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A Modified Benders Decomposition Algorithm for Supply Chain Network Design under Risk Consideration

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Abstract— *In today's competitive business environment, the design and management of supply chain network is one of the most important challenges that managers encounter. The supply chain network should be designed for satisfying of customer demands as well as minizing the total system costs. This paper presents a multi-period multi-stage supply chain network design problem under demand uncertainty. The problem is formulated as a two-stage stochastic program. In the first-stage, strategic location decisions are made, while the second-stage contains the tactical decisions. In our developed model, conditional value-at-risk (CVaR) as an effective risk measure is used to produce first-stage decisions in which the loss cost in the second-stage is minimized. In addition, a modified Benders decomposition algorithm is developed to solve the model exactly. The computational results on a set of randomly generated problem instances demonstrate the effectiveness of the proposed algorithm in terms of the solution quality.*

Keywords: *Benders decomposition, Conditional Value-at-Risk, Supply chain network design, Two-stage stochastic programming, Uncertain demand.*

I. INTRODUCTION

Supply chain management (SCM) is one of the research areas has attracted much attention over the past decades. The main objective in SCM is to integrate variety of entities including suppliers, manufacturers, distribution centers, and retailers to produce merchandises and distribute them in an efficient way. These entities constitute a network that has to be designed appropriately. In supply chain network design, important decisions should be made with multiple objective functions. The decision levels can be categorized as long-term decision (strategic level), mid-term decision (tactical level) and short-term decision (operational level) [1].

In the strategic level, three important decisions, i.e. the number, location, and the capacity of facilities are determined. Supply chain network design (SCND) is one of the most important strategic decisions in SCM that includes the aforementioned decisions. The tactical decisions contain the production quantity and the volume of transportation between entities. In the operational level, all material flows are scheduled based on the decisions made in the two previous levels [2].

Facility location decisions have been a well-established research area in the strategic design of supply chain networks. A general facility location problem contains a set of customers and a set of facilities to serve customer demands and some objectives such as distances, times or costs between customers and facilities should be minimized (see, for example [3-5]). Melo et al. [6] review the recent literature on facility location models in the context of SCM. Klose and Drexl [7] provide a comprehensive review on facility location within the design of distribution systems.

The dynamic nature of supply chain imposes a high degree of uncertainty in supply chain decisions and significantly influences on designing supply chain networks. Therefore, uncertainty as one of the most challenging in SCM has been received research attentions over the last two decades. Many works have considered customer demands as an uncertain parameter in their developed model. For example, Georgiadis et al. [8] address SCND problem as a strategic activity including multiproduct production facilities with production resources, warehouses, distribution centers and customer zones under time varying demand uncertainty. Pishvaee et al. [9] present a stochastic mixed-integer linear programming model for single period, single product, multi-stage integrated forward/reverse logistics network design considering uncertainty in the quantity and quality of returned products, demands and variable costs.

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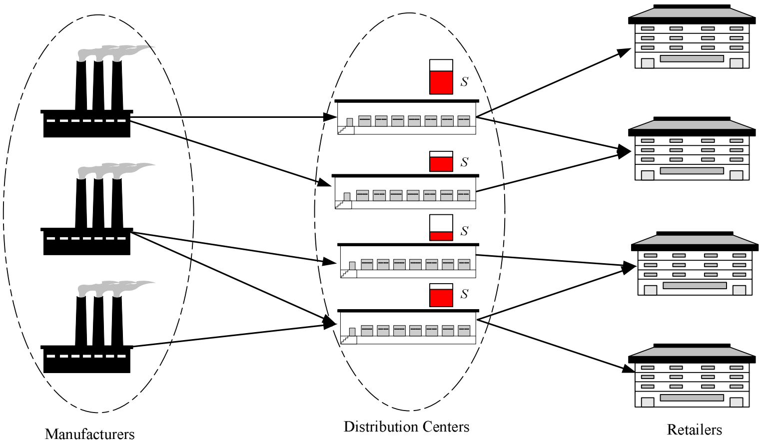

Fig. 1- The considered supply chain structure.

