Journal of Quality Engineering and Production Optimization
Vol. 1, No. 1, PP. 43-54, 2015

Simultaneous Monitoring of Multivariate-Attribute Process Mean and Variability Using Artificial Neural Networks

Mohammad Reza Maleki and Amirhossein Amiri
Industrial Engineering Department, Shahed University, Tehran, Iran

Corresponding Author: Amirhossein Amiri (E-mail: amiri@shahed.ac.ir)

Abstract- *In some statistical process control applications, the quality of the product is characterized by the combination of both correlated variable and attributes quality characteristics. In this paper, we propose a novel control scheme based on the combination of two multi-layer perceptron neural networks for simultaneous monitoring of mean vector as well as the covariance matrix in multivariate-attribute processes whose quality characteristics are correlated. The proposed neural network-based methodology not only detects separate mean and variance shifts, but also can efficiently detect simultaneous changes in mean vector and covariance matrix of multivariate-attribute processes. The performance of the proposed neural network-based methodology in detecting separate as well as simultaneous changes in the process is evaluated thorough a numerical example based on simulation in terms of average run length criterion and the results are compared with a statistical method based on the combination of two control charts that are developed for monitoring the mean vector and covariance matrix of multivariate-attribute processes, respectively. The results of model implementation on numerical example show the superior detection performance of the proposed NN-based methodology rather than the developed combined statistical control charts.*

Keywords: *Average run length, Covariance matrix, Mean vector, Multi-layer perceptron neural network, Multivariate-attribute process.*

I. INTRODUCTION

In many statistical control applications, the quality of the product or the process is characterized in terms of several correlated quality characteristics. In order to monitor such processes, different multivariate or multi-attribute control charts are developed separately by quality engineering researchers. Some of the newest multivariate and multi-attribute control schemes are listed as follows.

Yeh et al. (2012) proposed a control chart based on the penalized likelihood estimation of the precision matrix in order to monitor multivarite process variability when the individual observations are available. They compared the proposed control chart with the competing MaxMEWMV, MEWMS and MEWMC control charts in terms of average run length criterion.

Shang et al. (2013) proposed a new approach in order to model multistage processes with binomial data and developed corresponding monitoring and diagnosis schemes by utilizing a hierarchical likelihood approach and directional information based on the Binary State Space Model (BSSM). Aparisi et al. (2014) proposed some new control schemes for simultaneous monitring of several Poisson variables. Their proposed method can use a multiple scheme, i.e. one control chart for monitoring each quality characteristic, or can use a multivariate scheme, based on monitoring all the attributes with a single control chart. Li et al. (2014) investigated the use of log-linear models for characterizing the relationship among categorical factors in multivariate binomial and multivariate multinomial processes. Bersimis et al. (2007) have reviewed different procedures for construction of the multivariate control charts. Topalidou and Psarakis (2009) have also reviewed the multinomial and multi-attribute control charts.

Nowadays, artificial neural networks are considered as the effective alternatives of control charts in monitoring different processes. Recently, many researches have been devoted to the application of artificial neural networks in

Manuscript Received: 2 June 2014 and revised: 20 Aug 2014 *ISSN: 2423-3781*
Accepted: 28 Dec 2014

monitoring multivariate and multi-attribute processes due to the superior performance of artificial neural networks in comparison with control charts.

Niaki and Abbasi (2005) proposed an artificial neural network based model for diagnosing faults in out-of-control states as well as identifying aberrant variables when multivariate Hotelling's T^2 control chart is used. Aparisi et al. (2006) introduced a neural network based procedure in order to identify which variable have been shifted in situations where the T^2 control chart shows an out-of-control state. Hwarng (2008) presented a neural network based identifier to detect mean shifts in the multivariate processes as well as indicate the variable(s) that cause out-of-control signals. Niaki and Abbasi (2008) designed a neural network for detecting out-of-control states in multi-attribute processes as well as diagnosing attribute(s) that cause the signals. They presented three numerical examples and compared the performance of the proposed methodology with multi-attribute control charts. They found that their neural network based methodology outperforms the multi-attribute control charts in detecting different step shifts in the multi-attribute process mean.

Yu and Xi (2009) introduced a learning-based model for monitoring and diagnosing out-of-control signals in a bivariate process. They proved that the proposed model outperforms the conventional multivariate control scheme in terms of average run length (ARL) criterion. Yu et al. (2009) proposed a method based on the joint use of several selected neural networks to classify source(s) of out-of-control signals in multivariate processes. Hwarng and Wang (2010) proposed a neural-network-based identifier (NNI) for monitoring multivariate auto-correlated processes as well as to identify the source of the shifts in such processes. Cheng and Cheng (2011) presented a neural network-based approach for detecting variance shifts in multivariate processes. They also investigated some important implementation issues of neural networks such as window size, number of training examples, sample size and training algorithm.

Ahmadzadeh (2011) suggested two approaches including maximum likelihood estimator as well as the artificial neural network in order to estimate the time of change in the mean parameters of multivariate processes. Salehi et al. (2012) proposed a model with two modules for on-line analysis of out of control signals in multivariate processes. In the first module, they used a support vector machine classifier to recognize mean and variance shifts. Then in the second module, they applied two neural networks in order to identify magnitude of mean and variance shifts.

Sometimes in real manufacturing systems, the combination of both variable and attribute quality characteristics that are correlated expresses the quality of the product or the process. For example, in the production process of LED lamps, the number of nonconformities on a product and its weight are discrete and continuous quality characteristics, respectively that are correlated with each other. Despite of various statistical control schemes as well as artificial neural networks that are proposed separately for monitoring multivariate as well as multi-attribute processes, only few methods are available in the literature about monitoring multivariate-attribute processes. These few researches are listed as follows:

Kang and Brenneman (2011) provided a bootstrap methodology to construct a confidence bound for the overall defect rate of a product whose quality assessment involves multiple pass/fail binary data and multiple continuous data. They supposed that the quality characteristics are independent. However in most real systems, the independence assumption of multivariate-attribute quality characteristics is violated. Doroudyan and Amiri (2011) developed a multivariate T^2 control chart based on the root transformation method for monitoring the mean vector of multivariate-attribute quality characteristics.

