β" and "δ". For "β", the expected profit function is ascending; but, for "δ" it is descending. Thus, we can conclude that the supplier's behavior in terms of different "β" and "δ" is not the same. Fig. 4 shows the simultaneous changes of the whole supply chain profit in terms of "β" and "δ" for the case of Quantity flexibility contract. It indicates how total supply chain profit looks like when "β" and "δ" change simultaneously. Therefore, from the table and figures it is obvious that the contract helps the SC members to achieve coordination by adjusting the order quantity and sharing risks and profits for the compensation of the adjustment in order quantity. Due to the same profit allocation, this contract leads to lower risks for respective supply chain members.

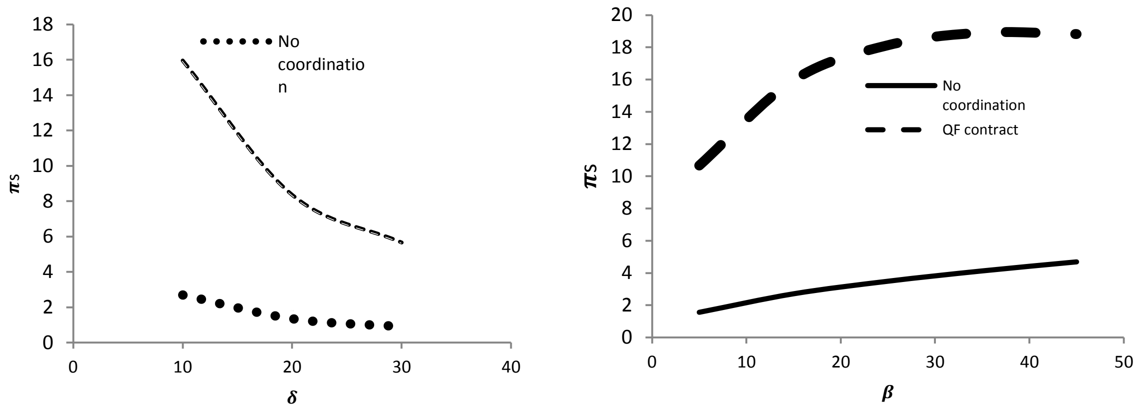

Fig. 3. The expected supplier's profit function in terms of "β" and "δ"

