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Online Fault Detection and Isolation Method Based on Belief Rule Base for Industrial Gas Turbines

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Abstract *−Real time and accurate fault detection has attracted an increasing attention with a growing demand for higher operational efficiency and safety of industrial gas turbines as complex engineering systems. Current methods based on condition monitoring data have drawbacks in using both expert knowledge and quantitative information for detecting faults. On account of this reason, this paper proposes an online fault detection and isolation method based on belief rule base (BRB), which can deal with modeling behavior of complex systems when semi-quantitative information is available. Although it is difficult to obtain accurate and complete quantitative information, some expert knowledge can be collected and represented by a BRB, which is essentially an expert system. As such, a new BRB based diagnosis model is proposed to detect and isolate faults of the system in real-time when aiming at isolating various damages and determining the severity of each. Moreover, a recursive algorithm is developed for online updating the parameters of the fault diagnosis model. Equipped with the recursive algorithm, the proposed diagnosis model can determine the severity of fault in real-time when two types of faults are dependent and competitive. To prove its potential application, experimental results demonstrate that the proposed model can track the fault severity very well, and the faults can be diagnosed accurately in real time. Thus,* R^2 *values of 0.99782 and 0. 99782 were obtained for the fault estimation of fouling and erosion, respectively, indicating the accurate performance of the proposed model.*

Keywords − *Belief rule base, Fault diagnosis, Industrial two-shaft gas turbine, Fouling and erosion faults.*

I. INTRODUCTION

Demand for higher operational efficiency and safety in industries is growing more than ever. Researchers in the field of condition-based maintenance (CBM) have focused on understanding the principles of system fault and predicting the time and the cause of their possible failure (Jardine, Lin and Banjevic, 2006; Ahmad and Kamaruddin, 2012).

Professional instruments, such as sensors, controllers, computational devices and a wide range of meters, facilitate machine diagnostics. The root cause of fault can be determined by using these instruments and implementing sophisticated diagnostic methodologies. In the real industry, some complex systems may subject to more than one failure mode due to deterioration. For example, the gas turbine as the most sophisticated mechanical structure that human has ever made is composed of a core engine and auxiliary equipment. Accurate detection, isolation, and estimation of core engine problems can be done through analyzing the measurements of gas path. Noticeable deviations

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in gas path measurements from a desired healthy engine are used to identify changes in gas turbine performance (Sadough Vanini, Khorasani and Meskin, 2014). Tracking changes in measured variables, such as engine speed, temperature, pressure, fuel flow, vibration, etc., provide the information required for detecting gas engine malfunctions. The overall goal of the gas engine diagnosis system is to mitigate the consequence of performance deterioration over time, which may vary from partial loss of stability and availability to catastrophic failure of the system. Among several factors that can affect the performance of the gas path modules of a gas turbine, fouling and erosion are usually considered as the most common cause of performance deterioration (Zwebek and Pilidis, 2003) (Mohammadi and Montazeri-Gh, 2016). When each of these two degradations exceeds the predefined threshold, specific actions must be taken to mitigate or eliminate each of these faults. It is noteworthy to mention that some corrective or preventive actions mitigating a specific form of deterioration may accelerate another form of deterioration or even worsen the healthy condition of gas turbines. Therefore, isolation and identification of type and severity of these deteriorations have become an important issue for engaged engineers and an interesting subject for researchers to develop as accurate model as possible to reach this goal.

The contribution of this paper lies in these following aspects: First, it presents a method that can isolate, detect, and estimate the severity of the fault when two fault types exist simultaneously and have the same impact on health parameter of the system. Second, it presents a knowledge-based approach by gathering different types of information integrating quantitative and qualitative data that can be used to establish the fault diagnosis model. Third, once new information becomes available, the proposed method can detect the type and severity of faults in real-time without waiting for all information to be provided, which can greatly save time and is of great practical significance. The above features allow the new real-time fault diagnosis algorithm to be applied widely in real engineering applications, as shown in this paper.

The reminder of this paper is organized as follows. The diagnosis approaches are classified and described in Section 2. Section 3 explains the mathematical foundation of the belief rule base and the proposed online BRB using semi-quantitative information. Section 4 presents the overall model of this two-shaft gas turbine jet engine, and proposes and describes the problem of fault detection and isolation structure and strategy in detail. Results and a conclusion based on using real data are presented in Section 6.

II. CLASSIFICATION OF DIAGNOSTICS APPROACHES

Fault diagnosis approach can be broadly classified into three major categories, namely model-based approach, data-driven approach, and knowledge-based approach, which have been extensively studied in the literature for prognostics and health management of rotary machinery.

A. Model-based approaches

Model-based approaches, also known as physics-based approaches, use traditional mathematical or physical models for system monitoring. It becomes more favorable when precise and rich data by sensors are available and the system behavior is well understood under healthy or faulty conditions (Esperon-Miguez, John and Jennions, 2013). Since the physical model describes the behavior of damaged systems, this approach can predict long-term behaviors of components under damage, which can be considered as an advantage; the model assumptions, limitations, and approximation should also be taken into consideration as well (An, Kim and Choi, 2015). Physics-based approaches utilize domain knowledge of a system, as well as operational and environmental conditions to identify potential failure and estimate the remaining useful life (Pecht and Jaai, 2010). In model-based approaches, Kalman filters are a quite popular use of this method for fault isolation and assessment in engine performance diagnosis started in the late 1970’s (Kobayashi and Simon, 2003; Meskin, Naderi and Khorasani, 2010; Urban, 1973). A drawback of model-based approach is that the system reliability decreases with increasing nonlinear complexities and modeling uncertainties (Sadough Vanini, Khorasani and Meskin, 2014). A model based approach proposed for detection and isolation of compressor efficiency, mass flow capacity, and actuator faults of the gas turbine as a nonlinear system with unknown inputs and disturbances (Kazemi and Yazdizadeh, 2020). Another model-based approach that relies on derivative-free nonlinear Kalman Filter for diagnosing faults (or cyberattacks) of the gas turbine is provided (Rigatos et al., 2019). They show that differences between the measured outputs of the power generation unit and the estimated outputs follows the $\chi 2$ distribution.

B. Data-driven approaches

Data-driven approaches learn system behaviors based on monitored data and transform these data into valuable information for makers. Unlike model-based approaches, they can be used without specific knowledge of system and treated as a black box. By training data based on this assumption in which statistical characteristics of system data remain relatively unchanged until a failure occurs in the system, they can learn the trend of patterns and detect system anomalies. These detected anomalies lead to determining the health state of system and degree of system degradation. Neural networks are most popular due to their proven success in system identification and learning nonlinear transformations that map a set of input to a set of output. A modular diagnostic system for a single shaft and dual spool gas turbine is proposed in the literature (Urban, 1973), (Kanelopoulos, Stamatis and Mathioudakis, 1997). Dynamic neural network or dynamic multi-layer perceptron have been recently utilized for problem identification of systems due to their capabilities in modeling nonlinear dynamic systems. By using dynamic neurons and having a feed-forward architecture, such networks achieve dynamic properties (Sadough Vanini, Khorasani and Meskin, 2014). The need for a bunch of data, training data to determine correlation, and establishment of patterns are the limitations of data-driven approaches. These limitations lead to a lack of precision compared to model-based approaches (Muller, Suhner and Iung, 2008; Medjaher, Moya and Zerhouni, 2009). A hierarchical symbolic analysis and convolutional neural network model was proposed to diagnose the faults of rotatory machinery (Yang et al., 2019). Their 3-step model decomposes original time series data to a series of low/high-frequency component, extracts preliminary feature, and eventually learns relationships between these information and health condition automatically. Their model is adaptive to different operating conditions and even different equipment.

