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An Intuitionistic Fuzzy DEA Cross-Efficiency Methodology with an Application to Production Group Decision-Making Problems

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Abstract *–In decision-making situations, the opinions expressed by decision-makers (DMs) are often vague. Using linguistic variables expressed in intuitionistic fuzzy numbers is a more realistic approach to describing DMs' judgments. The paper aims to develop a Group Decision Making (GDM) methodology based on the data envelopment analysis (DEA) method with intuitionistic fuzzy information. This method is utilized once a set of Decision-Making Units (DMUs) need to be ranked based on their efficiencies over a set of input and output measures considering DMs' weights in an intuitionistic fuzzy environment. In the proposed method, concerning the input and output measures, each DM utilizes membership and non-membership degrees to determine the degrees of satisfiability and non-satisfiability of each DMU, respectively. Besides, a new technique is presented to determine the DMs' weights. Different values of a DMU's efficiency obtained by individual DMs are converted into an aggregated efficiency based on the DMs' weights. Finally, the extended DEA method is used to rank the DMUs based on their efficiencies. A case study on a production company is done for illustration and verification of the proposed approach.*

Keywords–*Data envelopment analysis, Intuitionistic fuzzy number, Production group decision problems.*

I. INTRODUCTION

Managers often face situations where they need to involve a group of people in the decision-making process to incorporate different ideas and perspectives and improve the decision's quality. GDM approaches help managers to make conscious decisions. GDM considers the problem of selecting alternatives by a group of DMs (Meng and Pei, 2013, Vahdani et al., 2013, Ebrahimnejad et al., 2012, Xu and Chen, 2007, Gitinavard et al., 2018). In real-world problems, the decision-making process often has conflicting criteria and complexity (Ghaderi et al., 2019, Gitinavard et al., 2017, Ghaderi et al., 2017). Therefore, many GDM approaches have been developed to facilitate decision-making in engineering and management fields (Gitinavard and Akbarpour Shirazi, 2018, Gitinavard and Zarandi, 2016, Borujeni and Gitinavard, 2017).

In some decision-making problems, DMs' information about DMUs regarding the criteria is not precise (e.g., unquantifiable information or incomplete information about alternatives regarding attributes and the relative weight of each attribute) (Hashemi et al., 2013, 2014, 2018; Mousavi et al., 2019; Mohagheghi et al., 2015). In this situation,

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fuzzy set theory has been extensively applied in different fields since it was introduced by Zadeh (1965). Qin & Liu (2015) proposed an approach for GDM problems. They used combined ranking in an interval type-2 fuzzy environment. Interval type-2 fuzzy sets also have been used by Chen (2014) for developing an ELECTRE-based method for group decision-making. Mousavi et al. (2016) extended a VIKOR approach based on intuitionistic fuzzy weighted averaging with multi-attributes and multi-judges for solving the material handling selection and portfolio selection problems. Mousavi and Vahdani (2016) elaborated a hierarchical group decision-making methodology under an intuitionistic fuzzy set environment to solve the distribution system's cross-docking location selection problem.

Hesitant Fuzzy Sets (HFSs) represent several possible values for the membership degree of an element to a set, so it provides an appropriate means to consider different DMs groups' opinions. In this regard, Chen and Xu (2015) developed an approach that combines the HFS and ELECTRE methods to handle multiple-criteria decision-making (MCDM) problems. Zhang and Xu (Zhang and Xu, 2014) presented an interval programming method to solve GDM problems. They used hesitant fuzzy elements (HFEs) to evaluate the alternatives and interval numbers for all pair-wise comparisons of alternatives. Zadeh's fuzzy set theory is a suitable means of considering fuzziness in GDM problems (Salarpour et al., 2019; Davoudabadi et al., 2020; Hajighasemi and Mousavi, 2018). However, a fuzzy set is used to determine the fuzziness by membership degree. Atanassov (1986, 1989) further introduced an intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy sets, and it is a better means of considering fuzziness by adding non-membership degree.

Vague sets (VSs) as a kind of fuzzy set generalization were introduced by Gau and Buehrer (1993). Bustince and Burillo (1996) demonstrated that the notion of VSs is as good as that of IFSs. Xu and Yager (2006) proposed some geometric aggregation operators which extend the weighted geometric (WG) and ordered weighted geometric (OWG) operators to accommodate the intuitionistic fuzzy set environment. Zhang and Liu (2010) denoted the membership degree and the non-membership degree of the intuitionistic fuzzy numbers by the fuzzy triangular numbers and proposed the triangular intuitionistic fuzzy numbers. Then, they defined the weighted arithmetic average operator and proposed an approach for GDM problems with triangular intuitionistic fuzzy information. The fuzzy number intuitionistic fuzzy set was introduced by Wei et al. (2010). They also developed the induced fuzzy number intuitionistic fuzzy order weighted geometric operator and used the developed operator to propose the GDM. Du and Liu (2011) developed a comprehensive VIKOR MADM approach based on intuitionistic trapezoidal fuzzy numbers for attribute values.

