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Strategic Evaluation of Sustainable Projects based on Hybrid Group Decision Analysis with Incomplete Information

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Abstract – Sustainable evaluation of construction projects in strategy-focused condition is the main issue for municipalities to appropriately improve public sector services. In this respect, the group decision-making methods could help experts to select suitable sustainable projects and to schedule them regarding their ranking results. Therefore, the objective of this study is to present a hybrid group decision-making approach based on hesitant fuzzy sets theory to select the best strategic project for Tehran municipality. Hesitant fuzzy sets theory regarding the other modern fuzzy sets could assist the experts in assessing the candidate strategic projects based on evaluation criteria by assigning some membership degrees under a set to decrease the judgments' errors in vague environments. In this proposed approach, the weight of each decision maker (DM) is determined according to the proposed hesitant fuzzy collective wisdom weighting (HFCWW) method. Besides, the evaluation criteria weights are determined based on the presented hesitant fuzzy preference weighting (HFPW) technique. Hence, hesitant fuzzy utility index method is defined to rank the candidate strategic projects. Finally, a real case study about the sustainable strategic project selection for Tehran municipality is provided to represent the feasibility and applicability of the proposed framework.

Keywords – Strategic projects evaluation, Sustainable management, Group decision analysis, Uncertainty

I. INTRODUCTION

One of the essential factors for municipalities is sorting the construction projects which are planned for their strategic planning horizons. The project management and evaluation problems have been applied and discussed in different management functions, e.g., environmental energy management (Chen et al., 2010), quality management (Hariharan et al., 2004), research and development (Loch & Kavadias, 2002), and strategic project selection (Charoenngam, 2007). To address the issue, the multi-criteria group decision-making techniques could solve these problems regarding their conflicted criteria, suitability. In this respect, a few authors focused on this field to assess their project selection problems based on complete/certain information.

Thereby, San Cristóbal (2011), due to lack of ability of classical single-criterion decision making approaches, presented an integrated framework based on a compromise solution which was denoted by VIKOR and analytic hierarchy process (AHP) methods to evaluate the candidate renewable energy projects in Spain. In their study, the AHP method was used for weighting the evaluation criteria, and then, the VIKOR approach was utilized to rank the candidates. Polat et al. (2016) combined the PROMETHEE and AHP methods to assist construction companies in choosing a suitable urban renewal project. Bryce et al. (2017) as well as Ozer et al. (2017) presented a systematic

evaluation framework by reviewing the rating tools for road pavement projects regarding the sustainability criteria. Alwan et al. (2017) focused on strategic sustainable development in the UK construction industry to cleaner production and reduced the negative impacts of modern construction procedures. Zolfani et al. (2018) manipulated a hybrid decision-making framework based on complex proportional assessment (COPRAS) and step-wise weight assessment ratio analysis (SWARA) to apprise hotel construction projects regarding environmental sustainability.

In real complex decision-making problems, considering the crisp value for evaluating the candidates may lead to an inaccurate result. Moreover, it is easy for DMs or professional experts to judge the candidate strategic projects based on linguistic terms which are intrinsically fuzziness. Hence, fuzzy set theory is a powerful approach to assess the candidate under the conflicted criteria by copping with imprecise information. In this respect, hesitant fuzzy set (HFS) (Torra & Narukawa, 2009; Torra 2010) is one of an appropriate tool for handling the uncertainty with regard the classical and modern fuzzy set theories such as triangular fuzzy sets (Vahdani et al., 2011; Ebrahimnejad et al., 2012), intuitionistic fuzzy sets (Atanassov, 1986; Atanassov, 1989), fuzzy multisets (Miyamoto, 2000), type-2 fuzzy sets (Fotea, 2008; Erdogan, 2015), multi-hesitant fuzzy linguistic term sets (Wang et al., 2015), interval-valued fuzzy set (Mousavi et al., 2013; Vahdani et al., 2013). Thereby, many researchers focused on fuzzy group decision making approaches to select the best candidate under a vague environment.

Meanwhile et al. (2015) elaborated a three-stage hybrid technique based on data envelopment analysis (DEA) technique for the initial screening, the TOPSIS method for ranking the projects, and linear integer programming approach for selecting the most appropriate project portfolio under fuzzy environments regarding organizational objectives. Besides et al. (2015) prepared a decision-making process based on two various approaches as Yager's method and fuzzy AHP technique to evaluate the different reclamation project candidates which were provided by Magnesite Mine Company. Furthermore, Wu et al. (2018) manipulated a hybrid framework based on AHP and PROMETHEE II to sort the candidate large-scale rooftop photovoltaic projects under the triangular intuitionistic fuzzy numbers. Büyüközkan et al. (2018) presented a group decision making based on simple additive weighting (SAW) and technique for order performance by similarity to ideal solution (TOPSIS) methods under hesitant fuzzy linguistic term sets to select the suitable strategic renewable energy source.

Furthermore, some authors focused on determining the weights optimization or computation to increase the reliability of the results. For this sake, Torra and Narukawa (2007) investigated the weight selection techniques regarding the ordered weighted averaging and weighted mean relations. Moreover, Fan et al. (2002) proposed an optimization model to determine the importance of each criterion regarding the objective fuzzy decision matrixes and experts' fuzzy judgments. Hence, Wang and Parkan (2006) presented a general decision-making framework according to the objective information and subjective preferences to compute the significance of each criterion under a fuzzy environment. Chen and Lee (2011) elaborated a triangular fuzzy AHP technique for specifying the attributes importance of professional conference organizer. Furthermore, Xu and Zhang (2013) developed an optimization framework according to the maximizing deviation method to appraise the criteria influence under hesitant condition. Feng et al. (2014), to solve the hesitant fuzzy decision-making problems, utilized the TOPSIS method, in which the weights of each criterion are completely known. Also, Zhang et al. (2014) proposed an objective weighting approach based on Shannon information entropy and hesitant fuzzy information to obtain safety criteria weights.

The HFS theory is considered an influential tool in the literature to cover uncertain information in hesitant environments. In this respect, Yu et al. (2013) as well as Farhadinia (2013) defined that the HFSs could be taken in practical cases of decision-making problems to prevent or decrease the privacy, anonymity, and psychic contagion of experts. Wang et al. (2014) expressed that the HFSs are helpful for handling the decision-making problems which are mentioned under the imprecise conditions where experts are vague between several values before assigning their judgments. Zhang et al. (2014) mentioned that using the HFSs for decision-making problems could enhance the results in an efficient way when some fuzzy membership values are possible for a criterion or object. Meanwhile, Rodríguez et al. (2014) prepared an overview of hesitant fuzzy sets theory with the aim of providing a simple perspective on various trends, tools, and concepts. Hence, numerous relations, such as intersection and union, are extended for HFSs; Pei and

Yi (2015) investigated the algebraic structures and properties of these operations. Liu (2015) extended some aggregation operators to aggregate the hesitant fuzzy linguistic information for solving the electrical power system safety problem. Therefore, the HFS could be implemented as an appropriate tool for dealing with available uncertain information for strategic evaluation of sustainable projects problem.