Doroudyan and Amiri (2013) investigated the use of four transformation methods in order to monitor the multivariate-attribute processes. In the first approach, the distribution of multivariate-attribute quality characteristics is transformed to approximate multivariate Normal distribution and then the transformed data are monitored by multivariate control charts. In the second approach, multivariate-attribute quality characteristics are transformed such a way that the correlation between the quality characteristics becomes roughly equal to zero. Then the univariate control charts are used in order to monitor the transformed quality characteristics. In the third and fourth approaches, they used a method based on the combination of two transformation techniques in order to make the quality characteristics independent and transform them to Normal distribution. They mentioned that the difference between the third and fourth methods is the order of using the transformation techniques. Maleki et al. (2012) designed an artificial neural network for detecting mean shifts as well as diagnosing the source of out-of-control signals in multivariate-attribute processes.

Maleki et al. (2013) developed two exponentially weighted moving average (EWMA)-based control charts including

$MEWMS_{AS}$ as well as $MEWMS_{AT}$ for monitoring the covariance matrix of multivariate-attribute quality characteristics using Normal to anything (NORTA) inverse technique. Based on the comparison study, they pointed out that the developed control charts outperforms a traditional control chart in detecting variance shifts in the process. Amiri et al. (2014) proposed a neural network-based approach for monitoring the variability of multivariate-attribute processes as well as diagnosing the quality characteristic(s) responsible for the out-of-control states. We can conclude from the literature that there is no method about simultaneous monitoring of mean vector as well as covariance matrix of multivariate-attribute quality characteristics.

As the main contribution, in this paper first we design two multi-layer perceptron neural networks for monitoring mean and variance shifts in multivariate-attribute processes where the quality characteristics are correlated. Then, we use them simultaneously in order to provide a control scheme for simultaneous monitoring of the mean vector as well as the covariance matrix of multivariate-attribute processes. We also extend a combined control chart and use it for simultaneous monitoring of the multivarite-attribute process mean and variability that is the second contribution of our work. The neural network-based method as well as the extended control chart are both proposed under the assumption that the process is monitored in Phase II. Consequently, the distribution parameters of multivariate-attribute process data including the mean vector, covariance matrix as well as correlation coefficient between quality characteristics are known based on the Phase I analysis.

The rest of this paper is organized as follows: In section 2, the problem and assumptions of the multivariate-attribute model are briefly defined. In section 3, the extended multivariate-attribute control chart for simultaneous monitoring of the process mean and variability is discussed. In section 4, two multi-layer perceptron neural networks are designed for detecting mean and variance shifts in multivariate-attribute processes. Section 5 presents the proposed neural network-based methodology for simultaneous monitoring of mean vector as well as covariance matrix of multivariate-attribute quality characteristics. In section 6, through a simulated example, the performance of the proposed simultaneous neural networks-based procedure is evaluated and compared with a statistical method based on the extension of two control charts. Finally, the conclusions and the recommendations for future study are given in section 7.

II. PROBLEM STATEMENT AND ASSUMPTIONS

Consider a multivariate-attribute process with p variable and q attribute quality characteristics, where all quality characteristics are correlated and characterized by column vector of $\mathbf{X} = (x_1, x_2, ..., x_p, x_{p+1}, ..., x_{p+q})'$. In the vector **X,** the first p elements are variable and the last q elements are attribute quality characteristics. We assume that the correlation between multivariate-attribute quality characteristics is stable during the process. Both proposed NN-based as well as the extended multivariate-attribute control chart are involved in Phase II. Hence, the mean vector (μ_0) and the covariance matrix (Σ_0) of quality characteristics are known based on the Phase I analysis. The mean vector of the process at in-control state is $\boldsymbol{\mu}_0 = (\mu_1, ..., \mu_p, \mu_{p+1}, ..., \mu_{p+q})'$, where $\mu_i; i = 1, ..., p+q$ are the mean value of p+q quality characteristics. If a shift in the mean value of at least one quality characteristic occurs, the multivariate-attribute process mean considered to be out-of-control. The covariance matrix of the quality characteristics at in-control is determined as follows:

$$\boldsymbol{\Sigma}_0 = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1(p+q)} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2(p+q)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1(p+q)} & \sigma_{2(p+q)} & \cdots & \sigma_{p+q}^2 \end{pmatrix}, \tag{1}$$

where $\sigma_i^2; i = 1, ..., p+q$ is the variance of ith quality charactristic and $\sigma_{ij}; i \neq j$ is the covariance between ith and jth quality characteristics. If a variance shift in at least one quality characteristic occurs, the covariance matrix goes to an out-of-control state.

III. THE EXTENDED T^2-MEWMS$_{AS}$ CONTROL CHART

In this section the extended T^2-MEWMS$_{AS}$ for simultaneous monitoring of mean vector and covariance matrix of correlated multivariate-attribute data is briefly explained. The first multivariate control chart is T^2 control chart that is proposed by Hotelling (1947) for monitoring the mean vector of the processes with multivariate Normal data. The statistic of this control chart in which the number of samples in each subgroup is equal to n is calculated as follows:

$$T^2 = n(\bar{\mathbf{x}} - \bar{\bar{\mathbf{x}}})' \mathbf{S}^{-1} (\bar{\mathbf{x}} - \bar{\bar{\mathbf{x}}}). \tag{2}$$