DEA, proposed by Charnes et al. (1978) and developed by Banker et al. (1984), is a decision-making tool used to measure the efficiency of DMUs when there is no judgment on the weights of criteria (Ghaderi et al., 2019). In other words, for measuring the efficiency of DMUs, DEA specifies the weights of input and output indicators by an optimization model (Toloo, 2015). Also, in this method, the normalization of the indicators with different dimensions is not needed. Concerning the efficiency scores of DMUs, they are categorized into two groups: (1) efficient and (2) inefficient. Additionally, there are six categories for DMU ranking methods: (1) cross-efficiency, (2) super-efficiency, (3) benchmark, (4) ranking with multivariate statistics in the DEA context, (5) ranking inefficient DMUs, and (6) DEA and MCDM (Adler et al., 2002).

There is a vast literature involving the DEA method in an extensive range of fields. For instance, Lim et al. (2014) introduced a way for portfolio selection using DEA cross-efficiency evaluation. They also improved using of cross-efficiency in portfolio selection. Mansouri et al. (2014) proposed an integrated (TOPSIS-DEA) approach to ranking active companies in the cement industry. A game cross-efficiency based on DEA was presented by Ma et al. (2014) to assess suppliers' performances. Their proposed method can get an exceptional efficiency, which is a Pareto solution. Daneshvar Rouyendegh (2011) presented a two-stage model, an integrated fuzzy TOPSIS and DEA, for selecting DMUs with the most efficiency. Arya and Yadav (2019) manipulated intuitionistic fuzzy-DEA and dual intuitionistic fuzzy-DEA models based on alpha and beta cuts. Besides, they proposed an intuitionistic fuzzy correlation coefficient to validate the proposed methodologies. Otay et al. (2017) presented a novel multi-expert fuzzy approach by integrating DEA methodology and analytic hierarchy process under an intuitionistic fuzzy set environment.

Crisp information is one of the deficiencies of the classic DEA models. In this regard, Hajiagha et al. (2013) extended a DEA model in which intuitionistic fuzzy numbers represent some inputs and outputs. Amiri et al. (2010) presented an eigenvector–DEA–TOPSIS technique to assess the risk of the number of related portfolios to the FOREX spot market through determining the weights of criteria, computing the value of linguistic terms, and aggregating portfolio risks. Wu et al. (2019a) designed a production planning regarding the DEA approach and production stability. The authors (2019b) presented an EBD-DEA model to assess Chinese coal enterprises' geographical location and production efficiency. Xia et al. (2020) elaborated a DEA approach based on empirical analysis to appraise the coke production chain's dynamic performance. However, in many complex real-world problems, the information about DMUs regarding the input and output measures is incomplete or insufficient. Therefore, the decisions made by an individual DM could not be effective. In these cases, more opinions about the DMUs regarding the input and output measures are needed. GDM can lead to more accurate and useful decisions by having a larger number of individuals' perspectives. Therefore, the goal of this study is to develop a GDM approach based on a DEA cross-efficiency method with intuitionistic fuzzy information.

The organization of the paper is as follows. The mathematical preliminaries of IFSs are provided in Section 2. Section 3 develops a GDM method based on a DEA method with intuitionistic fuzzy information. Section 4 provides an application of the developed methodology. Finally, Section 5 concludes the paper with some future research directions.

II. PRELIMINARIES

Definition. 1. Let X be a universe set, then a fuzzy set is defined as follows (Zadeh, 1965):

$$A = \{\langle x, S_A(x)\rangle | x \in X\} \tag{1}$$

where the function$S_A : X \to [0,1]$denotes the membership degree and $T_A : X \to [0,1]$ denotes the non-membership degree of the element $x \in X$ to $A \subset X$. A generalized fuzzy set called IFS is defined as follows (Atanassov, 1986, Atanassov, 1999):

$$A = \{\langle x, S_A(x), T(x), P_A(x)\rangle \ | x \in X\ \} \tag{2}$$

where the membership function $S_A : X \to [0,1]$and non-membership function $T_A : X \to [0,1]$ satisfy the condition $0 \leq S_A(x) + T_A(x) \leq 1,\ \forall x \in X$. In the definition of IFS, $P_A(x)$ is called the non-determinacy degree of *x* in the set A and is defined as follows (Atanassov, 1999):

$$P_A(x) = 1 - S_A(x) - T_A(x) \tag{3}$$

For ordinary fuzzy sets, $P_A(x) = 0, \forall x \in A$.

Definition. 2. Let *A* and *B* be two IFNs that are denoted as $A = (S_A, T_A)$and $B = (S_B, T_B)$, then the following relations hold for every real number $\lambda > 0$ (Xu and Yager, 2006, Xu, 2007).