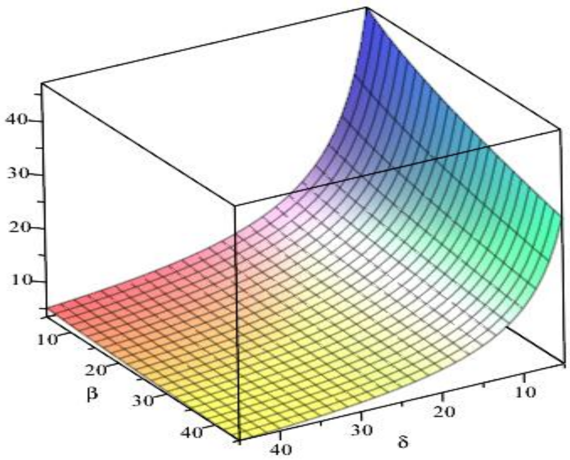

Fig. 4. The whole supply chain profit in terms of "β" and "δ" simultaneously

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we presented a decentralized two-level retailer-supplier supply chain with price and advertising dependent demand. A contract is assessed and different scenarios of coordination may be simulated, which may help in quantifying the performance measures and the effectiveness of coordination. The first scenario was "no coordination", and the other scenario was "coordination with quantity flexibility contract". It clearly shows that the contracts can increase the profit of supply chain members. The proposed models are solved using game theory approach, cooperative and Nash equilibrium. Eventually, the numerical results showed that contracts can make the profit functions much better and the average expected profit in the second case-coordination with quantity flexibility- was more than the other one. The supplier's behavior in terms of different "β" and "δ" is not the same, by increasing of "β" it rises and by increasing of "δ" it falls. This model is an extension of classical newsboy model, which can be applied to not only newspaper or books industry but such contracts are used by automobile and contract manufacturers, and are quite common in fuel oil and natural gas delivery markets. There are various opportunities for future research. This model is applicable to a single period; but, it can also be extended to multi-period. also in addition, in this paper the demand is assumed to be dependent upon the retail price and advertisement; we may relax this assumption in future and analyze supply chains where demand depends on price, quality and some other factors. In this paper, we have used the uniform function for demand; so, in the future researchers we can employ other functions, such as normal and exponential. Researchers can extend these models to three-echelon supply chains. They can consider the supply uncertainty that is a very attractive and unfamiliar assumption in this field.

REFERENCES

1. Arshinder, A. K. (2009). A framework for evaluation of coordination by contracts: A case of two-level supply chains .*Computers & Industrial Engineering, 56,* 1177-1191.
2. Chen, J, Bell, P. (2011). Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy, *European Journal of Operational Research, 212*, 293–300.
3. Connors, A. F., et al. (1995). A controlled trial to improve care for seriously iII hospitalized patients: The study to understand prognoses and preferences for outcomes and risks of treatments (SUPPORT). *Jama, 274(20),* 1591-1598.
4. Emmons, H., (1998), The role of returns policies in pricing and inventory decisions for catalogue goods *Management Science, 44*, 276-283.
5. Gan, X., Sethi, S. P., Yan, H. (2005). Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer. *Production and Operations Management, 14(1)*, 80-89.
6. Kim, W. (2011). Order quantity flexibility as a form of customer service in a supply chain contract model. *Flexible Service and Manufacturing Journal, 23*, 290–315.
7. Knoblich K. , Heavey, C., Williams, P., . (2015). Quantitative analysis of semiconductor supply chain contracts with order flexibility under demand uncertainty: A case study, *Computers & Industrial Engineering, 87*, 394–406.
8. Lovejoy, W.S., Tsay, A. (1999). Quantity flexibility contracts and supply chain performance. *Manufacturing & Service Operations Management*, *1(2),* 89-111.
9. Mahajan, S, (2010). A quantity flexibility contract in a supply chain with price dependent demand. *IIMB working paper*, 304-323.
10. Padmanabhan, V., Png, I.P.L.(1995). Returns policies: Make money by making good. *Sloan Management Review Fall*, 65–72.
11. Pasternack, B., (1985). Optimal pricing and returns policies for perishable commodities. *Marketing Science, 4*, 166–176.
12. SeyedEsfahani, M. M., Biazaran, M., Gharakhani, M. (2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains. *European Journal of Operational Research, 211(2)*, 263-273.
13. Tibben-Lembke, R.S. (2004). < i> N</i>-period contracts with ordering constraints and total minimum commitments: Optimal and heuristic solutions. European, *Journal of Operational Research, 156(2)*, 353-374.
14. Tsay, A. (1999). Quantity–flexibility contract and supplier–customer incentives. *Management Science, 45*, 1339–1358.
15. Wang, Tie. (2007). Coordination mechanisms of supply chain systems. *European journal of operational research, 179(1*), 1-16.
16. Wang, Tie, Hu, Qiying. (2010), Coordination of supply chain with advertise-setting newsvendor, *Management Science, 51*, 30-44

17. Xie, J., & Wei, J. C. (2009). Coordinating advertising and pricing in a manufacturer–retailer channel. *European Journal of Operational Research,197(2)*, 785-791.

APPENDIX

For two scenarios and cooperative channel, we prove the optimality of the obtained solution in 3 steps:

1. The value of "q" (for No coordination case and cooperative channel)/p (quantity flexibility contract case) is obtained by differentiating retailer's expected profit function, and it is optimal when the retailer's expected profit function is concaved to "q/p".

The value of "q/p" is replaced in retailer's expected profit function, so "p/q" and "e" is obtained by differentiating retailer's expected profit function. They are optimal solution when:

2. The retailer's expected profit function is concaved to "p/q".
3. The hessian matrix determinant to "p/q" and "e" is positive.