By implementing machine learning tools a fault diagnosis model proposed that does not need a lot of expert knowledge and fault data (Shi-sheng and Song, 2019). Another fault sensitivity analysis provided to evaluate the performance of fault detection of the gas turbine (Amirkhani, Chaibakhsh and Ghaffari, 2019). By using three types of neural networks, researchers identify and model the adaptive thresholds. They show that the adaptive threshold has a positive effect on the proposed method and is more robust than a simple threshold.

A multi-model fusion system is proposed to integrate three single data-driven diagnostic models, namely bi-level BRB model, bi-level ANN model, and evidential reasoning rule model, to identify wear faults of marine diesel engine (Xu et al., 2019). A fault diagnosis method was suggested for an aero-engine based on kernel principal component analysis and wavelet neural network (Cui et al., 2019). A safety assessment model was proposed for diesel engine based on the conditional generalized minimum variance (CGMV) and the belief rule base (BRB) (Li et al., 2017).

C. Knowledge-based approaches

Knowledge-based approaches often allow overlooked information available from human experts and are appropriate for designing a condition monitoring system. They have also been used in miscellaneous fields connected to condition monitoring outside of gas turbine diagnosis such as structural monitoring (Lin and Cheng, 2010) and patient priority scheduling as part of an asset management optimization strategy at hospital emergency departments (Kırış *et al.*, 2010). For condition monitoring of a gas turbine engine, it is important to provide a unique optimized maintenance scheduling allowing maintenance decisions to play a key part in driving forward operations strategies and present a business winning advantage. Expert system (ES) is also a major player in knowledge-based approaches, which has been used since the middle of 1960s (Garga *et al.*, no date). By embodying human expertise and using hybrid information, expert systems are capable a robust and reliable condition monitoring. A belief rule base (BRB) as a more generalized expert system has been developed recently, which uses evidential reasoning (RIMER) approach (Yang *et al.*, 2006). BRB provides informative and flexible scheme and extends traditional IF-THEN rules; furthermore, it is capable of capturing vagueness, incompleteness, and nonlinear causal relationships. In contrast to neural networks, which learn from input and output data, ES programming expert knowledge into computer and solves particular problems. A limitation of rule-based ES is the exponential growing number of rules when an increase occurs in the number of variables (Jardine, Lin and Banjevic, 2006). An uncertain rule-base fuzzy logic and hybrid dimensionally reduction was proposed for the gas turbine FDI (Yazdani and Montazeri-gh, 2019). In this recent proposed model, raw data are pre-processed by a sequel of feature extraction via SOM, and feature selection by the multi-objective NSGA-II algorithm. Selected feature subsets are utilized as the inputs of the uncertain rule-based health estimators.

A BRB system was introduced to model the complicated nonlinear relationship between the alarm evidence support degrees and the combination weight using the expert knowledge and training data at the same time (Xu *et al.*, 2018). A belief rule-based inference methodology using the evidential reasoning (RIMER) was developed to estimate remaining useful life (RUL) of turbofan engines (You et al., 2019). An infinite irrelevance BRB model proposed to estimate health condition of CNC machine (Yin et al., 2018). They confirmed that initial values of the parameters in their model given by experts might not be accurate. A safety assessment approach for regional railway using the hybrid Belief Rule Base (BRB) is proposed (Chang et al., 2019), which can model varied types of aspects/factors attributed under conjunctive, disjunctive or hybrid assumptions. They found that, if conjunctive rules made most of the hybrid BRB, the hybrid BRB would have to face the combinatorial explosion problem.

Table I lists the reviewed literature on BRB models used in diagnostic and prognostics. These models are compared in terms of multivariate quality, quantitative and qualitative data, online and offline types, and fault detection or prediction. Our proposed multivariate model, noted in the last row, considers quantitative, qualitative, and online data to detect turbine defects.

Table I. Literature on BRB models used in diagnostic and prognostics

Ref.	Univariate	Multivariate	Quantitative data	Qualitative data	Offline	Online	Diagnosis	Prognosis
(Xu *et al.*, 2007)		•	•		•		•	
(Z. J. Zhou *et al.*, 2009)		•	•	•	•		•	
(Zhou *et al.*, 2010)		•	•			•	•	
(Liu *et al.*, 2013)		•	•		•		•	
(Zhou *et al.*, 2011)		•		•		•	•	
(Jiang *et al.*, 2014)	•		•		•			•
(Zhang *et al.*, 2013)	•		•		•			•
(Chang *et al.*, 2015)	•		•		•			•
(Si *et al.*, 2011)	•		•		•			•
(Li *et al.*, 2017)	•		•	•		•	•	
(Yin *et al.*, 2018)		•	•	•	•		•	
(Xu *et al.*, 2018)		•	•	•		•		
(Xu *et al.*, 2019)	•		•	•		•	•	
(You *et al.*, 2019)		•	•			•		•
Proposed model		•	•	•		•	•	

In the present work, an online model for mapping and isolating fault mode based on the BRB and semi-quantitative information is proposed and developed as a novel approach for fault detection and isolation of gas turbines. It will be shown that the problem of fault detection and isolation of two-shaft industrial gas turbines can be addressed quiet effectively using hybrid information originated from human domain knowledge and measured embedded parameters of sensors. For comparison of our developed methodology with other methods, the reader can refer to (Mohammadi and Montazeri-Gh, 2016). These details are not included here for the sake of brevity.

III. ONLINE BELIEF RULE BASE BY USING SEMI-QUANTITATIVE INFORMATION

BRB approach is based on decision making, Dempster-Shafer evidence theory, and fuzzy set theory. With the collection of belief rules and taking evidence reasoning algorithm as the reasoning machine, it describes the knowledge and is able to capture the dynamic nature of a system (Yang *et al.*, 2006).