$$A + B = (S_A + S_B - S_A.S_B, T_A.T_B) \tag{4}$$

$$A \times B = (S_A.S_B, T_A + T_B - T_A.T_B) \tag{5}$$

$$\lambda A = \left(1-(1-S_A)^{\lambda}, T_A^{\lambda}\right), \lambda > 0; \tag{6}$$

$$A^{\lambda} = \left(S_A^{\lambda}, 1-(1-T_A)^{\lambda}\right), \lambda > 0; \tag{7}$$

$$A \le B \text{ if and only if } S_A \le S_B \text{ and } T_A \ge T_B \tag{8}$$

Definition. 3. Let $A_1, A_2, \ldots A_n$ be n IFNs and $w = (w_1, w_2, \ldots, w_n)$ be the weight of IFNs $A_j = (S_j, T_j)$ where $\sum_{j=1}^{n} w_j = 1, w_j \ge 0,$ then the intuitionistic fuzzy weighted averaging (IFWA) operator is defined as follows (Xu, 2010):

$$A(w) = \sum_{j=1}^{n} w_j A_j \tag{9}$$

We can rewrite Eq. (9) based on definition 2 as follows (Yue, 2014):

$$A(w) = \left(1 - \prod_{j=1}^{n}(1-S_j)^{w_j}, \prod_{j=1}^{n} T_j^{w_j}\right) \tag{10}$$

Let $w_1 = w_2 = \ldots = w_n = 1/n$ or $w = \left(1/n, 1/n, \ldots, 1/n\right),$ then $A(w)$ is converted to the arithmetic aggregation operator:

$$A(w) = \left(1 - \prod_{j=1}^{n}(1-S_j)^{1/n}, \prod_{j=1}^{n} T_j^{1/n}\right) \tag{11}$$

Let $A = (S_A, T_A)$ and $B = (S_B, T_B)$ be two IFNs, then the Euclidean distance between A and B is defined as follows (Szmidt and Kacprzyk, 2000):

$$d(A,B) = \sqrt{\frac{1}{2}\left((S_A - S_B)^2 + (T_A - T_B)^2 + (P_A - P_B)^2\right)} \tag{12}$$

where $P_A = 1 - S_A - T_A$, $P_B = 1 - S_B - T_B$ and $0 \le d(A,B) \le 1.$

Definition. 4. Let $X = (x_{ij})_{m \times n}$ be a matrix, if all the matrix elements X be IFNs then X is an IF matrix. Let $X_1 = \left((S_{ij}^1, T_{ij}^1)\right)_{m \times n}$ and $X_2 = \left((S_{ij}^2, T_{ij}^2)\right)_{m \times n}$ be two IF matrices, then the distance measure between X_1 and X_2 is defined as follows (Yue, 2014):

$$d(X_1, X_2) = \sqrt{\frac{1}{2mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left(S_{ij}^1 - S_{ij}^2\right)^2 + \left(T_{ij}^1 - T_{ij}^2\right)^2 + \left(P_{ij}^1 - P_{ij}^2\right)^2\right)} \tag{13}$$

where $P_{ij}^1 = 1 - S_{ij}^1 - T_{ij}^1,\ P_{ij}^2 = 1 - S_{ij}^2 - T_{ij}^2\ (i = 1,\dots,m,\ j = 1,\dots,n)$ and $0 \le d\left(X_1, X_2\right) \le 1.$

III. PROPOSED IF-DEA-GDM METHOD

The procedure of the proposed IF-DEA-GDM method is presented in this section. The indices, parameters, and variables of the proposed method are defined as follows.

j: index of the DMUs $j = 1, 2, \cdots, n$

i: index of the inputs $i = 1, 2, \cdots, m$

r: index of the outputs $r = 1, 2, \dots, s$

d : index of the DMs $d = 1, 2, \dots, t$

$x^d{}_{ij}$: the *i*th input of the *j*th DMU determined by the *d*th DM

$y^d{}_{rj}$: the *r*th output of the *j*th DMU determined by the *d*th DM

v_i : weight of the *i*th input

u_r : weight of the *r*th output

A. Determine the weights of DMs.

Step 1. Establish the individual intuitionistic fuzzy decision matrices for all DMs.

$$A^d = \begin{matrix} & DMU_1 & \cdots & DMU_n \\ X_1 & A^d{}_{11} & \cdots & A^d{}_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ X_m & A^d{}_{m1} & \cdots & A^d{}_{mn} \\ Y_1 & A^d{}_{(m+1)1} & \cdots & A^d{}_{(m+1)n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_s & A^d{}_{(m+s)1} & \cdots & A^d{}_{(m+s)n} \end{matrix} = \begin{bmatrix} x^d{}_{11} & \cdots & x^d{}_{1n} \\ \vdots & \ddots & \vdots \\ x^d{}_{m1} & \cdots & x^d{}_{mn} \\ \hline y^d{}_{11} & \cdots & y^d{}_{1n} \\ \vdots & \ddots & \vdots \\ y^d{}_{s1} & \cdots & y^d{}_{sn} \end{bmatrix} = \begin{bmatrix} (\mu^d_{x_{11}}, \upsilon^d_{x_{11}}) & \cdots & \left(\mu^d_{x_{1n}}, \upsilon^d_{x_{1n}}\right) \\ \vdots & \ddots & \vdots \\ (\mu^d_{x_{m1}}, \upsilon^d_{x_{m1}}) & \cdots & (\mu^d_{x_{mn}}, \upsilon^d_{x_{mn}}) \\ \hline (\mu^d_{y_{11}}, \upsilon^d_{y_{11}}) & \cdots & (\mu^d_{y_{1n}}, \upsilon^d_{y_{1n}}) \\ \vdots & \ddots & \vdots \\ (\mu^d_{y_{s1}}, \upsilon^d_{y_{s1}}) & \cdots & (\mu^d_{y_{sn}}, \upsilon^d_{y_{sn}}) \end{bmatrix} \tag{14}$$