As it can be seen in Fig. 1, the considered supply chain in this paper has three-stages: manufacturers, distribution centers and retailers in which the location and capacity of manufacturers and distribution centers are determined. In the assumed network the finished products are transferred from the manufacturers to the fixed and pre-determined retailers through distribution centers. In this regard, our aim is to satisfy the uncertain demands of the retailers in the multi-period of the planning horizon.

In this paper, demand uncertainty is incorporated into supply chain network design in which both strategic and tactical decisions should be determined simultaneously. The main issues to be addressed related to the strategic decisions contain the number, location and the capacity of the production and distribution centers and the base-stock levels. In the tactical level, the quantity of production, transportation, and the level of inventory in the distribution centers are determined.

This paper presents a multi-period multi-stage supply chain network design problem under demand uncertainty. The problem is formulated as a two-stage stochastic program. In the first-stage, strategic location decisions are made, while the second-stage contains the operational decisions.

In the SCND problems under demand uncertainty in which the optimization model is constructed using two-stage linear programming, the risk can be calculated based on the recourse function where the costs of second-stage are dependent to the demand realization. In our developed model, Conditional Value-at-Risk (CVaR) as a well-known risk measure is used to produce robust first-stage decisions in which the loss cost in the second-stage is to be minimized. To the best of authors' knowledge, this is the first time that CVaR is embedded into the optimization model in the SCND problem.

The remainder of this paper is structured as follows: Section II presents the optimization model of the considered problem. Section III characterizes the solution approach, based on a Benders decomposition. Section IV deals with demand uncertainty. Computational results and an analysis of these results are presented in Section V. Finally, Section VI makes concluding remarks and offers guidelines for future researches.

II. IPTIMIZATION MODEL

In a stochastic optimization model the decisions could be taken in two stages. In the first stage, strategic decisions are determined as here-and-now decisions that should be made before the demand realization and the tactical decisions should be made in the second-stage. Moreover, the second-stage in our model considers multi-periods in which the tactical costs can be efficiently captured. This would be advantageous specifically for those supply chain networks whose demands differ from one period to another period [10].

A. Model formulation

The following notations are used for the mixed integer linear programming (MILP) as a two-stage stochastic programming formulation:

Sets:

I	Set of potential manufacturer locations $i \in I$
J	Set of potential distribution center locations $j \in J$
T	Set of periods in planning horizon $t, p \in T$
R	Set of retailers $r \in R$
S	Set of scenarios $s \in S$

Parameters, constants, and coefficients:

F_i^M	Fixed cost of locating manufacturer at location *i*
F_j^{DC}	Fixed cost of locating distribution center at location *j*
C_i^M	Cost for capacity of manufacturer *i* per unit of products
C_j^{DC}	Cost for capacity of distribution center *j* per unit of products
Tc_{ijt}^M	Cost of transporting per unit of product between manufacturer *i* and distribution center *j* at period *t*
Tc_{jrt}^{DC}	Cost of transporting per unit of product between distribution center *j* and retailer *r* at period *t*
Ic_{jt}	Cost of holding per unit of inventory in distribution center *j* at period *t*
D_{rts}	Product demand of retailer *r* in scenario *s* at period *t*
Cap_i^M	Maximum production capacity of manufacturer *i*
Cap_j^{DC}	Maximum capacity of distribution center *j*
B_j	Base-stock level coefficient to calculate the capacity of distribution center *j*
$\Pr_s$	Probability of scenario *s*

Decision variables:

x_i^M	Binary variable equals to 1 if a manufacturer is located at location *i*, 0 otherwise
x_j^{DC}	Binary variable equals to 1 if a distribution center is located at location *j*, 0 otherwise
w_i^M	Capacity of manufacturer *i*
w_j^{DC}	Capacity of distribution center *j*
b_j	Base-stock level of distribution center *j* at the beginning of each period
z_{ijts}^M	Quantity of production in manufacturer *i* transported to distribution center *j* at period *t* in scenario *s*
z_{jrts}^{DC}	Quantity of products transported from distribution center *j* to retailer *r* at period *t* in scenario *s*
v_{jts}	Inventory level at distribution center *j* at the end of period *t* in scenario *s*