A. Fundamental structure of BRB

A BRB represents the relationship between antecedent attributes and consequents. In comparison with the simple IF-THEN rule-based structure, BRB is a more versatile scheme, consisting of belief rule base to reflect system dynamic nature as follow:

$$R_k: \quad \begin{array}{l} IF\ x_1(t)\ is\ A_1^k \wedge x_2(t)\ is\ A_2^k \wedge \dots \wedge x_{M_k}(t)\ is\ A_{M_k}^k\ , \\ THEN\ \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \dots, (D_N, \beta_{N,k})\}, \sum_{n=1}^{N} \beta_{n,k} \le 1\ , \end{array} \tag{1}$$

with rule weight θ_k and attribute weight$\delta_{1,k}, \delta_{2,k}, \dots, \delta_{M_k,k}$.

where $\boldsymbol{X}(t) = [x_1(t)\ \ x_2(t) \dots x_{M_k}(t)]$ denotes the input vector of the k-th rule, A_i^k $(i = 1, \dots, M, k = 1, \dots, L)$ is the referential value of the i-th antecedent attribute in the k-th rule, $A_i^k\ \in A_i = \{A_{i,j}\ , j = 1, \dots, J_j\}$ is a set of referential values for the i-th antecedent attribute, and J_j is the number of referential values. θ_k $(k = 1, \dots, L)$ is the relative weight of k-th rule, and $\delta_{1,k}, \delta_{2,k}, \dots, \delta_{M_k,k}$ are the relative weight of M_k antecedent attributes used in the k-th rule. $\beta_{j,k}$ $(j = 1, \dots, N, k = 1, \dots, L)$ is the belief degree assessed to D_j denoting the j-th consequent. If $\sum_{n=1}^{N} \beta_{n,k} = 1$, the k-th rule is considered to be complete; otherwise, it is incomplete. Note that "∧" is a logical connective to represent the "AND" operator. In addition, M is supposed to be the total number of antecedent attributes used in BRB.

The belief degree, rule weights, and attribute weights in belief rule (1) must satisfy some equality and inequality constraints as follows:

I. A rule weight is normalized, so that it is between zero and one, i.e.,

$$0 \le \theta_k \le 1,\ \ k = 1, \dots, L \tag{1a}$$

II. An attribute weight is normalized, so that it is between zero and one, i.e.,

$$0 \le \delta_m \le 1,\ \ m = 1, \dots, M_k \tag{1b}$$

III. A belief degree (subjective probability) must not be less than zero or more than one, i.e.,

$$0 \le \beta_{j,k} \le 1,\ \ j = 1, \dots, N\ \ ,\ \ k = 1, \dots\ , L \tag{1c}$$

IV. If the *k*-th belief rule is complete, its total belief degree in the consequent will be equal to one, i.e.,

$$\sum_{j=1}^{N} \beta_{j,k} = 1 \quad k = 1, \dots, L \tag{1d}$$

otherwise the total belief degree is less than one.
Let **V**(t) denote a vector composed of the above parameters in BRB and is written as

$$\boldsymbol{V}(t) = \left[\theta_1, \dots, \theta_L, \delta_1, \dots, \delta_{M_k}, \beta_{1,1}, \dots, \beta_{N,L}\right]^T \tag{2}$$

There are different ways to establish a belief rule as described in (1). Extracting belief rules from expert knowledge, extracting belief rules by examining historical data, using the previous rule bases if available, random rules without any pre-knowledge, and in some cases, a combination of these ways are used to establish initial BRB.

B. BRB reasoning method using the ER algorithm

The analytical description relationship between antecedents and consequents can be described by the RIMER approach, which can be treated with discrete or continuous, complete or incomplete, linear or non-linear, or non-smooth, or their mixture in data (Yang *et al.*, 2006). Assume that the input and output data of the system can be represented in the form of a data pair (**x**(t),**y**(t)). **x**(t) represents the input vector at the time instant t, and **y**(t) is the output vector describing fault states. The relationship between vector of inputs and estimated outputs reflected by BRB is composed of a series of belief rule base that can be represented as nonlinear in the mapping function below:

$$\hat{\boldsymbol{y}}(t) = f(\boldsymbol{x}(t), \boldsymbol{Q}(t)) \tag{3}$$

where $\hat{\boldsymbol{y}}(t)$ is the model estimation of $y(t)$, $\boldsymbol{x}(t)$ is input vector, and $\boldsymbol{Q}(t)$ is vector of model parameters. A more precise estimated detection value can be obtained by additional basic attributes and rules. However, the complexity burden and cost of time consumption increased obviously by increasing the number of rules. The number of the attributes depends on the real situation of system. The ultimate goal of using ER algorithm is to identify the relationship between the input and the output. The approximation of the function f(·) is key to solving the detection problem.

As introduced in (1), the BRB model can describe performance degradation of a system. In order to apply ER, the input data should be transformed into the belief structure. The N distinctive evaluation grade for fault severity, represented by $\boldsymbol{D} = \{D_1, \dots, D_N\}$, is defined as a pre-knowledge or set by examining historical data of system. It is worth noting that the output set $\boldsymbol{D}$ provides a mutually exclusive and collectively exhaustive set.

For establishing ER analytical algorithm, suppose that $\alpha_{i,j}^{k}(t)$ is the matching degree of the *i*-th input to *j*-th referential value in the *k*-th rule of BRB at time instance *t*, where $i = 1, \dots, M_k$. Thus, when *k*-th rule is activated, the activation degree of the *k*-th rule at time instant *t*, $\omega_k(t)$, is calculated by (Yang, 2001):

$$\omega_k(t) = \frac{\theta_k \prod_{i=1}^{T_k} \left(\alpha_{i,n}^{k}(t)\right)^{\bar{\delta}_i}}{\sum_{l=1}^{L} [\theta_l \prod_{i=1}^{T_l} \left(\alpha_{i,n}^{l}(t)\right)^{\bar{\delta}_i}]} \quad and\ \bar{\delta}_i = \frac{\delta_i}{\max\limits_{i=1,\dots,T_k} \delta_i} \tag{4}$$

The estimation output $\hat{\boldsymbol{y}}(t)$ is generated by aggregating all belief rules of BRB, which are activated by the input vector $\boldsymbol{x}(t)$ represented as the following distribution:

$$O\left(\hat{\boldsymbol{y}}(t)\right) = \left\{\left(D_j, \beta_j(t)\right), \qquad j = 1, \dots, N\right\} \tag{5}$$

where $\beta_j(t)$ denotes the belief degree in D_j at time instant *t*, and is calculated as:

$$\beta_j(t) = \frac{\left[\prod_{k=1}^{L}\left(\omega_k\beta_{n,k} + 1 - \omega_k \sum_{i=1}^{N}\beta_{i,k}\right) - \prod_{k=1}^{L}\left(1 - \omega_k \sum_{i=1}^{N}\beta_{i,k}\right)\right]}{\sum_{j=1}^{N}\prod_{k=1}^{L}(\omega_k\beta_{n,k} + 1 - \omega_k \sum_{i=1}^{N}\beta_{i,k}) - (N-1)\prod_{k=1}^{L}\left(1 - \omega_k \sum_{i=1}^{N}\beta_{i,k}\right) - \prod_{k=1}^{L}(1-\omega_k)} \tag{6}$$

Intuitively, the overall output must be D_j to a certain degree if the consequent in the k-th rule includes D_j with $\beta_{j,k}(t) > 0$ and k-th rule is activated. This degree is affected by both the weight of k-th rule and the degree to which the antecedents of the k-th rule are activated by the vector of input.