Where A^d is the individual intuitionistic fuzzy decision matrix determined by the *d*th DM where the IFNs $x^d{}_{ij} = (\mu^d_{x_{ij}}, \upsilon^d_{x_{ij}})$ and $y^d{}_{ij} = (\mu^d_{y_{ij}}, \upsilon^d_{y_{ij}})$ indicate the value the *j*th DMU with respect to the *i*th input measure and the *r*th output measure, respectively. Besides, $\mu^d_{x_{ij}}$ and $\mu^d_{y_{ij}}$ express the satisfiability degrees of the *j*th DMU with respect to the *i*th input measure and the *r*th output measure, respectively. Also, $\upsilon^d_{x_{ij}}$ and $\upsilon^d_{y_{ij}}$ denote the non-satisfiability degrees of the *j*th DMU with respect to the *i*th input measure and the *r*th output measure, respectively.

Step 2. Calculate the intuitionistic fuzzy preference value for each DM as follows:

$$IFPV_d = \sqrt{\sum_{j=1}^{n}\sum_{k=1}^{m+s}\left(\left|A_{kj}^d - \bar{R}_{kj}\right|^2\right)} \quad \forall d \tag{15}$$

where $\bar{R}_{kj}$ denotes the elements of the mean matrix that is calculated as follows:

$$\bar{R} = (\bar{R}_{kj})_{k\times n}$$

where $$\bar{R}_{kj} = \frac{1}{t}\sum_{d=1}^{t} A^d{}_{kj} \quad \forall k, j \tag{16}$$

$$\bar{R} = \begin{bmatrix} \bar{R}_{11} & \cdots & \bar{R}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{R}_{(m+s)1} & \cdots & \bar{R}_{(m+s)n} \end{bmatrix} = \begin{bmatrix} (\mu^*{}_{11}, \upsilon^*{}_{11}) & \cdots & (\mu^*{}_{1n}, \upsilon^*{}_{1n}) \\ \vdots & \ddots & \vdots \\ (\mu^*{}_{(m+s)1}, \upsilon^*{}_{(m+s)1}) & \cdots & (\mu^*{}_{(m+s)n}, \upsilon^*{}_{(m+s)n}) \end{bmatrix} \tag{17}$$

where $\mu^*{}_{kj} = 1 - \prod_{d=1}^{t}(1-\mu^d{}_{kj})^{1/t}$ and $\upsilon^*{}_{kj} = \prod_{d=1}^{t}\upsilon^d{}_{kj}{}^{1/t}$.

Step 3. Compute the intuitionistic fuzzy overall preference value (θ_d) for each DM regarding the deviation (η_d) from $IFPV_d$ as follows.

$$\eta_d = 1 - IFPV_d \quad \forall d \tag{18}$$

$$\theta_d = \frac{\eta_d}{\sum_{d=1}^{t}\eta_d} \quad \forall d \tag{19}$$

Where $\sum_{d=1}^{t}\theta_d = 1 \ \forall d$.

Step 4. Establish the intuitionistic fuzzy preference selection (PS) matrix for each DM as follows.

$$PS^d = (PS_{kj}^d) = \left(\theta_d . R_{kj}^d\right) = \begin{matrix} \\ X_1 \\ \vdots \\ X_m \\ Y_1 \\ \vdots \\ Y_s \end{matrix} \begin{matrix} DMU_1 & \cdots & DMU_n \\ \left[\begin{matrix} PS^d{}_{11} \\ \vdots \\ PS^d{}_{m1} \\ PS^d{}_{(m+1)1} \\ \vdots \\ PS^d{}_{(m+s)1} \end{matrix}\right. & \begin{matrix} \cdots \\ \ddots \\ \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} PS^d{}_{1n} \\ \vdots \\ PS^d{}_{mn} \\ PS^d{}_{(m+1)n} \\ \vdots \\ PS^d{}_{(m+s)n} \end{matrix}\right] \end{matrix} \tag{20}$$

Step 5. Construct the minimum value matrix as follows.

$$h=\left(\mu_{kj}^{\min}\right)=\begin{matrix} \\ X_1 \\ \vdots \\ X_m \\ Y_1 \\ \vdots \\ Y_s \end{matrix}\begin{matrix} DMU_1 & \cdots & DMU_n \\ \left[\begin{matrix} \mu_{11}^{\min} \\ \vdots \\ \mu_{m1}^{\min} \\ \mu_{(m+1)1}^{\min} \\ \vdots \\ \mu_{(m+s)1}^{\min} \end{matrix}\right. & \begin{matrix} \cdots \\ \ddots \\ \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} \mu_{1n}^{\min} \\ \vdots \\ \mu_{mn}^{\min} \\ \mu_{(m+1)n}^{\min} \\ \vdots \\ \mu_{(m+s)n}^{\min} \end{matrix}\right] \end{matrix} \tag{21}$$

Where $\mu_{kj}^{\min}=\min_{d}\left\{\mu_{kj}^{d}\right\}$.