The review of the literature shows that focusing on strategic project selection problem as an exciting issue and obtaining the criteria weights received limited attention. Furthermore, there is no evidence on the literature of strategic project selection problem that systematically covers various aspects of decision-making levels for computing the criteria importance, experts' weights, and candidate evaluation. Although the studies on strategic project evaluation are rich by focusing on ranking process and criteria definition, it is not an integrated, advanced, and systematic framework. This is the most forceful motivation to present a hybrid framework by integrating the three levels of decision-making structure which is aimed at coping with this gap of the literature.

However, this paper proposes a new hybrid group decision-making approach by determining the DMs and criteria weights. For this sake, determining the degree of expertise for DMs could lead to precise results, especially in an uncertain condition. Hence, this study introduces a hybrid hesitant fuzzy group decision-making method based on presented hesitant fuzzy collective wisdom weighting (HFCWW) method to determine the experts' weights, elaborated hesitant fuzzy preference weighting (HFPW) method to compute the criteria weights, and hesitant fuzzy utility index method to sort the candidate strategic projects for Tehran municipality.

The structure of this paper is manipulated as follows: in section 2, the procedure of the extended approach is defined based on the criteria and DM weighting methods and the hesitant fuzzy utility index technique to rank the candidate strategic projects. In section 3, the proposed approach is implemented to a real case study to rank the candidate projects which are provided as a strategic vision for Tehran municipality. Finally, the obtained results with their analysis and also future suggestions to improve the proposed approach are presented in section 4.

II. PROPOSED HYBRID HESITANT FUZZY GROUP DECISION-MAKING FRAMEWORK

A. Structure of the proposed approach

In this section, an extended hybrid hesitant fuzzy group decision-making model is proposed to specify the weights of evaluation criteria, the importance of experts' opinions, and prioritize the candidate strategic projects. To address the issue, the HFCWW method and the HFPW technique is proposed to determine the experts and criteria weights, respectively. Then, the HFPW method is defined to rank the candidate strategic projects.

Linguistic variables for rating the candidates	HFEs for rating the candidates	Linguistic variables for evaluating the criteria	HFEs for evaluating the criteria	
Very very high (VVH)	0.900	Vorge important (VI)	0.900	
Very high (VH)	0.850	very important (vi)		
High (H)	0.750	Loop ant ant (I)	0.775	
Medium high (MH)	0.650	- Important (1)	0.775	
Medium (M)	0.550		0.525	
Medium low (ML)	0.450	Miedium (M)		
low (L)	0.325		0.375	
Very low (VL)	0.175	- Unimportant (UI)		
Very very low (VVL)	0.100	Very unimportant (VUI)	0.100	

Table I. Linguistic terms for rating the candidates and appraise the criteria importance

In the process of the extended approach, a group of decision makers $(DM_k; k=1, 2, ..., K)$ is founded to appraise the candidate strategic projects $(A_i; i=1, 2, ..., m)$ based on evaluation criteria $(C_j; j=1, 2, ..., n)$. In this respect, the hesitant fuzzy group assessment matrix and the evaluation criteria weights are determined based on experts' judgments with linguistic terms that their equivalence hesitant fuzzy elements (HFEs) are provided in Table 1. In this study, the risk preference of decision-makers (such as pessimistic, moderate, optimistic) is considered moderate.

B. Procedure of the proposed approach

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The procedure of the proposed hybrid hesitant fuzzy group decision-making approach is presented based on the following steps:

Step 1. Construct a group of DMs to appraise the strategic projects in terms of the evaluation criteria by the hesitant fuzzy group assessment matrix ($\wp_k = \left[\lambda_{ij}^k\right]_{m \times n}$).

Step 2. Determine the weight of each DM according to the proposed hesitant fuzzy collective wisdom weighting (HFCWW) method. The proposed approach is inspired by the collective wisdom concept that is knowledge sharing by the group of experts. Thus, the weight of experts with more deviation from the average judgments is less than the others. However, the procedure of the proposed weighting approach is defined based on the following sub-steps.

Step 2.1. Normalize the hesitant fuzzy group assessment matrix for each decision maker $(\mathcal{D}_k^{nor} = \begin{bmatrix} \lambda_{ij}^{nor(k)} \end{bmatrix}_{m \times n})$ regarding the normalized hesitant fuzzy operation as:

$$b_{ij} = \bigcup_{t_{ij} \in b_{ij}} = \begin{cases} \{\gamma_{ij}\} & \text{for positive criteria} \\ \{1 - \gamma_{ij}\} & \text{for negative criteria} \end{cases} \forall i = 1, ..., m; j = 1, ..., n$$

$$\wp_{k}^{nor} = \begin{bmatrix} \lambda_{ij}^{nor(k)} \end{bmatrix}_{m \times n} = \begin{bmatrix} A_{1} \\ \vdots \\ A_{m} \\ \lambda_{m1}^{nor(k)} \\ \lambda_{m2}^{nor(k)} \\ \vdots \\ \lambda_{m2}^{nor(k)} \\ \lambda_{mn}^{nor(k)} \\ \lambda_{mn}^{nor$$

Step 2.2. Specify the hesitant fuzzy individual wisdom worthiness $(HFIWW_k)$ for each decision maker regarding the mean of normalized value matrix $\left(\frac{1}{K}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\tau=1}^{k}\left(\lambda_{ij}^{nor(k)\sigma(\tau)}(x_i)\right)\right)\right)$ and hesitant fuzzy Euclidean distance measures.

$$HFIWW_{k} = \frac{1}{K} \sqrt{\frac{1}{l_{x_{i}}}} \left| K \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\tau=1}^{l_{x_{i}}} \lambda_{ij}^{nor(k)\sigma(\tau)}(x_{i}) \right) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{\tau=1}^{l_{x_{i}}} \left(\lambda_{ij}^{nor(k)\sigma(\tau)}(x_{i}) \right) \right|^{2}} \quad \forall k$$
(3)

Step 2.3. Compute the weight of each expert ($\hat{\partial}_k$) regarding the normalized deviation ($K - \sum_{k=1}^{K} HFIWW_k$) from the hesitant fuzzy individual wisdom worthiness.

$$\partial_{k} = \frac{1 - HFIWW_{k}}{K - \sum_{k=1}^{K} HFIWW_{k}} \quad \forall k$$
(4)
where $\sum_{k=1}^{K} \partial_{k} = 1$

Step 3. Determine the evaluation criteria weights based on presented hesitant fuzzy preference weighting (HFPW) method.