Furthermore, a study (Yang, 2001) shows that $\hat{\boldsymbol{y}}(t)$ in Eq.(3) can be calculated by averaging its distributed score as:

$$\hat{\boldsymbol{y}}(t) = \sum_{j=1}^{N} U\left(D_j\right)\beta_j(t) \tag{7}$$

The utility (or score) of an individual consequent in Eq. (7) is denoted by $U\left(D_j\right)$, where D_j can be either given using a predefined value or extracted from experts or decision makers' preferences.
The output of BRB, $\hat{\boldsymbol{y}}(t)$, can be calculated by knowing a given input $\boldsymbol{x}(t)$, which could be as close to real value of $\boldsymbol{y}(t)$ as possible. In normal state, the estimated output value $\hat{\boldsymbol{y}}(t)$ should approach (approximate) the actual value $\boldsymbol{y}(t)$ as much as possible.

C. Parameter updating using the recursive algorithm

The model accuracy is affected by the precision of parameter estimation if the parameters are not given a priori or only known partial model accuracy will drop down dramatically. Previous research (Yang *et al.*, 2007) proposed an offline procedure for parameter updating, which is based on the minimum average variance method. Another study (Zhou *et al.*, 2011) proposed a recursive algorithm based on the Bayesian reasoning approach. And an online algorithm was proposed for parameter updating of BRB under expert intervention (Z.-J. Zhou *et al.*, 2009).

The recursive model tracks the system change timely and quickly for updating the parameters. It is a realistic assumption that the behavior of real output as a random variable obeys the following Gaussian distribution:

$$f(\boldsymbol{y}(t)|\boldsymbol{x}(t), Q) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{\left(y(t) - \hat{y}(t)\right)^2}{2\sigma}\right) \tag{8}$$

where $f(\boldsymbol{y}(t)|\boldsymbol{x}(t), Q)$ is the conditional probability density function of **y**(t) at time instant t, $\boldsymbol{Q}(t) = [\boldsymbol{V}(t)^T, \sigma]^T$ is the vector of model parameters, σ denotes the variance of random variable, and **V**(t) denotes the parameter vector of BRB and k=1,…,L, m=1,…,M, j=1,…,N, assuming that the BRB inputs **x**(t) are relatively independent and so the output **y**(t), then the recursive equation below is valid:

$$\boldsymbol{Q}(t+1) = \prod_{H}\left\{\boldsymbol{Q}(t) + \frac{1}{t}\left[\Xi\left(\boldsymbol{Q}(t)\right)\right]^{-1}\Gamma\left(\boldsymbol{Q}(t)\right)\right\} \tag{9}$$

where Q, as mentioned earlier, consists of belief degrees, rule weights, attribute weights, and other possible parameters. H represents the inequality constraint set of BRB model, which is defined in (1a)–(1d) and other possible constraints that need to be added based on system behavior. Also, there are:

$$\Gamma\left(\boldsymbol{Q}(t)\right) = \nabla_Q \log f\left(y(t)\middle|x(t), Q(t)\right) \tag{10}$$

$$\Xi\left(\boldsymbol{Q}(t)\right) = E\left\{-\nabla_Q\nabla_Q^{\mathrm{T}} \log f\left(y(t)\middle|x(t), Q(t)\right)\right\} \tag{11}$$

$\Gamma\left(\boldsymbol{Q}(t)\right)$ and $\Xi\left(\boldsymbol{Q}(t)\right)$ introduced in equations above can be expressed as:

$$\Gamma(\boldsymbol{Q}(t)) = \left[\Gamma_1(Q(t))^T, \Gamma_2(Q(t))^T\right]^T \tag{12}$$

$$\Xi(\boldsymbol{Q}(t)) = \begin{bmatrix} \Xi_1(\boldsymbol{Q}(t)) & 0 \\ 0 & \Xi_2(\boldsymbol{Q}(t)) \end{bmatrix} \tag{13}$$

where $\Gamma_1(Q(t))$ and $\Xi_1(\boldsymbol{Q}(t))$ are the first order derivatives with respect to **V**(t), and $\Gamma_2(Q(t))^T$ and $\Xi_2(\boldsymbol{Q}(t))^{-1}$ are the second order derivatives with respect to σ. Obviously, there is

$$\Xi(\boldsymbol{Q}(t))^{-1} = \begin{bmatrix} \Xi_1(\boldsymbol{Q}(t))^{-1} & 0 \\ 0 & \Xi_2(\boldsymbol{Q}(t))^{-1} \end{bmatrix} \tag{14}$$

If we consider **V**(t) only, according to Eq.(14), the recursive Eq.(9) can be transformed into the following form:

$$\boldsymbol{V}(t+1) = \prod_H \{\boldsymbol{V}(t) + \frac{1}{t}[\Xi(\boldsymbol{Q}(t))]^{-1}\Gamma(\boldsymbol{Q}(t))\} \tag{15}$$

In Eq.(15), **V**(t) is known. The a-th element of the gradient vector of $\Gamma_1(Q(t))$ and the entries of $\Xi_1(\boldsymbol{Q}(t))$ at time instant t are:

$$[\Gamma_1(\boldsymbol{Q}(t))]_a = \frac{(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t))}{\sigma(t)} \sum_{j=1}^{N} \mu_j \frac{\partial \beta_j(t)}{\partial V_a}\bigg|_{\boldsymbol{V}=\boldsymbol{V}(t)} \tag{16}$$

$$\begin{aligned} [\Xi_1(\boldsymbol{Q}(t))]_{a,b} &= E\left\{\frac{1}{\sigma}\frac{\partial y(t)}{\partial V_a}\frac{\partial y(t)}{\partial V_b} - \frac{1}{\sigma}\frac{\partial^2 y(t)}{\partial V_a \partial V_b}(\partial y(t) - \partial \hat{y}(t))\,|\boldsymbol{Q}(t)\right\} \\ &= \frac{1}{\sigma(t)}\left[\sum_{j=1}^{N}\frac{\partial \beta_j(t)}{\partial V_a}\right]\left[\sum_{j=1}^{N}\frac{\partial \beta_j(t)}{\partial V_b}\right]\bigg|_{\boldsymbol{V}=\boldsymbol{V}(t)} \end{aligned} \tag{17}$$

σ in Eqs. (16) and **Error! Reference source not found.**) needs to be calculated by knowing $\boldsymbol{y}(t), \hat{\boldsymbol{y}}(t)$ and $\boldsymbol{V}(t)$ as follows:

$$\sigma(t) = \arg\max_{\sigma} log\, f(y(t)|x(t), \boldsymbol{Q}(t))\,|_{v=v(t)} = (y(t) - \hat{y}(t))^2\,|_{v=v(t)} \tag{18}$$

It is assumed the output observations of the system are independent, which is a common assumption in our case of real gas turbines. In some other engineering practice, however, the dependence of output leads to the use of other functions. Therefore, it seems to be necessary to propose a robust recursive algorithm that can handle the estimation of BRB based model parameters in future works.