Step 6. The relative significance of each DM is computed as follows.

$$w_d=\frac{\sqrt{\sum_{j=1}^{n}\sum_{k=1}^{m+s}\left(\left|PS_{kj}^{d}-h_{kj}^{\min}\right|^2\right)}}{\sum_{d=1}^{t}\sqrt{\sum_{j=1}^{n}\sum_{k=1}^{m+s}\left(\left|PS_{kj}^{d}-h_{kj}^{\min}\right|^2\right)}}\ \forall d \tag{22}$$

B. Rank the preference order of the DMUs

Following is the DEA classic model, also known as the CCR model, with precise inputs and outputs. This model that presupposes constant returns to scale is used to measure the efficiency of a set of DMUs. This method evaluates each DMU in terms of its ability to convert inputs into outputs (Andersen and Petersen, 1993).

$$efficiency=\frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} \tag{23}$$

CCR model in the multiplier form is represented as follows:

$$\begin{aligned} &\max\ E_0=\sum_{r=1}^{s}u_r y_{r0} \\ &s.t. \\ &\sum_{i=1}^{m}v_i x_{i0}=1 \\ &\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\le 0;\ \forall j \\ &v_i,u_r\ge 0;\ \forall i,r \end{aligned} \tag{24}$$

We run the DEA model for all of the DMUs to calculate the optimal weight of input and output measures such that the efficiency of the given DMU is maximized. This model (17) is input-oriented, and its associated dual form (envelopment model) is represented as follows:

$$\begin{aligned}
&Min\,\theta_0\\
&s.t.\\
&\sum_{j=1}^{J}\lambda_j x_{ij} \le \theta_0 x_{i0}\ \ \forall i\\
&\sum_{j=1}^{J}\lambda_j y_{rj} \ge y_{r0}\ \ \forall r\\
&\lambda_j \ge 0,\ \forall j\\
&\theta_0 : free
\end{aligned} \tag{25}$$

Where θ_0 is the efficiency of the DMU_0.

A DEA model could be developed under different input or output-oriented assumptions, variable returns to scale, additive. The model considered in this paper is an input-oriented and variable returns to scale model, which is called BCC. BCC was developed by Banker et al. (1984), and it is different from the CRS model by the constraint $\sum_{j=1}^{n}\lambda_j = 1$ related to the returns to scale assumption.

$$\begin{aligned}
&Min\,\theta_0\\
&s.t.\\
&\sum_{j=1}^{J}\lambda_j x_{ij} \le \theta_0 x_{i0}\ \ \forall i\\
&\sum_{j=1}^{J}\lambda_j y_{rj} \ge y_{r0}\ \ \forall r\\
&\sum_{j=1}^{n}\lambda_j = 1\\
&\lambda_j \ge 0,\ \forall j\\
&\theta_0 : free
\end{aligned} \tag{26}$$

DEA model (19) is a classic BCC model with crisp data for inputs (x_{ij}) and outputs (y_{rj}). Given this paper's aim, for all of the DMs, we assume inputs (${x^d}_{ij}$) and outputs (${y^d}_{rj}$) as intuitionistic fuzzy numbers. The envelopment model proposed by Hajiagha et al. (2013) is used to develop the multiplier model, which is presented as follows:

$$\begin{aligned}
&Min\,\theta_0\\
&s.t.\\
&\sum_{j=1}^{J}\lambda_j {x^d}_{ij} \le \theta_0 {x^d}_{i0}\ \ \forall i\\
&\sum_{j=1}^{J}\lambda_j {y^d}_{rj} \ge {y^d}_{r0}\ \ \forall r\\
&\sum_{j=1}^{J}\lambda_j = 1\\
&\lambda_j \ge 0,\ \forall j\\
&\theta_0 : free
\end{aligned} \tag{27}$$

where $x_{ij}^d = (\mu_{x_{ij}}^d, \upsilon_{x_{ij}}^d)$ and $y_{ij}^d = (\mu_{y_{ij}}^d, \upsilon_{y_{ij}}^d)$.

In the first constraint, $\sum_{j=1}^{J} \lambda_j x^d{}_{ij} \le \theta_0 x^d{}_{i0}\ \forall i,$ the left-hand side $\sum_{j=1}^{J} \lambda_j x^d{}_{ij}$, is replaced with $\left\langle \left(1-\prod_{j=1}^{J}\left(1-\mu_{x_{ij}^d}\right)^{\lambda_j}\right), \prod_{j=1}^{J} \upsilon_{x_{ij}^d}^{\lambda_j} \right\rangle$ by IFWA operator, and the right-hand side $\theta_0 x^d{}_{i0}$, is replaced with $\left\langle 1-\left(1-\mu_{x_{i0}^d}\right)^{\theta_0}, \upsilon_{x_{i0}^d}^{\theta_0} \right\rangle$ by Eq. 6. Thus, the first constraint can be replaced with the following constraint.

$$\left\langle \left(1-\prod_{j=1}^{J}\left(1-\mu_{x_{ij}^d}\right)^{\lambda_j}\right), \prod_{j=1}^{J} \upsilon_{x_{ij}^d}^{\lambda_j} \right\rangle \le \left\langle 1-\left(1-\mu_{x_{i0}^d}\right)^{\theta_0}, \upsilon_{x_{i0}^d}^{\theta_0} \right\rangle \tag{28}$$