Step 3.1. Construct the normalized hesitant fuzzy group assessment matrix for each criterion $(\wp_j^{nor} = \left[\lambda_{ij}^{nor(k)}\right]_{m \times n})$.

$$\mathcal{D}M_{1} \quad \mathcal{D}M_{2} \quad \cdots \quad \mathcal{D}M_{K}$$

$$\mathcal{D}_{j}^{nor(j)} = \begin{bmatrix} \lambda_{ik}^{nor(j)} \end{bmatrix}_{m \times k} = \begin{bmatrix} A_{1} \\ A_{11} \\ \vdots \\ A_{m} \\ \end{bmatrix}_{m \times k} \begin{pmatrix} \lambda_{11}^{nor(j)} & \lambda_{12}^{nor(j)} \\ \vdots \\ \lambda_{m1}^{nor(j)} & \lambda_{m2}^{nor(j)} \\ \vdots \\ \lambda_{m2}^{nor(j)} \\ \vdots \\ \lambda_{mk}^{nor(j)} \\ \end{bmatrix}_{m \times k} \forall j$$

$$(5)$$

Step 3.2. Compute the hesitant fuzzy preference worthiness (ξ_j) for each criterion based on hesitant fuzzy Hamming distance measure.

$$\xi_{j} = \left| 1 - \left| \frac{1}{l_{x_{i}}} \sum_{i=1}^{m} \sum_{k=1}^{K} \sum_{\tau=1}^{l_{x_{i}}} \lambda_{ik}^{nor(j)\sigma(\tau)} - \frac{1}{nl_{x_{i}}} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{\tau=1}^{l_{x_{i}}} \left(\lambda_{ik}^{nor(j)\sigma(\tau)} \left(x_{i} \right) \right) \right) \right| \quad \forall j$$
(6)

Step 3.3. Calculate the evaluation criteria weights (ω_j) regarding to decision makers' judgments about the relative importance of each criterion and the hesitant fuzzy weighted geometric (HFWG) relation.

$$\omega_{j} = \frac{\prod_{k=1}^{K} \xi_{j} \left(\boldsymbol{\varpi}_{j}^{k} \right)^{\partial_{k}}}{\sum_{j=1}^{n} \left(\prod_{k=1}^{K} \xi_{j} \left(\boldsymbol{\varpi}_{j}^{k} \right)^{\partial_{k}} \right)} \quad \forall j$$

$$(7)$$

where $\boldsymbol{\sigma}_{j}^{k}$ is the significance of *j*th criterion which is defined by *k*th decision maker.

Step 4. In the following, the hesitant fuzzy utility index method is presented to rank the candidate strategic projects.

Thus, this step provides the aggregated weighted normalized hesitant fuzzy group assessment matrix ($[\varphi]_{m \times n}$) based on the HFWG relation.

Step 5. Determine the hesitant fuzzy separation measure from the best and worst score values based on hesitant fuzzy Hamming distance measure that are denoted by ψ_i^* and ψ_i^- , respectively.

$$\psi_{i}^{*} = \frac{1}{l_{x_{i}}} \left| \sum_{j=1}^{n} \sum_{\tau=1}^{l_{x_{i}}} \left(\varphi_{ij}^{\sigma(\tau)}(x_{i}) - \max_{i} \left\{ \varphi_{ij}(x_{i}) \right\} \right) \right| \quad \forall i$$

$$\psi_{i}^{-} = \frac{1}{l_{x_{i}}} \left| \sum_{j=1}^{n} \sum_{\tau=1}^{l_{x_{i}}} \left(\varphi_{ij}^{\sigma(\tau)}(x_{i}) - \min_{i} \left\{ \varphi_{ij}(x_{i}) \right\} \right) \right| \quad \forall i$$
(10)

where J is the positive criterion, in which the ψ_i^* and ψ_i^- must be computed based on $\min_i \{\varphi_{ij}(x_i)\}$ and $\max_i \{\varphi_{ij}(x_i)\}$ for negative criterion.

Step 6. Compute the hesitant fuzzy utility index (\mathcal{G}_i) based on the HFWG relation.

$$\mathcal{G}_{i} = \frac{\left(\prod_{i}^{m} \left(\boldsymbol{\psi}_{i}^{-}\right)\right)^{min\left\{\boldsymbol{\psi}_{i}^{-}\right\}}}{\left(\prod_{i}^{m} \left(\frac{\min\left\{\boldsymbol{\psi}_{i}^{-}\right\}}{\boldsymbol{\psi}_{i}^{-}}\right)\right)^{\boldsymbol{\psi}_{i}^{-}}} + \boldsymbol{\psi}_{i}^{*} \left(\prod_{i}^{m} \left(\boldsymbol{\psi}_{i}^{-}\right)\right)^{min\left\{\boldsymbol{\psi}_{i}^{-}\right\}} \quad \forall i$$

$$(11)$$

Step 7. Rank the candidate strategic projects by decreasing the sorting of hesitant fuzzy utility index values.

C. Outline of the proposed approach

In this section, the aforementioned proposed hybrid hesitant fuzzy group decision-making approach is summarized as follows:

Phase 1. Construct the hesitant fuzzy group assessment matrix based on Eq. (1) to appraise the candidate strategic projects in terms of the evaluation criteria.

Phase 2. Determine the weight of each DM based on proposed hesitant fuzzy collective wisdom weighting (HFCWW) method by Eqs. (2)-(4).

Phase 3. Determine the evaluation criteria weights based on proposed hesitant fuzzy preference weighting (HFPW)

method by Eqs. (5)-(7).

Phase 4. Rank the candidate strategic projects based on presented hesitant fuzzy utility index method by Eqs. (8)-(11).

III. CASE STUDY

In this section, a real practical case study about the strategic project evaluation problem in Tehran municipality is provided to show the implementation procedure of the proposed hybrid hesitant fuzzy group decision-making approach. Tehran municipality regarding the sustainable development of the city is implementing and planning many expansions projects and functions in the fields of highways, bridges, buildings, and tunnels. The technical and development deputy of Tehran municipality supervises these projects and functions. To handle these expansion projects, an efficient and special engineering system is required within a responsible organization which is called Engineering and Development Organization of the City of Tehran (EDOCT).

The EDOCT was established in 1990 to implement these expansion projects and functions. EDOCT by having four asphalting operations group and asphalt factories, consisting of a great number of construction and development machinery is known as the most prominent independent organization that is working under the technical and development affairs deputy of Tehran Municipality. In this sake, rating the five candidate strategic projects (A_i , i=1, 2, ..., 5) and defining the importance of ten evaluation criteria (C_j , j=1,2,...,10) are done based on three experts' judgments (DM_k , k=1, 2, 3) by linguistic terms which are represented in Table 1. However, the candidate strategic projects and the evaluation criteria are defined as follows:

 A_1 : Bridge construction; A_2 : Highway expansion; A_3 : Urban tunnels construction; A_4 : Subway extension; A_5 : Urban improvement.

and;

 C_1 : Costs; C_2 : Environmental competencies; C_3 : Social aspects; C_4 : Technical capability; C_5 : Local experience; C_6 : Duration; C_7 : Expandability; C_7 : Expandability; C_8 : Sanctions effects; C_9 : Strategic risk; C_{10} : Project execution risk.

In this respect, DMs appraise the strategic project selection problem and the importance of each evaluation criterion based on linguistic terms that are indicated in Tables 2 and 3, respectively. Then, the constructed hesitant fuzzy group assessment matrix and the relative importance of each evaluation criterion are converted to HFEs based on Table 1.

In this respect, the proposed hesitant fuzzy collective wisdom weighting (HFCWW) method is applied to compute the experts' weights. Thereby, the normalized hesitant fuzzy group assessment matrix for each decision maker is established regarding Eq. (2). Then, the hesitant fuzzy individual wisdom worthiness ($HFIWW_k$) for each decision maker is calculated by Eq. (3). Finally, the weight of each expert is obtained based on Eq. (4). The results are given in Table 4.