IV. GAS TURBINE DIAGNOSIS BASED ON BRB

A. Gas turbine description

To improve the reliability and precision of fault detection of gas turbines, online diagnosis is presented in this paper using the BRB model. The gas turbine, known as IGT25, studied here is a small-size industrial two-shaft gas turbine. The main module of the examined gas turbine consists of a two-stage gas generator turbine and power turbine, annular combustion chamber, and a ten-stage axial compressor. For more details about the engine specifications, the reader can refer to Ref. (Siemens Industrial Turbomachinery Inc., 2005). A scheme of the examined gas turbine is shown in Figure 1 (Mohammadi and Montazeri-Gh, 2016).

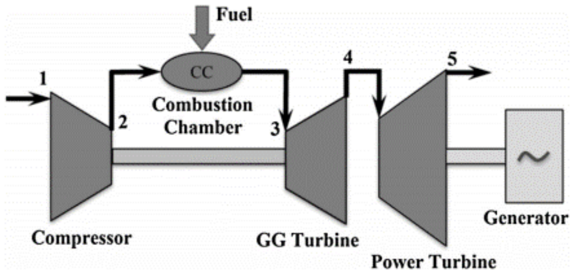

Fig. 1. The schematic of a two-shaft industrial gas turbine

B. The design of a fault detection model for the gas turbine

Typically, a two-shaft gas turbine has five modules, viz. fan, compressor, combustion chamber (CC), gas generator turbine (GG), and power turbine (PT). The typical fingerprint chart concerning changes in measurement deltas for four basic parameters with the faulty module are given in (Ganguli, 2003). Although it is useful to determine the location (module) of damage, isolating the type of damages and determining their severity are of more interest in real practice. Most physical damages to engine affect the health parameters of turbines such as module efficiency or flow capacity. Therefore, monitoring changes in these health parameters can lead engineers to detect the existence and severity of physical damages, such as erosion and fouling, to turbine blades, which are the most conventional physical damages to gas turbines. To detect physical damages (erosion and fouling) of gas turbines, it is necessary to turn off the turbine and examine the compressor blades. Accordingly, the main question is whether or not the type and severity of physical faults can be detected through online examination of changes in the health parameters (e.g. mass flow or efficiency) from nominal values without the need for other information or halting the turbine. This study concentrates on detecting and isolating physical damages to the compressor and estimating their severity. Based on the severity of occurrence, each damage incurs a specific amount of perturbation on the efficiency or mass flow of faulty modules and eventually the whole turbine. To reduce the fouling severity, compressor washing can be used during operation of the turbine, but these maintenance and repair actions will worsen the erosion fault; thus, the compressor blades need replacing or repairing. The maintenance and repair actions are quite different for the two physical faults, and making wrong decisions on the turbine maintenance and repair will worsen the other fault. It is, therefore, necessary to accurately detect the fault type and estimate its severity with a high precision. The existence and severity of faults in the turbine can be modeled and simulated by zero-dimension models of a gas turbine to accurately measure the effects of damages on health parameters. These behavioral data seem to be correct and reliable enough as they are derived from precise computerized models tuned accurately to represent the real condition of a gas turbine. This study has shown how a BRB model can determine the severity of measurement delta using expert knowledge, while accounting for uncertainty.

V. ESTABLISHMENT OF FAULT DETECTION MODEL

In this section, the procedure of designing the proposed BRB model is presented for the examined gas turbine. For this purpose, the model is first established by introducing the main input-output of BRB model, and then some explanations are provided about the procedure of creating a bank of rules and expert scheme. Finally, the overall structure of proposed BRB model is presented in detail.

A. Design of BRB for gas turbine fault detection

The gas turbine faults considered in this paper correspond to changes in two health parameters, namely the compressor efficiency and flow capacity. Inputs and outputs of the BRB model are two health parameter deltas and severity of faults, respectively. Five health parameter deltas, represented by $y(t)$, and five degrees of severity of damage, represented by $x(t)$, are considered in proposed model. The objective is to find a dynamic functional mapping between $x(t)$ and $y(t)$. Mathematically, this can be represented as the equation below:

$$\boldsymbol{y}(t) = f(\boldsymbol{x}(t), \boldsymbol{Q}(t)) \tag{19}$$

where $y(t) = \{Fouling\ or\ Erosion\}$ and $x(t) = \{\Delta m, \Delta\eta\}$. Δm and $\Delta\eta$ are representatives of delta of mass flow and delta of efficiency, respectively. Here, fouling and erosion are two sets denoting the severity of each damage type. Each health parameter has uncertainty, ambiguity, and damage types. Additionally, this proposed model can handle the problem of the incompleteness of data when faced with the lack or hardness of access to information due to the computational burden of simulations or even the lack of domain knowledge of experts. The health parameter deltas Δm and $\Delta\eta$ and degree of severity of damages are treated as a set of terms. To get a high degree of resolution for the dynamic mapping, the health parameters are further split into linguistic variables as shown in Table I.

Table II. The refrential point of delta mass flow, delta efficiency, and erosion or fouling severity

		Fouling (F)					Erosion (E)				
Delta mass flow (Δm)	Linguistic term	VL	L	M	H	VH	VL	L	M	H	VH
	Numerical value	0	-%2	-%4	-%6	-%8	0	-%1	-%2	-%3	-%4
Delta efficiency ($\Delta\eta$)	Linguistic term	VL	L	M	H	VH	VL	L	M	H	VH
	Numerical value	0	-%2	-%4	-%6	-%8	0	-%1	-%2	-%3	-%4

In Table I, delta mass flow is considered as a linguistic variable that can be decomposed into a set of terms: $\Delta m = \{Very\ Low\ (VL), Low(L), Medium(M), High(H), Very\ High(H)\}$. For instance, mass flow may decrease from zero to 8 percent below its nominal value in presence of fouling defect, and it may decrease from zero to 4 percent of its nominal value in presence of erosion defect. Other linguistic variables mentioned in Table I are defined as well, where each term in linguistic variables is characterized by a set of referential values and is well defined to include the vicinity of the range of changes in variables. These are considered as referential values for input variables. In order to detect the severity of damage, the referential values should cover the range of health parameter deltas. However, since these deltas are limited to predefine referential values, it will indicate a large fault occurrence of catastrophic fault for gas turbine in situations where the health parameter deltas exceed these ranges.

In order to detect the fault based on health parameter deltas subject to the fouling fault mode, the k-th belief rule in BRB is described as:

$$R_k: \quad IF\ \Delta m(t)\ is\ A_1^k \wedge \Delta\eta(t)\ is\ A_2^k\ THEN\ F\ \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \dots, (D_5, \beta_{5,k})\} \tag{20}$$

with a rule weight θ_k and attribute weight δ_1, δ_2 where $A_i^k \in \{VL, L, M, H, VH\}$. There are 25 belief degrees in BRB, i.e., k=1,2,…,25. D_j $(j = 1, \dots, 5)$ denotes the j-th consequent used for evaluating the severity of fouling or erosion fault. $\beta_{j,k}$ $(j = 1, \dots, 5, k = 1, \dots, 25)$ denotes the belief degree assessed to the j-th consequent D_j in the k-th belief rule.
Here, determination of D_j may affect the performance of the BRB based model. However, D_j is given by experts and represents a special meaning. If it is assigned randomly, it will be disadvantageous to interpret the result. Belief rules in

BRB for estimating the severity of erosion can be defined the same as fouling fault mode.