On the other hand, MEWMS$_{AS}$ control chart is proposed by Memar and Niaki (2011) based on the squared deviation of observations from target for monitoring the covariance matrix of multivariate processes with Normal data. In order to apply both T^2 and MEWMS$_{AS}$ control charts for monitoring the mean vector as well as the covariance matrix of multivariate-attribute processes, first using NORTA Inverse method, we transform the distribution of original data to a multivariate Normal distribution. After using NORTA Inverse transformation, the joint distribution of the transformed data follows a standardized multivariate Normal distribution in which the quality characteristics are correlated. In MEWMS$_{AS}$ control chart, after using the NORTA Inverse transformation, the correlated data should be independent. For this purpose, we use a transformation that is proposed by Golnabi and Houshmand (1999). The independent standardized Normal quality characteristics are computed according to Equation (3):

$$\mathbf{x}_{tk} = \boldsymbol{\Sigma}_0^{-1/2} (\mathbf{y}_{tk} - \boldsymbol{\mu}_0), \tag{3}$$

where μ_0 and Σ_0 are the mean vector and covariance vector of transformed quality characteristics, respectively and obtained in Phase I analysis $y_{tk} = (y_{tk1,} y_{tk2,\dots,} y_{tk(p+q)})'$, is the tth transformed observation (based on NORTA Inverse transformation) in kth subgroup where $t = 1,2, \dots$ and $k = 1,2 \dots, n$. Note that, y_{tkj} is jth element (quality characteristic) of matrix y_{tkj} where. $j = 1,2, \dots, p + q$ As noted, the MEWMS$_{AS}$ control chart is based on the S_t statistic that is proposed by Yeh et al. (2005). Hence, in this paper we use the S_t statistic based on transformed data with smoothing parameter of λ according to the following equation:

$$S_t = (1-\lambda) S_{t-1} + \frac{\lambda}{n} \sum_{k=1}^{n} \mathbf{x_{1k}} \mathbf{x'_{1k}}, S_0 = \frac{1}{n} \sum_{k=1}^{n} \mathbf{x_{1k}} \mathbf{x'_{1k}}. \tag{4}$$

The control statistic of MEWMS$_{AS}$ statistic is the sum of the elements of matrix S_t with the following control limits:

$$UCL_{MEWMS_{AS}} = \frac{p+q}{\nu} \chi^2_{\frac{\alpha_{AS}}{2}}(\nu), \tag{5}$$

$$LCL_{MEWMS_{AS}} = \frac{p+q}{\nu} \chi^2_{1-\frac{\alpha_{AS}}{2}}(\nu) \ , \tag{6}$$

where p and q are the number of variables and attribute quality characteristics, respectively and $\nu = (n(2-\lambda))/\lambda$.

The extended T^2-MEWMS$_{AS}$ control chart alarms an out-of-control signal when at least one statistic falls outside its corresponding control limits. Based on simulation, the control limits of the extended T^2 and MEWMS$_{AS}$ control charts based on the transformed data with standardized multivariate normal distribution are determined such that: (1) the value of ARL_0 obtained separately for each control chart be the same, (2) The desired overall ARL_0 is obtained by simultaneous application of these control charts. Note that after transforming the original data into multivariate Normal distribution by NORTA Inverse method, the mean vector and the covariance matrix of quality characteristics are independent. Consequently, after transforming the data, the control limits of the extended control charts are determined based on the mutivarte Normal distribution.

IV. MONITORING MULTIVARIATE-ATTRIBUTE PROCESSES USING NEURAL NETWORKS

In this section, the structure and the training procedure of two artificial neural networks proposed for monitoring the mean vector and covariance matrix of multivariate-attribute processes are illustrated.

A. Detecting mean shifts in multivariate-attribute processes

In order to monitor the mean vector of multivariate-attribute processes, a three-layer feed-forward neural network which uses back-propagation training algorithm with following structure is suggested:

The number of the nodes in the input layer of neural network A that is suggested for detecting mean shifts in multivariate-attribute processes is considered equal to total quality characteristics. For instance, in a process whose quality is represented by the combination of p variable and q attributes with the correlation matrix of $\boldsymbol{\rho} = [\rho]_{(p+q)\times(p+q)}$, $p+q$ nodes will be considered in the input layer of neural network A. The input vector of the neural network A is the column vector of $\overline{\mathbf{X}_{\mathrm{j}}} = [\bar{x}_{1j}, \bar{x}_{2j}, ..., \bar{x}_{p+q,j}]^T$, where $\bar{x}_{ij}, i = 1, 2, ..., p+q$ is the mean value of ith quality characteristics in jth subgroup. The neural network A has one node in its output layer, where its observed value determines the process mean state.

It is pointed out in the literature that one or two hidden layer in most engineering applications will be enough (Cheng, 1995). It is also mentioned that only one hidden layer can properly approximate any continuous mapping from the input patterns to the output patterns in a back propagation network. Because of lacking any systematic method, the number of nodes in the hidden layer that is highly problem-dependent is set based on trial and error experiments. The sigmoid function is used as the transfer function in the designed neural network A for detecting mean shifts in the process. The sigmoid transfer function that is the mostly used functions put the outputs values of the neural network A in the range of [0,1]. Figure (1) depicts the proposed neural network for detecting mean shifts:

In order to train the neural network A after designing its structure, the proper training data sets which have a crucial effect on the performance of the neural network should be collected.

There are several methods for generating multivariate random numbers in the literature. The proposed methods in which the quality characteristics are correlated and follow a known marginal distribution are categorized into three main types including analytic, numeric and simulation based methods. In the analytic and numeric approaches, it is assumed that the joint distribution of quality characteristics is known. However, in most situations this assumption is violated. Moreover, these methods are mainly applicable in bivariate situations.