Similarly, the second constraint can be replaced with the following constraint.

$$\left\langle \left(1-\prod_{j=1}^{J}\left(1-\mu_{y_{rj}^d}\right)^{\lambda_j}\right), \prod_{j=1}^{J} \upsilon_{y_{rj}^d}^{\lambda_j} \right\rangle \ge \left\langle \mu_{y_{r0}^d}, \upsilon_{y_{r0}^d} \right\rangle \tag{29}$$

According to Eq. 8, relation (28) can be replaced with two following relations.

$$\prod_{j=1}^{J}\left(1-\mu_{x_{ij}^d}\right)^{\lambda_j} \ge \left(1-\mu_{x_{i0}^d}\right)^{\theta_0} \tag{30}$$

$$\prod_{j=1}^{J} \upsilon_{x_{ij}^d}^{\lambda_j} \ge \upsilon_{x_{i0}^d}^{\theta_0} \tag{31}$$

Similarly, relation (29) can be replaced with two following relations.

$$\prod_{j=1}^{J}\left(1-\mu_{y_{rj}^d}\right)^{\lambda_j} \le 1-\mu_{y_{r0}^d} \tag{32}$$

$$\prod_{j=1}^{J} \upsilon_{y_{rj}^d}^{\lambda_j} \le \upsilon_{y_{r0}^d} \tag{33}$$

The non-linear relations (30)-(33) are transformed to linear relations by using the ln(x) function as follows (Hajiagha et al., 2013). Accordingly, the relations (34)-(37) are the linear relations associated with the relations (30)-(33), respectively.

$$\sum_{j=1}^{J} \lambda_j .\ln\left(1-\mu_{x_{ij}^d}\right) \ge \theta_0 .\ln\left(1-\mu_{x_{i0}^d}\right) \tag{34}$$

$$\sum_{j=1}^{J} \lambda_j .\ln\left(\upsilon_{x_{ij}^d}\right) \ge \theta_0 .\ln\left(\upsilon_{x_{i0}^d}\right) \tag{35}$$

$$\sum_{j=1}^{J}\lambda_j.\ln\left(1-\mu_{y_{rj}^d}\right) \le \ln\left(1-\mu_{y_{r0}^d}\right) \tag{36}$$

$$\sum_{j=1}^{J}\lambda_j.\ln\left(\upsilon_{y_{rj}^d}\right) \le \ln\left(\upsilon_{y_{r0}^d}\right) \tag{37}$$

According to the above relations, the IF-DEA model for each DM is as follows:

$$\begin{aligned}
&Min\,\theta_0\\
&\sum_{j=1}^{J}\lambda_j.\ln\left(1-\mu_{x_{ij}^d}\right) \ge \theta_0.\ln\left(1-\mu_{x_{i0}^d}\right)\\
&\sum_{j=1}^{J}\lambda_j.\ln\left(\upsilon_{x_{ij}^d}\right) \ge \theta_0.\ln\left(\upsilon_{x_{i0}^d}\right)\\
&\sum_{j=1}^{J}\lambda_j.\ln\left(1-\mu_{y_{rj}^d}\right) \le \ln\left(1-\mu_{y_{r0}^d}\right)\\
&\sum_{j=1}^{J}\lambda_j.\ln\left(\nu_{y_{rj}^d}\right) \le \ln\left(\nu_{y_{r0}^d}\right)\\
&\sum_{j=1}^{J}\lambda_j = 1\\
&\lambda_j \ge 0,\ \forall j\\
&\theta_0 : free
\end{aligned} \tag{38}$$

The multiplier model is the dual of the envelopment model (38); hence, the multiplier model is represented as follows:

$$\begin{aligned}
&E_0^d = \sum_{r=1}^{s}\left(-\ln\left(1-\mu_{y_{r0}^d}\right).u_{r1} - \ln\left(\upsilon_{y_{r0}^d}\right).u_{r2}\right) - u_0\\
&\sum_{i=1}^{m}\left(\ln\left(1-\mu_{x_{ij}^d}\right).v_{i1} + \ln\left(\upsilon_{x_{ij}^d}\right).v_{i2}\right) - \sum_{r=1}^{s}\left(\ln\left(\upsilon_{y_{rj}^d}\right).u_{r1}\right) - u_0 \le 0 \quad \forall j\\
&\sum_{i=1}^{m}\left(-\ln\left(1-\mu_{x_{i0}^d}\right).v_{i1} - \ln\left(\upsilon_{x_{i0}^d}\right).v_{i2}\right) = 1\\
&v_{i1}, v_{i2}, u_{r1}, u_{r2} \ge 0\\
&u_0\ free
\end{aligned} \tag{39}$$

Then, for each DM, we run the IF-DEA model for all DMUs to construct the individual cross-efficiency matrices as follows:

$$\theta^d = \begin{pmatrix} E^d{}_{11} & \cdots & E^d{}_{1n}\\ \vdots & \ddots & \vdots \\ E^d{}_{n1} & \cdots & E^d{}_{nn}\end{pmatrix}$$

For instance, E^{d}_{1n} is the efficiency of the *n*th DMU under the weights which are obtained to maximize the efficiency of the 1st DMU.