As shown in Table , the initial belief degrees of BRB are given by an expert. Some authors proposed a detailed method that can be used to determine the initial belief degrees (Xu *et al.*, 2007). The initial values of $\theta_1, \theta_2, \dots, \theta_{25}$ and sum of $\delta_1, \dots, \delta_3$ are set to 1, thereby, constructing the initial fault diagnosis model. In order to accurately detect and isolate the whole system fault, it is necessary to update online the initial fault diagnosis model using the available observation data of health parameter deltas.

Table III: Initial belief degrees of BRB

	Deviation in health parameters	**Distribution of consequence** $\{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \dots, (D_5, \beta_{5,k})\}$	
Rule number	$\Delta m(t)$ ***and*** $\Delta\eta(t)$	**Fouling**	**Erosion**
1	VL and VL	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}
2	VL and L	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
3	VL and M	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
4	VL and H	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}
5	VL and VH	{(D1,1),(D2,0),(D3,0),(D4,0),(D5,0)}	{(D1,1),(D2,0),(D3,0),(D4,0),(D5,0)}
6	L and VL	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
7	L and L	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}
8	L and M	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
9	L and H	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
10	L and VH	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}
11	M and VL	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
12	M and L	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
13	M and M	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}
14	M and H	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
15	M and VH	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
16	H and VL	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}
17	H and L	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
18	H and M	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
19	H and H	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}
20	H and VH	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
21	VH and VL	{(D1,1),(D2,0),(D3,0),(D4,0),(D5,0)}	{(D1,1),(D2,0),(D3,0),(D4,0),(D5,0)}
22	VH and L	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}	{(D1,0.1),(D2,0.8),(D3,0.1),(D4,0),(D5,0)}
23	VH and M	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}	{(D1,0),(D2,0.1),(D3,0.8),(D4,0.1),(D5,0)}
24	VH and H	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}	{(D1,0),(D2,0),(D3,0.1),(D4,0.8),(D5,0.1)}
25	VH and VH	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}	{(D1,0),(D2,0),(D3,0),(D4,0),(D5,1)}

According to Table III, the expert cannot initially distinguish the severity of each two different faults (erosion and fouling) in faulty gas turbines by observing the deviations (mass flow and efficiency deviations) in health parameters. Hence, it seems to be rational that the belief degree of each consequence is the same for both two fault modes. Otherwise stated, if health parameters of the gas turbine deviate from their nominal values, it will be hard for the operator to recognize whether the fault type one (erosion) or type two (fouling) has happened to the gas turbine. It is important to distinguish between the two faults and detect their severity accurately because the maintenance action is unique for mitigating each type of fault, and the health condition of the gas turbine might be worsened if a wrong maintenance action is implemented.

B. Recursive algorithm for updating the BRB based diagnostic model

The recursive algorithm for updating the diagnostic model is described in this section. It is necessary to develop a method that can optimally estimate BRB parameters and track the system dynamic timely and quickly. Observations on system input and output are required to update the parameter vector Q. Based on the set of observation pairs $(x(t), y(t))$ of the diagnostic model, the history input and output of data must be accounted because this approach applies mathematical statistics method to estimate parameters, as described earlier in Section 2.3.

Beliefs are considered as a probability in the ER algorithm, and the first 30 observable data of health parameters (inputs) and the corresponding severity degree of faults (outputs) are used as the training data set. By implementing the recursive algorithm, the updated rule weight and belief degrees are shown in Table IV.

Table IV: Updated rule weight and belief degrees in BRB for fouling and erosion faults

		Deviation in health parameters	**Fouling**
Rule number	**Rule weight**	$\boldsymbol{\Delta m(t)\ and\ \Delta\eta(t)}$	**Distribution of consequence** $\{(\boldsymbol{D_1}, \boldsymbol{\beta_{1,k}}), (\boldsymbol{D_2}, \boldsymbol{\beta_{2,k}}), \dots, (\boldsymbol{D_5}, \boldsymbol{\beta_{5,k}})\}$
1	0.927656	VL and VL	{(D1,0.1306),(D2,0.1274),(D3,0.0319),(D4,0.0578),(D5,0.6524)}
2	0.052992	VL and L	{(D1,0.1877),(D2,0.1282),(D3,0.004),(D4,0.4956),(D5,0.1845)}
3	0.926395	VL and M	{(D1,0.0991),(D2,0.0542),(D3,0.5526),(D4,0.2913),(D5,0.0028)}
4	0.487461	VL and H	{(D1,0.0078),(D2,0.3474),(D3,0.1255),(D4,0.5192),(D5,0)}
5	0.000451	VL and VH	{(D1,0.4965),(D2,0.0896),(D3,0.0243),(D4,0.2343),(D5,0.1553)}
6	1.33E-06	L and VL	{(D1,0.1962),(D2,0.0337),(D3,0.0006),(D4,0.4634),(D5,0.3062)}
7	0.88523	L and L	{(D1,0.0194),(D2,0.0577),(D3,0.0066),(D4,0.0093),(D5,0.9069)}
8	0.693815	L and M	{(D1,0.1081),(D2,0.2173),(D3,0.6107),(D4,0),(D5,0.0639)}
9	0.017678	L and H	{(D1,0.0578),(D2,0.2602),(D3,0.0065),(D4,0.3426),(D5,0.3329)}
10	0.861722	L and VH	{(D1,0.0426),(D2,0.3838),(D3,0.5599),(D4,0),(D5,0.0137)}
11	0.756567	M and VL	{(D1,0.0005),(D2,0.072),(D3,0.7029),(D4,0.0143),(D5,0.2102)}
12	0	M and L	{(D1,0.4193),(D2,0.001),(D3,0.1962),(D4,0.0018),(D5,0.3818)}
13	0.969499	M and M	{(D1,0.0764),(D2,0.1516),(D3,0.1352),(D4,0.4008),(D5,0.236)}
14	0.00074	M and H	{(D1,0.0511),(D2,0.0126),(D3,0.0285),(D4,0.6095),(D5,0.2984)}
15	0.999266	M and VH	{(D1,0.0077),(D2,0.1856),(D3,0.317),(D4,0.3046),(D5,0.1851)}
16	0.233453	H and VL	{(D1,0.0553),(D2,0.4909),(D3,0.0246),(D4,0.0697),(D5,0.3595)}
17	0.864655	H and L	{(D1,0.2314),(D2,0.2669),(D3,0),(D4,0.1475),(D5,0.3542)}
18	0.630124	H and M	{(D1,0.0906),(D2,0.0099),(D3,0.0283),(D4,0.8593),(D5,0.0119)}
19	0.7406	H and H	{(D1,0.0098),(D2,0.196),(D3,0.0185),(D4,0.0056),(D5,0.7701)}