In the simulation based methods, the random vectors can be generated only by having the marginal distributions of the quality characteristics as well as their correlation matrix and knowing the joint distribution of quality characteristics is not required. This approach is based on the transformation of Normal random vectors into our desirable random vectors. In this paper, we use the normal to anything (NORTA) method which is a simulation-based method to generate the multivariate-attribute quality characteristic. In order to generate the vector **X** in a multivariate-attribute process where F_{i} is the marginal distribution of ith; $i = 1, ..., p+q$ quality characteristic the following steps should be applied. Suppose the correlation matrix between the quality characteristics is known according to Equation (7):

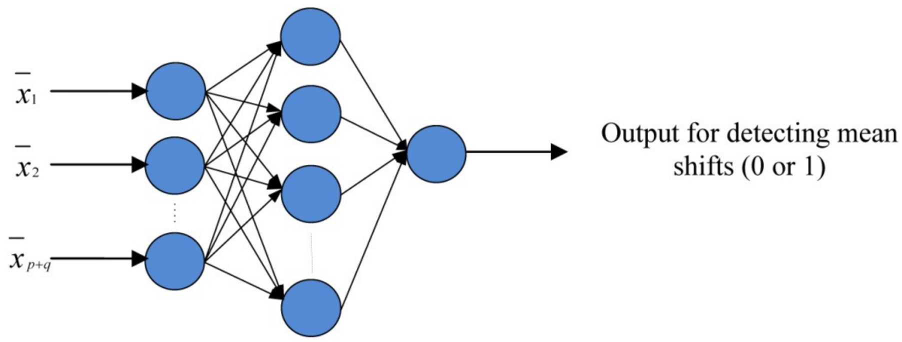

Fig. 1: The structure of the proposed neural network for monitoring the process mean

$$\mathbf{R_X} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1,(p+q)} \\ \rho_{12} & 1 & \cdots & \rho_{2,(p+q)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1,(p+q)} & \rho_{2,(p+q)} & \cdots & 1 \end{pmatrix}. \quad (7)$$

1. The random vector of $Z=(z_1,...,z_{p+q})$ form a multivariate standardized Normal distribution with following correlation matrix:

$$\mathbf{R_Z} = \begin{pmatrix} 1 & \rho'_{12} & \cdots & \rho'_{1,(p+q)} \\ \rho'_{12} & 1 & \cdots & \rho'_{2,(p+q)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho'_{1,(p+q)} & \rho'_{2,(p+q)} & \cdots & 1 \end{pmatrix}. \quad (8)$$

2. The vector **X** is calculated by using the following equation:

$$\mathbf{X} = \begin{pmatrix} F_{x_1}^{-1}(\Phi(z_1)) \\ \vdots \\ F_{x_{p+q}}^{-1}(\Phi(z_{p+q})) \end{pmatrix}, \quad (9)$$

where $\Phi(.)$ is the cumulative distribution function of a standard Normal distribution. Note that, the correlation matrix of $\mathbf{R_z}$ in step 1 depends on the original correlation matrix of $\mathbf{R_X}$. Finding the elements of the $\mathbf{R_z}$ matrix to achieve the original correlation matrix between quality characteristics are usually done through simulation or Newton methods which are very time consuming. This issue is important especially in situations that the number of quality characteristics increases because the elements of correlation matrix increases. In this paper, the Gaussian copula function in MATLAB software is used to find the best values of $\Phi(z_i)$ which lead to obtaining the original coirrelation matrix of quality characteristics in both proposed ANN-based method as well as the extended T^2-$MEWMS_{AS}$ control chart. This facilitates using the NORTA method in generating multivariate-attribute quality characteristics. For more information about the Gaussian copula method, refer to Cherubini et al. (2004).

In the training process of neural network A, first for each state that the multivariate-attribute process mean is out-of-control, we generate 200 random samples of size *n*. After that, the in-control random samples of size *n* are prepared as equal as the total generated out-of-control random samples. It should be added that, the increasing the number of data sets in training process do not have a significant effect on the neural network performance. On the other hand, in the situations that the number of training data sets is not adequate, learning performance of the neural network will not be satisfactory.

After generating all in-control and out-of-control data sets, the sample mean value of each quality characteristic in the generated dataset is computed and used as the input value of the designed neural network A. Finally, the neural network A for recognizing the process mean state is trained via the generated input vectors as well as their corresponding target values. Note that target value for in-control and all out-of-control random samples are considered equal to zero and one, respectively. In each iteration of training process, the mean square error (MSE) that is based on the difference between observed values of output layer and the target value is propagated backward from output towards the input layer. Then the weights associated to the connections are modified. This iterated process is continued until the MSE criterion decreases adequately. Note that all the simulations including generating data sets as well as training the neural networks are done in MATLAB computer package.

B. Detecting variance shifts in multivariate-attribute process

In order to detect different variance shifts in multivariate-attribute processes, a three layer perceptron neural network with back-propagation training algorithm is suggested. The structure of the neural network B that is designed for monitoring the process variability including the number of the nodes in the input and output layer, number of hidden layers as well as the transfer function is similar to the neural network A that is presented for monitoring the process mean. The number of nodes in the hidden layer is also determined based on trial an error procedure. We use the column vector of $\mathbf{S}_j=[S_{1j},S_{2j},\ldots,S_{p+qj}]^T$ as the input vector of the neural network B, where $S_{ij}, i=1,2,\ldots,p+q$ is the standard deviation of *i*th quality characteristics in *j*th subgroup. The structure of neural network B is represented in Figure (2):

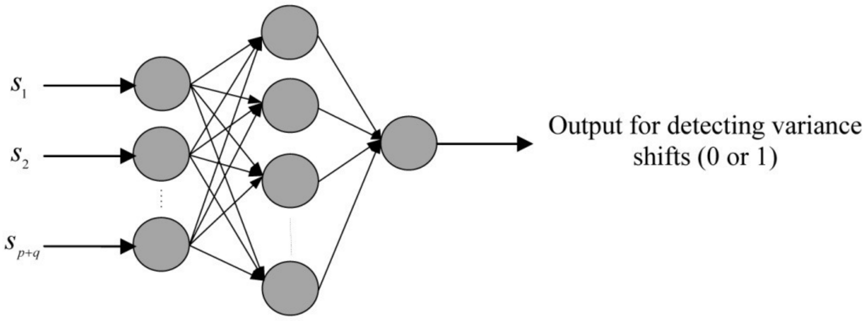

Fig. 2: The structure of the proposed neural network for monitoring the process variability