Valuation erterna projects DM, DM, DM, A, H H WH VH A, VH VH VH A, VH VH VVH A, VH VH VVH A, VH H H A, WH H M A, WH H WH A, WH H MI A, WH WH H A, WH WH H A, WH WH H A, WH WH WH A, WH WH WH A, WH WH WH A, WH WH WH		Candidate strategic projects	Decision makers			
A ₁ H H H VH VH A ₂ H VH VH VH VH A ₄ VVH VH VH VH VH A ₄ VVH VH H H H A ₂ WH H H H A ₂ WH H H WH A ₄ VVH VVH VVH VH A ₄ WH H H H H A ₄ WH WH H H H H A ₄ WH VH VH VH VH H H H H H H H H H H H H H H H H H H	Evaluation criteria		DM ₁	DM ₂	DM ₃	
A2 H VH VH VH A1 VH VH VH VVH A4 VVH H H H A5 VH H H H A2 VH H H VH A4 VVH VVH VVH VH A4 VVH VVH VVH VH A4 VVH H H H A4 VVH VH H H A4 VVH H H H A4 VVH VH VH H A4 VH H MH MH A4 VH VH VH H A2 WH VH VH H A4		A_{l}	Н	Н	VH	
C ₁ A ₂ VH VH VH VVH A ₁ VVH H H H A ₂ VVH H H H A ₂ VVH H H M A ₂ VVH H H VH A ₂ VVH VVH VVH A ₂ VVH H H VH A ₃ M ML ML ML A ₄ VVH VVH VVH VH A ₃ H WH H H A ₄ VH VH VH WH A ₄ VH VH VH VH A ₄ WH H MH MH A ₄ WH VH VH VH A ₄ VH VH WH H A ₄ VH VH WH M A ₄ VH VH WH M	C_{I}	A_2	Н	VH	VH	
A ₄ VVH VH VH VH WH A ₁ M MH M M A ₂ VH H WH M A ₂ VH H WH WH A ₂ VH H WH VH A ₄ VVH VVH VVH VVH A ₄ VVH VVH VVH VVH A ₄ VVH WH H MI A ₄ WH H H MI A ₄ WVH VVH VVH VVH A ₄ WH H H MI A ₄ WH H MI MI A ₄ MH H MI MI A ₄ WH WH WH WH A ₄ WH WH WH WH A ₄ WH WH WH WH A ₄ WH WH WH <td>A_3</td> <td>VH</td> <td>VH</td> <td>VVH</td>		A_3	VH	VH	VVH	
A3 VH H H H A1 M MH MH M A2 VH H VH VH A2 VH H WH VH A4 VVH VVH VVH VVH A3 M ML ML A3 M MH H H A4 VVH VH H H A4 VH H MH H A4 VH WH H MH A4 VH VH VH VH A4 MH MH MH MH A4 WH VH VH VH A4 WH WH MH MH A2 MH MH MH MH A4 WH VH VH VH A4 H WH WH MH A4 WH		A_4	VVH	VH	VVH	
A ₁ M MH MH MH A ₂ VH H H VH A ₄ VVH VVH VVH VVH A ₄ VVH VVH VVH VVH A ₄ VVH VVH VVH MI. A ₁ H MH H MI. A ₂ MH H MH H A ₂ MH H MH MH A ₄ VVH VVH VVH VVH A ₄ VVH VVH VH MH A ₄ VVH VH MH MH MH A ₄ WH H MH MI MI MI MI MI MI MI MI <td< td=""><td></td><td>A_5</td><td>VH</td><td>Н</td><td>Н</td></td<>		A_5	VH	Н	Н	
A2 VH H W VH A3 H H WH VVH VVH A4 WVH VVH VVH VVH A2 M ML ML A2 MH H H A2 MH H MH A2 MH H MH A3 VH VVH VVH A4 VVH VVH VVH A4 VVH VVH VVH A4 VVH VVH VVH A4 VH VH WH A4 VH VH H A4 VH VH VH A4 VH VH VH A4 WH MH M A4 WH WH WH A4 H WH M A4 WH WH M A4 WH WH <td< td=""><td></td><td>A_{I}</td><td>М</td><td>MH</td><td>М</td></td<>		A_{I}	М	MH	М	
A_1 H H H VH VH A_4 VH VVH VVH VVH VVH A_2 M ML ML ML A_2 MH H MH H A_2 WH H MH H A_2 WH VH WH H A_3 WH H MH H A_3 MH H WH VH A_3 H VVH VH VH A_4 WH H MH MH A_4 VH VVH VH VH A_4 VH VVH VH VH A_4 VH VH VH VH A_4 VH VH H H A_4 VH VH VH H A_4 VH VH VH H A_4 H <td< td=""><td></td><td>A_2</td><td>VH</td><td>Н</td><td>VH</td></td<>		A_2	VH	Н	VH	
A_i VVH VVH VVH VVH A_i M ML ML A_j H MH H A_j WH H MH A_j VH H H A_i WH VVH VVH A_i WH VH WH A_i H VH WH A_i H WH WH A_i H WH WH A_i H WH WH A_i WH VVH VVH A_i WH VH VVH A_i WH VH VH A_i WH VH VH A_i WH VH H A_i WH WH H A_i WH WH H A_i WH WH WH A_i WH WH H <td>C_2</td> <td>A_3</td> <td>Н</td> <td>Н</td> <td>VH</td>	C_2	A_3	Н	Н	VH	
A_i M ML ML A_i H MH H MH H A_j VH H MH MH MH A_j VH H H MH H A_j VH H H H VVH A_j MH H WH VH A_j MH H MH H A_j MH H MH H A_j MH MH H H A_j H VH VH VH A_j H WH H H A_j VH VVH VH VH A_j VH VH H H A_j H WH H H A_j VH VH H H A_j VH VH H H A_j WH		A_4	VVH	VVH	VVH	
A_1 H MH H A_2 MH H MH A_3 VH H H A_4 VVH VVH VVH A_4 VVH VVH VVH A_4 VVH VVH VVH A_2 M M MH A_2 M M MH A_2 M M MH A_2 M M MH A_2 M VH VH A_4 VH VVH VH A_4 VH VH VH A_2 VH VH VH A_2 VH VH H A_4 MH MH M A_4 VH VH H A_4 WH VH H A_4 VH VH VH A_4 VH VH VH		A_5	М	ML	ML	
$C_3 = egin{array}{ c c c c c c } \hline A_2 & MH & H & MH & MH \\ \hline A_3 & VH & WH & WH & MH \\ \hline A_4 & VVH & VVH & VVH \\ \hline A_5 & H & VH & VH & MH \\ \hline A_2 & M & M & MH & MH \\ \hline A_2 & M & M & MH & MH \\ \hline A_2 & M & M & MH & MH \\ \hline A_4 & VH & VVH & VVH & VVH \\ \hline A_5 & MH & M & M & M \\ \hline A_2 & VH & VVH & VH & VH \\ \hline A_2 & VH & VVH & VH & VH \\ \hline A_2 & VH & VVH & VH & H \\ \hline A_2 & VH & VVH & H & H \\ \hline A_2 & MH & MH & M & M \\ \hline A_2 & VH & VVH & VH \\ \hline A_3 & H & VH & VH \\ \hline A_2 & VH & VVH & VH \\ \hline A_4 & VVH & VH & H \\ \hline A_4 & VH & VH & VH \\ \hline A_5 & M & M & ML \\ \hline A_4 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VH \\ \hline A_2 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_5 & VH & VH & VH \\ \hline A_4 & H & VH & VH \\ \hline A_4 & VH & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_1 & WH & H & H \\ \hline A_4 & VVH & VH & VH \\ \hline A_2 & VVH & VH & VH \\ \hline A_5 & VL & U & U \\ \hline A_2 & VVH & VH & VH \\ \hline A_5 & L & VL & L \\ \hline A_2 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & VL \\ \hline A_5 & L & VL & VL \\ \hline A_2 & VU & VH & H & VH \\ \hline A_4 & VVH & H & H \\ \hline A_4 & VVH & H & H \\ \hline A_4 & VVH & H & H \\ \hline A_4 & VVH & VH & VV \\ \hline \hline A_5 & L & VL & VL \\ \hline \hline A_5 & L & VL & VL \\ \hline $		A_{l}	Н	MH	Н	
A_2 VH H H H A_4 VVH VVH VVH VVH A_5 H VH VH VH A_2 M MH H MH A_2 M M MH H A_2 M M MH H A_4 VH VH VH H A_4 VH VH VH H A_4 VH VH VH VH A_4 VH VH VH VH A_2 VH VH VH VH A_2 VH VH VH H A_4 MH MH M M A_2 VH VH VH H A_2 VH VH VH H A_2 VH VH VH M A_2 VH VH VH M </td <td></td> <td>A_2</td> <td>MH</td> <td>Н</td> <td>MH</td>		A_2	MH	Н	MH	
A_4 VVH VVH VVH VVH A_3 H VH VH VH A_4 MH H MH MH A_2 M M MH MH A_3 H VH VH H A_4 VH VVH VVH H A_4 VH VH VH VH A_4 WH VH H H A_4 MH MH M M A_4 WH VH VH H A_4 WH H H H A_4 VH VH VH H A_4 VH VH VH H A_4 VH VH VH V	C_3	A_3	VH	Н	Н	
A_1 H VH VH VH A_1 MH H MH MH A_2 M M MH H A_3 H VH VH VVH A_4 VH VVH VVH VVH A_4 VH VH VVH VVH A_5 MH M M M A_2 VH VVH VH VH A_4 MH MH M M A_4 NH NH H H A_4 MH MH M M A_4 NH H H H A_4 NH NH M M A_4 VH VH VH VH A_4 VH VH VH NH A_4 VH VH VH VH A_4 VH VH VH		A_4	VVH	VVH	VVH	
$C_4 = egin{array}{c c c c c c c c c c c c c c c c c c c $		A_5	Н	VH	VH	