Then, the efficiency of each DMU is obtained by the following equation for all of the DMs.

$$E_j^d = \frac{\sum_{i=1}^{m} E_{ij}^d}{n} \ \forall j; \tag{40}$$

Where E_j^d is the efficiency of the jth DMU concerning the dth DM's opinions.

Finally, there will be *t* efficiency value for each DMU, and E^d is the efficiency vector of the DMUs determined based on the *d*th DM's opinions.

$$E^d = \left(E_1^d, E_2^d, ..., E_n^d\right), \forall d$$

The final efficiency of each DMU is obtained by the weighted average operator as follows:

$$E_j = \sum_{d=1}^{D} w_d . E_j^d \ , \forall j \tag{41}$$

Where w_d is the weight of the *d*th DM, and E_j is the final efficiency of the *j*th DMU.

IV. CASE STUDY

Table I. Linguistic variables for the rating of the DMUs

Linguistic variables	***Intuitionistic fuzzy number***
Extremely high (EH)	(1.00,0.00)
Very very high (VVH)	(0.90,0.10)
Very high (VH)	(0.80,0.10)
High (H)	(0.70,0.20)
Medium high (MH)	(0.60,0.30)
Medium (M)	(0.50,0.40)
Medium low (ML)	(0.40,0.50)
Low (L)	(0.25,0.60)
Very low (VL)	(0.10,0.75)
Very very low (VVL)	(0.10,0.90)

The application of the proposed approach is illustrated by a case study related to a production company. There are five instructors for teaching the technical production issues as DMUs to be evaluated here. A group composed of five interns as DMs are responsible for judgment. Two input measures, including teaching experience and level of knowledge, and two output measures, including interns' satisfaction and executive achievements, are used to determine DMUs' efficiencies. Linguistic variables express the judgments, and then they are converted to IFNs according to Table I. Table II, and Table III summarizes DMs' judgments on the instructors' input and output measures based on linguistic variables and IFNs, respectively. The weights of DMs are determined based on the method presented in Section 3.1, and the results are shown in Table IV. Table V summarizes the efficiency vectors determined by the DMs. Table VI shows the final efficiency of each DMU, which is obtained by the weighted average operator. It also shows the ranking of the instructors based on their efficiencies. The proposed IF-DEA-GDM approach is compared with the approach presented by Liu et al. (2019) to demonstrate the proposed method verification. Liu et al. (2019) defined an interval transform function to transform intuitionistic fuzzy preference relations into interval preference relations. Afterward, they developed a GDM approach based on DEA with intuitionistic fuzzy preference relations to find the efficiencies and ranking of DMUs. They used goal programming to obtain DMs' weights. However, the comparison between the ranking results of the two methods shows that both approaches result in the same ranking. The results are presented in Table VII.

Table II. DMs' judgments on the DMUs based on linguistic variables

DMs	***Measure type***		***Instructor 1***	***Instructor 2***	***Instructor 3***	***Instructor 4***	***Instructor 5***
DM 1	Inputs	Teaching experience	L	VH	H	M	VVH
		Level of knowledge	VVH	VH	ML	VH	VL
	Outputs	Interns' satisfaction	VL	VVL	M	ML	H
		Executive achievements	VH	VVL	VH	ML	ML
DM 2	Inputs	Teaching experience	H	ML	ML	VH	VVH
		Level of knowledge	VH	VVH	MH	VH	H
	Outputs	Interns' satisfaction	M	VL	ML	VVL	VVH
		Executive achievements	MH	VH	M	VVL	M
DM 3	Inputs	Teaching experience	VVH	ML	MH	H	VVH
		Level of knowledge	VH	MH	MH	ML	VL
	Outputs	Interns' satisfaction	VH	ML	VVH	L	H
		Executive achievements	VVH	M	L	L	ML
DM 4	Inputs	Teaching experience	VL	VVH	VH	L	ML
		Level of knowledge	MH	VL	VH	VVH	ML
	Outputs	Interns' satisfaction	L	H	VVL	VL	M
		Executive achievements	H	ML	VVL	VH	H
DM 5	Inputs	Teaching experience	H	M	ML	VVH	H
		Level of knowledge	VH	VH	MH	H	MH
	Outputs	Interns' satisfaction	M	L	ML	VVH	H
		Executive achievements	MH	L	M	M	MH