No coordination

Step 1.

$$\frac{d^2\pi_r}{dq^2} < 0 \rightarrow \frac{-p}{b-a} < 0$$

So, the retailer's expected profit function is concaved to "q"

Step 2.

$$\frac{d^2\pi_r}{dp^2} < 0 \rightarrow \frac{-1}{p^3(a-b)}(3\beta^2p^4 + w^2b^2 + w^2a^2 - 2\beta\delta ep^3 - 2b\beta p^3 - 2\alpha\beta p^3 + 4\beta ap^3 - 2baw^2) < 0$$

It is enough to prove the phrase in brackets is negative. According to our assumptions: "$a < q < b$" and "$w\gamma < p$"
The following condition is concluded from equation above:

$$\Rightarrow \frac{\beta p^3}{bw^2} < \frac{a-b}{3\beta p - 2\delta e}$$

So, when the above condition is satisfied, we can say the expected profit function is concaved to "p".

Step 3.

$$\frac{-\delta^2(\beta p^2 + 2pa - pb - p\alpha - wb + wa - p\delta e)(\beta p^2 + 2pa - pb - p\alpha + wb - wa - p\delta e)}{p^2(a-b)^2}$$

It is enough to prove that one of the brackets is positive and the other one is negative.

$$\beta p^2 + 2pa - pb - p\alpha - wb + wa - p\delta e < 0$$
$$\beta p^2 + 2pa - pb - p\alpha + wb - wa - p\delta e > 0$$

If the first phrase is multiplied by "minus one", and added to the second phrase, then:

$$2wb - 2wa > 0 \rightarrow b > a$$

This is one of our assumptions. Hence, the hessian matrix determinant in terms of "p" and "e" is positive.

Coordination with quantity flexibility contract

Step 1.

$$\frac{d^2\pi_r}{\mathrm{dp}^2} < 0 \rightarrow \frac{-2\beta(\mathrm{q}-\mathrm{a})}{a-b} < 0$$

So, the retailer's expected profit function is concaved to "p".

Step 2.

$$\frac{d^2\pi_r}{\mathrm{d}q^2} < 0 \rightarrow \frac{-1}{8\beta(a-b)(a-q)^3}(84aw\gamma\beta q^2 - 84w\gamma q\beta a^2 - 72w\beta a\gamma^2 q^2 - 8w\gamma\beta ba^2 + 72w\beta q\gamma^2 a^2 + 12q\delta ea^2 + 24w\beta\gamma^2 q^3 - 12\delta eaq^2 - 28w\gamma\beta q^3 + 36w\gamma\beta a^3 - 24w\beta a^3\gamma^2 + 12\alpha qa^2 - 4\delta ea^3 - 12\alpha aq^2 - 12abq^2 + 12qba^2 + 4\delta eq^3 - 3q^4 + a^4 + 4aq^3 + 6a^2q^2 - 4\alpha a^3 - 12qa^3 + 4bq^3 + 4\alpha q^3 + 4ba^3 - 4a^2\beta^2\gamma^2 w^2 - 4a^2b^2) < 0$$

The amount of the above function by letting "$q = 0$" equals to the following:

$$\frac{-(-4\delta ea^3 - 4\alpha a^3 + 4ba^3 - 4a^2b^2 - 8w\gamma\beta ba^2 + a^4 + 36w\gamma\beta a^3 - 24w\beta\gamma^2 a^3 - 4a^2\beta^2\gamma^2 w^2)}{8\beta a^3(a-b)}$$

If "$\delta e + \alpha - \frac{a}{4} > 6w\gamma\beta(1-\gamma)$" then the result of "$\frac{d^2\pi_r}{\mathrm{d}q^2}$" for "$q = 0$" is negative.