Uncertain demand in our mathematical formulation is introduced by ζ. ζ_s is a given realization of uncertain demand and E_ζ represents the expected value with respect to ζ. The actual value of ζ becomes known in the second stage in which recourse decisions can be calculated. Therefore, decisions related to the first-stage are made by taking the future uncertain effects into account. These effects are measured by the recourse function, $\mathcal{Q}(x,w,b) = E_\zeta\left(Q\left(x,w,b,\zeta^s\right)\right)$, where $\mathcal{Q}(x,w,b)$ is the expected value of the second-stage costs.

$$\text{Min } Z = \sum_{i \in I} F_i^M x_i^M + \sum_{i \in I} C_i^M w_i^M + \sum_{j \in J} F_j^{DC} x_j^{DC} + \sum_{j \in J} C_j^{DC} w_j^{DC} + \mathcal{Q}(x, w, b) \tag{1}$$

Subject to:

$$w_i^M \le x_i^M \times Cap_i^M \qquad \forall i \in I, \tag{2}$$

$$w_j^{DC} \le x_j^{DC} \times Cap_j^{DC} \qquad \forall j \in J, \tag{3}$$

$$w_j^{DC} \ge B_j \times b_j \qquad \forall j \in J, \tag{4}$$

$$x_i^M, x_j^{DC} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \tag{5}$$

$$b_j \ge 0 \qquad \forall j \in J, \tag{6}$$

where $Q(x,w,b)$ being the solution of the following second-stage problem:

$$\text{Min } Q(x,w,b) = E_\zeta\left(Q\left(x,w,b,\zeta^s\right)\right) = \sum_{s\in S}\Pr_s\left(\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} z_{ijts}^M Tc_{ijt}^M + \sum_{j\in J}\sum_{r\in R}\sum_{t\in T} z_{jrts}^{DC} Tc_{jrt}^{DC} + \sum_{j\in J}\sum_{t\in T} v_{jts} Ic_{jt}\right) \tag{7}$$

Subject to:

$$\sum_{i\in I}\sum_{p\le t} z_{ijps}^M - \sum_{r\in R}\sum_{p\le t} z_{jrps}^{DC} = v_{jts} \qquad \forall j \in J, \forall t \in T, \forall s \in S, \tag{8}$$

$$\sum_{r\in R}\sum_{p\le t} z_{jrps}^{DC} - \sum_{i\in I}\sum_{p\le t} z_{ijps}^M \le 0 \qquad \forall j \in J, \forall t \in T, \forall s \in S, \tag{9}$$

$$\sum_{i\in I}\sum_{p\le t} z_{ijps}^M - \sum_{r\in R}\sum_{p<t} z_{jrps}^{DC} = b_j \qquad \forall j \in J, \forall t \in T, \forall s \in S, \tag{10}$$

$$\sum_{j\in J} z_{jrts}^{DC} \ge D_{rts} \qquad \forall r \in R, \forall t \in T, \forall s \in S, \tag{11}$$

$$\sum_{j\in J} z_{ijts}^M \le w_i^M \qquad \forall i \in I, \forall t \in T, \forall s \in S, \tag{12}$$

$$z_{ijts}^M, z_{jrts}^{DC}, w_i^M, w_j^{DC}, v_{jts} \ge 0 \qquad \forall i \in I, \forall j \in J, \forall r \in R, \forall t \in T, \forall s \in S. \tag{13}$$

Relation (1) is the objective function that minimizes the sum of the first-stage costs and the expected second-stage costs. The first-stage costs represent the costs of locating and capacity of the manufacturers and distribution centers. The objective function of the second-stage, i.e. equation (7), includes two types of costs: firstly, the transportation costs, and secondly, the inventory holding costs. Constraints (2) and (3) ensure the capacity restrictions for each manufacturer and distribution center, respectively. Constraint (4) guarantees that the capacity of each distribution center should be at least equals to a coefficient of the corresponding base-stock level. Equation (8) calculates the inventory level at the end of period t by subtracting the total output flow of products to the retailers in scenario s from all input flows to each distribution center until period t. Constraint (9) assures that the products output flow of each distribution center is less than or equal to the product input flow and the inventory level of that distribution center in each scenario and period. Relation (10) calculates the base-stock level for each distribution center in scenario s at period t. This constraint refers to the push-based strategy concept in addressed mathematical formulation. Constraint (11) ensures that the demands of all retailers are satisfied in scenario s at period t. Constraint (12) assures that products are not produced more than manufacturers' capacities in each scenario and period. Constraints (5), (6) and (13) define the feasible sets for corresponding decision variables.