$C_4 = egin{array}{c c c c c c } A_i & M & M & MH \\ \hline A_i & H & VH & H \\ \hline A_i & H & VH & VH & VVH \\ \hline A_i & MH & M & M \\ \hline A_i & MH & M & M \\ \hline A_i & VH & VH & VH & VH \\ \hline A_2 & VH & VH & H \\ \hline A_2 & VH & VH & H \\ \hline A_2 & H & VH & MH & M \\ \hline A_i & H & H & H \\ \hline A_i & H & H & H \\ \hline A_2 & VH & VVH & VH \\ \hline A_i & H & H & H \\ \hline A_2 & VH & VVH & VH \\ \hline A_i & H & H & H \\ \hline A_2 & VH & VH & VH \\ \hline A_i & MH & MH & M \\ \hline A_i & VH & VH & VH \\ \hline A_i & MH & VH & VH \\ \hline A_i & MH & VH & VH \\ \hline A_i & VH & VH & VH \\ \hline A_i & M & M & ML \\ \hline A_i & M & M & ML \\ \hline A_i & M & MH & VH \\ \hline A_i & M & MH & VH \\ \hline A_i & M & MH & VH \\ \hline A_i & M & MH & VH \\ \hline A_i & M & MH & ML \\ \hline A_i & M & MH & ML \\ \hline A_i & VVH & VVH & VH \\ \hline A_i & M & MH & M \\ \hline A_i & M & MH & M \\ \hline A_i & VH & VVH & VH \\ \hline A_i & M & ML & VH \\ \hline A_i & M & ML & ML \\ \hline A_i & M & ML & ML \\ \hline A_i & M & ML & ML \\ \hline A_i & M & ML & ML \\ \hline A_i & M & MH & H \\ \hline A_i & M & MH & H \\ \hline A_i & VVH & VH & VH \\ \hline A_i & M & MH & M \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & M & MH & M \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline A_i & VVH & VH & VH \\ \hline \hline A_i & VVH & VH & VH \\ \hline \hline A_i & VVH & VH & VH \\ \hline $		A_l	MH	Н	MH	
C_4 A_2 H VH VH VH A_4 VH VH VVH VH A_5 MH M M M A_1 VVH VH VH VH A_2 VH VH VH VH A_2 VH VH VH H A_3 H VH H H A_4 MH MH M M A_3 VH VH VH H A_3 VH VH VH H A_4 MH H H H A_2 VH VH VH H A_4 VH VH H A_2 VH VH VH H A_2 VH VH H H A_4 VVH VH H A_4 VVH VH VH H A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 H VH VH VH A_5 VH VH VH VH A_5 VH VH VH H A_5 VL L L A_5 VL L L A_5 VVH VH VH VH A_5 VVH VH VH VH A_5 VVH VH VH VH A_5 VVH VH VH VH A_5 VL L L L A_4 H VH VH VH A_5 VL VH VH VH A_5 VL VH VH VH A_5 VVH VH VH VH A_5 VL VH VH VH VH A_5 VVH VH VH VH VH VH A_5 VVH VH VH VH VH VH VH A_4 VVH VH VH VH VH VH VH V		A_2	М	М	MH	
A_{s} VH VVH VVH A_{s} MH M M A_{j} VVH VH VH A_{2} VH VVH VH A_{2} VH VVH VH A_{3} H VH VH A_{4} MH MH M A_{4} MH VH H A_{4} MH VH VH A_{4} VH VH VH A_{2} VH VVH VH A_{4} VH VH H A_{4} VH VH VH A_{4} VVH VH VH A_{4} VH VH VH A_{4} M M ML A_{4} VH VH VH A_{4} VH VH VH A_{4} VH VH VH A_{5} VL L<	C_4	A_3	Н	VH	Н	
A_5 MH M M A_1 VVH VH VH VVH A_2 VH VVH VH VH A_3 H VH VH H A_4 MH MH MH M A_4 MH MH MH M A_5 VH VVH VH A_2 VH VH H A_2 VH VH H A_2 VH VH H A_2 VH VH H A_2 VH VH VH A_3 VH VH VH A_4 VVH VH VH A_2 VH VH VH A_4 VH VVH VH A_4 VH VH VH A_5 VH VH VH A_4 H H H		A_4	VH	VVH	VVH	
$C_{5} = egin{array}{ c c c c c c c c c c c c c c c c c c c$		A_5	MH	М	М	
$C_5 = egin{array}{ c c c c c c c c c c c c c c c c c c c$		A_{l}	VVH	VH	VVH	
C_5 A_3 H VH H A_4 MH MH MH MH M A_5 VH VVH VH VH A_2 VH H H H A_2 VH VH H H A_4 VVH VH VH H A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 VH VH VH VH A_4 H VH VH VH A_4 VH VVH VH VH A_4 VH VH VH VH A_4 VL L <t< td=""><td></td><td>A_2</td><td>VH</td><td>VVH</td><td>VH</td></t<>		A_2	VH	VVH	VH	
A_4 MH MH MH MH MH M A_5 VH VVH VVH VH VH A_2 VH H H H A_2 VH VH VH H A_4 VVH VH VH VH A_4 VVH VH VH VH A_4 VVH VH VH VH A_5 M M ML 0 A_2 VVH VVH VH VH A_2 VVH VVH VH VH A_4 H VH VH VH A_4 VH VH VH VH A_4 H VH VH VH A_4 H VH VH VH A_4 H VH VH NL L C_8 A_3 MH H H H	C_5	A_3	Н	VH	Н	
A_5 VH VVH VH VH A_1 H H H H A_2 VH VH H H A_2 VH VH VH H A_3 VH VH VH VH A_4 VVH VH VH VH A_4 VVH VH VH VH A_2 VVH VH VH VH A_2 VVH VH VH VH A_2 VVH VVH VH VH A_2 VVH VVH VH VH A_2 VH VH VH VH A_4 VH VH VH ML ML C_8 A_4 H H H H A_4 H VH VH H H C_9 A_3 H MH H H		A_4	MH	MH	М	
$C_6 = egin{array}{c c c c c c c c c c c c c c c c c c c $		A_5	VH	VVH	VH	
$C_6 = egin{array}{ c c c c c c c } \hline A_2 & VH & VH & VH & VH \\ \hline A_3 & VH & VH & VH & VH \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & M & M & ML \\ \hline A_5 & M & M & ML \\ \hline A_2 & VVH & VVH & VVH \\ \hline A_2 & VVH & VVH & VVH \\ \hline A_3 & M & MH & M \\ \hline A_4 & VH & VVH & VH \\ \hline A_5 & VH & VVH & VH \\ \hline A_5 & VH & VH & VH \\ \hline A_2 & L & L & ML \\ \hline A_2 & L & L & ML \\ \hline A_2 & L & L & ML \\ \hline A_4 & H & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_4 & H & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_4 & H & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_4 & VVH & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_4 & VVH & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & VH \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & L \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & L \\ \hline A_4 & VVH & VH & VVH \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & VL \\ \hline A_4 & VVH & VH & VH \\ \hline A_4 & VVH & VH & VVL \\ \hline A_5 & L & VL & VL \\ \hline A_4 & VH & H & VH \\ \hline A_4 & VH & H & VH \\ \hline \end{array}$		A_{I}	Н	Н	Н	
$C_{6} = \begin{array}{c ccccccccccccccccccccccccccccccccccc$		A2	VH	VH	Н	
$egin{array}{ c c c c c c c } \hline A_4 & VVH & VVH & VVH \ \hline A_5 & M & M & ML \ \hline A_1 & H & VH & VH \ \hline A_2 & VVH & VVH & VVH \ \hline A_2 & VVH & VVH & VVH \ \hline A_3 & M & MH & M \ \hline A_4 & VH & VVH & VVH \ \hline A_5 & VH & VH & VVH \ \hline A_5 & VL & L & ML \ \hline A_4 & H & VH & VH \ \hline A_5 & VL & L & ML \ \hline A_4 & H & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_2 & M & MH & M \ \hline A_2 & M & MH & H \ \hline A_4 & VVH & VH & VH \ \hline A_5 & L & VL & L \ \hline A_3 & H & MH & H \ \hline A_4 & VVH & VH & VH \ \hline A_5 & L & VL & UL \ \hline A_3 & M & MH & MH \ \hline A_4 & VVH & VH & VH \ \hline A_3 & M & MH & MH \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & VH \ \hline A_5 & L & VL & VL \ \hline A_3 & M & MH & MH \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & VH \ \hline \hline A_5 & L & VL & VL \ \hline A_5 & VL & VL & VL \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_4 & VH & H & H \ \hline A_5 & L & VL & VL \ \hline A_5 & VL & VL & VL \ \hline A_5 & VL & VL & VL \ \hline A_5 & VL & VL & VL \ \hline A_5 & VL & VL & VL \ \hline A_5 & VL & VH & H & VH \ \hline \hline \hline A_5 & VL & VH & H & VH \ \hline $	C_6	A_3	VH	VH	VH	
$egin{array}{ c c c c c c c } \hline A_5 & M & M & ML & ML \\ \hline A_1 & H & VH & VH & VH \\ \hline A_2 & VVH & VVH & VVH & VVH \\ \hline A_2 & VVH & VVH & VVH & VVH & ML & M$		A_4	VVH	VH	VVH	