Continue Table IV: Updated rule weight and belief degrees in BRB for fouling and erosion faults

		Deviation in health parameters	Fouling
Rule number	**Rule weight**	$\Delta m(t)$ *and* $\Delta\eta(t)$	**Distribution of consequence** $\{(D_1,\beta_{1,k}),(D_2,\beta_{2,k}),\ldots,(D_5,\beta_{5,k})\}$
20	0.468249	H and VH	{(D1,0.0673),(D2,0.6183),(D3,0.021),(D4,0.2896),(D5,0.0039)}
21	0.328892	VH and VL	{(D1,0.2659),(D2,0.1602),(D3,0.393),(D4,0.1635),(D5,0.0174)}
22	0.634017	VH and L	{(D1,0.2951),(D2,0.3772),(D3,0.2214),(D4,0.0339),(D5,0.0724)}
23	0.796462	VH and M	{(D1,0.1999),(D2,0.0088),(D3,0.4381),(D4,0),(D5,0.3532)}
24	0.581102	VH and H	{(D1,0.0058),(D2,0.0295),(D3,0.0273),(D4,0.3219),(D5,0.6156)}
25	0.989349	VH and VH	{(D1,0.1448),(D2,0.1662),(D3,0.0485),(D4,0.0016),(D5,0.6388)}
Erosion			
Rule number	**Rule weight**	$\Delta m(t)$ *and* $\Delta\eta(t)$	**Distribution of consequence** $\{(D_1,\beta_{1,k}),(D_2,\beta_{2,k}),\ldots,(D_5,\beta_{5,k})\}$
1	0.959028	VL and VL	{(D1,0.1823),(D2,0.3772),(D3,0.02),(D4,0.3801),(D5,0.0404)}
2	0.133091	VL and L	{(D1,0.3078),(D2,0.3543),(D3,0.0383),(D4,0.0503),(D5,0.2493)}
3	0.359858	VL and M	{(D1,0.007),(D2,0.4526),(D3,0),(D4,0.0723),(D5,0.468)}
4	0.926344	VL and H	{(D1,0.1897),(D2,0.2747),(D3,0),(D4,0.2611),(D5,0.2745)}
5	0.652867	VL and VH	{(D1,0.073),(D2,0.3687),(D3,0.3708),(D4,0.0118),(D5,0.1757)}
6	0.216586	L and VL	{(D1,0.7374),(D2,0.1965),(D3,0.0641),(D4,0.0002),(D5,0.0018)}
7	0.492547	L and L	{(D1,0.0001),(D2,0),(D3,0.9999),(D4,0),(D5,0)}
8	0.004929	L and M	{(D1,0.0212),(D2,0),(D3,0.8526),(D4,0.1249),(D5,0.0013)}
9	0.992435	L and H	{(D1,0.1922),(D2,0.1766),(D3,0.2502),(D4,0.2101),(D5,0.1709)}
10	0.008446	L and VH	{(D1,0.0252),(D2,0.2672),(D3,0.0001),(D4,0.4331),(D5,0.2743)}
11	0.524159	M and VL	{(D1,0.3189),(D2,0.6575),(D3,0.0091),(D4,0.0067),(D5,0.0079)}
12	0.985483	M and L	{(D1,0.1635),(D2,0.2916),(D3,0.0973),(D4,0.3024),(D5,0.1451)}
13	0.568838	M and M	{(D1,0.0775),(D2,0),(D3,0.0001),(D4,0.5072),(D5,0.4151)}
14	0.238724	M and H	{(D1,0.0916),(D2,0.2293),(D3,0.2312),(D4,0.2313),(D5,0.2166)}
15	0.943735	M and VH	{(D1,0.8257),(D2,0.0004),(D3,0.1194),(D4,0.0498),(D5,0.0048)}
16	0.005655	H and VL	{(D1,0.4064),(D2,0.0198),(D3,0.1029),(D4,0.1101),(D5,0.3607)}
17	0.000000	H and L	{(D1,0.2835),(D2,0.1473),(D3,0.2842),(D4,0.2848),(D5,0.0001)}
18	0.166113	H and M	{(D1,0.1444),(D2,0.2008),(D3,0.2026),(D4,0.2247),(D5,0.2275)}
19	0.098938	H and H	{(D1,0.0184),(D2,0),(D3,0.4264),(D4,0.0983),(D5,0.4569)}
20	0.028062	H and VH	{(D1,0.2618),(D2,0.1713),(D3,0.2362),(D4,0.1146),(D5,0.216)}
21	0.485207	VH and VL	{(D1,0.139),(D2,0),(D3,0.3195),(D4,0.2141),(D5,0.3274)}
22	0.942964	VH and L	{(D1,0.2769),(D2,0.2992),(D3,0.1123),(D4,0.2794),(D5,0.0321)}
23	0.993697	VH and M	{(D1,0.1278),(D2,0.2563),(D3,0.3167),(D4,0.0008),(D5,0.2984)}
24	0.882621	VH and H	{(D1,0.1515),(D2,0.0504),(D3,0.2748),(D4,0.2493),(D5,0.274)}
25	0.485165	VH and VH	{(D1,0),(D2,0.0002),(D3,0.4658),(D4,0.247),(D5,0.287)}

VI. IMPLEMENTATION OF UPDATED BRB MODEL

The updated BRB based model data of erosion and fouling faults and their estimations are shown in Figures 2-a and 2-b, respectively. In the case studied in this paper, delta mass flow and delta efficiency are the two continuous variables considered as antecedent attributes in the BRB system, and degrees of erosion and fouling are the system output.