B. Risk consideration

Decision making under uncertainty is usually based on the expected value criterion. However, this criterion might be misleading in many applications specifically where the random events are considerably varying. Moreover, the risk measurement is a vital issue that should be chosen with specific precaution based on the application under study.

In the SCND problems under demand uncertainty in which the optimization model is constructed using two-stage linear programming, the risk can be calculated based on the recourse function where the costs of the second-stage are dependent to the demand realization. In fact, the amount of investment for the first-stage can depend on the type of risk measurement and the level of risk importance denoted by risk-aversion factor. In our developed MILP optimization model, CVaR as a well-known and effective risk measure is used to produce robust first-stage decisions in which the loss cost in the second-stage is to be minimized. To the best of authors' knowledge, this is the first time that CVaR is embedded into the optimization model in the SCND problem.

The model is formulated according to [11] in which the objective function is changed into Equation (14) and Constraints (15) and (16) are added to the mathematical formulation.

$$\text{Min } Z = \sum_{i\in I} F_i^M x_i^M + \sum_{i\in I} C_i^M w_i^M + \sum_{j\in J} F_j^{DC} x_j^{DC} + \sum_{j\in J} C_j^{DC} w_j^{DC} + Q(x,w,b) + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S}\Pr_s \times \mu_s\right)\right], \quad (14)$$

$$Q(x,w,b) - \xi - \mu_s \le 0 \qquad \forall s \in S, \quad (15)$$

$$\mu_s, \xi \ge 0 \qquad \forall s \in S, \quad (16)$$

where the possible loss for each scenario s and the level of confidence are denoted by μ_s and α, respectively, ξ is the Value at Risk (VaR) and γ is risk-aversion factor.

CVaR can be computed by the expected value of costs that exceeding the VaR. CVaR can be calculated as a linear function as follows:

$$CVaR_\alpha = \xi + \frac{1}{1-\alpha}\left(\sum_{s\in S}\Pr_s \times \mu_s\right) \quad (17)$$

II. A MODIFIED BENDERS DECOMPOSITION-BASED SOLUTION ALGORITHM

Benders decomposition algorithm is a classical solution approach for combinatorial optimization problems, which was firstly presented to solve MILP problems by Benders [12]. This type of decomposition is one of wide commonly used techniques in the SCND problems (see for example [13] and [14]).

A. Solution approach

As it is clear, the proposed two-stage optimization model is formulated as an MILP. In applying Benders decomposition, the original problem is decomposed into a master problem and several subproblems.

For any determined capacity of manufacturer and base-stock level of distribution center, i.e. $\{\widehat{w}_{i,k}^M, \widehat{b}_{j,k}\}$, the original problem can be reduced to the following linear program called subproblem (SP) which includes z_{ijt}^M, z_{jrt}^{DC} and v_{jt} continuous variables:

$$\text{Min } Z_{s,k}^{SP} = \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} z_{ijts}^M Tc_{ijt}^M + \sum_{j\in J}\sum_{r\in R}\sum_{t\in T} z_{jrts}^{DC} Tc_{jrt}^{DC} + \sum_{j\in J}\sum_{t\in T} v_{jts} Ic_{jt} \quad (18)$$

Subject to:

Constraints (9)-(13)

$$w_i^M = \widehat{w}_{i,k}^M \qquad \forall i \in I \quad (19)$$

$$b_j = \widehat{b}_{j,k} \qquad \forall j \in J, \quad (20)$$

$$b_j, w_i^M \ge 0 \qquad \forall i \in I, \forall j \in J. \quad (21)$$

In the above formulation, $Z_{s,k}^{SP}$ is the objective function of the SP related to scenario s at iteration k of Benders decomposition algorithm. Note that each SP is constructed such that only one scenario is considered and there is no dependency between all subproblems. By introducing the additional continues variables θ and f, master problem (MP) as an MILP problem can be written as follows:

$$Min\ Z^{MP} = \theta + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S}\Pr_s \times \mu_s\right)\right] \tag{22}$$

Subject to:
Constraints (2)-(6)

$$\theta \ge f + \sum_{s\in S}\Pr_s\left(Z_{s,k}^{sp} + \sum_{i\in I}\pi_{s,k,i}^{w^M}(w_i^M - \hat{w}_{i,k}^M) + \sum_{j\in J}\pi_{s,k,j}^{b}(b_j - \hat{b}_{j,k})\right) \qquad k = 1,2,... \tag{23}$$

$$f = \sum_{i\in I}F_i^M x_i^M + \sum_{i\in I}C_i^M w_i^M + \sum_{j\in J}F_j^{DC}x_j^{DC} + \sum_{j\in J}C_j^{DC}w_j^{DC} \tag{24}$$

$$\theta - f - \xi - \mu_s \le 0 \qquad \forall s\in S, \tag{25}$$

$$x_i^M, x_j^{DC} \in \{0,1\} \qquad \forall i\in I, \forall j\in J, \tag{26}$$

$$b_j \ge 0 \quad \forall j\in J, \tag{27}$$

Relation (22) is the objective function of the MP. Constraint (23) is the optimality cut added to the MP at each iteration of Benders decomposition algorithm. It is worth mentioning that in constraint (23), $\hat{w}_{i,k}^M$ and $\hat{b}_{j,k}$ are the optimal values of w_i^M and b_j, respectively, obtained from the previous iteration. In addition, $\pi_{s,k,i}^{w^M}$ and $\pi_{s,k,j}^{b}$ are the optimal values of the dual variables associated with the constraints (19) and (20) of the SP of Benders decomposition algorithm, respectively. Relation (24) calculates the amount of f that is the sum of the first-stage costs. Constraint (25) is equivalent to constraint (15), which will be proved in Lemma.

B. Valid inequalities

When solving the MP, some valid inequalities can be added to the formulation. These constraints can improve the convergence rate by hopefully reducing the associated feasible space of MP. In our problem, following constraints can be added to the MP to avoid from infeasibility in subproblems:

$$\sum_{i\in I}w_i^M \ge \sum_{j\in J}b_j \tag{28}$$

$$\sum_{j\in J}b_j \ge \max_{s\in S, t\in T}\{\sum_{r\in R}D_{rts}\} \tag{29}$$

Constraint (28) guarantees that total capacity of all manufacturers should be greater than or equal to the summation of base-stock level of all distribution centers. In addition, Constraint (29) ensures that total base-stock level of all distribution centers must be at least equal to the maximum of summation of product demands of all retailers through each scenario and each period.

The upper and lower bounds of the objective function of the original problem can be calculated at each iteration of the Benders decomposition algorithm as follows [15]:

$$Z_k^{Upper} = f + \sum_{s\in S}\Pr_s\left(Z_{s,k}^{SP}\right) + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S}\Pr_s \times \mu_s\right)\right] \tag{30}$$

$$Z_k^{Lower} = \theta + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S} \Pr_s \times \mu_s\right)\right] \tag{31}$$

After computing the lower and upper bounds of the optimal value of the objective function of the original problem, the following constraint is checked:

$$Z_k^{Upper} - Z_k^{Lower} < \varepsilon \qquad k = 1,2,... \tag{32}$$

In Constraint (32), ε is a small tolerance value to control the convergence of the algorithm. The details of Benders decomposition algorithm is presented in the following pseudo-code:

Step 0. **Initialization**

i. $Z_0^{Upper} = +\infty$.

ii. $Z_0^{Lower} = -\infty$.

iii. k =0.

iv. Solve the initial master problem to obtain $w_{i,k}^M, b_{j,k}$.