$C_7 = egin{array}{ c c c c c c c } \hline A_1 & H & VH & VH & VH \ \hline A_2 & VVH & VVH & VVH \ \hline A_3 & M & MH & MH & M \ \hline A_4 & VH & VVH & VVH \ \hline A_5 & VH & VH & VVH \ \hline A_5 & VH & ML & ML \ \hline A_2 & L & L & ML \ \hline A_4 & H & VH & H & H \ \hline A_4 & H & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_2 & M & MH & H & H \ \hline A_4 & H & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_5 & VL & L & L \ \hline A_1 & VVH & VH & VH \ \hline A_2 & M & MH & H & H \ \hline A_4 & VVH & VH & VH \ \hline A_5 & L & VL & L \ \hline A_2 & M & MH & MH \ \hline A_4 & VVH & VH & VH \ \hline A_2 & M & MH & MH \ \hline A_2 & M & MH & MH \ \hline A_2 & M & MH & MH \ \hline A_2 & M & MH & MH \ \hline A_2 & M & MH & MH \ \hline A_2 & VVH & VH & VVH \ \hline A_5 & L & VL & VL \ \hline A_2 & VL & VL & VL \ \hline A_3 & M & MH & MH \ \hline A_4 & VVH & VH & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_4 & VH & H & VH \ \hline A_5 & L & ML & I \ \hline A_4 & VH & H & VH \ \hline A_5 & L & ML & I \ \hline A_4 & VH & H & VH \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & ML & I \ \hline A_5 & L & L & L & L \ \hline A_5 & L & L & L & L & L \ \hline A_5 & L & L & L & L & L & L & L & L & L & $		A ₅	M	M	ML	
$C_7 = egin{array}{ c c c c c c c } A_2 & VVH & VVH & VVH \\ \hline A_3 & M & MH & MH \\ \hline A_4 & VH & VVH & VH \\ \hline A_4 & VH & VVH & VH \\ \hline A_5 & VH & VH & VH \\ \hline A_5 & VH & ML & ML \\ \hline A_2 & L & L & ML \\ \hline A_2 & L & L & ML \\ \hline A_2 & L & L & ML \\ \hline A_3 & MH & H & H \\ \hline A_4 & H & VH & VH \\ \hline A_5 & VL & L & L \\ \hline A_5 & VL & L & L \\ \hline A_1 & VVH & VH & VVH \\ \hline A_2 & M & MH & M \\ \hline A_2 & M & MH & M \\ \hline A_2 & M & MH & H \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & L \\ \hline A_4 & VVH & VH & VVH \\ \hline A_5 & L & VL & L \\ \hline A_4 & VVH & VH & VVH \\ \hline A_4 & VVH & VH & VVL \\ \hline A_4 & VVH & VL & VVL \\ \hline A_4 & VVH & MH & MH \\ \hline A_4 & VH & H & VVH \\ \hline A_4 & VH & H & VVL \\ \hline A_5 & L & VL & VL \\ \hline A_4 & VH & H & VVH \\ \hline A_4 & VH & H & VVH \\ \hline \end{array}$			Н	VH	VH	
$C_7 = egin{array}{c c c c c c c c c c c c c c c c c c c $		<u>A2</u>	VVH	VVH	VVH	
$egin{array}{ c c c c c c c c c c c c c c c c c c c$	C_7	<u>A</u> 3	M	MH	M	
$egin{array}{ c c c c c c c c c c c c c c c c c c c$		A_4	VH	VVH	VH	
$C_8 = egin{array}{ c c c c c c c c c c c c c c c c c c c$		A ₅	VH	VH	VVH	
$C_8 = egin{array}{ c c c c c c c c c c c c c c c c c c c$			M	ML	ML	
$C_8 = egin{array}{cccccccccccccccccccccccccccccccccccc$	-	A2		L	ML	
$C_{9} = \begin{bmatrix} A_{4} & H & VH & VH \\ \hline A_{5} & VL & L & L \\ \hline A_{4} & VVH & VH & VH \\ \hline A_{2} & M & MH & M \\ \hline A_{2} & M & MH & M \\ \hline A_{3} & H & MH & H \\ \hline A_{4} & VVH & VH & VH \\ \hline A_{5} & L & VL & L \\ \hline A_{5} & L & VL & L \\ \hline A_{2} & VL & VL & VL \\ \hline A_{2} & VL & VL & VV \\ \hline A_{4} & VH & MH & MH \\ \hline A_{4} & VH & H & VH \\ \hline A_{5} & L & VL & VV \\ \hline A_{5} & L & VL & VV \\ \hline A_{5} & L & VL & VV \\ \hline A_{5} & VL & VV \\ \hline A_{5} & VL & VV \\ \hline A_{5} & VV \\ \hline A_{5} & VV & VV \\ \hline A_{5} & V$	C_8	<u>A</u> 3	MH	Н	Н	
$C_{9} = \begin{bmatrix} A_{5} & VL & L & L \\ A_{1} & VVH & VH & VVH \\ \hline A_{2} & M & MH & M \\ \hline A_{2} & M & MH & M \\ \hline A_{3} & H & MH & H \\ \hline A_{4} & VVH & VH & VVH \\ \hline A_{5} & L & VL & L \\ \hline A_{5} & L & VL & UL \\ \hline A_{2} & VL & VL & VL \\ \hline A_{2} & VL & VL & VVL \\ \hline A_{3} & M & MH & MH \\ \hline A_{4} & VH & H & VH \\ \hline A_{5} & L & MH & MH \\ \hline A_{5} & L & MH & MH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & L & MH & H \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & H & VH \\ \hline A_{6} & VH & VH & VH \\ \hline A_{6} & VH & VH & VH \\ \hline A_{6} & VH & VH & VH \\ \hline A_{6} & VH & VH & VH \\ \hline A_{6} & VH & VH \\ \hline A_{7} & VH \\ \hline A_{7} & VH & VH \\ \hline A_{7} & VH \\ \hline A_$		A	П	νп	νп	
$C_{9} = \begin{bmatrix} A_{I} & VH & VH & VH \\ \hline A_{2} & M & MH & M \\ \hline A_{3} & H & MH & H \\ \hline A_{4} & VVH & VH & VVH \\ \hline A_{5} & L & VL & L \\ \hline A_{6} & L & VL & VL \\ \hline A_{1} & L & VL & VL \\ \hline A_{2} & VL & VL & VVL \\ \hline A_{3} & M & MH & MH \\ \hline A_{4} & VH & H & VH \\ \hline A_{5} & L & ML & L \\ \hline \end{array}$		A5				
$C_{9} = \begin{bmatrix} A_{2} & M & MH & M \\ MH & M & H \\ \hline A_{3} & H & MH & H \\ \hline A_{4} & VVH & VH & VVH \\ \hline A_{5} & L & VL & L \\ \hline A_{5} & L & VL & L \\ \hline A_{2} & VL & VL & VL \\ \hline A_{2} & VL & VL & VVL \\ \hline A_{3} & M & MH & MH \\ \hline A_{4} & VH & H & VH \\ \hline A_{5} & L & ML & L \\ \hline \end{array}$			VVH M	νп	v v п М	
$C_{9} = \begin{array}{c cccc} A_{3} & \mathbf{n} & \mathbf{M}\mathbf{n} & \mathbf{H} \\ \hline A_{4} & \mathbf{V}\mathbf{V}\mathbf{H} & \mathbf{V}\mathbf{H} & \mathbf{V}\mathbf{V}\mathbf{H} \\ \hline A_{5} & \mathbf{L} & \mathbf{V}\mathbf{L} & \mathbf{L} \\ \hline A_{5} & \mathbf{L} & \mathbf{V}\mathbf{L} & \mathbf{L} \\ \hline A_{2} & \mathbf{V}\mathbf{L} & \mathbf{V}\mathbf{L} & \mathbf{V}\mathbf{U} \\ \hline A_{3} & \mathbf{M} & \mathbf{M}\mathbf{H} & \mathbf{M}\mathbf{H} \\ \hline A_{4} & \mathbf{V}\mathbf{H} & \mathbf{H} & \mathbf{V}\mathbf{H} \\ \hline A_{5} & \mathbf{L} & \mathbf{M}\mathbf{L} & \mathbf{L} \end{array}$	C_9	A2		МП		
$C_{10} = \begin{bmatrix} A_4 & VH & VH & VH \\ \hline A_5 & L & VL & L \\ \hline A_6 & L & VL & VL \\ \hline A_1 & L & VL & VL \\ \hline A_2 & VL & VL & VL \\ \hline A_3 & M & MH & MH \\ \hline A_4 & VH & H & VH \\ \hline A_5 & L & ML & L \\ \end{bmatrix}$		A3 A				
$C_{10} \qquad \begin{array}{c ccccccccccccccccccccccccccccccccccc$		A4 A	ννп		т	
C_{10} A_{1} C_{10} A_{2} C_{10} A_{2} C_{10} A_{3} M MH MH MH MH MH MH MH		A5 A	L T			
$C_{10} \qquad \begin{array}{c cccc} A_2 & VL & VL & VVL \\ \hline A_3 & M & MH & MH \\ \hline A_4 & VH & H & VH \\ \hline A_5 & L & ML & L \\ \end{array}$						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	C	A12				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	C_{10}	A.	VH	ИП	VH	
		A =	I.	ML	I.	