In order to train the neural network B for recognizing variance shifts, first we prepare 200 random samples of size n for each state that covariance matrix of quality characteristics is out-of-control. Then as equal as the number of out-of-control random samples, the in-control random samples each of size n are generated. After that, the sample standard deviation of each quality characteristic in the generated dataset are calculated and used as the input vectors of the designed neural network. Similar to neural network A, the target value for in-control and out-of-control states are considered equal to zero and one, respectively. After generating all input vectors as well their corresponding target values, the neural network B is trained using back-propagation training algorithm. Note that as similar to neural network A, the mean square error (MSE) criterion is used for evaluation of training the neural network B.

V. PROPOSED NN-BASED METHOD FOR SIMULTANEOUS MONITORING OF PROCESS MEAN AND VARIABILITY

After designing the structure of neural networks A and B as well as their training steps, the proposed neural network-based control scheme can be implemented for simultaneous monitoring of mean vector as well as covariance matrix of multivariate-attribute quality characteristics. For this purpose, we determine the threshold values for the output neurons of both neural networks A and B. The process state is determined based on the simultaneous comparison between the outputs of two proposed neural networks and their corresponding threshold values. The threshold values of the neural networks A and B are calculated such that:

1. When the neural networks A and B are applied separately for detecting mean and variance shifts, respectively; the in-control average run length (ARL_0) obtained by them should be approximately equal.
2. When the neural networks A and B are applied simultaneously, the ARL_0 value obtained by them should be approximately equal to ARL_0 obtained by extended combinatory control charts.

The procedure of calculating the threshold value for the output neuron of both neural networks A and B that are trained for detecting mean and variance shifts is similar. In order to calculate the threshold value for the output neuron of neural network, which is designed for monitoring the mean vector (covariance matrix) of the multivariate-attribute processes, the following steps should be applied:

1. Generating 10000 random samples each of size n form a multivariate-attribute process whose mean vector (covariance matrix) is in-control.
2. Calculating the sample mean (sample standard deviation) value of quality characteristics in each sample taken and entering the input vector that is comprised of $p+q$ elements to the neural network A (B) which is proposed for detecting mean (variance) shifts.
3. Sorting the observed values of output neuron of neural network A (neural network B) in ascending order and saving them in a vector like $\mathbf{c_1}$ ($\mathbf{d_1}$).
4. Determining an element of the vector $\mathbf{c_1}$ ($\mathbf{d_1}$) as the threshold value of output neuron of neural network A (neural network B) such that the ARL_0 obtained by the neural network A (B) becomes equal to a predetermined value.

In order to identify the process state we set two rules as follows:

1. If the observed values of output neurons of both designed neural networks are equal or less than their corresponding thresholds, the process is introduced as an in-control state.
2. Otherwise, if the observed values of at least one output neuron of any neural networks A or B are greater than their corresponding thresholds, the process will be out-of-control.

We also extend a combinatory control charts and compare the results of the proposed neural network-based methodology with it. For this purpose, based on NORTA Inverse technique we extend multivariate T^2 and $MEWMS_{AS}$ control charts and apply them for monitoring mean vector and covariance matrix of multivariate-attribute processes where the quality characteristics are correlated. Then, we apply the extended T^2-$MEWMS_{AS}$ control charts for simultaneous monitoring of mean vector and covariance matrix of multivariate-attribute processes.

VI. NUMERICAL EXAMPLE

In this section a numerical example based on simulation is presented to illustrate the high performance of the proposed neural network-based method and then the results are compared with developed multivariate-attribute T^2-$MEWMS_{AS}$ control chart. In the numerical example the quality of product is considered to be expressed by the combination of a Poisson attribute and a Normal variable. The parameters of quality characteristics are known based on Phase I analysis. Accordingly, the parameter of Poisson attribute (x_1) is equal to 4 and the mean and variance of Normal variable (x_2) are equal to 3 and 4, respectively. The coefficient constant between quality characteristics is considered equal to 0.357 and the random samples of size 10 are used for monitoring the process.

In order to provide a control scheme for simultaneous monitoring of the mean vector and covariance matrix of this process, two three-layer perceptron neural networks should be designed. According to subsections A and B, both neural networks have two (number of quality characteristics) and one nodes in their input and output layers, respectively. Based on trial and error experiments, the designed neural networks A and B that are designed for monitoring the mean vector and covariance matrix of the process have one hidden layer with 12 and 20 nodes, respectively.

According to subsection A, in order to train the neural network A for detecting mean shifts, first for each out-of-control states of the process mean we generate 200 random samples of size n=10. Because, there are three out-of-control states, we prepare totally 600 out-of-control random samples. After that, 600 in-control random samples of size n=10 are also generated. Then, the sample mean values of both quality characteristics in all 1200 generated data sets are calculated and used as the input vectors of the neural network A. The input vectors of the proposed neural network A for detecting mean shifts is the column vector of $\overline{\mathbf{X}}_{\mathbf{j}} = [\bar{x}_{1j}, \bar{x}_{2j}]^T,\ j = 1,2,...,1200$, which $\bar{x}_{1j}$ nd $\bar{x}_{2j}$ are the mean values of Poisson and Normal quality characteristics in the jth training input vector. It should be mentioned that in order to generate out-of-control random samples, the shift of $\mu_i' = \mu_i + 2\sigma_i$ is used in which μ_i and μ_i' are the mean value of ith quality characteristic before and after shift, respectively and σ_i is the standard deviation of ith quality characteristic under in-control state. Table (1) represents the required information in training process of the first neural network:

The first neural network designed for monitoring the multivariate-attribute process mean is trained with the generated input vectors as well as their corresponding target values using back-propagation algorithm. Finally, the MSE value that is obtained from training step is obtained equal to 0.001.