While ($Z_k^{Upper} - Z_k^{Lower} > \varepsilon$)

Step 1. Solving the subproblems

For each ***s*** **∈*S***

Solve the subproblem by determined $\{\hat{w}_{i,k}^M, \hat{b}_{j,k}\}$.

End for

Step 2. Updating the lower and upper bounds

Step 3. Solving the master problem

i. Add optimality cut to the master problem.

ii. k=k+1.

iii. Solve the master problem to obtain $w_{i,k}^M, b_{j,k}$.

End while

As mentioned before, the following lemma proves that constraint (26) is equivalent to constraint (16). This Lemma was employed in the construction of master problem in the structure of Benders decomposition algorithm.

Lemma. *Constraint* $\theta - f - \xi - \mu_s \le 0, \forall s \in S$ *in the MP of the proposed Benders decomposition algorithm is equivalent to constraint* $Q(x,w,b) - \xi - \mu_s \le 0, \forall s \in S$ *when the algorithm is converged.*

Proof. When the convergence of algorithm is occurred, $Z_k^{Upper} \cong Z_k^{Lower}$. So, according to Relations (28) and (29) we have:

$$f + \sum_{s\in S} \Pr_s \left(Z_{s,k}^{SP} \right) + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S} \Pr_s \times \mu_s\right)\right] \cong \theta + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S} \Pr_s \times \mu_s\right)\right] \rightarrow$$

$$f + \sum_{s\in S} \Pr_s \left(Z_{s,k}^{SP} \right) \cong \theta \rightarrow \sum_{s\in S} \Pr_s \left(Z_{s,k}^{SP} \right) \cong \theta - f ,$$

On the other hand, in the Benders algorithm always we can write $Z_k^{Lower} \le Z \le Z_k^{Upper}$. As a consequence

$$Z \cong Z_k^{Upper} \rightarrow f + Q(x,w,b) + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S} \Pr_s \times \mu_s\right)\right] \cong f + \sum_{s\in S} \Pr_s \left(Z_{s,k}^{SP} \right) + \gamma\left[\xi + \frac{1}{1-\alpha}\left(\sum_{s\in S} \Pr_s \times \mu_s\right)\right]$$

Hence, when the Benders algorithm is converged $\theta - f - \xi - \mu_s \le 0, \forall s \in S$ is approximately equivalent to $Q(x,w,b) - \xi - \mu_s \le 0, \forall s \in S$. □

III. DEMAND UNCERTAINTY

In order to deal with demand uncertainty in multiple periods, the demand at a particular period can be described through an uncertain parameter. On the other hand, a proper time series should be utilized to consider time dependencies in each scenario. In this regard, Schütz et al. [16] employed an autoregressive (AR) to model demand uncertainty. In our case, the predicted demand at period t + 1 and scenario s by considering the error term εr,t+1,s, denoted by $D_{r,t+1,s}$, is given by the following relation:

$$D_{r,t+1,s} = \alpha + \sum_{i=1}^{P} \beta_i \hat{D}_{r,t+j-i} + \varepsilon_{r,t+1,s} \tag{33}$$

where α is a constant, βi is the autoregressive parameters, $\hat{D}_{r,t+1-i}$ is the historical demand of retailer r at (t + 1 – i) period, and $\varepsilon_{r,t+1,s}$ is the error term at (t + 1)th period and sth scenario. In addition, the Latin hypercube sampling (LHS) method is used to cover domain space of stochastic variables in this paper to generate ε and consequently different scenarios [17].

IV. COMPUTATIONAL RESULTS

To evaluate the performance of the proposed Benders decomposition algorithm in terms of the solution quality, we performed some numerical experiments on a set of randomly generated problem instances. The algorithm was implemented in GAMS using CPLEX solver. All experiments were run with an Intel Pentium IV dual core 2.1 GHz CPU PC at 1 GB RAM under a Microsoft Windows XP environment.

TABLE I. THE VALUES OF THE PARAMETERS USED IN THE PROBLEM INSTANCES.