Table II. The hesitant fuzzy group assessment matrix

Frankradian anitania	Decision makers				
Evaluation criteria	DM ₁	DM_2	DM_{3}		
C_I	VI	VI	VI		
C_2	Ι	Ι	VI		
C_3	VI	VI	VI		
C_4	Ι	Ι	Ι		
C_5	VI	Ι	VI		
C_6	М	М	М		
C_7	Ι	VI	Ι		
C_8	VI	Ι	VI		
C_9	Ι	VI	VI		
<i>C</i> 10	VI	VI	Ι		

Table III. The importance of evaluation criteria based on linguistic terms

Table IV. Obtained results from the proposed HFCWW method

	Decision makers					
	DM_1 DM_2 DM_3					
$HFIWW_k$	0.025	0.25	0.225			
∂_k	0.39	0.30	0.31			

Hence, the presented hesitant fuzzy preference weighting (HFPW) method is considered to determine the evaluation criteria weights. In this respect, the normalized hesitant fuzzy group assessment matrix for each criterion is founded based on Eq. (5). Then, the hesitant fuzzy preference worthiness (\Im_j) for each criterion is calculated based on Eq. (6). Finally, the evaluation criteria weight (ω_j) regarding to decision makers' judgments about the relative importance of each criterion is achieved by Eq. (7). The computational results of proposed HFPW method are reported in Table 5.