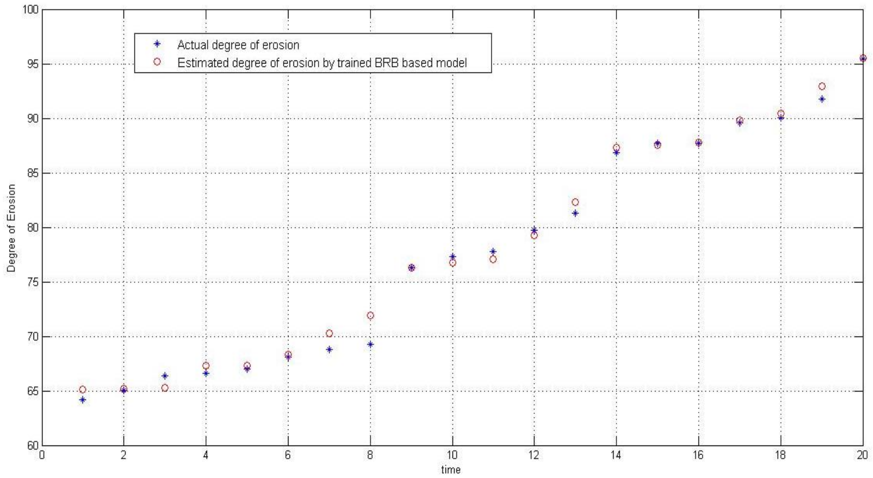

Fig. 2-a

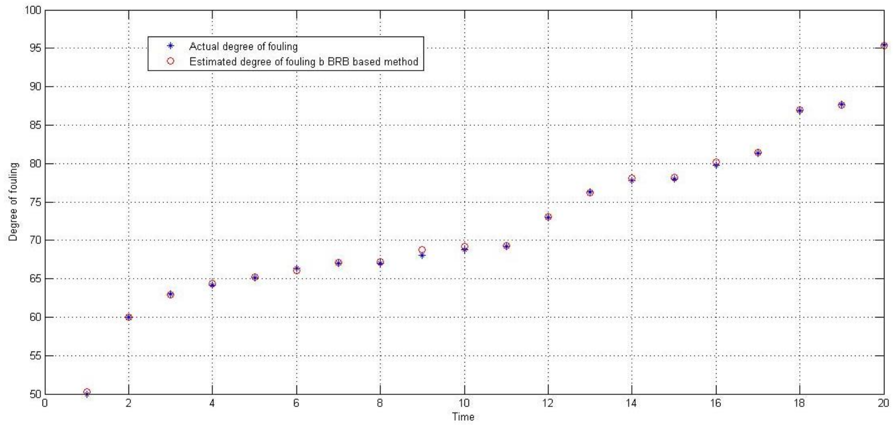

Fig. 2-b

Fig. 2. The testing data and estimates of fault detection (2-a erosion and 2-b fouling) generated by the updated BRB based model

VII. RESULT ANALYSIS AND DISCUSSION

A discussion is presented here on the results of implementing the proposed model in the detection of the type and severity of each fault. These results together with real values of each fault severity obtained through single-dimensional models for the gas turbine simulation are represented in figure 2Following training the BRB model, the values estimated for the severity of faults are more accurate and closer to the simulation model data than those of before model training. After time t, the BRB model parameters are also set according to Formula 9. After few steps, the initial values offered by the expert will change to those adjusted based on the model describing the turbine fault status, which can be observed by examination of changes in the distribution of consequences.

According to the comparison of Tables 2 and 3, the rules 2, 5, 6, 9, 12, and 14 have gained lower weights than the other BRB rules at a given time-period t for the estimation of fouling fault. The reduction/elevation in the weight of a rule and changes in the distribution of consequences are dependent on the fouling fault severity. Obviously, if another simulation is done for the fouling fault severity, these weights will also change accordingly. In addition, a comparison of the erosion fault values in Tables 2 and 3 indicates that the rules 2, 8, 10, 14, 16, 17, and 20 have gained lower weights than the other BRB rules.

To clarify the issue, the initial weights proposed by the expert for the fault model are shown in Figure 3-a. The mass flow and efficiency differences from the nominal values are observed on the horizontal and vertical axes, respectively. As shown in the figure, increases in the turbine health parameters lead to elevations in the nominal values of the fault severity presented numerically in the range of [0-1]. Figures 3-b and 3-c also display the distribution of consequences for the fouling and erosion faults, respectively, at time-period t after updating the BRB model parameters. In both figures, an important point is that the BRB model considers all weighted rules and the distribution of consequences to estimate the fouling severity at any time t. Hence, the weights updated by the BRB model are different from those provided initially by the expert. In other words, the expert applies only one rule to estimate the turbine fault status, while the BRB model uses all the rules to do this at any time. As shown in Figures 2-a and 2-b, the values estimated by the BRB model correspond to the real values determined through the simulation.

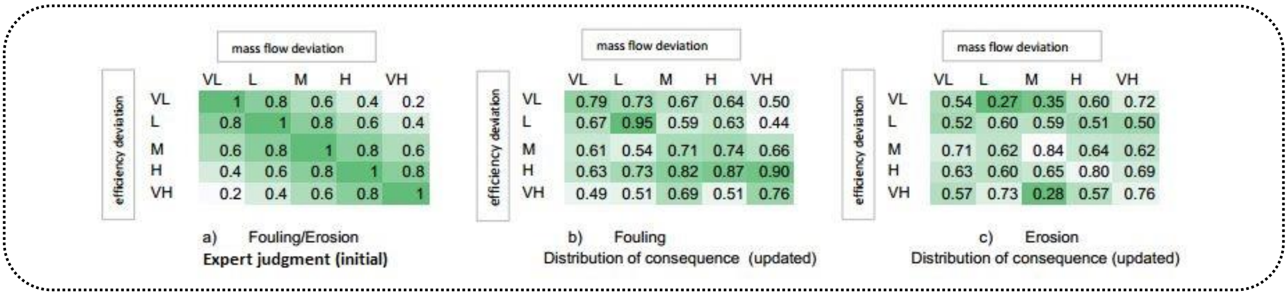

Fig. 3. Distribution of consequences for the 3-a expert judgment, 3-b fouling and 3-c erosion faults

The estimation accuracy of the BRB model was verified using the R^2 and RMSE indicators, the values of which for both erosion and fouling (0.99782 and 0.06585, respectively) faults are shown in Table V, suggesting a high precision of the proposed model in estimating the fouling fault; the same is true for estimating the erosion fault severity.

Table V: Value of R^2 and RMSE indicators for erosion and fouling faults

	Fouling	Erosion
RMSE	0.06585	0.01771
R^2	0.99782	0.99887

VIII. CONCLUSION

The present research proposed a knowledge-based approach to detect and isolate fouling and erosion faults in gas turbines. The BRB based model proposed here uses simultaneously both quantitative (expert opinion) and qualitative (from system health parameters) information for detection and isolation of the faults to solve the diagnosis problem. It is formulated by a real-world system when the two types of faults are dependent and competitive. A recursive algorithm for the online update of the BRB based model is developed on the basis of recursive algorithms for estimating the BRB parameters. Finally, a new BRB based method for online fault diagnosis is proposed for condition-based maintenance (CBM) of industrial gas turbines, which have the main contributions to this paper. An experimental case study is examined to demonstrate the implementation and potential applications of the proposed method for online fault isolation and diagnosis. There are several features in the proposed fault diagnosis method. Firstly, the proposed method can isolate, detect, and estimate the severity of the fault when two fault types exist simultaneously and have the same impact on health parameters of the system. Secondly, the quantitative or qualitative information can be used to establish the fault prognosis model. This is inherited from the BRB, ensuring that different types of information can be considered in order to improve the forecasting precision of faults. This feature is also one of the main contributions of this paper. Finally, once new information becomes available, the proposed method can detect the type and severity of faults in real-time without waiting for all information to be provided, which can greatly save time and is of great practical significance. As shown in this paper, the above features allow the new real-time fault diagnosis algorithm to be applied widely in real engineering applications. Moreover, our results demonstrate a high precision of the proposed mode for detection and isolation of the faults. Future research will extend the proposed model not only for fault diagnosis but also for fault prognosis under uncertain conditions. Another interesting research topic can be a combination of historical failure data (leading to system shut down) with fault data (reducing system performance).

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