Category	Parameter	Range
Different types of costs	F_i^M	~ Uniform (500000, 1000000)
	F_j^{DC}	~ Uniform (200000, 450000)
	C_i^M	~ Uniform (1500, 2000)
	C_j^{DC}	~ Uniform (800, 1200)
	$Tc_{ijt}^M, Tc_{jrt}^{DC}$	~ Uniform (20, 30)
	Ic_{jt}	~ Uniform (15, 20)
Maximum capacities	Cap_i^M	Small sizes: ~ Uniform (150, 250)
		Medium sizes: ~ Uniform (300, 400)
	Cap_j^{DC}	Small sizes: ~ Uniform (300, 400)
		Medium sizes: ~ Uniform (400, 500)
Demand of retailers	D_{rts}	$AR(1): D_{rts} = \alpha + \beta_1 D_{r,t-1,s} + \varepsilon_{rts}$
		α ~ Uniform (20, 40)
		β_i ~ Uniform (0.15, 0.2)
		$\varepsilon_{r,t,s}$ ~ N(0, Uniform (20, 35))
		$\hat{D}_{r,t-1}$ ~ Uniform (30, 50)
Base-stock level coefficients	B_j	3

TABLE II. PROBLEM INSTANCES' SIZES

Size of problem instances	No. of potential Manufacturer	No. of potential distribution centers	No. of retailers
Small	4	6	8
	4	8	10
Medium	8	15	20
	8	20	30

A. Data generation and settings

The problem data can be characterized by four factors: different types of costs, two types of maximum capacities, product demand of retailers and base-stock level coefficients. The required data for random generation of problem instances drawn from the probability distributions and equations are shown in TABLE I. Afterward, using the generated parameters, six problem instances with different sizes are constructed. TABLE II specifies the features of problem instances used in this section.

We assume that the number of periods is equal to 12 for all problem instances, which is related to the number of months in each year. Moreover, in order to make an appropriate trade-off between the first- and second-stage costs, the costs relevant to the second-stage is multiplied by a constant value equal to 10.

B. Experimental results

In this subsection, we present numerical experiments by means of the proposed methodology for solving some supply chain design problem instances using the characteristics described in the previous subsection.

TABLEs III and IV reports the first- and second-stage costs, the objective function, the CPU times and CVaR vales for pre-defined problem instances with γ=0 and γ=10. We solved each problem instance for three sets of scenarios, i.e. 20, 40 and 60. Fig. 2 demonstrates the convergence checking of the upper and lower objective functions (Z_k^{Upper} and Z_k^{Lower}) for a small problem instance. In addition, Fig. 3 illustrates the first- and second-stage costs and objective function with respect to different values of γ for a medium problem instance.As it is shown in Fig. 3, increasing the value of γ would increase the objective function value.

TABLE III. SUMMARY OF TEST RESULTS FOR γ=0

Size of problem instances		No. of scenarios	Objective ($\times 10^6$)	First-stage ($\times 10^6$)	Second-stage ($\times 10^6$)	Time (Seconds)	Optimality gap
Small	1	20	4.82	3.29	1.53	211	0.00%
		40	4.83	3.34	1.49	408	0.00%
		60	4.89	3.35	1.55	247	0.00%
	2	20	6.00	4.13	1.87	200	0.00%
		40	5.99	4.12	1.87	392	0.00%
		60	6.03	4.11	1.93	593	0.00%
Medium	3	20	12.9	9.01	3.88	1353	0.29%
		40	12.5	8.67	3.80	2643	0.38%
		60	12.6	8.74	3.91	3865	0.44%
	4	20	18.9	13.3	5.62	1833	0.76%
		40	18.4	13.2	5.19	2737	0.83%
		60	18.6	13.0	5.64	6213	1%

TABLE IV. SUMMARY OF TEST RESULTS FOR γ=10

Problem instances		No. of scenarios	Objective ($\times 10^7$)	First-stage ($\times 10^6$)	Second-stage ($\times 10^6$)	Time (Seconds)	Optimality gap	CVaR
Small	**1**	20	1.95	3.79	1.44	171	0.01%	1431680
		40	1.95	3.72	1.44	351	0.07%	1430800
		60	1.98	3.71	1.46	522	0.00%	1459080
	2	20	2.37	5.04	1.85	196	0.66%	1676670
		40	2.42	4.54	1.80	371	0.07%	1781880
		60	2.48	4.42	1.92	815	0.28%	1842010