Now, we should prove that the derivative of "$\frac{d^2\pi_r}{dq^2}$" in terms of "q" is negative.

$$\frac{3(-2ab + 3a^2 - 2qa + q^2 - 2a\beta\gamma w)(-2ab + a^2 + 2qa - q^2 - 2a\beta\gamma w)}{8\beta(a-b)(a-q)^4}$$

It is enough to prove that both brackets are negative.

$-2ab + 3a^2 - 2qa + q^2 - 2a\beta\gamma w < 0$

$-2ab + a^2 + 2qa - q^2 - 2a\beta\gamma w < 0$

If the first phrase is added to the second phrase, the result is as following:

$a < b + \beta\gamma w$

This condition is always true. Thus, the phrase "$\frac{d^2\pi_r}{\mathrm{d}q^2}$" has a negative and descending trend and we proved that the result of "$\frac{d^2\pi_r}{\mathrm{d}q^2}$" as per "$q = 0$" is negative. Hence we can say "$\frac{d^2\pi_r}{\mathrm{d}q^2}$" is negative, and the expected profit function is concaved to "q".

Step 3.

$$\frac{-\delta^2}{16(-q+a)^2\beta^2(-b+a)^2}(16\delta eaw\gamma q\beta + 4q^2b^2 - 4bq^3 - 4\alpha q^3 - 4ba^3 + 4aq^3 - 2a^2q^2 + 4\alpha^2 q^2 + 4\alpha^2 a^2 - 12\alpha a^3 - 12qa^3 - 16\alpha q\delta ea - 8\alpha w\gamma\beta q^2 + 76aw\gamma\beta q^2 - 92w\gamma q\beta a^2 - 72wa\beta q^2\gamma^2 - 8\alpha w\gamma\beta a^2 + 72wq\beta a^2\gamma^2 - 8qaw^2\gamma^2\beta^2 - 16bw\gamma\beta a^2 - 16q\delta eab - 8w\gamma\beta bq^2 + q^4 + 5a^4 + 8\alpha\delta ea^2 + 20q\delta ea^2 - 8qa\delta^2e^2 + 4w^2\gamma^2q^2\beta^2 + 24w\beta\gamma^2 q^3 - 16\alpha qab + 8\delta ebq^2 - 4\delta eaq^2 - 20w\gamma\beta q^3 + 8\alpha\delta eq^2 + 8\delta eba^2 + 44w\gamma\beta a^3 - 24w\beta\gamma^2 a^3 + 20\alpha qa^2 - 4abq^2 + 4\delta^2e^2a^2 - 4\alpha aq^2 - 4\delta eq^3 + 8abq^2 - 12\delta ea^3 - 8qa\alpha^2 + 4\delta^2e^2q^2 + 20qba^2 - 8qab^2 + 8\alpha ba^2 - 8\delta ew\gamma\beta a^2 + 16w\gamma q\beta ba + 16\alpha aw\gamma q\beta - 8\delta ew\gamma\beta q^2)$$

The result of the above function as per "$p = 0$" and "$e = 0$" equals to the following:

$$\frac{-\delta^2}{16\beta^2(a-b)^2}(-4ba+4\alpha^2-12\alpha a-8\alpha w\gamma\beta-16bw\gamma\beta+5a^2+44w\gamma\beta a-24w\beta\gamma^2 a+8\alpha b)$$

If "$\frac{\alpha}{a} > \frac{w\gamma\beta(5-6\gamma)+\frac{a}{4}}{3a-\alpha-2b}$" then the result of the hessian matrix determinant as per "$p = 0$" and "$e = 0$" is positive.

Now, we should prove that the derivative of the hessian matrix determinant in terms of "p" and "e" is positive.

$$\frac{\delta^3}{4\beta^2(a-b)^2} > 0$$

This condition is always true. Thus, the hessian matrix determinant in terms of "p" and "e" has a positive and ascending trend. And we proved that the amount of the hessian matrix determinant as per "$p = 0$" and "$e = 0$" is positive. Hence, we can say the hessian matrix determinant in terms of "p" and "e" is positive.