TABLE I. DETAILS OF TRAINING THE DESIGNED NN FOR MONITORING THE PROCESS MEAN

Shifted quality characteristic	Mean process state	Number of data sets	Mean value of x_1	Mean value of x_2	Target value
-	in-control	600	$\lambda = 4$	$\mu = 3$	0
x_1	out-of-control	200	$\lambda = 8$	$\mu = 3$	1
x_2	out-of-control	200	$\lambda = 4$	$\mu = 7$	1
x_1 and x_2	out-of-control	200	$\lambda = 8$	$\mu = 7$	1

TABLE II. DETAILS OF TRAINING THE DESIGNED NN FOR MONITORING THE PROCESS VARIABILITY

Shifted quality characteristic	Mean process state	Number of data sets	Variance of of x_1	Variance value of x_2	Target value
-	in-control	600	$\lambda = 4$	$\sigma^2 = 4$	0
x_1	out-of-control	200	$\lambda = 16$	$\sigma^2 = 4$	1
x_2	out-of-control	200	$\lambda = 4$	$\sigma^2 = 16$	1
x_1 and x_2	out-of-control	200	$\lambda = 16$	$\sigma^2 = 16$	1

The procedure and the number of data sets required for training second neural network that is designed for detecting variance shifts is almost identical to the first neural network. The only difference is that the input vectors of the second neural network is the column vector of $S_j=[S_{1j},S_{2j}]^T$, j=1,2,…,1200, which S_{1j} and S_{2j} are the sample standard deviations of Poisson and Normal quality characteristics in the jth training data sets. In order to generate out-of-control data sets in training process of the second neural network, the shift of $\sigma_i' = 2\sigma_i$ is used which σ_i and σ_i are the standard deviation of ith quality characteristic before and after shift in its variability. The information required for training the second neural network is summarized in Table (2).

The second neural network for variance shifts is trained with the all 1200 input vectors as well as their corresponding target values using back-propagation algorithm. Finally, the MSE value of the training process is obtained equal to 0.0615.

In order to compare the performance of the proposed neural network-based methodology with the combined T^2-$MEWMS_{AS}$ control chart, we set the threshold values of the designed neural networks as well as the parameters of the extended control charts such that the ARL_0 obtained by both methods become roughly equal to 200. The ARL_0 obtained by simultaneous application of the designed neural networks will be equal to 200 if the ARL_0 obtained by separate application of each neural network become equal to 400. For this purpose, according to section 3 the output thresholds of the first and second neural networks are considered equal to 0.460 and 0.900, respectively. Consequently, based on 10000 replicates the ARL_0 obtained by the first and second neural networks in monitoring the process mean and the process variability are to 395.48 and 400.48, respectively. Then, the overall ARL_0 based on the simultaneous using of the designed neural networks is calculated equal to 194.21. After calculating the threshold values of both neural networks, the process state can be determined by simultaneous comparison between outputs values and their threshold values according to section 3.

Note that, the mean and the variance of the Poisson quality characteristic are dependent. Hence, the proposed NNs will be dependent. It is clear that the dependency between the proposed NNs improves the detection performance of the proposed NN-based method. Because it can increase the probability of detecting shifts, even very small shifts in the parameter of Poisson quality characteristic by at least one of the designed NNs.

The results of the proposed neural network-based method in simultaneous monitoring of mean vector and covariance matrix of the process in terms of out-of-control average run length (ARL_1) criterion are presented in Table (3) and the results are compared with the combined T^2-$MEWMS_{AS}$ multivariate-attribute control chart. For each simultaneous shift, the standard deviations of run lengths are also presented in bracket. The results of both combined methods are obtained based on 10000 replicates. The results of Table (3) show high detection performance of the proposed neural network-based approach in detecting different simultaneous shifts in the multivariate-attribute process. The first column (first six simultaneous shifts) of Table (3) are shifts in the mean of x_1 and the variance of x_2, while the others are shifts in the variance of x_1 and the mean of x_2. We can conclude that in the first six simultaneous shifts, the proposed methodology outperforms the extended combined T^2-$MEWMS_{AS}$ control chart, however in the last six shifts; the detection performance of both methods is almost the same. The results of Table (3) show that the standard deviations of run lengths for both NN-based approach and the extended control chart are small. Consequently, the performance of both methods in detecting simultaneous shifts in the mean vector and covariance matrix of the process in terms of ARL_1 criterion are valid.

TABLE III. ARL1 VALUES UNDER DIFFERENT SIMULATNEOUS SHIFTS

simultaneous shift	ANN	T^2-MEWMS$_{AS}$	simultaneous shift	ANN	T^2-MEWMS$_{AS}$
$(\mu_1' = \mu_1 + \sigma_1, \sigma_2' = 1.1\sigma_2)$	1.52 (0.95)	1.56 (1.30)	$(\sigma_1' = 1.3\sigma_1, \mu_2' = \mu_2 + 0.25\sigma_2)$	1.17 (0.46)	1.20 (0.53)
$(\mu_1' = \mu_1 + \sigma_1, \sigma_2' = 1.2\sigma_2)$	1.55 (0.95)	1.57 (1.18)	$(\sigma_1' = 1.35\sigma_1, \mu_2' = \mu_2 + 0.25\sigma_2)$	1.04 (0.22)	1.05 (0.26)
$(\mu_1' = \mu_1 + 1.25\sigma_1, \sigma_2' = 1.1\sigma_2)$	1.18 (0.47)	1.23 (0.61)	$(\sigma_1' = 1.4\sigma_1, \mu_2' = \mu_2 + 0.25\sigma_2)$	1.01 (0.10)	1.01 (0.12)
$(\mu_1' = \mu_1 + 1.25\sigma_1, \sigma_2' = 1.2\sigma_2)$	1.19 (0.47)	1.20 (0.55)	$(\sigma_1' = 1.3\sigma_1, \mu_2' = \mu_2 + 0.5\sigma_2)$	1.21 (0.56)	1.17 (0.49)
$(\mu_1' = \mu_1 + 1.5\sigma_1, \sigma_2' = 1.1\sigma_2)$	1.05 (0.24)	1.07 (0.30)	$(\sigma_1' = 1.35\sigma_1, \mu_2' = \mu_2 + 0.5\sigma_2)$	1.05 (0.30)	1.05 (0.26)
$(\mu_1' = \mu_1 + 1.5\sigma_1, \sigma_2' = 1.2\sigma_2)$	1.05 (0.24)	1.06 (0.27)	$(\sigma_1' = 1.4\sigma_1, \mu_2' = \mu_2 + 0.5\sigma_2)$	1.02 (0.13)	1.02 (0.14)