Table III. DMs' judgments on the DMUs based on IFNs

DM Number	*Measure Type*		*Instructor 1*	*Instructor 2*	*Instructor 3*	*Instructor 4*	*Instructor 5*
DM 1	Inputs	Teaching experience	(0.25,0.6)	(0.80,0.10)	(0.70,0.20)	(0.50,0.4)	(0.90,0.10)
		Level of knowledge	(0.90,0.10)	(0.80,0.10)	(0.40,0.50)	(0.80,0.10)	(0.10,0.75)
	Outputs	Interns' satisfaction	(0.10,0.75)	(0.10,0.90)	(0.50,0.4)	(0.25,0.60)	(0.70,0.20)
		Executive achievements	(0.80,0.10)	(0.10,0.90)	(0.80,0.10)	(0.25,0.60)	(0.40,0.50)
DM 2	Inputs	Teaching experience	(0.70,0.20)	(0.25,0.60)	(0.40,0.50)	(0.80,0.10)	(0.90,0.10)
		Level of knowledge	(0.80,0.10)	(0.90,0.10)	(0.60,0.30)	(0.80,0.10)	(0.70,0.20)
	Outputs	Interns' satisfaction	(0.50,0.40)	(0.10,0.75)	(0.40,0.50)	(0.10,0.90)	(0.90,0.10)
		Executive achievements	(0.60,0.30)	(0.80,0.10)	(0.50,0.40)	(0.10,0.90)	(0.50,0.40)
DM 3	Inputs	Teaching experience	(0.90,0.10)	(0.40,0.50)	(0.60,0.30)	(0.70,0.20)	(0.90,0.10)
		Level of knowledge	(0.80,0.10)	(0.60,0.30)	(0.60,0.30)	(0.40,0.50)	(0.10,0.75)
	Outputs	Interns' satisfaction	(0.80,0.10)	(0.40,0.50)	(0.90,0.10)	(0.25,0.6)	(0.70,0.20)
		Executive achievements	(0.90,0.10)	(0.50,0.40)	(0.25,0.6)	(0.25,0.6)	(0.40,0.50)
DM 4	Inputs	Teaching experience	(0.10,0.75)	(0.90,0.10)	(0.80,0.10)	(0.25,0.6)	(0.40,0.50)
		Level of knowledge	(0.60,0.30)	(0.10,0.75)	(0.80,0.10)	(0.90,0.10)	(0.40,0.50)
	Outputs	Interns' satisfaction	(0.25,0.6)	(0.70,0.20)	(0.10,0.90)	(0.10,0.75)	(0.50,0.40)
		Executive achievements	(0.70,0.20)	(0.40,0.50)	(0.10,0.90)	(0.80,0.10)	(0.70,0.20)
DM 5	Inputs	Teaching experience	(0.70,0.20)	(0.50,0.40)	(0.40,0.50)	(0.90,0.10)	(0.70,0.20)
		Level of knowledge	(0.80,0.10)	(0.80,0.10)	(0.60,0.30)	(0.70,0.20)	(0.60,0.30)
	Outputs	Interns' satisfaction	(0.50,0.40)	(0.25,0.6)	(0.40,0.50)	(0.90,0.10)	(0.70,0.20)
		Executive achievements	(0.60,0.30)	(0.25,0.6)	(0.50,0.40)	(0.50,0.40)	(0.60,0.30)

Table IV. DMs' weights

DM Number	*Weight*
DM 1	0.158179
DM 2	0.208263
DM 3	0.276812
DM 4	0.1808
DM 5	0.1808

Table V. Efficiency vectors determined by the DMs

	j=1	*j=2*	*j=3*	*j=4*	*j=5*
E^1	0.441	0.439	0.744	0.456	0.539
E^2	0.766	0.748	0.762	0.748	0.818
E^3	0.962	0.772	0.838	0.727	0.796
E^4	0.289	0.889	0.166	0.166	0.560
E^5	0.698	0.6702	0.686	0.809	0.733

Table VI. Final efficiencies and rankings of DMUs

DMUs	E_j	*Ranking*
Instructor 1	0.674	3
Instructor 2	0.721	1
Instructor 3	0.662	4
Instructor 4	0.606	5
Instructor 5	0.710	2

Table VII. Summarized comparative analysis of the proposed approach versus Liu et al.'s (2019) approach

DMUs	*Ranking by IF-DEA-GDM approach*	*Ranking by Liu et al.'s (2019) approach*
Instructor 1	3	3
Instructor 2	1	1
Instructor 3	4	4
Instructor 4	5	5
Instructor 5	2	2

V. CONCLUSIONS AND MANAGERIAL INSIGHTS

In real-world problems, due to increasing complexity, conflicting criteria, and incomplete or insufficient information about the DMUs, more opinions and perspectives are needed for making a practical decision. This paper developed a GDM methodology based on DEA cross-efficiency for ranking DMUs in an intuitionistic fuzzy setting. First, DMs' opinions about the DMUs regarding the input and output measures are expressed as linguistic variables transformed into IFNs. To do so, each DM uses a membership degree and a non-membership degree to determine the satisfiability degree and non-satisfiability degree of each DMU regarding the input and output measures, respectively. Also, a new technique is presented to determine the DMs' weights. The advanced DEA method is used to calculate a set of efficiencies for each DMU based on all DMs' opinions. Then, the efficiency set of each DMU is converted into an aggregated efficiency based on the DMs' weights. Finally, the rankings of the DMUs are determined according to their associated efficiencies. The proposed approach is verified by a real case study and a comparison analysis. However, production managers can be considered the proposed approach to assess their DMUs regarding each expert's relative importance. Considering other types of uncertainty (e.g., stochastic uncertainty) for the values of DMUs regarding input and output measures could be an exciting direction for future research.

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