Table V. Obtained results from the proposed HFPW method

Evaluation criteria	\mathfrak{I}_{j}	$arnothing_j$
C_{I}	4.1625	0.197522
C_2	1.9375	0.082926
C_3	2.9375	0.139392
C_4	2.3125	0.094493
C_5	3.2875	0.149157
C_6	3.2125	0.088924
<i>C</i> ₇	3.3375	0.142634
C_8	0.2625	0.011910
C_9	1.7875	0.080016
C10	0.2875	0.013025

Moreover, the hesitant fuzzy utility index method is implemented to rank the candidate strategic projects. Meanwhile, the aggregated weighted normalized hesitant fuzzy group assessment matrix (\mathcal{P}) is provided based on Eq. (8). Also, the hesitant fuzzy separation measure from the best (ψ_i^*) and worst score (ψ_i^-) values are specified by Eqs. (9) and (10), respectively. Finally, the candidate strategic projects are ranked by decreasing sorting of the hesitant fuzzy utility index (ϑ_i) values which are computed based on Eq. (11). As represented in Table 6, the subway extension (A_4) and the urban improvement (A_5) are selected as the best and worst candidate strategic projects regarding the evaluation criteria, respectively.

Candidates	ψ_i^*	ψ_i^-	\mathcal{G}_{i}	Rank the candidate strategic projects
A_{I}	0.748297	0.859366	21.40	3
A_2	0.844086	0.763578	14.81	4
A_3	0.654135	0.953528	30.91	2
A_4	0.156928	1.450736	223.35	1
A_5	1.249064	0.3586	3.65	5

Table VI. Obtained ranking results from the proposed approach

However, a comparative analysis is considered to analyze the verification of the proposed hybrid hesitant fuzzy group decision-making framework. In this respect, the proposed approaches of Zhang & Wei (2013), Xu & Zhang (2013), Onar (2014), and Öztayşi and Kahraman (2017) are implemented in the case study for comparing the obtained ranking results with this study. Hence, as reported in Table 7, all five approaches have the same ranking results which confirmed the verification of the proposed approach of this study.

Candidates	Proposed approach	Zhang & Wei (2013)' approach	Xu & Zhang (2013)' method	Onar (2014)' framework	Öztayşi and Kahraman (2017)' methodology	Rank the candidate strategic projects
A_{I}	21.40	0.481730	0.534544	0.703924	0.775916	3
A_2	14.81	0.411575	0.474961	0.639915	0.690863	4
A_3	30.91	0.658540	0.593114	0.800263	0.826374	2
A_4	223.35	0.916670	0.902388	0.846745	0.884925	1
A_5	3.65	0.243947	0.223057	0.398422	0.590344	5
Standard deviation	92.51	0.256631	0.244286	0.17586	0.11578	

Table VII. Comparative analysis based on related literature

Furthermore, the standard deviation measure is considered for five approaches to show the dispersion rate of ranking values. Indeed, the standard deviation measure could help the experts to select the best strategic project among the different candidates in an uncertain environment. Thereby, the proposed hybrid hesitant fuzzy group decision-making framework has a higher standard deviation regarding the four other approaches. The trend of ranking results for each approach is depicted in Figure 1. Consequently, the proposed approach of this study could reach to precise results in solving the sustainable strategic project selection problem. It is appropriate to denote that different fuzzy decision-making techniques have unique features prevalently. Therefore, it is unsuitable to express that one fuzzy decision tool is powerful or better than one another generally because every fuzzy decision-making method has distinctive characteristics underlying an assertion or theory. However, the proposed approach of this study leads the results more precise/reliable than Zhang & Wei (2013), Xu & Zhang (2013), Onar (2014), and Öztayşi and Kahraman (2017)'

approaches. It is because that the presented approach is constructed based on proposed hesitant fuzzy collective wisdom weighting (HFCWW) method to determine the degree of expertise for each DM. In addition, the weight of each criterion is computed based on elaborated hesitant fuzzy preference weighting (HFPW) method. In sums, the advantages of the presented approach versus the four other methods are explained as follows:

I. The hesitant fuzzy collective wisdom weighting (HFCWW) is developed to determine the experts' weights;

II. The hesitant fuzzy preference weighting (HFPW) method is extended to compute the criteria weights;

III. The hesitant fuzzy utility index is presented to rank the candidates in a reliable manner by considering the different aspects of the proposed hybrid approach.



Figure 1. Schematically representation of comparative analysis

IV. CONCLUSIONS

Construction projects assessment based on sustainability factors in strategy-focused condition of municipalities is an important issue to enhance public services. Hence, selecting the best construction project among the candidate strategic projects is a decision-making problem that could be solved based on group decision-making techniques. In this paper, a new hybrid hesitant fuzzy group decision-making framework is presented to appraise the candidate strategic projects based on evaluation criteria and imprecise information. In this respect, the weight of each expert and criterion is determined based on proposed hesitant fuzzy collective wisdom weighting (HFCWW) and hesitant fuzzy preference weighting (HFPW) method, respectively. Moreover, the ranking procedure for sorting the candidate strategic projects is manipulated based on proposed hesitant fuzzy utility index method. Meanwhile, as mentioned before, the hesitant fuzzy set theory is provided for establishing the proposed hybrid approach to assign some membership degrees for a candidate strategic project based on evaluation criteria under a set to cover the uncertainty condition and decrease the judgments' errors. Finally, the proposed hybrid hesitant fuzzy group decision-making approach is implemented in a real case study about the strategic construction project selection for Tehran municipality to show the applicability and feasibility of this study. In this respect, the results indicated that the subway extension is selected as the most suitable strategic project in the planning horizon of Tehran municipality. In this case, urban improvement alternative is also chosen as the worst candidate for implementing among the other candidate strategic projects. Furthermore, comparing the obtained ranking results from the proposed approach and two other studies of related literature confirmed the verification of the presented hybrid hesitant fuzzy group decision-making framework. In addition, the suitable performance of the proposed

approach is indicated regarding two other approaches based on the standard deviation measure. For future directions, the elaborated framework could be enhanced, considering the interval-valued hesitant fuzzy set. Besides, scheduling the candidate strategic projects regarding the obtained ranking results from the proposed approach is more interesting. Moreover, defining the hierarchical structure for assessment criteria can improve the obtained results of a sustainable strategic project selection problem.

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