We also evaluate the performance of the proposed method in detecting separate mean and variance shifts and compare it with T^2-MEWMS$_{AS}$ control chart. The results of detecting different step shifts in the mean vector and the covariance matrix by both methods in terms of average run length as well as the standard deviation of run lengths criteria are given in Tables (4) and (5), respectively. The results of Tables (4) and (5) represent the satisfactory implementation of both methods in detecting mean and variance shifts, respectively. It can be also concluded from Tables (4) and (5) that the standard deviations values of run lengths in NN-based and the extended control chart are adequately small. Obviously, the results of ARL_1 values in detecting different mean shifts as well as the variance shifts are valid.

TABLE IV. ARL1 VALUES UNDER DIFFERENT MEAN SHIFTS

Mean shift	ANN	T^2-MEWMS$_{AS}$	Mean shift	ANN	T^2-MEWMS$_{AS}$
$(\mu_1' = \mu_1 + 0.75\sigma_1)$	2.76 (2.24)	3.47 (3.74)	$(\mu_1' = \mu_1 + 1.25\sigma_1)$	1.19 (0.48)	1.27 (0.65)
$(\mu_2' = \mu_2 + 0.75\sigma_2)$	9.28 (9.26)	3.36 (3.55)	$(\mu_2' = \mu_2 + 1.25\sigma_2)$	1.50 (0.87)	1.16 (0.45)
$(\mu_1 + 0.75\sigma_1, \mu_2 + 0.75\sigma_2)$	5.44 (5.02)	1.91 (1.67)	$(\mu_1 + 1.25\sigma_1, \mu_2 + 1.25\sigma_2)$	1.09 (0.76)	1.04 (0.22)
$(\mu_1' = \mu_1 + \sigma_1)$	1.58 (0.98)	1.78 (1.41)	$(\mu_1' = \mu_1 + 1.5\sigma_1)$	1.05 (0.24)	1.09 (0.33)
$(\mu_2' = \mu_2 + \sigma_2)$	2.90 (2.46)	1.63 (1.16)	$(\mu_2' = \mu_2 + 1.5\sigma_2)$	1.11 (0.37)	1.03 (0.17)
$(\mu_1 + \sigma_1, \mu_2 + \sigma_2)$	2.35 (1.84)	1.19 (0.54)	$(\mu_1 + 1.5\sigma_1, \mu_2 + 1.5\sigma_2)$	1.09 (0.32)	1.01 (0.06)

TABLE V. ARL1 VALUES UNDER DIFFERENT VARIANCE SHIFTS

Variance shift	ANN	T^2-MEWMS$_{AS}$	Variance shift	ANN	T^2-MEWMS$_{AS}$
$(\sigma_1' = 1.15\sigma_1)$	4.02 (3.44)	5.18 (6.01)	$(\sigma_1' = 1.3\sigma_1)$	1.10 (0.33)	1.15 (0.44)
$(\sigma_2' = 1.5\sigma_2)$	5.76 (5.43)	4.19 (4.93)	$(\sigma_2' = 2\sigma_2)$	1.66 (1.07)	1.52 (1.14)
$(\sigma_1' = 1.15\sigma_1, \sigma_2' = 1.5\sigma_2)$	2.71 (2.16)	2.04 (1.95)	$(\sigma_1' = 1.3\sigma_1, \sigma_2' = 2\sigma_2)$	1.04 (0.22)	1.06 (0.16)
$(\sigma_1' = 1.2\sigma_1)$	1.95 (1.43)	2.35 (2.15)	$(\sigma_1' = 1.35\sigma_1)$	1.02 (0.16)	1.05 (0.22)
$(\sigma_2' = 1.75\sigma_2)$	2.58 (2.04)	2.16 (2.14)	$(\sigma_2' = 2.25\sigma_2)$	1.37 (0.70)	1.23 (0.70)
$(\sigma_1' = 1.2\sigma_1, \sigma_2' = 1.75\sigma_2)$	1.44 (0.80)	1.24 (0.69)	$(\sigma_1' = 1.35\sigma_1, \sigma_2' = 2.25\sigma_2)$	1.01 (0.09)	1.01 (0.07)

VII. CONCLUSION AND FUTURE RESEARCH

In this paper, we proposed a neural network-based approach control scheme for simultaneous monitoring of the multivariate-attribute process mean and variability. For this purpose, first we designed two three-layer perceptron neural networks for detecting mean and variance shifts, respectively. Then, in order to determine the multivariate-attribute process state, we applied both neural networks simultaneously. Because of lacking any method in the literature for comparison study, we also developed two multivariate control charts including T^2 and $MEWMS_{AS}$ that are proposed for monitoring multivariate process mean and variability, respectively. Then we combined and applied them for simultaneous detecting mean and variance shifts in multivariate-attribute processes. The results of comparison showed that the proposed neural network-based method outperforms the T^2-$MEWMS_{AS}$ control chart in most simultaneous shifts. The results also confirmed the satisfactory performance of both methods in detecting separate mean and variance shifts in the process. As a future research, identifying the magnitude of the simultaneous shifts in multivariate-attribute processes can be investigated using statistical methods as well as artificial neural networks.