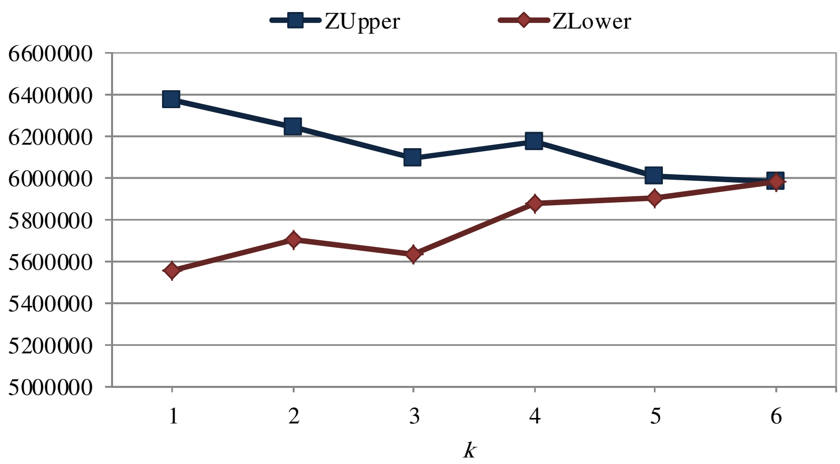

Fig. 2. Convergence evolution of the upper and lower objective functions (ZUpper and ZLower) for a small problem instance

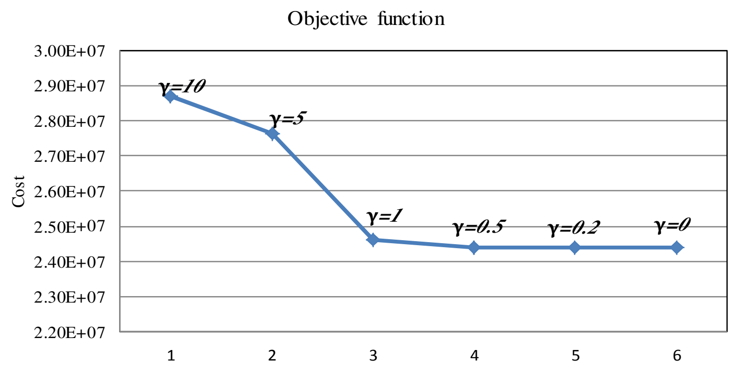

Fig. 3. The obtained results for the objective function for a medium problem instance

V. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE STUDIES

The configuration of the supply chain network has an important effect on the overall performance of the supply chain. Supply chain management operates at three decision levels, strategic, tactical and operational. The strategic level generally relates to the optimization of network resources such as designing networks, location and determination of the number of facilities. Tactical decisions deal with the mid-term, including inventory levels, production levels at all plants and lot sizes. Operational decisions are related to how to make the tactical decisions happen in the short-term, such as production planning and scheduling. This paper mainly discusses how to incorporate the strategic and tactical decisions together in the supply chain. For this purpose, a novel two-stage mixed-integer stochastic programming model was developed in this paper under demand uncertainty. The proposed mathematical model considered the optimum supply chain network, the location and capacity of manufacturers and distribution centers and base-stock level of distribution centers. In order to solve the problem, a Benders decomposition algorithm was employed. Finally for illustrative purposes, computational experiments on randomly generated problem instances were presented to demonstrate that how incorporating the conditional-value-at-risk (CVaR) as a risk measure affects the optimal solutions of the stochastic model.

We believe this paper provides a good starting point in this research area. Considering routing decisions in the addressed problem are a valuable future research. Furthermore, since the computational time increases significantly when problem size and the number of scenarios increase, developing heuristic/meta-heuristic methods to solve large-sized problems is a challenging area for future studies.

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