ACKNOWLEDGEMENT

The authors are grateful to the respectful referees for their precious comments which led to improvement in the paper.

REFERENCES

1. Ahmadzadeh, F. (2011). Change point detection with multivariate control charts by artificial neural network. *The International Journal of Advanced Manufacturing Technology*, 1-12, published online. DOI: 10.1007/s00170-009-2193-6
2. Amiri, A., Maleki, M. R., & Doroudyan, M. H. (2015). Monitoring Variability of multivariate-attribute processes using artificial neural network. *Production and Operations Management*, 5 (2) 21-36.
3. Aparisi, F., Avendaño, G., & Sanz, J. (2006). Techniques to interpret T^2 control chart signals. *IIE Transactions*, 38(8) 647-657.
4. Aparisi, F., García-Bustos, S., & Epprecht, E. K. (2014). Optimum multiple and multivariate Poisson statistical control charts. *Quality and Reliability Engineering International*, 30(2) 221-234.
5. Bersimis, S., Psarakis, S., & Panaretos, J. (2007). Multivariate statistical process control charts: an overview. *Quality and Reliability Engineering International*, 23(5) 517-543.
6. Brian Hwarng, H., & Wang, Y. (2010). Shift detection and source identification in multivariate autocorrelated processes. *International Journal of Production Research*, 48(3) 835-859.
7. Cheng, C. S. (1995). A multi-layer neural network model for detecting changes in the process mean. *Computers & Industrial Engineering*, 28(1) 51-61.
8. Cheng, C. S., & Cheng, H. P. (2011). Using neural networks to detect the bivariate process variance shifts pattern. *Computers & Industrial Engineering*, 60(2) 269-278.
9. Cherubini, U., Luciano, E., & Vecchiato, W. (2004). Copula methods in finance. John Wiley & Sons.
10. Doroudian, M. H., Amiri, A., Root transformation method for monitoring correlated variable and attribute quality characteristics. Proceedings of 11th Islamic Countries Conference on Statistical Sciences (ICCS-11), Lahore, Pakistan, December 19-21, 2011.
11. Doroudyan, M. H., & Amiri, A. (2013). Monitoring multivariate–attribute processes based on transformation techniques. *The International Journal of Advanced Manufacturing Technology*, 69(9-12) 2161-2172.
12. Golnabi, S., & Houshmand, A. A. (1999). *Multivariate shewhart x-bar chart*. Inter Stat, 4.

13. Hotelling, H. (1947). Multivariate quality control. *Techniques of statistical analysis*. New York: McGraw-Hill, 111-184.

14. Hwarng, H. B. (2008). Toward identifying the source of mean shifts in multivariate SPC: a neural network approach. *International Journal of Production Research*, 46(20) 5531-5559.

15. Kang, L., & Brenneman, W. A. (2011). Product defect rate confidence bound with attribute and variable data. *Quality and Reliability Engineering International*, 27(3) 353-368.

16. Li, J., Tsung, F., & Zou, C. (2014). Multivariate binomial/multinomial control chart. *IIE Transactions*, 46(5) 526-542.

17. Maleki, M. R., Amiri, A., & Doroudyan, M. H. (2012). Monitoring multivariate-attribute processes using artificial neural network. 42th conference on Computers and Industrial Engineering, Cape Town, South Africa. (CIE42).

18. Maleki, M. R., Amiri, A., & Rasouli, M. (2013). Monitoring Variability of Multivariate-attribute Processes Using EWMA Control Charts Based on NORTA Inverse Technique. 3rd International Conference on Production and Industrial Engineering, Jalanhar, India, (CPIE-2013), pp. 1390-1393.

19. Memar, A.O. & Niaki, S.T.A. (2011). Multivariate variability monitoring using EWMA control charts based on squared deviation of observations from target. *Quality and Reliability Engineering International*, 27(8) 1069-1086.

20. Niaki, S. T. A., & Abbasi, B. (2005). Fault diagnosis in multivariate control charts using artificial neural networks. *Quality and Reliability Engineering International*, 21(8) 825-840.

21. Niaki, S. A., & Abbasi, B. (2008). Detection and classification mean-shifts in multi-attribute processes by artificial neural networks. *International Journal of Production Research*, 46(11) 2945-2963.

22. Salehi, M., Kazemzadeh, R. B., & Salmasnia, A. (2012). On line detection of mean and variance shift using neural networks and support vector machine in multivariate processes. *Applied Soft Computing*, 12(9) 2973-2984.

23. Shang, Y., Tsung, F., & Zou, C. (2013). Statistical process control for multistage processes with binary outputs. *IIE Transactions*, 45(9) 1008-1023.

24. Topalidou, E., & Psarakis, S. (2009). Review of multinomial and multiattribute quality control charts. *Quality and Reliability Engineering International*, 25(7) 773-804.

25. Yeh, A. B., Huwang, L., & Wu, C. W. (2005). A multivariate EWMA control chart for monitoring process variability with individual observations. *IIE Transactions*, 37(11) 1023-1035.

26. Yeh, A. B., Li, B., & Wang, K. (2012). Monitoring multivariate process variability with individual observations via penalised likelihood estimation. *International Journal of Production Research*, 50(22) 6624-6638.

27. Yu, J. B., & Xi, L. F. (2009). A neural network ensemble-based model for on-line monitoring and diagnosis of out-of-control signals in multivariate manufacturing processes. *Expert Systems With Applications*, 36(1) 909-921.

28. Yu, J., Xi, L., & Zhou, X. (2009). Identifying source (s) of out-of-control signals in multivariate manufacturing processes using selective neural network ensemble. *Engineering Applications of Artificial Intelligence*, 22(1) 141-152.